Updates to information and delayed decision making

Daniel Chung

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Abstract

This paper develops a model of rational behaviour under uncertainty when behaviour involves constrained consumption over multiple time periods. The key features are that rational agents not only respond to updated information but are aware of their ability to do so. The effects of such a model are that agents intentionally delay decisions in order to benefit from updated information, consuming under the premise that they may respond to information they expect to receive in the future.

1 Introduction

When it comes to economic decision-making, uncertainty and information are inseperble. Uncertainty serves as a burden to economic agents, one which interferes with their ability to plan for the future. Information serves as the antidote to uncertainty, reducing uncertainty and thus permitting agents to make decisions under greater foresight. As uncertainty is present in every economic decision, it is of little surprise that much of economic theory has been dedicated to analyzing how agents interact with uncertainty and information.

Relevant to uncertainty and information are two significant areas of analysis. The first is precautionary saving, introduced by Friedman (1957). Precautionary saving concerns the rise in savings that occurs in response to uncertainty and has been heavily examined theoretically and empirically. The standard explanation for precautionary savings stems from the assumption that utility is a concave function of consumption. Under such assumptions, agents benefit by spreading consumption roughly evenly, such that marginal utility is equal for all periods. In the presence of uncertainty, such consumption smoothing involves maintaining a savings reserve, lest agents become unable to consume (at prior levels) in the aftermath of a negative income shock. If a consumer fails to maintain savings and cannot consume during misfortune, the consumer suffers greatly. However, even a relatively small amount of savings allows the agent to avoid a significant loss in utility if a negative income shock occurs.

Another field, search theory, treats information as a variable partially within an agent's control. First developed by Stigler (1960), search theory analyzes the behaviour of buyers and sellers in markets with incomplete information. When information is incomplete, this permits the existence of heterogeneous prices, in turn making it advantageous to scope the market to find the lowest price. Even when searching the market comes at a cost, search theory finds that such searching will still occur to the extent that the expected price reduction exceeds the cost of looking for lower prices.

When applied to household consumption, the treatment of uncertainty has been wholly different. The amount of information available is considered exogenous, and in standard models, agents cannot, through any efforts, know with greater certainty their future income or needs. This is a realistic treatment, as the information relevant to a consumer is, by nature, highly difficult to attain. There is no feasible method by which people can efficiently gather information about how their lives will be in the future. It can be assumed that any information that may be useful (for example, life-cycle trends such as retirement) is already known. Therefore, no search will occur.

While consumers may not express their preferences for information through search, this does not rule out the possibility of other means of expressing such preferences. According to search theory, an agent intentionally moves from a state of low information to high information through searching. When searching is unfeasible, it does not mean agents lose their preference for more information. Consumers have another means by which they can receive information; they may wait for updates to information to naturally occur. By delaying decisions until updates to information arrive, agents may allocate them to periods in which they are the most informed.

Such a mechanism is best illustrated with the analogy of a military general. Suppose this general is preparing for the attack of a neighbouring territory. By attacking immediately, they benefit from the element of surprise. Despite this, they delay the attack by a few days. They do so for a good reason, as they expect spies to return with information of strategic value by then. The general has opted to delay decisions, deferring them to when information is more abundant.

Consider another analogy, one of an ill patient choosing between different procedures. Currently, there are two options; the first is a well-established procedure which has been performed for decades but requires a lengthy, expensive and painful process. On the other hand, an experimental treatment shows considerable promise, yet data is relatively sparse, and trials are still being conducted. This patient may opt to postpone treatment until more data is collected, although this may come at a cost, given the tendency for illnesses to increase in severity over time. As with the general, our patient is an agent delaying decisions under the premise that updated information will permit them to make better decisions.

Applying this idea to a consumer, it follows that in periods of high uncertainty, households will delay spending decisions (saving) periods. For example, during an economic downturn, households may decrease consumption in response to the increased likelihood of a negative income shock.

Our analysis differs from standard precautionary saving theory in a few key areas. First, we do not assume that the payoffs/utility of agents are necessarily determined by a concave function. Payoff functions may be concave, and it is realistic to treat them as concave in most scenarios, but it is not an assumption which affects our conclusions.

Second, and much more importantly, greater focus is placed on the importance of updates to information. The standard interpretation of precautionary saving is that such saving occurs due to a consumer's desire to smooth consumption. Little emphasis is usually placed on the importance of the ability of agents to receive updates to information. However, without agents' ability to receive information updates, it is doubtful that any precautionary saving even occurs. The theory of precautionary savings rests on the premise that in the event of an income shock, a consumer may adjust their consumption in response. However, if knowledge of such an income shock is not made available to a consumer, they would be unable to respond to it, eliminating any purpose of saving for such an event. It would not be sensible for a consumer to prepare in anticipation for an event they cannot be made aware of if it happens. Therefore, we propose an alternative causal explanation of precautionary saving.

The objectives of this paper are two-fold. First, a basic consumption model is developed. This consumption model is then augmented to capture consumer behaviour under various forms of uncertainty. Second, these models are then compared to each other to reach certain conclusions about the effects of uncertainty and information. We analyze both its effect on welfare as well as its effect on consumption and saving. In this analysis, we achieve multiple findings. First, it is possible to prove the existence of precautionary savings under a reasonably broad range of parameters without using computed approximations. More interestingly, we find that the impact of updates to information on a consumer's welfare and decision-making is dependent on how they interact with other parties, namely creditors.

2 The nature of information

The principal issue examined in this paper concerns the effects that updates to information have on the level of consumption. In examining this, we are comparing agents which receive updates to information, and those that do not. Consequentially, as a precursor, we must answer what effect the level of information has on an agent.

It is intuitively understood by most that information allows an agent to make better, more informed decisions. This is a sufficient way of thinking for most purposes. However, this is an approximation of how information benefits agents, one which cannot directly be applied to quantify uncertainty in a consumption model. One major complication making such task difficult is that information neither expands the set of choices available to an agent nor alters the relationship between a particular choice and its payoffs. Therefore, the exact mechanism by which information benefits its recipient is unclear. If there are two identical agents, the only difference between them being that one holds a given set of information, and the other a different one; if they pursue the same actions, their outcomes should be the same.

To resolve this difficulty, I propose the following interpretation of the effects on information: In an uncertain environment where multiple possible states are possible, information allows its recipient to (at least to some degree) discriminate between each possibility and behave differently depending on what is actually true.

To illustrate such an interpretation, we shall use a simple arithmetic example. Suppose an agent must select between two options, A and B. The payoffs associated with selecting A and B are uncertain yet with a known distribution; the possibilities are binary: there is a 50% chance that the payoffs are 75 and 25 and a 50% chance that the payoffs are 25 and 75 for options A and B, respectively. An agent lacking information about which option is more valuable cannot differentiate between a scenario in which A is more valuable and one where B is more valuable. Therefore, regardless of which is actually more valuable, they will have the same preference to each: they will be indifferent between A and B. As such, their expected payoff is 50.

In comparison, an agent informed about the true payoffs of each option will select A in the case that A has a payoff of 75, and select B in the case that B has a payoff of 75. Using their knowledge of which is more valuable, they are able to differentiate between the two possibilities and behave accordingly.

A Basic mathematical demonstration: Let us construct a one-period decision model to represent this. Suppose an agent's decisions are represented by a vector $C \in \mathbb{A}$, where $\mathbb{A}$ is the set of possible decisions.The agent's environment is uncertain and is in one of n possible discrete states, each with a probability $P(-)$. The set of possible states is X, and an unspecified member of X is denoted by X_i. The agent has a payoff function $U(-)$, for which C is an input. The payoff function for the state X_i is denoted by $U(-)_{X_i}$, and the consumption which occurs if X_i occurs is denoted by C_{X_i}. First, consider an agent who knows their state. Their payoffs are given by:

$$\max_{C_{X_i}}[\sum_{i=1}^{n} U_{X_i}(C_{X_i}) \cdot P(X_i)]$$
$$\text{s.t. } C_{X_i} \in A$$

Next, consider an agent who is not aware of their state. Such an agent faces an additional constraint:

$$\max_{C_{X_i}}[\sum_{i=1}^{x} U_{X_i}(C_{X_i}) \cdot P(X_i)]$$
$$\text{s.t. } C_{X_i} \in A, |\{C_X\| = 1$$

Where $C_{(a)}$ is a constant. In other words, decisions for the uninformed agent must be equal regardless of their actual state.

We can easily verify that the expected payoffs for the agent with greater information are higher by observing that the set of possible consumption values for the informed agent is a superset of that for the uninformed agent. From this, it follows that the expected payoffs for the informed agent are greater or equal to that of the uninformed agent. The reverse is not true, for if there exists some optimal strategy for the informed agent, the uninformed agent may not be able to replicate it on account of their additional constraints.

In subsequent sections, we shall construct a consumption model in which the consumer's environment is separated into several states, each with a given probability of occurrence. Depending on the consumer's level of knowledge, we shall place various restrictions on the consumer's consumption which require consumption to be equal regardless of what state consumption occurs in (i.e. a consumer with no information cannot spend at different rates depending on what the information that consumer lacks would have told them).

We shall apply this thinking to the environment of a consumer, one who makes decisions through multiple time periods. If a consumer possesses a set of consumption opportunities which is a superset of those of another, and the mapping from a given consumption opportunity to the resulting payoffs is the same for both, then we can immediately verify that such a consumer will have higher expected payoffs.

3 Guide

In the following few sections, we shall introduce many non-standard notations. This section compiles them; it may be helpful to use this section as a guide when going through them.

Time and state variables	
Symbol	Description
t	Unspecified time period
n	Last time period
(X)	State
(Z)	History (series of states)
z	Last of a set of states or histories (specifics depends on context)
(X_{ti})	The ith of z possible states which may occur during period t
(Z_{ti})	The ith of z possible histories which go up to period t

Some important notes:

1. Recall that the number of possible histories which go up to period t is the number of unique combinations of states up to t which can be made.

Other variables and functions	
Symbol	Description
$\mu_{t(s)}$	Culmunative utility from period t onward given state (s)
$C_{t(s)}$	Consumption at period t given state (s)
$U_{t(s)}(C)$	Utility of C at period t given state (s)
δ	Rate of time preference
$A_{t(s)}$	Assets at period t given state (s)
r	Interest rate
$Y_{t(s)}$	Income (exlc. interest) at period t given state (s)
$Y^P_{t(s)}$	Permanant income at period t given state (s)
$E_s[a]$	Expected value of a given state (s)
$P_s(a)$	Probability of a given state (s)

Details of the late resolution, early resolution, and temporal models are given below:

1. The late resolution model represents an environment in which information is static, such that an agent must rely entirely on information available before a certain point in time. Such a scenario is analogus to being blindfolded and deprived of all information received after they are done so.

2. The early resolution model also represents an environment in which information is static. However, it is differentiated from the late resolution model by.

3. The temporal model represents an environment in which information is recieved regularly. This model intends to most closely represent reality.

4 Set up

We shall consider the following abstraction of an uncertain environment. An agent lives through a finite sequence of discrete time periods $1, 2, ..., n$, with unspecified time periods indicated by the subscript t. For each time period $t > 1$, there is a finite number of possible states $X_{t1}, X_{t2}, ..., X_{tx}$ (the state at period 1 is certain). Each state X_{ti} has a corresponding utility/payoff function $U_{ti}(-)$, and a corresponding income flow Y_{ti}. The set of past and present sequences of states (referred to as "histories" henceforth) is denoted by $Z_{t1}, Z_{t2}, ..., Z_{tz}$, each history Z_{ti} has a corresponding sequence of utility functions $U_1(-), U_{2i}(-), ..., U_{ti}(-)$, and a corresponding sequence of income flows $Y_1, Y_{2i}, ..., Y_{ti}$.

It should be noted that multiple different sequences of states may have the same history if the scope of time is restricted. For example, consider the sequences $X_1, X_{21}, X_{31}, X_{41}$ and $X_1, X_{21}, X_{31}, X_{42}$: the sequences are identical from periods 1 to 3, but not for periods 1 to 4. Therefore, the sequence of states from periods 1 to 3 may be represented using a single history Z_{31}. However, the sequence for periods 1 to 4 must be represented using two different histories, Z_{41} and Z_{42}. Each state X_{ti} has a probability $P(X_{ti})$, which is conditional on Z_{t-1}, denoted by $P_{t-1}(X_{ti})$. Similarly, each history Z_{ti} has a probability $P(Z_{ti})$ which at period 1 is:

$$P_1(Z_{ni}) = \prod_{t=2}^{n} P_{(X_{t-1i})}(X_{ti}) \tag{1}$$

5 Consumption under certainty

First, consider consumer behaviour in the absence of uncertainty, where borrowing is restricted. Under perfect foresight, utility is given by:

$$\mu_1 = \max_{C_t} \left[\sum_{t=1}^{n} U_t(C_t) \right] \cdot (1+\delta)^{1-t} \tag{2}$$

μ_t = Culmunative utility from period t onwards

C_t = Consumption value at t, which is a non-negative real number

$U_t(C_t)$ = Utility of C_t at period t

n = Number of time periods

δ = Rate of time preference, which is a non-negative real number

The inability to borrow restricts consumption to current assets:

$$C_t \leq A_t \tag{3}$$

A_t = Assets at period t

Where current assets evolve according to the following rule:

$$A_t = A_{t-1} - C_{t-1} + Y_t \tag{4}$$

Y_t = Income (exluding interest) at period t

Consumption with Borrowing: We shall now introduce the ability to borrow and lend in this model. For simplicity, we assume that there is a single rate of interest at which the consumer may both borrow and lend money. With borrowing and lending, current assets evolve according to this updated rule:

$$A_t = (A_{t-1} - C_{t-1})(1 + r) + Y_t \tag{5}$$

$r =$ Rate of interest, which is a non-negative real number

The ability to borrow also affects constraints on consumption. In the absence of borrowing, consumption is constrained by current assets. With borrowing, creditors are willing to lend money to consumers up to the point at which they are no longer able to pay back the initial sum plus interest payments:

$$C_1 \leq Y_1^P = A_1 + \sum_{t=2}^{n} Y_t + \sum_{t=1}^{n} A_t r \tag{6}$$

$Y_t^P =$ Permanant income at period t

Here, note that constraints on present consumption is derived from the value of future consumption (as A_t is derived from consumption in periods $1 - t$). We express constraints on consumption in this manner because there is perfect information (and thus, future consumption for a rational agent is predictable).

The introduction of borrowing transforms our consumption model from one in which an agent interacts solely with their environment to one in which multiple agents interact with both each other and the environment. While the emphasis is placed on the consumption agent, creditors are agents of their own, with their own objectives and behaviour affected by the consumer. This must be considered in the next section, where our certainty model is augmented to represent behaviour under various forms of uncertainty. As the intention of this paper is to analyze the effects of changes in consumer information, including a mechanism by which creditors also receive changes in their information that will disrupt such intentions. Including such a mechanism will make it difficult to distinguish whether a particular result is caused by changes in consumer information or by changes in creditor information.

For that reason, we shall consider three different scenarios. The first is a scenario in which borrowing is restricted. In the second scenario, the consumer receives updates to information; however, the creditor does not. In the last scenario, both the consumer and creditor receive updates to information. Considering these three scenarios, it can be reasonably concluded that any differences between the first and second scenarios can be attributed to the ability to borrow. Likewise, any differences between the second and third scenarios can be attributed to the updates to information received by creditors.

6 Consumption under uncertainty

When analyzing choice under uncertainty, economists frequently distinguish between two categories of risk: timeless and temporal uncertainty (Chavas and Larson, 1994). The former concerns an environment in which any updates to information occur after all decisions have been made, such that the expected value of any variable is unchanged throughout all decision-making periods. On the other hand, the latter concerns problems where decisions are made incrementally, in between updates to information. Consequently, expectations evolve over time, meaning choices must also be re-evaluated over time. Mossin (1969) points out that in the context of consumption, realistically risk is temporal, as opposed to timeless. Updates to information occur incrementally, in between each time period. This is in tandem with consumption decisions, which are also made incrementally.

Despite this, I shall briefly consider two hypotheticals in which risk is timeless. These hypotheticals serve two purposes. First, constructing a temporal model is easier if aspects from timeless models are borrowed. Second, comparisons between timeless and temporal models are helpful in determining the effects of updates to information in a temporal environment.

Late-resolution environment: First, consider a timeless environment where there are no updates to information. Henceforth, this environment shall be called a late-resolution environment. In a late-resolution model, we explore the behavior of an agent which has been blindfolded and deprived of any information available afterwards. More precisely, in a late resolution environment, state X_1 is known to the agent at period 1. However, in all subsequent periods, no updates to information occur, such that the resolution of any subsequent states, even those in the past or the present, are never known to the agent. Agents are blindfolded in periods 2 to n and must rely solely on information available from period 1. The only information known in periods 2 to n by the consumer is past and present consumption, internal information. Since the external information available to the agent does not evolve, neither does the perceived probabilities of any state. In a late resolution model, expected utility is given by the following:

$$E_1[\mu_1] = \sum_{t=1}^{n} E_1[U_t(C_t)] \cdot (1+\delta)^{1-t} \tag{7}$$

$E_t[a]$ Expected value of variable a at period t

In a late resolution environment, the agent cannot know constraints on consumption, for such information counts as updates to information. Therefore, we employ a trick to prevent the agent from learning of such constraints while still being bound to them. We restrict the consumption to whichever is less between current assets and the agent's intended consumption level. Any consumption which exceeds current assets is considered invalid. If desired consumption exceeds current assets, the consumer will consume at current assets, although they will not be made aware of this fact:

$$E_1[U_t(C_t)] = \sum_{i=1}^{z} U_{(X_{ti})}(\min[\{C_t, A_{t(Z_{ti})}\}]) \cdot P_1(Z_{ti}) \tag{8}$$

$U_{(X_{ti})}(-)$ Utility function for state X_{ti}

$A_{t(Z_{ti})}$ Assets at period t if history Z_{ti} is true

Current assets evolve under the following rule:

$$A_{t(Z_{ti})} = (A_{t-1(Z_{t-1i})} - \min[\{C_{t-1}, A_{t-1(Z_{t-1i})}\}])(1+r) + Y_{(X_{ti})} \tag{9}$$

Early Resolution Environment: Second, consider a timeless environment in which all uncertainty is resolved at t=1, before any consumption decisions. We shall label this an "early resolution" environment. Consumers are given perfect information, such that for any possible histories they can consume optimally. In the early resolution model, expected utility is given by the following:

$$E_1[\mu_1] = \sum_{i=1}^{z} \sum_{t=1}^{n} U_{ti}(C_{ti}) \cdot (1+\delta)^{1-t} \cdot P_1(Z_{ni}) \tag{10}$$

In this model, agents fully anticipate the sequence of states they experience. Therefore, each history Z_{ni} has a unique set of consumption values $C_{1(Z_{ni})}, C_{2(Z_{ni})}, ..., C_{n(Z_{ni})}$ whose values may differ from consumption values of different histories.

Before we advance, note the significance of the different subscripts used for consumption values. In the late resolution model, the subscript t is used with every consumption value, which indicates that any consumption occurring in time period t, regardless of the state, must be the same amount C_t. In the late resolution model, the subscript $t(Z_{ni})$ is used, which indicates that any consumption value which occurs in period t and occurs in the history Z_{ni} must be of the same amount.

Constraints are straightforward, consumption may not exceed current assets:

$$\sum_{t=1}^{n} C_{ti} \leq A_{t(Z_{ni})} \tag{11}$$

Two-period Temporal Environment: Features of both models are directly applicable to a two-period, temporal risk scenario. Recall that in a temporal model, information is received in between each time period. In a two-period case, information is received once, in between periods 1 ad 2. It is, therefore, logical that decision-making in the first period follows the late resolution model, where all information is received ex-post. In contrast, decision-making in the second period is characterized by a lack of uncertainty which is characteristic to the early resolution model. Applying features of both models, we get a two-period model, where utility is given by:

$$E_1[\mu_1] = U_1(C_1) + \sum_{i=1}^{z} \mu_{2i} \cdot P_1(Z_{2i}) \tag{12}$$

$$\mu_{2i} = U_{2i}(C_{2(Z_{2i})}) \cdot (1+\delta)^{-1} \tag{13}$$

Consumption constrained by:

$$C_1 \leq A_1 \tag{14}$$

$$C_{2(Z_{2i})} \leq (A_1 - C_1)(1+r) + Y_{(2i)} \tag{15}$$

In constructing this two-period model, we achieve one similar to that constructed by Dreze and Modigliani (1972) (which is itself an augmentation of Dreze and Modigliani (1966)), which also considers a two-period temporal consumption problem. The model is as follows:

$$\mu_1 = \int U_1(C_1) + U_2((Y_1 + Y_2 - C_1)(1 + r))d\phi(Y_2) \tag{16}$$

$\phi(Y_2)$ is a probability density function for second-period income.

Our model differs in that it considers a set of discrete possible states rather than a continuous probability distribution of second period income. This difference is irrelevant, for they do not concern the treatment of decision making. A more consequential difference is that the model from Dreze and Modigliani (1966) sets second period consumption as the difference between income and first period consumption $y - c_1$. In other words, the agent spends the entirety of what remains after period 1. This is a very clever construction which restricts actual decision-making to a single period, making the model mathematically tractable. The model is functional as long as the agent's utility function is monotonically increasing, in which case the optimal decision is to spend as much as possible in period 2 in any case. However, it does not function in cases where utility is not necessarily monotonic. More importantly, it cannot easily be generalized to an n-period temporal consumption model, for even when it is assumed that the consumer spends everything left in the last period, the minimum number of decision-making periods is $n - 1$, which exceeds 1 for $n \geq 3$.

General Temporal Environment: From here, it is appropriate to generalize this two-period model to cases where the number of decision-making periods exceeds two. One complication with this is the introduction of the time periods 2 to $n - 1$, for which neither of the timeless models developed earlier is directly applicable. These in-between periods are characterized by neither the lack of updated information found in period 1, nor the complete elimination of uncertainty found in period n. Instead, periods 2 to $n-1$ lie between updates to information and are thus characterized by partially improved foresight. Our solution is to develop a recursively defined model (in which second period consumption from our 2 period model serves as a base case). Cumulative utility from period 1 is given by:

$$E_1[\mu_1] = U_1(C_1) + \sum_{i=1}^{z} E_{(Z_{1i})}[\mu_{2(Z_{2i})}] \cdot P_1(Z_{2i}) \tag{17}$$

The general rule for any period and history $t(Z_{ti})$:

$$E_{(Z_{ti})}[\mu_{t(Z_{ti})}] = U_{t(Z_{ti})}(C_{t(Z_{ti})}) \cdot (1 + \delta)^{1-t} + \sum_{i=1}^{z} E_{(Z_{ti})}[\mu_{t+1(Z_{t+1i})}] \cdot P_{(Z_{ti})}(Z_{t+1i}) \tag{18}$$

The base case at period and history $n(Z_{ni})$:

$$\mu_{n(Z_{ni})} = U_{n(Z_{ni})}(C_{n(Z_{ni})}) \cdot (1 + \delta)^{1-n} \tag{19}$$

At any period of time, agents know the value of their current assets. Therefore, constraints are simple, unlike in the late resolution model.

$$C_{t(Z_{ti})} \leq A_{t(Z_{ti})} \tag{20}$$

The construction of this recursive model is based upon the premise that agents are aware of and respond to their ability to receive updates to information. Whenever the agent considers a possible future state, they consider the decisions available to them in such a scenario, with the recognition that with such state arrives useful information exclusive to that state. For example, if an agent considers the case of a possible future recession, they will acknowledge that this may bring reduced income and thus a reduced ability to consume. However, the agent also recognizes that in such a scenario, there will be updates to information which will allow the agent to be aware that such a recession has occurred in the first place. Because this a recursive model, the agent further speculates on the many branching possible states which may occur once a recession has occured. For example, the recession may be prolonged, or it may be brief, and so on. This recursive process repeats until the agent reaches period n, where their horizon ends. At n, the agent may cease this process, for there is no period beyond it. Therefore, it is here where our recursive process ends.

7 Delayed decision making under temporal risk

In the previous section, three consumption models representing behaviour under three differing levels of uncertainty have been constructed. The first, the late resolution model, captures behaviour when there is uncertainty but no updates to information. The second, the early resolution model, captures behaviour when there is neither uncertainty nor information updates. The third, the temporal model, captures behaviour when there is uncertainty, as well as regular updates to information. Given that probability distributions of payoffs and income streams in each model is the same, when comparing each model with the others, if it can be proved that more precautionary savings will occur in the temporal model than in the late-resolution model, it will follow that updates to information is the condition necessary for such savings to occur. Likewise, suppose it can be proved that agents will not necessarily consume less in the late-resolution model than in the early-resolution model. In that case, it follows that uncertainty in the absence of updates to information are not a sufficient condition for precautionary savings.

Before we proceed with the proofs, it is first useful to determine the domain of consumption in each environment.

Recall that it has been stated in the previous section that the subscripts following consumption values indicate the amount of equality constraints placed on consumption.

The subscript t implies that all consumption values in period t, regardless of history or state, must be the same value. This is appropriate for use in a late-resolution model. The subscript $t(Z_{ti})$ implies that all the consumption values in period t which occur in the history (Z_{ti}) must be equal. This is appropriate for use in a temporal model. The subscript $t(Z_{ni})$ implies that all the consumption values in period t which occur in the history (Z_{ni}) must be equal. This is appropriate for use in an early-resolution model.

The crucial detail is that $t(Z_{ni})$ is a element of $t(Z_{ti})$, which itself is an element of t. Therefore, given the domain of consumption for the set of all consumption decisions $\{C_{t(Z_{ni})}\}$, we can express the constraints (excluding those based on income) imposed in a late resolution model with the restriction $C_{t(Z_{ni})} = C_t$. We can express the constraints imposed in a temporal model with the restriction $C_{t(Z_{ni})} = C_{t(Z_{ti})}$. Based on this, we can express the domain of consumption for each environment as follows:

$$\mathbb{D}_1^L = \{C_{t(Z_{ni})} \in \mathbb{R} \ : \ 0 \leq C_{t(Z_{ni})} = \min[\{C_t, A_{t(Z_{ni})}\}]\} \tag{21}$$

$$\mathbb{D}_1^E : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq A_{t(Z_{ni})}\} \tag{22}$$

$$\mathbb{D}_1^T : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq A_{t(Z_{ni})}, C_{t(Z_{ni})} = C_{t(Z_{ti})}\} \tag{23}$$

Lemma 1: Under rational behavior, maximum utility in the early resolution model is greater than in the temporal model, which is in turn greater than utility in the late resolution model.

Let us first note that $\mathbb{D}_1^E \supseteq \mathbb{D}_1^T$. This is trivial, as the domain for each is identical except for the additional constraint $C_{t(Z_{ni})} = C_{t(Z_{ti})}$ for the temporal environment.

Let us next prove that $\mathbb{D}_1^T \supseteq \mathbb{D}_1^T$. We can do so by considering 2 cases:

<u>Case 1</u>: $C_t \leq A_{t(Z_{ni})}$

$$\mathbb{D}_1^L = \{C_{t(Z_{ni})} \in \mathbb{R} \ : \ 0 \leq C_{t(Z_{ni})} = C_t\} \tag{24}$$

$$\mathbb{D}_1^T : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq A_{t(Z_{ni})}, C_{t(Z_{ni})} = C_{t(Z_{ti})}\} \tag{25}$$

Since $C_{t(Z_{ti})} \in C_t$, $\mathbb{D}_1^T \supseteq \mathbb{D}_1^T$.

<u>Case 2</u>: $A_{t(Z_{ni})} < C_t$

Assume $C_{t(Z_{ti})} = A_{t(Z_{ni})}$ (in other words, consumers in a temporal environment spend all of their current assets):

$$\mathbb{D}_1^L = \{C_{t(Z_{ni})} \in \mathbb{R} \ : \ 0 \leq C_{t(Z_{ni})} = A_{t(Z_{ni})}\} \tag{26}$$

$$\mathbb{D}_1^T : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} - A_{t(Z_{ni})}\} \tag{27}$$

Since the domains are the same, $\mathbb{D}_1^T \supseteq \mathbb{D}_1^T$.

Regardless of case, $\mathbb{D}_1^T \supseteq \mathbb{D}_1^T$. Given any consumption choice possible in an early environment is possible in a temporal environment, and any consumption choice possible in a temporal environment is possible in an early resolution environment, $\max[E_1[\mu_1^E]] \geq \max[E_1[\mu_1^T]] \geq \max[E_1[\mu_1^L]]$.

The intuition behind this lemma is quite straighforward. An agent with greater levels of knowledge can use such knowledge to make better consumption decesions. As a matter of fact, it is intuitively impossible for updates to information to negatively affect a rational agent. Even when updates to information are percieved to less accurate than that which preceded it, an agent may ignore it, pretend as if such updates to information never occured. In the worst case scenario, a temporal consumer may simply act as if they had not recieved those updates, and behave as if they are in a late resolution environment. The understanding that a temporal agent has the same access to information as a late resolution agent, plus more, is crucial to this lemma.

Lemma 2: Regardless of environment, maximum future cumulative utility is greater with more current assets than less current assets.

Let us prove Lemma 2 in the case of the late resolution environment. The structure of this proof is as follows: Suppose we are given two values of A_1, $A_1^a > A_1^b$. If lemma 2 is true, then we must have that $\mathbb{D}_1^{Lb} \supseteq \mathbb{D}_1^{Lb}$. This is equivalent to the identity $\mathbb{D}_1^{Lb} - \mathbb{D}_1^{Lb} = \emptyset$, and it is this equivalency we shall directly prove.

We shall prove that for all time values τ such that $0 \leq \tau \leq n$, the upper bound of consumption (given by $A_{t-1(Z_{ni})}$) is not greater if $A_1 = A_1^b$.

Let $\{C_{t(Z_{ni})}\}$ be the set of consumption choices made in history $Z_{\tau i}$ up to period $\tau - 1$. Furthermore, assume $\{C_{t(Z_{ni})}\}$ is a valid set of consumption values given A_1^b (i.e. no restrictions on consumption are violated).

Using the definition of $A_{t(Z_{ni})}$ from Section 5:

$$A_{t(Z_{ni})} = (A_{t-1(Z_{ni})} - C_{t(Z_{ni})})(1+r) + Y_{t(Z_{ni})}$$

We can express $A^b_{t(Z_{ni})}$ as $A^b_{t(Z_{ni})} + (A^a_{t(Z_{ni})} - A^b_{t(Z_{ni})})(1+r)^{\tau-1}$. Since $A^a_{t(Z_{ni})} - A^b_{t(Z_{ni})} > 0$, we can take for granted that $A_{t(Z_{ni})}$ is also possible given A_1^a. Therefore, the set of consumption values possible given A_1^b but not possible given A_1^a is as follows:

$$\mathbb{D}_1^{Lb} - \mathbb{D}_1^{La} = \{C_{t(Z_{ni})} \in \mathbb{R} \ : \ 0 \leq \min[\{C_t, (A_1^b - A_1^a)(1+r)^{\tau i}]\} \tag{28}$$

Since $A_1^b - A_1^a < 0$, we are given the condition $0 < 0$, which is never met, meaning $\mathbb{D}_1^{Lb} - \mathbb{D}_1^{La} = \emptyset$. This means that $\mathbb{D}_1^{Lb} \supseteq \mathbb{D}_1^{Lb}$, and thus $\max[E_\tau[\mu_\tau^{La}]] \geq \max[E_\tau[\mu^{Lb}]]$.

The proofs for the temporal and early resolution model follow a near-identical format, and thus shall be ommitted.

Lemma 3: When current savings increase, the marginal increase in future utility is greater in an early resolution environment than in a temporal resolution environment, which is greater than future utility in a late resolution environment. Formally, this lemma states the following:

$$\frac{\partial \max[E_1[\mu_2^E]]}{\partial C_1} \leq \frac{\partial \max[E_1[\mu_2^T]]}{\partial C_1} \leq \frac{\partial \max[E_1[\mu_2^L]]}{\partial C_1} \tag{29}$$

First, from lemma 2 we know that all of the above derivatives are negative. Since increased consumption in period 1 reduces current assets in period 2, from lemma 2 we know that maximum utility from period 2 also decreases.

This lemma concerns the set of consumption decisions made available due to the result of increased saving. If, in an early resolution environment, newly available consumption choices made available through saving is a superset of the corresponding set in a temporal environment (with the same comparison being true with a temporal and late resolution environment), then this lemma is true.

Once again, let $\{C_{t(Z_{ni})}\}$ be the set of consumption choices made in history Z_{ni} up to period $\tau - 1$. Furthermore, assume $\{C_{t(Z_{ni})}\}$ is a valid set of consumption values given A_1^b. Per lemma 2, we know that those same choices are possible given A_1^a, and thus the following

$$\mathbb{D}_2^{La} - \mathbb{D}_2^{Lb} = \{C_{2(Z_{ni})} \in \mathbb{R} \mid A_{2b(Z_{ti})} \leq C_{2(Z_{ni})} = \min[\{C_2, A_{2a(Z_{ti})}\}]\} \tag{30}$$

$$\mathbb{D}^E_{(2-3)a-(2-3)b} : \{C_{2(Z_{ni})} \in \mathbb{R} \mid A_{2b(Z_{ti})} \leq C_{2(Z_{ni})} \leq A_{2a(Z_{ni})}\} \tag{31}$$

$$\mathbb{D}^T_{(2-3)a-(2-3)b} : \{C_{2(Z_{ni})} \in \mathbb{R} \mid A_{2b(Z_{ti})} \leq C_{2(Z_{ni})} \leq A_{2a(Z_{ni})}, C_{2(Z_{ni})} = C_{2(Z_{ti})}\} \tag{32}$$

By inspection, we can conclude that $\mathbb{D}^E_{(2-3)a-(2-3)b} \supset \mathbb{D}^T_{(2-3)a-(2-3)b} \supset \mathbb{D}^L_{(2-3)a-(2-3)b}$. Additionally, note that $\min[C^E_{2(Z_{ni})}] \leq \min[C^T_{2(Z_{ni})}] \leq \min[C^L_{2(Z_{ni})}]$, thus this conclusion can be repeated for periods $3 - n$. Therefore, $\frac{\partial \max[E_1[\mu_2^E]]}{\partial C_1} \leq \frac{\partial \max[E_1[\mu_2^T]]}{\partial C_1} \leq \frac{\partial \max[E_1[\mu_2^L]]}{\partial C_1}$.

This lemma is a natural extention of lemmas 1 and 2. Lemma 1 states that utility in a temporal model is greater than utility in a late resolution environment. This is due to the greater ascess to information in a temporal model which permits a consumer to make improved decisions. Lemma 2 states that by saving in the present, a consumer creates greater consumption opportunities for the future, allowing them to dervie higher utiility in the future. If these two premises are correct, it follows that when agents save at a given level, a temporal consumer can use their greater availability of information to allocate these savings to more productive uses than can an agent in a late resolution environment. It thus follows that marginal future utility from increasing current savings is greater in a temporal model than in a late resolution model.

Result: As first period consumption decreases, period 1 utility in a temporal environment does not decrease at a greater rate than period 1 utility in a late resolution environment.

This lemma is trivial. In both the temporal and late resolution environment, consumption takes on a single variable C_1, regardless of the history (this is because the level of information at period 1 is equal in the temporal and late resolution environments). Constraints imposed are equal, $C_1 \leq A_1$, thus marginal utility in the first period is equal in both models.

From here, we can directly prove that precautionary savings is greater in a temporal environment. First, note that by increasing savings (decreasing consumption), marginal future utility is greater in a temporal model but marginal current utility is not lesser compared to a late resolution model. Therefore, we can conclude that marginall culmunative utility (from period 1) from saving is greater in a temporal model:

$$\frac{\partial \max[E_1[\mu_2^T]]}{\partial C_1} \leq \frac{\partial \max[E_1[\mu_2^L]]}{\partial C_1} \wedge \frac{\partial U_1^T(C_1)}{\partial C_1} = \frac{\partial U_1^L(C_1)}{\partial C_1} \implies \frac{\partial \max[E_1[\mu_1^T]]}{\partial C_1} \leq \frac{\partial \max[E_1[\mu_1^L]]}{\partial C_1} \tag{33}$$

A rational agent saves at the level where marginal utility from increasing saving is equal to zero. Therefore, if marginal utility is greater in a temporal model for all consumption values, this means that an agent will save more in a temporal model as compared to a late resolution model, for the point where marginal utility is zero is at a greater level of savings.

8 Delayed decision making with creditors

In the previous section, we have explored the behavior of consumers without access to credit. We have established that precautionary savings occurs for such a consumer in a temporal environment. In this section, we shall augment the model of consumption with creditors to capture scenarios in which creditors are in a late resolution environment. While we explore consumer behavior in a late resolution, early resolution, and temporal environment, we shall keep the environment of the creditor constant to avoid conflating the different outcomes of each model to one particular change or the other.

Recall the consumption with borrowing model developed for a certain environment:

$$\mu_1 = \max_{C_t}[\sum_{t=1}^{n} U_t(C_t)] \cdot (1+\delta)^{1-t} \tag{34}$$

$$C_1 \leq Y_1^P \tag{35}$$

$$A_t = (A_{t-1} - C_{t-1})(1+r) + Y_t \tag{36}$$

With the introduction of borrowing, upper constraints on consumption are determined by permanant income, not current assets. This introduces a possible issue when borrowing is introduced to an uncertain environment. Recall the definition of permanant income from section 3:

$$Y_1^P = A_1 + \sum_{t=2}^{n} Y_t + \sum_{t=1}^{n} A_t r \tag{37}$$

Observe that permanant income is dependent on future liquid assets, as interest is earned off it. As liquid assets are determined by consumption, this in turn makes permanant income an endogenous variable. In a certain environment, permanant income is defined because consumption is predictable. There are no updates to information, thus consumption can be planned ahead with no need to modify plans. In an uncertain variable, consumption is unpredictable, and so is future income, meaning there is no single value of permanant income. However, since an uncertain environment is the aggregate of many possible histories, within which both consumption and income is defined, the expected value of permanant income is also defined, and can be used without issue. In an uncertain environment, creditors lend to consumers based on expected permanant income, with the nature of the expectations depending on the type of uncertain environment the creditor is in.

In augmenting this model to each type of uncertain environment, we need not modify the consumption functions, only its constraints, for the effects of saving and lending are restricted to a change in the constraints. Recall the constraints held by each environment when saving and lending are restricted:

$$\mathbb{D}_1^L : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} = \min[\{C_t, A_{t(Z_{ni})}\}]\} \tag{38}$$

$$\mathbb{D}_1^E : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq A_{t(Z_{ni})}\} \tag{39}$$

$$\mathbb{D}_1^T : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq A_{t(Z_{ni})}, C_{t(Z_{ni})} = C_{t(Z_{ti})}\} \tag{40}$$

In a late resolution environment, the creditor lends to consumers solely on the basis of information held at period 1. Assuming risk-neutral creditors, they lend up to the expected value of permanant income at period 1. Interestingly, this is not neccesarily the upper bound for consumption. As the creditor does not recieve updates to information, the agent and creditors' expectations of permanant income may diverge. In extreme cases, current assets may exceed expected permanant income, leading to a scenario where agents can immediately spend an amount greater than what is offered by creditors. Therefore, maximum consumption is the greater of either expected permanant income or current assets:

$$\mathbb{D}_1^{L,L} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} = \min[\{C_t, \max[\{E_1[Y_t^P], A_{t(Z_{ti})}\}]\}]\} \tag{41}$$

$$\mathbb{D}_1^{E,L} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq \max[\{E_1[Y_t^P], A_{t(Z_{ti})}\}]\} \tag{42}$$

$$\mathbb{D}_1^{T,L} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq \max[\{E_1[Y_t^P], A_{t(Z_{ti})}\}], C_{t(Z_{ni})} = C_{t(Z_{ti})}\} \tag{43}$$

The first superscript indicates the environment of the consumer, the second indicates the environment of the creditor.

In an early resolution environment, the creditor has perfect foresight, and lends based on a certain value of permanant income. This is reflective of all past, present, and future states, thus there is no possibility of current assets exceeding creditors' expectations of permanant income:

$$\mathbb{D}_1^{L,E}L : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} = \min[\{C_t, Y^P_{t(Z_{ni})}\}]\} \tag{44}$$

$$\mathbb{D}_1^{E,E} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq Y^P_{t(Z_{ni})}\} \tag{45}$$

$$\mathbb{D}_1^{T,E} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq Y^P_{t(Z_{ni})}, C_{t(Z_{ni})} = C_{t(Z_{ti})}\} \tag{46}$$

In a temporal environment, creditors base expectations of permanant income off knowledge of past and present states:

$$\mathbb{D}_1^{L,T} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} = \min[\{C_t, E_{(Z_{ti})}[Y^P_{t(Z_{ti})}]\}]\} \tag{47}$$

$$\mathbb{D}_1^{E,T} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq E_{(Z_{ti})}[Y^P_{t(Z_{ti})}]\} \tag{48}$$

$$\mathbb{D}_1^{T,T} : \{C_{t(Z_{ni})} \in \mathbb{R} \mid 0 \leq C_{t(Z_{ni})} \leq E_{(Z_{ti})}[Y^P_{t(Z_{ti})}], C_{t(Z_{ni})} = C_{t(Z_t)}\} \tag{49}$$

With our consumption model augmented to capture creditor behavior under various forms of uncertainty, we can repeat the same analysis done with creditless consumption in the previous section. We shall not repeat the process on paper, for the process is identical, however, we reach the same conclusions made with the creditless model. We conclude that $C_1^{L,T} \leq C_1^{L,L}$, $C_1^{T,E} \leq C_1^{L,E}$, and $C_1^{T,T} \leq C_1^{L,T}$. We also reach the same welfare implications from lemma 1, namely, $\max[E_1[\mu_1^{E,L}]] \geq \max[E_1[\mu_1^{T,L}]] \geq \max[E_1[\mu_1^{L,L}]]$, $\max[E_1[\mu_1^{E,E}]] \geq \max[E_1[\mu_1^{T,E}]] \geq \max[E_1[\mu_1^{L,E}]]$, and $\max[E_1[\mu_1^{E,T}]] \geq \max[E_1[\mu_1^{T,T}]] \geq \max[E_1[\mu_1^{L,T}]]$.

When holding the environment of creditors constant, . In other words, when all else is constant,

A more interesting analysis occurs when updates to information occur both to the consumer and creditor. Here, we compare behavior and welfare between an environment where both agents are in a late resolution environment, both are in an early environment, and both are in a temporal one. When repeating the analysis done to prior comparisons, we observe that lemmas 1 and 3 fail. When both the consumer and creditor recieves updates to information, such a scenario is not neccesarily preferred to one where both are in a late resolution environment. Likewise, precautionary saving does not neccesarily occur.

Lemma 1. Under utility-maximizing behavior, maximum utility in the early resolution model is greater than in the temporal model, which is in turn greater than utility in the late resolution model.

The following

Lemma 3. When current savings increase, the marginal increase in future utility is greater in an early resolution environment than in a temporal resolution environment, which is in turn greater than future utility in a late resolution environment.

Once again, consider two consumption values $C_{1a} \leq C_{1b}$. Per lemma 2, we know that $A_{2b(Z_{2i})} \leq A_{2b(Z_{2i})}$. Repeating the same analysis, but for permanant income, we note that $Y^P_{2(Z_{ni})} = (A_1 - C_1)(1 + r) + \sum_{t=2}^n Y_{t(X_{ti})} + \sum_{t=2}^n A_t r$, observing that permanant income decreases as first period consumption increases. Therefore, $Y^P_{2b(Z_{ni})} \leq Y^P_{2a(Z_{ni})}$, $Y^P_{2b(Z_{ti})} \leq Y^P_{2a(Z_{ti})}$, and $Y^P_{2b} \leq Y^P_{2a}$. The following are the domains of period 2 consumption for a and b:

$$\mathbb{D}^{L,L}_{(2-3)a-(2-3)b} : \{C_{2(Z_{ni})} \in \mathbb{R} \mid Y^P_{2b} \leq C_{2(Z_{ni})} = \min[\{C_2, Y^P_{2a}\}]\} \tag{50}$$

$$\mathbb{D}^{E,E}_{(2-3)a-(2-3)b} : \{C_{2(Z_{ni})} \in \mathbb{R} \mid Y^{P}_{2b(Z_{ni})} \leq C_{2(Z_{ni})} \leq Y^{P}_{2a(Z_{ni})}\} \tag{51}$$

$$\mathbb{D}^{T,T}_{(2-3)a-(2-3)b} : \{C_{2(Z_{ni})} \in \mathbb{R} \mid Y^{P}_{2b(Z_{ti})} \leq C_{2(Z_{ni})} \leq A_{2a(Z_{ti})}, C_{2(Z_{ni})} = C_{2(Z_{ti})}\} \tag{52}$$

Most crucially, it is not always true. Therefore, lemma 3 is not true for this comparison. This in turn means that the rest of the analysis fails, and precautionary saving does not necesarily occur when both the consumer and creditor recieves updates to information.

Like with other findings.

9 Generalizations to any uncertain environment

In the second section, we provide a generalized proof that for any static decision, certainty is preferred to uncertainty. Here, we shall generalize the findings of the previous section to any scenario in which there is uncertainty, decision-making, and the possibility of recieving updates to information. Specifically, when holding the information of other parties constant, the agent prefers to recieve additional information. If such information is recieved over time, the agent will delay decisions. When the information of other parties is not constant, conclusions are indeterminate. First, consider a generali . Because of the infrastructure developed in previous sections, the proof is straightfoward, and we can largely reuse models developed for consumption, but without the specifications

Late resolution:

$$E_1[\mu_1] = \sum_{i=1}^{z} P_1(Z_{ni}) \cdot U_{ni}(A_1, A_2, ..., A_n) \tag{53}$$

Early resolution:

$$E_1[\mu_1] = \sum_{i=1}^{z} P_1(Z_{ni}) \cdot U_{ni}(A_{1(Z_{ni})}, A_{2(Z_{ni})}, ..., A_{n(Z_{ni})}) \tag{54}$$

Temporal resolution:

$$E_1[\mu_1] = \sum_{i=1}^{z} P_1(Z_{ni}) \cdot U_{ni}(A_{1(Z_{ti})}, A_{2(Z_{ti})}, ..., A_{n(Z_{ti})}) \tag{55}$$

Temporal resolution:

$$E_1[\mu_1] = \sum_{i=1}^{z} P_1(Z_{ni}) \cdot U_{ni}(A_1, A_2, ..., A_n, B_1, B_2, ..., B_n) \tag{56}$$

One finding which is made for our consumption models but cannot be generalized is that when other agents are static in a temporal or early resolution model, the primary agent prefers moving from a

We interpret this

. For example, our . It is possible that a rational creditor may recognize this, reason that they are expected to lose money, and pull out of the credit market. As another example, suppose you live under an authorotarian regime where information is tightly controlled. The secret police know whenever a particular citizen recieves information they are not supposed to have access to. In such a scenario, certain updates to information, those which compromise the regime, may be detrimental. These discrepancies raise the possibility that our particular results may have arisen as a consequence of the particular assumptions made, not as a inevitable consequence of the nature of information.

10 Conclusion

This paper employs a unique interpretation of information, which attributes its positive effect on its ability to allow possessors of information to act differently depending on the contents of such information. From this, a simple consumption model is augmented to capture the consumer's optimization problem under different forms of uncertainty. When constructing the augmented models, effort has been placed to accurately incorporate our interpretation of information into them. Once the models have been constructed, they have been compared to each other to isolate the effects of the differences in assumptions made in each model.

This paper concludes that in the absence of saving and borrowing, precautionary savings occurs under all parameters. When saving and borrowing is introduced, along with creditors who faciliate such borrowing, conclusions are more complicated. When the information available to creditors is static, as is under a late resolution or early resolution environment, precautionary saving still occurs under all parameters. When creditors and consumers recieve information in tandem, as is the case in a temporal environment, precautionary saving does not neccesarily occur.

Policy implications of these findings are immediate. Policies designed to stimulate increases in consumption will be less effective if they simutaneously contribute to higher uncertainty. For example, beneficiaries of a potential stimulus package may delay major consumption decisions until it is revealed whether such package will be pursued or not.

Another promising direction these findings may be extended is to apply them to other areas of analysis which involve one or more agents making decisions throughout time. Although this paper explores the narrow focus of consumption, it is likely that many of the conclusions can be generalized, for none of the proofs require assumptions restricted to the area of consumption. Although we have explored many different models of uncertainty, with many forms of interaction between different agents, in truth the most relevant to real life is the interaction of many agents in a temporal environment. Coincidentally, it is in such an environment where we have been unable to prove the strict existence of precautionary savings. In such an environment, to what extent agents delay decisions will depend on the particulars of their relationships with other agents. For example, the delaying of decisions will likely occur more frequently when agents have cooperative interactions with other agents, rather than hostile ones.

11 References

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