

# Modelling Propositional Logic

## Propositions

We model propositions as having five basic types:

- **AtomicProposition**: Meant to model the atomic propositions of the language,  $p, q, r \dots$
- **ConjunctiveProposition**: Represents the conjunction  $p_1 \wedge \dots \wedge p_n$  of  $n$  propositions.
- **DisjunctiveProposition**: Similarly  $p_1 \vee \dots \vee p_n$ .
- **ComplementProposition**: Represents the proposition  $\neg p$ .
- **ImplicativeProposition**: Represents the proposition  $p \Rightarrow q$ .

## Deduction Rules

A deduction rule is modelled as one of the introduction or elimination rules for Natural Deduction.

$$\frac{A \quad B}{A \wedge B} \wedge\text{-int}$$

Deduction rules have a weird property - they are universally quantified over propositions, but nobody ever writes them this way. When we write the above rule, it is simply understood that the propositions  $A$  and  $B$  may be replaced by any other definite proposition we like. But then we risk running into ambiguity if we *do* have definite propositions called  $A$  and  $B$ .

This can probably just be modelled by the fact that the particular deduction is wrapped up inside of a rule.

## Proof

A proof is a sequence of propositions, each of which is a consequence of the earlier hypotheses, according to one of the rules of deduction. In order to simplify the check for validity, we implement a **ProofStep** object, which is supposed to model a single application of a deduction rule. A **Proof** is then a sequence of **ProofSteps**, and the proof is valid if and only if all of its steps are valid.

There is an interesting