Contentſ

INTRODUCTION.

THE ELEMENTS OF EUCLID

BOOK I.

DEFINITIONS.

I.
A point if that which haſ no partſ.

II.
A line if length without breadth.

III.
The extremitieſ of a line are pointſ.

IV.
A ſtraight or right line if that which lieſ evenly between itſ extermitieſ.

V.
A ſurface if that which haſ length and breadth only.

VI.
The extremitieſ of a ſurface are lineſ.

VII.

A plane ſurface iſ that which lieſ evenly between itſ extremitieſ.

VIII.

A plane angle iſ the inclination of two lineſ to one another, in a plane, which meet together, but are not in the ſame direction.

IX.

A plane rectilinear angle iſ the inclination of two ſtraight lineſ to one another, which meet together, but are not in the ſame ſtraight line.

X.

When one ſtraight line ſtanding on another ſtraight linem makeſ the adjacent angleſ equal, each of theſe angleſ iſ called a right angle, and each of theſe lineſ iſ ſaid to be perpendicular to the other.

XI.

An obtuſe angle iſ an angle greater than a right angle.

XII.

An acute angle iſ an angle leſſ than a right angle.

XIII.

A term of boundary iſ the extremity of any thing.

XIV.

A figure iſ a ſurface encloſed on all ſideſ by a line or lineſ.

XV.

A circle ſ a plane figure, bounded by one continued line, called itſ circumference or periphery; and having a certain point within t, from which all ſtraight lineſ drawn to itſ circumference are equal.

XVI.

The point (from which the equal lineſ are drawn) ſ called the centre of the circle.

XVII.

A diameter of a circle ſ a ſtraight line drawn through the centre, terminating both wayſ in the circumference.

XVIII.

A ſemicircle ſ the figure contained by the diameter, and the part of the circle cut off by the diameter.

XIX.

A ſegment of a circle ſ a figure contained by a ſtraight line, and the part of the circumference which cutſ it off.

XX.

A figure contained by ſtraight lineſ only, ſ called a rectilinear figure.

XXI.

A triangle ſ a rectilinear figure encloſed by three ſideſ.

XXII.

A quadrilateral figure ſ one which ſ bounded by four ſideſ.

The ſtraight lineſ and connecting the verticeſ of the oppoſite angleſ of a quadrilateral figure, are called itſ diagonalſ.

XXIII.

A polygon iſ a rectilinear figure bounded by more than four ſideſ.

XXIV.

A traingle whoſe three ſideſ are equal, iſ ſaid to be equilateral.

XXV.

A triangle which haſ only two ſideſ equal iſ called an iſoſceleſ triangle.

XXVI.

A ſcalene triangle iſ one which haſ no two ſideſ equal.

XXVII.

A right angled triangle iſ that which haſ a right angle.

XXVIII.

An obtuſe angled triangle iſ that which haſ an obtuſe angle.

XXIX.

An acute angled triangle iſ that which haſ three acute angleſ.

XXX.

Of four-ſided figureſ, a ſquare iſ that which haſ all itſ ſideſ equal, and all itſ angleſ right angleſ.

XXXI.

A rhombuſ iſ that which haſ all itſ ſideſ equal, but itſ angleſ are not right angleſ.

XXXII.

An oblong is that which has all its angles right angles, but has not all its sides equal.

XXXIII.

A rhomboid is that which has its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.

XXXIV.

All other quadrilateral figures are called trapeziums.

XXXV.

Parallel straight lines are such as are in the same plane, and which being produced continually in both directions, would never meet.

POSTULATES.

I.

Let it be granted that a straight line may be drawn from any one point to any other point.

II.

Let it be granted that a finite straight line may be produced to any length in a straight line.

III.

Let it be granted that a circle may be described with any centre at any distance from that centre.

AXIOMS.

I.

Magnitudeſ which are equal to the ſame are equal to each other.

II.

If equalſ be added to equalſ the ſumſ will be equal.

III.

If equalſ be taken away from equalſ the remainderſ will be equal.

IV.

If equalſ be added to unequalſ the ſumſ will be unequal.

V.

If equalſ be taken away from unequalſ the remainderſ will be unequal.

VI.

The doubleſ of the ſame or equal magnitudeſ are equal.

VII.

The halveſ of the ſame or equal magnitudeſ are equal.

VIII.

Magnitudeſ which coincide with one another, or exactly fill the ſame ſpace, are equal.

IX.

The whole iſ greater than itſ part.

X.

Two ſtraight lineſ cannot include a ſpace.

XI.

All right angleſ are equal.

XII.

If two ſtraight lineſ () meet at a third ſtraight line ( ) ſo aſ to make the two interior angleſ ( and ) on the ſame ſide leſſ than two right angleſ, theſe two ſtraight lineſ will meet if they be produced on that ſide on which the angleſ are leſſ than two right angleſ.

ELUCIDATIONS.

The twelfth axiom may be expreſſed in any of the following ways

1. Two diverging ſtraight lineſ cannot be both parallel to the ſame ſtraight line.
2. If a ſtraight line interſect one of the two parallel ſtraight lineſ it muſt alſo interſect the other.
3. Only one ſtraight line can be drawn through a given point, parallel to a given ſtraight line.

Geometry haſ for itſ principal object the expoſition and explanation of the propertieſ of figure, and figure iſ defined to be the relation which ſubſiſtſ between the boundarieſ of ſpace. Space or magnitude iſ of three kindſ, linear, ſuperficial, and ſolid.

Angleſ might properly be conſidered aſ a fourth ſpecieſ of magnitude. Angular magnitude evidently conſiſtſ of partſ, and muſt therefore be admitted to be a ſpecieſ of quantity. The ſtudent muſt not ſuppoſe that the magnitude of an angle iſ affected by the length of the ſtraight lineſ which include int, and of whoſe mutual divergence it iſ the meaſure. The vertex of an angle iſ the point where the ſideſ or the legſ of the angle meet, aſ A.

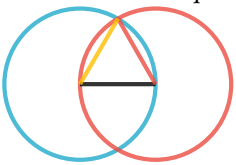
An angle iſ often deſignated by a ſingle letter when itſ legſ are the only lineſ which meet together at itſ vertex. Thuſ the red and blue lineſ form the yellow angle, which in other ſyſtemſ would be called the angle A. But when more than two lineſ meet in the ſame point, it waſ neceſſary by former methodſ, in order to avoid confuſion, to emplot three letterſ to deſignate an angle about that point, the letter which marked the vertex of the angle being alwayſ placed in the middle. thuſ the black and red lineſ meeting together at C, form the blue angle, and haſ been uſually denominated the angle FCD or DCF. The lineſ FC and DC are the legſ of the angle; the point C iſ itſ vertex. In like manner the black angle would be deſignated the angle DCB or BCD. The red and blue angleſ added together, or the angle HCF added to FCD, make the angle HCD; and ſo of other angleſ.

1.5. FAULTS TO BE CORRECTED BEFORE READING THIS VOLUME.9

Faultſ to be corrected before reading thiſ Volume.

PROPOSITIONS.

Propoſition 1 (problem). On a given finite ſtraight line (——) to deſcribe an equlateral triangle.



Proof. Deſcribe ◯ and ◯ (3); draw —— and —— (1). then will △ be equilateral.

For —— = ——(15)
and —— = ——(15)
∴ —— = ——(1)

Q.E.D.

Propoſition 2 (Problem). From a given point (——), to draw a ſtraight line equal to a given finite ſtraight line (——).

Proof. Draw ------ (1), deſcribe △ (pr. 1.), produce —— (2), deſcribe ◯ (3), and ◯ (3); produce —— (2), then —— if the line required.
For —— = —— (15), and —— = —— (conſt.), ∴ —— =

 (3), but (15) = = ; ∴ drawn from the given point (), if equal the given line .

Q.E.D.

Propoſition 3 (Problem). From the greater of two ſtraight lineſ (), to cut a part off equal to the leſſ ( )

Proof. Draw = (pr. 2.)

Q.E.D.