Modelling non-commuting trips: estimation outline

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April 30, 2025

Estimation

- The number of non-commuting trips for a resident commuting i to j is:

$$n_{ijk} = \frac{a_k \tau_{ijk}^{-\delta}}{\sum_t a_t \tau_{ijk}^{-(\delta-1)}} \sum_t n_{ijt} \tau_{ijt}$$

- The model implies a gravity equation for non-commuting trips between locations. Residents will travel more often to relatively attractive and close locations.
- Assuming that we observe n_{ijk} and τ_{ijk}, we need to find values for a_k and δ.

Estimation

- A possible way to estimate δ is to define the ratio of non-commuting trips to a destination t as

$$= \frac{a_k \tau_{ijk}^{-\delta}}{a_t \tau_{ijk}^{-\delta}}$$

- Taking logs and expanding the equation to account for deviations

$$\ln\left(\frac{n_{ijk}}{n_{ijt}}\right) = \ln\left(\frac{a_k}{a_t}\right) - \delta \ln\left(\frac{\tau_{ijk}}{\tau_{ijt}}\right) + \epsilon_{ijkt}$$

  which can be estimated with the fixed effect estimator or with an instrument.
- Question: Is there a way to use the PPML estimator?

Estimation

- Once we have an estimate for δ, the next step consists in solving for a_k in every location.
- Assume that we observe n_{ijk}, τ_{ijk} and the value for δ from the previous step. Define the following excess demand function for each location k

$$\mathbb{D}_k(\mathbf{a}) = \sum_i \sum_j n_{ijk}\tau_{ijk} - \sum_i \sum_j \frac{a_k \tau_{ijk}^{-\delta}}{\sum_t a_t \tau_{ijt}^{-(\delta-1)}} \sum_t n_{ijt}\tau_{ijt}$$

- Using the procedure outlined in Monte et al., we can show that there is a (to scale) unique solution.

Estimation

- The excess demand function has the following properties:
  1. $\mathbb{D}(\mathbf{a})$ is continuous.
  2. $\mathbb{D}(\mathbf{a})$ is homogenous of degree zero.
  3. $\sum_k \mathbb{D}_k(\mathbf{a}) = 0$
  4. $\mathbb{D}(\mathbf{a})$ exhibits gross substitution.
- (1), (2) & (3) together with Brouwer's Fixed Point theorem imply that there is at least one solution in the simplex.
- (4) implies that there is at most one solution in the simplex.

Estimation

- Once we know the values for a_k and δ, we can estimate the commuting gravity equation

$$R_{ij} = \frac{X_i E_j A_{ij}^{\eta} v_{ij}^{\nu}}{\sum_r \sum_s X_r E_s A_{rs}^{\eta} v_{rs}^{\nu}}$$

where $A_{ij} = \sum_t a_t \tau_{ijt}^{-(\delta-1)}$ and $v_{ij} = \frac{w_j W - c_{ij}}{W + t_{ij}}$

- Taking logs to the previous equation and expanding the equation to account for deviations

$$\ln R_{ij} = \varphi_i + \phi_j + \eta \ln A_{ij} + \nu \ln v_{ij} + \epsilon_{ij}$$

which can be estimated with the fixed effect estimator or the PPML estimator.

Estimation

- The outline procedure allow us to estimate

$$\delta - 1 = \frac{\rho(1-\alpha)}{1-\rho}$$

$$\eta = \frac{(1-\rho)(1-\beta)\theta}{\rho}$$

$$\nu = ((1-\alpha)(1-\beta)+\beta)\theta$$

which doesn't allow us to identify each parameter individually.

- We must calibrate a parameter using values from the literature or we must find an extra estimation equation.

Estimation

- Workplace amenities can be recover solving the following system of non-linear equations

$$R_j = \sum_i \pi_{j|i} R_i = \sum_i \frac{E_j A_{ij}^{\eta} v_{ij}^{\nu}}{\sum_s X_s A_{is}^{\eta} v_{is}^{\nu}} R_i$$

- Residence amenities can be recover solving the following system of non-linear equations assuming that we observe rents

$$R_i = \sum_j \pi_{i|j} R_j = \sum_j \frac{X_i q_i^{(\gamma-1)} A_{ij}^{\eta} v_{ij}^{\nu}}{\sum_r X_r q_r^{(\gamma-1)} A_{rj}^{\eta} v_{rj}^{\nu}} R_j$$

Estimation

- The profit maximization for firms define an equation that relates wages to labour demand measured in hours

$$\mu A_j L_j^{\mu-1} - w_j = 0$$

where

$$L_j = \sum_i l_{ij} R_{ij}$$

and l_{ij} is the individual labour supply of a resident commuting i to j

$$l_{ij} = \frac{\beta}{\alpha(1-\beta)+\beta} \frac{T - \sum_k n_{ijk} t_{ik}}{W + t_{ij}} + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\beta} \frac{\sum_k n_{ijk} c_{ik}}{w_j W - c_{ij}}$$