

Contentf

INTRODUCTION.		iii
I BOOK I.		I
I.1 DEFINITIONS.		I
I.2 POSTULATES.		5
I.3 AXIOMS.		6
I.4 ELUCIDATIONS.		7
I.5 Faults to be corrected before reading this Volume.		9
I.6 PROPOSITIONS.		10
I.7 Proposition 1		10
I.8 Proposition 2		10
I.9 Proposition 3		11

INTRODUCTION.

THE ELEMENTS OF EUCLID

BOOK I.

DEFINITIONS.

I.

A point is that which has no part.

II.

A line is length without breadth.

III.

The extremities of a line are points.

IV.

A straight or right line is that which lies evenly between its extremities.

V.

A surface is that which has length and breadth only.

VI.

The extremities of a surface are lines.

VII.

A plane surface is that which lies evenly between its extremities.

VIII.

A plane angle is the inclination of two lines to one another, in a plane, which meet together, but are not in the same direction.

IX.

A plane rectilinear angle is the inclination of two straight lines to one another, which meet together, but are not in the same straight line.

X.

When one straight line standing on another straight line makes the adjacent angles equal, each of these angles is called a right angle, and each of these lines is said to be perpendicular to the other.

XI.

An obtuse angle is an angle greater than a right angle.

XII.

An acute angle is an angle less than a right angle.

XIII.

A term of boundary is the extremity of any thing.

XIV.

A figure is a surface enclosed on all sides by a line or lines.

XV.

A circle is a plane figure, bounded by one continued line, called its circumference or periphery; and having a certain point within it, from which all straight lines drawn to its circumference are equal.

XVI.

The point (from which the equal lines are drawn) is called the centre of the circle.

XVII.

A diameter of a circle is a straight line drawn through the centre, terminating both ways in the circumference.

XVIII.

A semicircle is the figure contained by the diameter, and the part of the circle cut off by the diameter.

XIX.

A segment of a circle is a figure contained by a straight line, and the part of the circumference which cuts it off.

XX.

A figure contained by straight lines only, is called a rectilinear figure.

XXI.

A triangle is a rectilinear figure enclosed by three sides.

XXII.

A quadrilateral figure is one which is bounded by four sides.

The straight line and connecting the vertices of the opposite angle of a quadrilateral figure, are called its diagonals.

XXIII.

A polygon is a rectilinear figure bounded by more than four sides.

XXIV.

A triangle whose three sides are equal, is said to be equilateral.

XXV.

A triangle which has only two sides equal is called an isosceles triangle.

XXVI.

A scalene triangle is one which has no two sides equal.

XXVII.

A right angled triangle is that which has a right angle.

XXVIII.

An obtuse angled triangle is that which has an obtuse angle.

XXIX.

An acute angled triangle is that which has three acute angles.

XXX.

Of four-sided figures, a square is that which has all its sides equal, and all its angles right angles.

XXXI.

A rhombus is that which has all its sides equal, but its angles are not right angles.

XXXII.

An oblong is that which has all its angles right angles, but has not all its sides equal.

XXXIII.

A rhomboid is that which has its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.

XXXIV.

All other quadrilateral figures are called trapeziums.

XXXV.

Parallel straight lines are such as are in the same plane, and which being produced continually in both directions, would never meet.

POSTULATES.

I.

Let it be granted that a straight line may be drawn from any one point to any other point.

II.

Let it be granted that a finite straight line may be produced to any length in a straight line.

III.

Let it be granted that a circle may be described with any centre at any distance from that centre.

AXIOMS.

I.

Magnitudes which are equal to the same are equal to each other.

II.

If equals be added to equals the sum will be equal.

III.

If equals be taken away from equals the remainder will be equal.

IV.

If equals be added to unequals the sum will be unequal.

V.

If equals be taken away from unequals the remainder will be unequal.

VI.

The double of the same or equal magnitudes are equal.

VII.

The half of the same or equal magnitudes are equal.

VIII.

Magnitudes which coincide with one another, or exactly fill the same space, are equal.

IX.

The whole is greater than its part.





X.

Two straight lines cannot include a space.

XI.

All right angles are equal.

XII.

If two straight lines () meet at a third straight line () so as to make the two interior angles ( and ) on the same side less than two right angles, these two straight lines will meet if they be produced on that side on which the angles are less than two right angles.

ELUCIDATIONS.

The twelfth axiom may be expressed in any of the following ways

1. Two diverging straight lines cannot be both parallel to the same straight line.
2. If a straight line intersects one of the two parallel straight lines it must also intersect the other.
3. Only one straight line can be drawn through a given point, parallel to a given straight line.

Geometry has for its principal object the exposition and explanation of the properties of figure, and figure is defined to be the relation which subsists between the boundaries of space. Space or magnitude is of three kinds, linear, superficial, and solid.

Angles might properly be considered as a fourth species of magnitude. Angular magnitude evidently consists of parts, and must therefore be admitted to be a species of quantity. The student must not suppose that the magnitude of an angle is affected by the length of the straight lines which include it, and of whose mutual divergence it is the measure. The vertex of an angle is the point where the sides or the legs of the angle meet, as A.

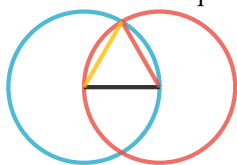
An angle is often designated by a single letter when its legs are the only lines which meet together at its vertex. Thus the red and blue lines form the yellow angle, which in other systems would be called the angle A. But when more than two lines meet in the same point, it was necessary by former methods, in order to avoid confusion, to employ three letters to designate an angle about that point, the letter which marked the vertex of the angle being always placed in the middle. Thus the black and red lines meeting together at C, form the blue angle, and has been usually denominated the angle FCD or DCF. The lines FC and DC are the legs of the angle; the point C is its vertex. In like manner the black angle would be designated the angle DCB or BCD. The red and blue angles added together, or the angle HCF added to FCD, make the angle HCD; and so of other angles.

1.5. FAULTS TO BE CORRECTED BEFORE READING THIS VOLUME.9

Faultf to be corrected before reading thif Volume.

PROPOSITIONS.

Proposition 1 (problem). On a given finite straight line (—) to describe an equilateral triangle.



Proof. Describe (—) and (—) (3); draw — and —

(1). then will  be equilateral.


For — = — (15)

and — = — (15)

\therefore — = — (1)

Q.E.D.

Proposition 2 (Problem). From a given point (—), to draw a straight line equal to a given finite straight line (—).

Proof. Draw (1), describe  (pr. 1.), produce —

(2), describe (—) (3), and (—)

(3); produce — (2), then — if the line required.

For — = — (15), and — = — (conft.), \therefore — =

--- (3), but (15) $\text{---} = \text{---} = \text{---}$; \therefore --- drawn from the given point (---), if equal the given line --- .
Q.E.D.

Proposition 3 (Problem). From the greater of two straight lines (---), to cut a part off equal to the less (---)

Proof. Draw $\text{---} = \text{---}$ (pr. 2.)

Q.E.D.