CSSS 510: Lab 3

Logistic Regression

2017-10-13

0. Agenda

1. Deriving a likelihood function for the logistic regression model
2. Fitting a logit model using optim() and glm()
3. Simulating predicted values and confidence intervals
4. Simulating first differences

1. Deriving a likelihood function for the logistic regression model

Recall from lecture the logit model:

$$
\begin{aligned}
y_i &\sim \text{Bern}(\pi_i) \\
\pi_i &= \text{logit}^{-1}(\boldsymbol{x}_i\boldsymbol{\beta}) \\
\pi_i &= \frac{\exp(\boldsymbol{x}_i\boldsymbol{\beta})}{1+\exp(\boldsymbol{x}_i\boldsymbol{\beta})} = \frac{1}{1+\exp(-\boldsymbol{x}_i\boldsymbol{\beta})}
\end{aligned}
$$

In the simple case, this stems from the latent variable model:

$$y^* = \beta_0 + \beta_1 x + \epsilon$$

where the relationship between latent variable y^* and the explanatory variable x is modeled using simple linear regression, and the binary outcome y is a function of the sign of y^*:

$$y = \begin{cases} 1, & \text{if } y^* > 0 \\ 0, & \text{if } y^* \leq 0 \end{cases} \tag{1}$$

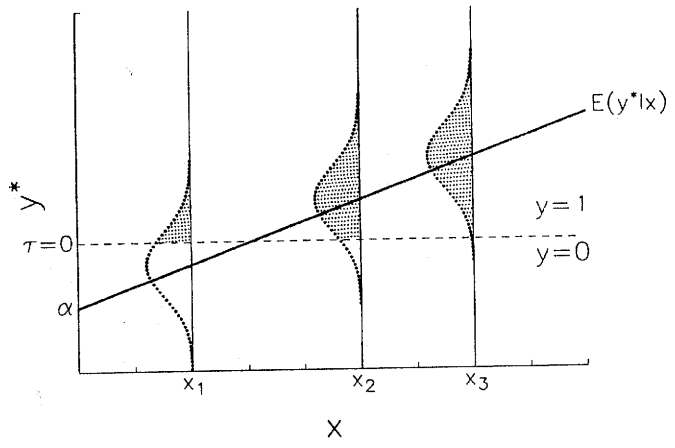


Figure 3.2. The Distribution of y^* Given x in the Binary Response Model

The logistic regression model is obtained if we assume the errors of this latent variable model follow a standard logistic distribution. Recall that the pdf and cdf of the standard logisitic distribution are as follows:

$$f(t) = \frac{\exp(t)}{(1+\exp(t))^2}$$
$$F(t) = \frac{\exp(t)}{1+\exp(t)}$$

We therefore have the following:

$$\begin{aligned}
\Pr(y=1|x) &= \Pr(y^* > 0|x) \\
&= \Pr(\beta_0 + \beta_1 x + \epsilon > 0|x) \\
&= \Pr(\epsilon > -(\beta_0 + \beta_1 x)) \\
&= \Pr(\epsilon < \beta_0 + \beta_1 x) \\
&= F(\beta_0^L + \beta_1^L x)
\end{aligned}$$

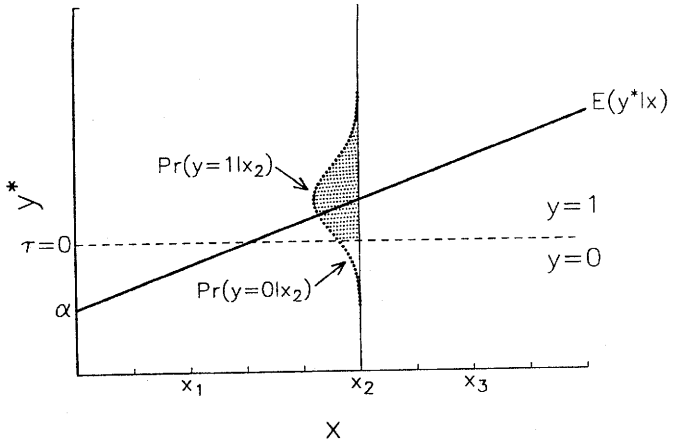


Figure 3.4. Probability of Observed Values in the Binary Response Model

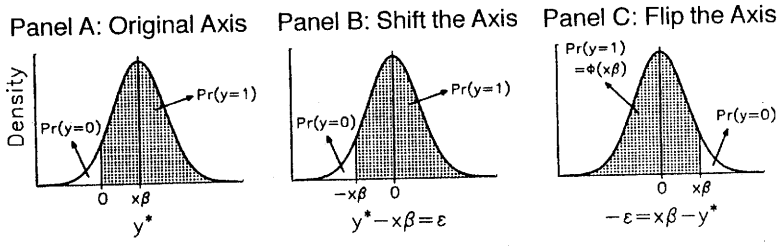


Figure 3.5. Computing $\Pr(y = 1 \mid x)$ in the Binary Response Model

Since we assume the errors follow a standard logistic distribution, we have

$$\Pr(y = 1|x) = F(\beta_0^L + \beta_1^L x) = \frac{\exp(\beta_0^L + \beta_1^L x)}{1 + \exp(\beta_0^L + \beta_1^L x)}$$

and $\mathrm{E}(\epsilon)$=0 and $\mathrm{Var}(\epsilon) = \frac{\pi^2}{3}$.

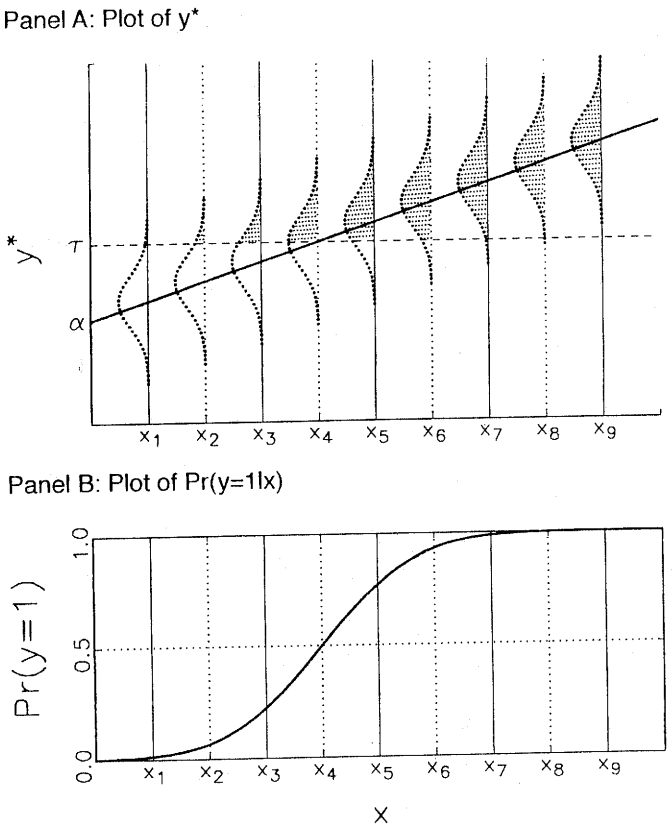


Figure 3.6. Plot of y^* and $\Pr(y = 1 \mid \mathbf{x})$ in the Binary Response Model

The logit function is the inverse of the logistic function:

$$\mathrm{logit}(p) = \log\frac{p}{1-p}$$

or

$$\text{logit}^{-1}(p) = \frac{\exp(x)}{1+\exp(x)}$$

We therefore have the following

$$\Pr(y=1|x) = \text{logit}^{-1}(\beta_1^L + \beta_1^L x)$$

or

$$\text{logit}(\Pr(y=1|x)) = \beta_1^L + \beta_1^L x$$

or

$$\log\frac{\Pr(y=1|x)}{\Pr(y=0|x)} = \beta_0^L + \beta_1^L x.$$

Recall from lecture that a Bernoulli distribution has the following pdf:

$$\Pr(y_i = 1|\pi_i) = \pi_i^{y_i}(1-\pi_i)^{1-y_i}$$

And the likelihood function can be derived from the joint probability:

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\pi}|\boldsymbol{y}) &\propto \prod_{i=1}^{n} \pi_i^{y_i}(1-\pi_i)^{1-y_i} \\
\mathcal{L}(\boldsymbol{\beta}|\boldsymbol{y}) &\propto \prod_{i=1}^{n} \left(\frac{1}{1+\exp(-\boldsymbol{x}_i\boldsymbol{\beta})}\right)^{y_i} \left(1-\frac{1}{1+\exp(-\boldsymbol{x}_i\boldsymbol{\beta})}\right)^{1-y_i} \\
\mathcal{L}(\boldsymbol{\beta}|\boldsymbol{y}) &\propto \prod_{i=1}^{n} (1+\exp(-\boldsymbol{x}_i\boldsymbol{\beta}))^{-y_i}(1+\exp(-\boldsymbol{x}_i\boldsymbol{\beta}))^{-(1-y_i)} \\
\log\mathcal{L}(\boldsymbol{\beta}|\boldsymbol{y}) &\propto \sum_{i=1}^{n} -y_i\log(1+\exp(-\boldsymbol{x}_i\boldsymbol{\beta})) - (1-y_i)\log(1+\exp(\boldsymbol{x}_i\boldsymbol{\beta}))
\end{aligned}$$

2. Fitting a logit model using **optim**() and **glm**()

```
rm(list = ls()) # clear up the memory

#install and load the packages needed
#from CRAN: install.packages("MASS", dependencies = TRUE)
library(MASS)
library(RColorBrewer)

#download simcf and tile packages from- http://faculty.washington.edu/cadolph/software
# don't unzip the archive (tar) file
library(simcf)
library(tile)
```

```
## Loading required package: grid
```

```r
# Load data
file <- "nes00a.csv"
data <- read.csv(file, header=TRUE)

# attach(data)

# Estimate logit model using optim()
# Construct variables and model objects
y <- data$vote00
x <- cbind(data$age,data$hsdeg,data$coldeg)

# Likelihood function for logit
llk.logit <- function(param,y,x) {
  os <- rep(1,length(x[,1])) # constant
  x <- cbind(os,x) # constant+covariates
  b <- param[ 1 : ncol(x) ]
  # number of parameters to be estimated equals number of columns in x
  # (i.e, one for constant and one for each covariates : total 4)
  xb <- x%*%b
  sum( y*log(1+exp(-xb)) + (1-y)*log(1+exp(xb))) # log-likelihood function for logit model
  # (based on our choice of standard logistic cdf as the systematic component)
               # optim is a minimizer, so use -lnL here
}

# Fit logit model using optim
ls.result <- lm(y~x)  # use ls estimates as starting values (for convenience)
stval <- ls.result$coefficients  # initial guesses
logit.result.opt <- optim(stval,llk.logit,method="BFGS",hessian=T,y=y,x=x)
                   # call minimizer procedure or max by adding control=list(fnscale=-1)
pe.opt <- logit.result.opt$par   # point estimates
vc.opt <- solve(logit.result.opt$hessian)  # var-cov matrix
se.opt <- sqrt(diag(vc.opt))    # standard errors
ll.opt <- -logit.result.opt$value  # likelihood at maximum

logit.optim<-data.frame(cbind(round(pe.opt,3), round(se.opt,3)))
rownames(logit.optim)<-c("intercept", "age", "highschool" , "college")
colnames(logit.optim)<-c("pe", "std.err")
logit.optim
```

```
##                pe std.err
## intercept  -2.149   0.257
## age         0.031   0.003
## highschool  1.213   0.179
## college     1.102   0.130
```

```r
#p-value based on t-statistics
2*pt(abs(logit.optim$pe/logit.optim$std.err), df=length(y)-length(pe.opt) , lower.tail = FALSE)
```

```
## [1] 1.229175e-16 2.393414e-24 1.668155e-11 4.780627e-17
```

```r
# Estimate logit model using glm()

# Run logit & extract results using glm.
# GLM solves the likelihood equations with a common numeric algorithm
# called iteratively re-weighted least squares (IRWLS).
```

```r
logit.result <- glm(vote00~age +hsdeg+coldeg, family=binomial, data=data)
# family "binomial" calls logit transformation (log(pi/1-pi)) as a "link function"
# corresponding to logistic distribution.
# Link function transforms pi so that it follows a linear model.
# (So although pi itself is dependent on covariates in a non-linear way,
# logit transformed pi is dependent on covariates in a linear way.)

summary(logit.result)
```

```
##
## Call:
## glm(formula = vote00 ~ age + hsdeg + coldeg, family = binomial,
##     data = data)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.4487  -1.1347   0.6360   0.8973   1.8854
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -2.147997   0.256608  -8.371  < 2e-16 ***
## age          0.030885   0.003386   9.121  < 2e-16 ***
## hsdeg        1.212882   0.179447   6.759 1.39e-11 ***
## coldeg       1.102465   0.130426   8.453  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 2293.5  on 1782  degrees of freedom
## Residual deviance: 2076.0  on 1779  degrees of freedom
## AIC: 2084
##
## Number of Fisher Scoring iterations: 4
```

```r
# now a new model adding age^2
model <- vote00 ~ age + I(age^2) + hsdeg + coldeg
mdata <- extractdata(model, data, na.rm=TRUE) # needs library(simcf)

logit.result <- glm(model, family=binomial, data=mdata)
summary(logit.result)
```

```
##
## Call:
## glm(formula = model, family = binomial, data = mdata)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.2045  -1.1145   0.6335   0.8743   1.9841
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.0193891  0.4181899  -7.220 5.19e-13 ***
## age          0.0747252  0.0168440   4.436 9.15e-06 ***
```

```
## I(age^2)    -0.0004427  0.0001655  -2.674  0.00749 **
## hsdeg        1.1243908  0.1800069   6.246 4.20e-10 ***
## coldeg       1.0795702  0.1312113   8.228  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 2293.5  on 1782  degrees of freedom
## Residual deviance: 2069.0  on 1778  degrees of freedom
## AIC: 2079
##
## Number of Fisher Scoring iterations: 4
```

```
pe <- logit.result$coefficients  # point estimates
vc <- vcov(logit.result)         # var-cov matrix

pe
```

```
##   (Intercept)           age      I(age^2)         hsdeg        coldeg
## -3.0193890720  0.0747251922 -0.0004427014  1.1243907749  1.0795702357
```

```
sqrt(diag(vc))
```

```
##  (Intercept)          age     I(age^2)        hsdeg       coldeg
## 0.4181899367 0.0168440337 0.0001655466 0.1800068893 0.1312113265
```

3. Simulating predicted values and confidence intervals

```
# Simulate parameter distributions
sims <- 10000
simbetas <- mvrnorm(sims, pe, vc) #needs library(MASS) # draw 10000 sets of simulated
# parameter (beta) estimates from a multivariate normal distribution with
# mean pe and variance-covariance vc

# Now let's plan counterfactuals: We will have three sets of counterfactuals based
# on education lavel (less than hs edu, hs edu, college or higher edu), and for each set
# we will make age varies between 18 years old and 97 years old.

# Set up counterfactuals:  all ages, each of three educations
xhyp <- seq(18,97,1)  # create age vector
nscen <- length(xhyp)  # we will have total 80 different age scenarios for each education level

nohsScen <- hsScen <- collScen <- cfMake(model, mdata, nscen)  #this is just to initialize
# 80 scenarios for each education level. As default, all covariate values are set at the mean.

# Create three sets of education counterfactuals


for (i in 1:nscen) {
  # No High school scenarios (loop over each age, total 80 scenarios)
  nohsScen <- cfChange(nohsScen, "age", x = xhyp[i], scen = i)
  nohsScen <- cfChange(nohsScen, "hsdeg", x = 0, scen = i) # no hs degree
```

```
  nohsScen <- cfChange(nohsScen, "coldeg", x = 0, scen = i) # no college degree

  # HS grad scenarios (loop over each age, total 80 scenarios)
  hsScen <- cfChange(hsScen, "age", x = xhyp[i], scen = i)
  hsScen <- cfChange(hsScen, "hsdeg", x = 1, scen = i) # has hs degree
  hsScen <- cfChange(hsScen, "coldeg", x = 0, scen = i)  # no college degree

  # College grad scenarios (loop over each age, total 80 scenarios)
  collScen <- cfChange(collScen, "age", x = xhyp[i], scen = i)
  collScen <- cfChange(collScen, "hsdeg", x = 1, scen = i) # has hs degree
  collScen <- cfChange(collScen, "coldeg", x = 1, scen = i) # has college degree
}

# # Now given the counterfactual covariates (nohsScen/hsScen/collScen) and simulated
# parameters (simbetas), we can calculate expected value of the response.
# In this case, expected probability of voting!

head(nohsScen$x) #we will fit the counterfactual data
```

```
##      vote00 age hsdeg coldeg
## 1 0.6567583  18     0      0
## 2 0.6567583  19     0      0
## 3 0.6567583  20     0      0
## 4 0.6567583  21     0      0
## 5 0.6567583  22     0      0
## 6 0.6567583  23     0      0
```

```
nohsScen$model # in the model specification
```

```
## vote00 ~ age + I(age^2) + hsdeg + coldeg
```

```
nohsSims <- logitsimev(nohsScen, simbetas, ci=0.95) # using simulated betas to get expected values.
# Built-in function "logitsimev" calculates the expected value
# for every individual scenario you created.

nohsSims #reports lower and upper confidence intervals as well as expected probabilities
```

```
## $pe
##  [1] 0.1418490 0.1489419 0.1562245 0.1636892 0.1713270 0.1791283 0.1870827
##  [8] 0.1951788 0.2034047 0.2117480 0.2201954 0.2287334 0.2373481 0.2460254
## [15] 0.2547507 0.2635096 0.2722877 0.2810704 0.2898436 0.2985932 0.3073054
## [22] 0.3159669 0.3245647 0.3330864 0.3415198 0.3498537 0.3580770 0.3661795
## [29] 0.3741514 0.3819837 0.3896679 0.3971960 0.4045607 0.4117553 0.4187735
## [36] 0.4256099 0.4322591 0.4387165 0.4449781 0.4510399 0.4568988 0.4625516
## [43] 0.4679959 0.4732293 0.4782499 0.4830558 0.4876458 0.4920184 0.4961727
## [50] 0.5001078 0.5038228 0.5073173 0.5105907 0.5136427 0.5164729 0.5190811
## [57] 0.5214671 0.5236309 0.5255723 0.5272914 0.5287881 0.5300625 0.5311146
## [64] 0.5319446 0.5325526 0.5329388 0.5331034 0.5330467 0.5327692 0.5322713
## [71] 0.5315534 0.5306163 0.5294607 0.5280876 0.5264980 0.5246932 0.5226747
## [78] 0.5204439 0.5180030 0.5153540
##
## $lower
##  [1] 0.09390292 0.10010630 0.10662098 0.11301051 0.11931962 0.12585331
##  [7] 0.13295794 0.14055653 0.14783534 0.15529577 0.16263450 0.17015526
## [13] 0.17732029 0.18507903 0.19278234 0.20016104 0.20764642 0.21534336
```

```
## [19] 0.22297627 0.23043678 0.23814250 0.24539771 0.25337365 0.26102848
## [25] 0.26871205 0.27590341 0.28300376 0.28992749 0.29699218 0.30383416
## [31] 0.31066360 0.31721771 0.32421343 0.33120906 0.33809704 0.34466352
## [37] 0.35137581 0.35795037 0.36448926 0.37055772 0.37630981 0.38215196
## [43] 0.38823399 0.39366754 0.39903840 0.40413149 0.40884316 0.41352037
## [49] 0.41761033 0.42121597 0.42475501 0.42846488 0.43115630 0.43349251
## [55] 0.43629860 0.43842020 0.43983954 0.44072062 0.44163902 0.44199760
## [61] 0.44176694 0.44133236 0.44009831 0.43818752 0.43594678 0.43307395
## [67] 0.43008276 0.42600465 0.42141441 0.41677185 0.41197745 0.40638676
## [73] 0.40009728 0.39310625 0.38599656 0.37856523 0.37071606 0.36267408
## [79] 0.35409970 0.34563443
##
## $upper
##  [1] 0.2019016 0.2096653 0.2175677 0.2256691 0.2338949 0.2428245 0.2515584
##  [8] 0.2605789 0.2695783 0.2786635 0.2880380 0.2976895 0.3070052 0.3169506
## [15] 0.3270397 0.3367112 0.3464118 0.3559747 0.3657268 0.3754267 0.3854430
## [22] 0.3949564 0.4045318 0.4137376 0.4233257 0.4323296 0.4412183 0.4498831
## [29] 0.4587525 0.4665734 0.4745100 0.4821851 0.4897181 0.4967111 0.5038297
## [36] 0.5104048 0.5169559 0.5234201 0.5288516 0.5344348 0.5401356 0.5455378
## [43] 0.5502708 0.5547691 0.5588814 0.5632937 0.5672669 0.5710330 0.5745070
## [50] 0.5779951 0.5816414 0.5849904 0.5885923 0.5917727 0.5948650 0.5982617
## [57] 0.6014468 0.6041682 0.6077760 0.6109218 0.6139671 0.6172311 0.6207064
## [64] 0.6240185 0.6279355 0.6305912 0.6336502 0.6375042 0.6409336 0.6442131
## [71] 0.6473857 0.6510254 0.6543734 0.6585626 0.6622663 0.6658136 0.6699336
## [78] 0.6733632 0.6771774 0.6811384
```

```
# same thing for two other sets of sceneraios
hsSims <- logitsimev(hsScen, simbetas, ci=0.95)
collSims <- logitsimev(collScen, simbetas, ci=0.95)

# Get 3 nice colors for traces
col <- brewer.pal(3,"Dark2")

# Set up lineplot traces of expected probabilities

#Traces are elements of tile : lines, labels, legends...

# no hs
nohsTrace <- lineplot(x=xhyp, # age on x-axis
                      y=nohsSims$pe, #expected probability on y-axis
                      lower=nohsSims$lower, # lower confidence interval
                      upper=nohsSims$upper, #upper confidence interval
                      col=col[1], #color choice
                      extrapolate=list(data=mdata[,2:ncol(mdata)],
                                       #actual covariates (i.e., values in your data)
                                cfact=nohsScen$x[,2:ncol(hsScen$x)],#counterfactual covariates
                        omit.extrapolated=FALSE), #don't show extrapolated values
                      plot=1)

# hs but no college
hsTrace <- lineplot(x=xhyp,
                    y=hsSims$pe,
                    lower=hsSims$lower,
                    upper=hsSims$upper,
```

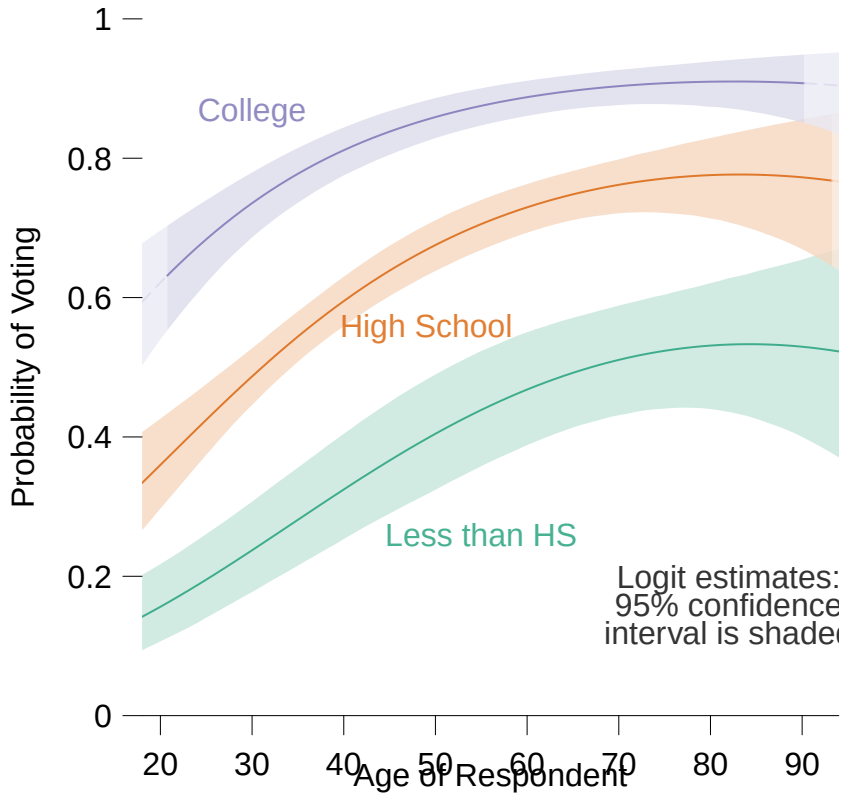
```
                    col=col[2],
                    extrapolate=list(data=mdata[,2:ncol(mdata)],
                                     cfact=hsScen$x[,2:ncol(hsScen$x)],
                                     omit.extrapolated=FALSE),
                    plot=1)

#college
collTrace <- lineplot(x=xhyp,
                      y=collSims$pe,
                      lower=collSims$lower,
                      upper=collSims$upper,
                      col=col[3],
                      extrapolate=list(data=mdata[,2:ncol(mdata)],
                                       cfact=collScen$x[,2:ncol(hsScen$x)],
                                       omit.extrapolated=FALSE),
                      plot=1)

# Set up traces with labels

labelTrace <- textTile(labels=c("Less than HS", "High School", "College"),
                       x=c( 55,    49,     30),
                       y=c( 0.26,  0.56,   0.87),
                       col=col,
                       plot=1)

# For legend

legendTrace <- textTile(labels=c("Logit estimates:", "95% confidence", "interval is shaded"),
                        x=c(82, 82, 82),
                        y=c(0.2, 0.16, 0.12),
                        plot=1)


#options(device="quartz")

# Plot traces using tile
voting<-tile(nohsTrace,
     hsTrace,
     collTrace,
     labelTrace,
     legendTrace,
     limits=c(18,94,0,1),
     xaxis=list(at=c(20,30,40,50,60,70,80,90)),
     yaxis=list(label.loc=-0.5, major=FALSE),
     xaxistitle=list(labels="Age of Respondent"),
     yaxistitle=list(labels="Probability of Voting"),
     width=list(null=5,yaxistitle=4,yaxis.labelspace=-0.5)
     #,output=list(file="educationEV",width=5.5)
)
```

```
## Loading required package: WhatIf
```

```
## Loading required package: lpSolve
```

```
## ##########################################################
```

```
## ##
## ##  WhatIf (Version 1.5-6, built 2014-01-06)
## ##  Complete documentation available from http://gking.harvard.edu/whatif
## ##
## #######################################################

## [1] "Running whatif"
## [1] "Preprocessing data ..."
## [1] "Performing convex hull test ..."
## [1] "Calculating distances ...."
## [1] "Calculating the geometric variance..."
## [1] "Calculating cumulative frequencies ..."
## [1] "Finishing up ..."
## [1] "Whatif finished; returning to tile"
## [1] "Running whatif"
## [1] "Preprocessing data ..."
## [1] "Performing convex hull test ..."
## [1] "Calculating distances ...."
## [1] "Calculating the geometric variance..."
## [1] "Calculating cumulative frequencies ..."
## [1] "Finishing up ..."
## [1] "Whatif finished; returning to tile"
## [1] "Running whatif"
## [1] "Preprocessing data ..."
## [1] "Performing convex hull test ..."
## [1] "Calculating distances ...."
## [1] "Calculating the geometric variance..."
## [1] "Calculating cumulative frequencies ..."
## [1] "Finishing up ..."
## [1] "Whatif finished; returning to tile"
```



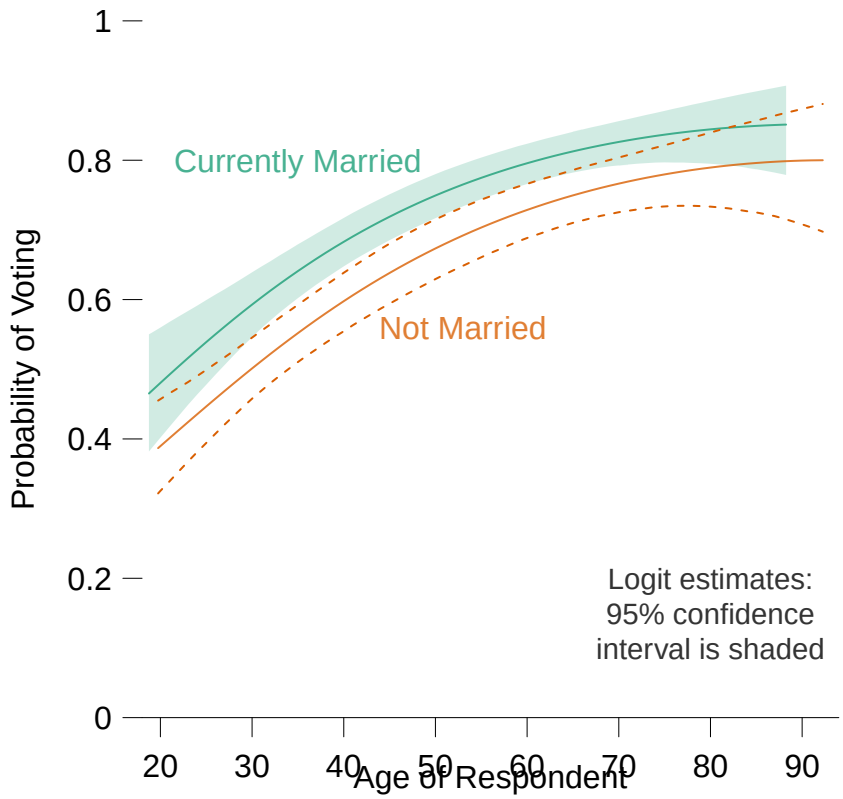
4. Simulating first differences

```
#####################################################################
#
# Now consider a new specification adding the variable
# "ever married", or marriedo
#
# We will estimate this new model with glm(), then
# simulate new scenarios for marrieds and non-marrieds


# Estimate logit model using glm()


# Set up a new model formula and model specific data frame

model2 <- vote00 ~ age + I(age^2) + hsdeg + coldeg + marriedo
mdata2 <- extractdata(model2, data, na.rm=TRUE)

# Run logit & extract results
logit.m2 <- glm(model2, family=binomial, data=mdata2)
pe.m2 <- logit.m2$coefficients  # point estimates
vc.m2 <- vcov(logit.m2)         # var-cov matrix


# Simulate parameter distributions
```

```
sims <- 10000
simbetas.m2 <- mvrnorm(sims, pe.m2, vc.m2)


# Set up counterfactuals:  all ages

xhyp <- seq(18,97,1)
nscen <- length(xhyp)
marriedScen <- notmarrScen <- cfMake(model2, mdata2, nscen)
for (i in 1:nscen) {


  #  - we will use the marriedScen counterfactuals in FDs and RRs as well as EVs
  # Note below the careful use of before scenarios (xpre) and after scenarios (x)
  # :i.e., use of the same age range (18-97) for both x and xpre, only marriedo values differ.

  # Married (loop over each age)
  marriedScen <- cfChange(marriedScen, "age", x = xhyp[i], xpre= xhyp[i], scen = i)
  marriedScen <- cfChange(marriedScen, "marriedo", x = 1, xpre= 0, scen = i)

  # Not Married (loop over each age)
  notmarrScen <- cfChange(notmarrScen, "age", x = xhyp[i], scen = i)
  notmarrScen <- cfChange(notmarrScen, "marriedo", x = 0, scen = i)
}

# Simulate expected probabilities for all age scenarios for married and not married respectively
marriedSims <- logitsimev(marriedScen, simbetas.m2, ci=0.95)
notmarrSims <- logitsimev(notmarrScen, simbetas.m2, ci=0.95)

# Simulate first difference of voting wrt marriage: E(y|married)-E(y|notmarried)
marriedFD <- logitsimfd(marriedScen, simbetas.m2, ci=0.95)

# Simulate relative risk of voting wrt marriage: E(y|married)/E(y|notmarried)
marriedRR <- logitsimrr(marriedScen, simbetas.m2, ci=0.95)


## Make plots using tile

# Get 3 nice colors for traces
col <- brewer.pal(3,"Dark2")

# Set up lineplot traces of expected probabilities
marriedTrace <- lineplot(x=xhyp,
                         y=marriedSims$pe,
                         lower=marriedSims$lower,
                         upper=marriedSims$upper,
                         col=col[1],
                         extrapolate=list(data=mdata2[,2:ncol(mdata2)],
                                          cfact=marriedScen$x[,2:ncol(marriedScen$x)],
                                          omit.extrapolated=TRUE),
                         plot=1)

notmarrTrace <- lineplot(x=xhyp,
```

```
                        y=notmarrSims$pe,
                        lower=notmarrSims$lower,
                        upper=notmarrSims$upper,
                        col=col[2],
                        ci = list(mark="dashed"),
                        extrapolate=list(data=mdata2[,2:ncol(mdata2)],
                                         cfact=notmarrScen$x[,2:ncol(notmarrScen$x)],
                                         omit.extrapolated=TRUE),
                        plot=1)


# Set up traces with labels and legend
labelTrace <- textTile(labels=c("Currently Married", "Not Married"),
                       x=c( 35,    53),
                       y=c( 0.8,  0.56),
                       col=col,
                       plot=1)

legendTrace <- textTile(labels=c("Logit estimates:", "95% confidence", "interval is shaded"),
                        x=c(80, 80, 80),
                        y=c(0.2, 0.15, 0.10),
                        cex=0.9,
                        plot=1)

# Plot traces using tile
tile(marriedTrace,
     notmarrTrace,
     labelTrace,
     legendTrace,
     limits=c(18,94,0,1),
     xaxis=list(at=c(20,30,40,50,60,70,80,90)),
     yaxis=list(label.loc=-0.5, major=FALSE),
     xaxistitle=list(labels="Age of Respondent"),
     yaxistitle=list(labels="Probability of Voting"),
     width=list(null=5,yaxistitle=4,yaxis.labelspace=-0.5)
     #,output=list(file="marriedEV",width=5.5)
)
```

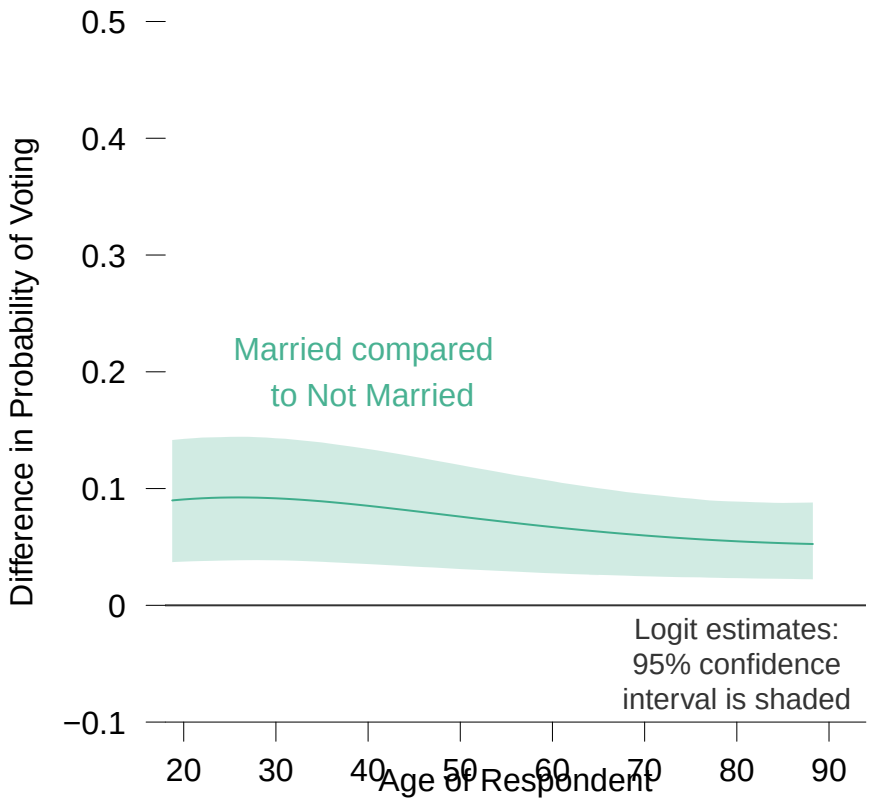
```
## [1] "Running whatif"
## [1] "Preprocessing data ..."
## [1] "Performing convex hull test ..."
## [1] "Calculating distances ...."
## [1] "Calculating the geometric variance..."
## [1] "Calculating cumulative frequencies ..."
## [1] "Finishing up ..."
## [1] "Whatif finished; returning to tile"
## [1] "Running whatif"
## [1] "Preprocessing data ..."
## [1] "Performing convex hull test ..."
## [1] "Calculating distances ...."
## [1] "Calculating the geometric variance..."
## [1] "Calculating cumulative frequencies ..."
## [1] "Finishing up ..."
## [1] "Whatif finished; returning to tile"
```




```
# Plot First Difference

# Set up lineplot trace of first difference

marriedFDTrace <- lineplot(x=xhyp,
                           y=marriedFD$pe,
                           lower=marriedFD$lower,
                           upper=marriedFD$upper,
                           col=col[1],
                           extrapolate=list(data=mdata2[,2:ncol(mdata2)],
                                            cfact=marriedScen$x[,2:ncol(marriedScen$x)],
                                            omit.extrapolated=TRUE),
                           plot=1)


# Set up baseline: for first difference, this is 0
baseline <- linesTile(x=c(18,94),
                      y=c(0,0),
                      plot=1)

# Set up traces with labels and legend
labelFDTrace <- textTile(labels=c("Married compared \n to Not Married"),
                         x=c( 40),
                         y=c( 0.20),
                         col=col[1],
                         plot=1)

legendFDTrace <- textTile(labels=c("Logit estimates:", "95% confidence", "interval is shaded"),
```

```
                          x=c(80, 80, 80),
                          y=c(-0.02, -0.05, -0.08),
                          cex=0.9,
                          plot=1)

# Plot traces using tile
tile(marriedFDTrace,
     labelFDTrace,
     legendFDTrace,
     baseline,
     limits=c(18,94,-0.1,0.5),
     xaxis=list(at=c(20,30,40,50,60,70,80,90)),
     yaxis=list(label.loc=-0.5, major=FALSE),
     xaxistitle=list(labels="Age of Respondent"),
     yaxistitle=list(labels="Difference in Probability of Voting"),
     width=list(null=5,yaxistitle=4,yaxis.labelspace=-0.5)
     #,output=list(file="marriedFD",width=5.5)
)
```

```
## [1] "Running whatif"
## [1] "Preprocessing data ..."
## [1] "Performing convex hull test ..."
## [1] "Calculating distances ...."
## [1] "Calculating the geometric variance..."
## [1] "Calculating cumulative frequencies ..."
## [1] "Finishing up ..."
## [1] "Whatif finished; returning to tile"
```



```
# Plot Relative Risk

# Set up lineplot trace of relative risk
marriedRRTrace <- lineplot(x=xhyp,
                           y=marriedRR$pe,
                           lower=marriedRR$lower,
                           upper=marriedRR$upper,
                           col=col[1],
                           extrapolate=list(data=mdata2[,2:ncol(mdata2)],
                                            cfact=marriedScen$x[,2:ncol(marriedScen$x)],
                                            omit.extrapolated=TRUE),
                           plot=1)


# Set up baseline: for relative risk, this is 1

baseline <- linesTile(x=c(18,94),
                      y=c(1,1),
                      plot=1)

# Set up traces with labels and legend
labelRRTrace <- textTile(labels=c("Married compared \n to Not Married"),
                         x=c( 55),
                         y=c( 1.25),
                         col=col[1],
                         plot=1)

legendRRTrace <- textTile(labels=c("Logit estimates:", "95% confidence", "interval is shaded"),
                          x=c(80, 80, 80),
                          y=c(0.98, 0.95, 0.92),
                          cex=0.9,
                          plot=1)

# Plot traces using tile
tile(marriedRRTrace,
     labelRRTrace,
     legendRRTrace,
     baseline,
     limits=c(18,94,0.9,1.5),
     xaxis=list(at=c(20,30,40,50,60,70,80,90)),
     yaxis=list(label.loc=-0.5, major=FALSE),
     xaxistitle=list(labels="Age of Respondent"),
     yaxistitle=list(labels="Relative Risk of Voting"),
     width=list(null=5,yaxistitle=4,yaxis.labelspace=-0.5)
     #,output=list(file="marriedRR",width=5.5)
)
```

```
## [1] "Running whatif"
## [1] "Preprocessing data ..."
## [1] "Performing convex hull test ..."
## [1] "Calculating distances ...."
## [1] "Calculating the geometric variance..."
## [1] "Calculating cumulative frequencies ..."
## [1] "Finishing up ..."
```

```
## [1] "Whatif finished; returning to tile"
```