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INTRODUCTION.

THE artſ and ſcienceſ have become ſo extenſive, that to facilitate their acquirement iſ of aſ much importance aſ to extend their boundarieſ. Illuſtration, if it doeſ not ſhorten the time of ſtudy, will at leaſt make it more agreeable. Thiſ work haſ a greater aim than mere illuſtration; we do not introduce colourſ for the purpoſe of entertainment, or to amuſe by certain combinationſ of tint and form, but to aſſiſt the mind in itſ reſearcheſ after truth, to increaſe the facilitieſ of inſtruction, and to diffuſe permanent knowledge. If we wanted authoritieſ to prove the importance and uſefulneſſ of geometry, we might quote every philoſopher ſince the dayſ of Plato. Among the Greekſ, in ancient, aſ in the ſchool of Peſtalozzi and otherſ in recent timeſ, geometry waſ adopted aſ the beſt gymnaſtic of the mind. In fact, Euclid'ſ Elementſ have become, by common conſent, the baſiſ of mathematical ſcience all over the civilized globe. But thiſ will not appear extraordinary, if we conſider that thiſ ſublime ſcience iſ not only better calculated than any other to call forth the ſpirit of inquity, to elevate the mind, and to ſtrengthen the reaſoning facultieſ, but alſo it formſ the beſt introduction to moſt of the uſeful and important vocationſ of human life. Arithmetic, land-ſurveying, menſuration, enginerring, navigation, mechanicſ, hydroſtaticſ, pneumaticſ, opticſ, phyſical aſtronomy, &c. are all dependent on the propoſitionſ of geometry.

Much however dependſ on the firſt communication of any ſcience to a learner, though the beſt and moſt eaſy methodſ are ſeldom adopted. Propoſitionſ are placed before a ſtudent, who though having a ſufficient underſtanding, if told juſt aſ much about them on entering at the very threſhold of the ſcience, aſ giveſ giveſ him a prepoſſeſſion moſt unfavourable to hiſ future ſtudy of thiſ delightful ſubject; or "the formalititeſ and paraphernalia of rigour are ſo oſtentatiouſly put forward, aſ almoſt to hide the reality. Endleſſ and perplexing repetitionſ, whih ſo not confer gfrater exactitude on the reaſoning, render the demonſtrationſ involved and obſcure, and conceal from the view of the ſtudent the conſecution of evidenve." Thuſ and averſion iſ created in the mind of the pupil, and a ſubject ſo calculated to improve the ewaſoning powerſ, and give the chabir of cloſe thinking, iſ degraded by a fry and rigid memory, To eaiſe the curioſuity, and to awaken the liſtleſſ and dormant powerſ of younger minddſ ſhould be the aim of every reacher; but where exampleſ of excellence are wanting, the attemptſ to attain it are but few, while eminence excitedſ attention and produceſ imitarion, The object ofthiſ Work iſ to introduce a method of teaching geometry, which haſ been much approveed of by many ſcientific men in thiſ country, aſ well aſ in France and America. The plan here adopted forciblu appealſ to the eye, the moſt ſenſitive and the moſt comprehenſive of our external organſ, and itſ pre-eminence to imprint it ſubject on the mind iſ ſupported by the incontrovertible maxim ſexpreſſed in the well known wordſ of Horace:-

THE ELEMENTS OF EUCLID

Book 1 BOOK 1.

DEFINITIONS.

I.
A point iſ that which haſ no partſ.

II.
A line iſ length without breadth.

III.
The extremitieſ of a line are pointſ.

IV.
A ſtraight or right line iſ that which lieſ evenly between itſ extermitieſ.

V.
A ſurface iſ that which haſ length and breadth only.

VI.
The extremitieſ of a ſurface are lineſ.

VII.

A plane ſurface iſ that which lieſ evenly between itſ extremitieſ.

VIII.

A plane angle iſ the inclination of two lineſ to one another, in a plane, which meet together, but are not in the ſame direction.

IX.

A plane rectilinear angle iſ the inclination of two ſtraight lineſ to one another, which meet together, but are not in the ſame ſtraight line.

X.

When one ſtraight line ſtanding on another ſtraight linem makeſ the adjacent angleſ equal, each of theſe angleſ iſ called a right angle, and each of theſe lineſ iſ ſaid to be perpendicular to the other.

XI.

An obtuſe angle iſ an angle greater than a right angle.

XII.

An acute angle iſ an angle leſſ than a right angle.

XIII.

A term of boundary iſ the extremity of any thing.

XIV.

A figure iſ a ſurface encloſed on all ſideſ by a line or lineſ.

XV.

A circle is a plane figure, bounded by one continued line, called its circumference or periphery; and having a certain point within t, from which all ſtraight lineſ drawn to itſ circumference are equal.

XVI.

The point (from which the equal lineſ are drawn) iſ called the centre of the circle.

XVII.

A diameter of a circle iſ a ſtraight line drawn through the centre, terminating both wayſ in the circumference.

XVIII.

A ſemicircle iſ the figure contained by the diameter, and the part of the circle cut off by the diameter.

XIX.

A ſegment of a circle iſ a figure contained by a ſtraight line, and the part of the circumference which cutſ it off.

XX.

A figure contained by ſtraight lineſ only, iſ called a rectilinear figure.

XXI.

A triangle iſ a rectilinear figure encloſed by three ſideſ.

XXII.

A quadrilateral figure iſ one which iſ bounded by four ſideſ.

The ſtraight lineſ and connecting the verticeſ of the oppoſite angleſ of a quadrilateral figure, are called itſ diagonalſ.

XXIII.

A polygon iſ a rectilinear figure bounded by more than four ſideſ.

XXIV.

A traingle whoſe three ſideſ are equal, iſ ſaid to be equilateral.

XXV.

A triangle which haſ only two ſideſ equal iſ called an iſoſceleſ triangle.

XXVI.

A ſcalene triangle iſ one which haſ no two ſideſ equal.

XXVII.

A right angled triangle iſ that which haſ a right angle.

XXVIII.

An obtuſe angled triangle iſ that which haſ an obtuſe angle.

XXIX.

An acute angled triangle iſ that which haſ three acute angleſ.

XXX.

Of four-ſided figureſ, a ſquare iſ that which haſ all itſ ſideſ equal, and all itſ angleſ right angleſ.

XXXI.

A rhombuſ iſ that which haſ all itſ ſideſ equal, but itſ angleſ are not right angleſ.

XXXII.

An oblong iſ that which haſ all itſ angleſ right angleſ, but haſ not all itſ ſideſ equal.

XXXIII.

A rhomboid iſ that which haſ itſ oppoſite ſideſ equal to one another, but all itſ ſideſ are not equal, nor itſ angleſ right angleſ.

XXXIV.

All other quadrilateral figureſ are called trapeziumſ.

XXXV.

Parallel ſtraight lineſ are ſuch aſ are in the ſame plane, and which being produced continually in both directionſ, would never meet.

POSTULATES.

I.

Let it be granted that a ſtraight line may be drawn from any one point to any other point.

II.

Let it be granted that a finite ſtraight line may be produced to any length in a ſtraight line.

III.

Let it be granted that a circle may be deſcribed with any centre at any diſtance from that centre.

AXIOMS.

I.

Magnitudeſ which are equal to the ſame are equal to each other.

II.

If equalſ be added to equalſ the ſumſ will be equal.

III.

If equalſ be taken away from equalſ the remainderſ will be equal.

IV.

If equalſ be added to unequalſ the ſumſ will be unequal.

V.

If equalſ be taken away from unequalſ the remainderſ will be unequal.

VI.

The doubleſ of the ſame or equal magnitudeſ are equal.

VII.

The halveſ of the ſame or equal magnitudeſ are equal.

VIII.

Magnitudeſ which coincide with one another, or exactly fill the ſame ſpace, are equal.

IX.

The whole iſ greater than itſ part.

X.

Two ſtraight lineſ cannot include a ſpace.

XI.

All right angleſ are equal.

XII.

If two ſtraight lineſ () meet at a third ſtraight line () ſo aſ to make the two interior angleſ ( and ) on the ſame ſide leſſ than two right angleſ, theſe two ſtraight lineſ will meet if they be produced on that ſide on which the angleſ are leſſ than two right angleſ.

ELUCIDATIONS.

The twelfth axiom may be expreſſed in any of the following ways

1. Two diverging ſtraight lineſ cannot be both parallel to the ſame ſtraight line.
2. If a ſtraight line interſect one of the two parallel ſtraight lineſ it muſt alſo interſect the other.
3. Only one ſtraight line can be drawn through a given point, parallel to a given ſtraight line.

Geometry haſ for itſ principal object the expoſition and explanation of the propertieſ of figure, and figure iſ defined to be the relation which ſubſiſtſ between the boundarieſ of ſpace. Space or magnitude iſ of three kindſ, linear, ſuperficial, and ſolid.

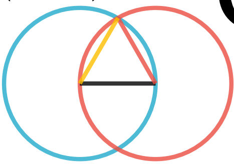
Angleſ might properly be conſidered aſ a fourth ſpecieſ of magnitude. Angular magnitude evidently conſiſtſ of partſ, and muſt therefore be admitted to be a ſpecieſ of quantity. The ſtudent muſt not ſuppoſe that the magnitude of an angle iſ affected by the length of the ſtraight lineſ which include int, and of whoſe mutual divergence it iſ the meaſure. The vertex of an angle iſ the point where the ſideſ or the legſ of the angle meet, aſ A.

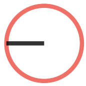
An angle iſ often deſignated by a ſingle letter when itſ legſ are the only lineſ which meet together at itſ vertex. Thuſ the red and blue lineſ form the yellow angle, which in other ſyſtemſ would be called the angle A. But when more than two lineſ meet in the ſame point, it waſ neceſſary by former methodſ, in order to avoid confuſion, to emplot three letterſ to deſignate an angle about that point, the letter which marked the vertex of the angle being alwayſ placed in the middle. thuſ the black and red lineſ meeting together at C, form the blue angle, and haſ been uſually denominated the angle FCD or DCF. The lineſ FC and DC are the legſ of the angle; the point C iſ itſ vertex. In like manner the black angle would be deſignated the angle DCB or BCD. The red and blue angleſ added together, or the angle HCF added to FCD, make the angle HCD; and ſo of other angleſ.

Faultſ to be correƈted before reading thiſ Volume.

PROPOSITIONS.

Propoſition 1 (problem). On a given finite ſtraight line ( ) to deſcribe an equilateral triangle.



Proof. Deſcribe  and  (3); draw and

(1). then will  be equilateral.

For $=$ (15)

and $=$ (15)

$\therefore$ $=$ (1)

Q.E.D.

Propoſition 2. From a given point ( ), to draw a ſtraight line equal to a given finite ſtraight line ( ).

Proof. Draw (1), deſcribe (pr. 1.), produce (2), deſcribe (3), and (3); produce (2), then iſ the line required. For = (15), and = (conſt.), ∴ = (3), but (15) = = ; ∴ drawn from the given point ( ), iſ equal the given line .

Q.E.D.

Propoſition 3. From the greater of two ſtraight lineſ ( ), to cut a part off equal to the leſſ ( )

Proof. Draw = (pr. 2.)

Q.E.D.