Informality, Inflation and Fiscal Progressivity in Developing Countries

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Abstract

The informal sector in developing economies has substantial implications for public finance. Its scale limits the effectiveness of standard tax instruments, often justifying the use of inflation as an alternative revenue source. Informality also has distributional consequences: informal businesses tend to be small, rely heavily on cash, and supply a larger share of goods to poorer households. In this paper, we present a general equilibrium model where firms decide on formality status, and households choose their consumption bundles, allowing us to examine these distributional aspects of informality. We use the model to study the trade-offs between different revenue-equivalent combinations of inflation and consumption taxes. We calibrate the model for Peru and find a notable disparity in effective tax rates across wealth levels under a benchmark 4% inflation rate and an 18% consumption tax: the bottom income quintile pays an effective tax rate equivalent to just 55% of that of the top quintile. Reducing inflation from 4% to 0% requires raising consumption taxes by 2.2 percentage points. This shift benefits the poorest 90.7% of households at the expense of the wealthiest 9.3%. This would increase the welfare of the bottom quintile by 0.25% in consumption-equivalent units, whereas the top quintile experiences a 0.01% decrease.

1 Introduction

Governments in developing countries struggle to raise revenue. Besley and Persson (2014) document that in low-income countries government revenue ranges between 10 and 20% of GDP, while for rich countries this number is closer to 40%. Limited state capacity and the presence of large informal sectors have long been identified as major barriers to governments' tax collection capacity (Besley and Persson 2009, 2014; Jensen 2022). This has led governments

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in developing countries to rely more heavily on alternative sources of revenue such as tariffs, in-kind labor payments (Olken and Singhal 2011), and seigniorage (Aisen and Veiga 2008).

This difficulty in raising revenue has been pointed out as one of the reasons why developing countries sustain higher levels of long-run inflation: seigniorage is a convenient way of raising revenue for governments with otherwise limited tax instruments at their disposal (Aruoba 2021; Koreshkova 2006; Nicolini 1998). Inflation is a particularly effective way of taxing the informal economy because it relies heavily on money for transactions to evade government oversight (La Porta and Shleifer 2014). However, certain features of the informal economy imply that the decision of whether to rely on the inflation tax has significant distributional implications. Indeed, informal firms tend to have unskilled owners and disproportionately hire unskilled workers. Poorer households are not only more likely to work in the informal sector, but also purchase a larger proportion of their consumption bundle from informal firms, a fact recently documented by Bachas, Gadenne, and Jensen (2023). The magnitude of this non-homoteticity is large: in Peru, the wealthiest 10% purchase around 30% of their consumption from informal firms—half the share purchased by the poorest 10%.

In this project we want to analyze the distributional consequences of public finance in economies with large informal sectors, taking into consideration the aforementioned empirical regularities. To do so, we develop a dynamic general equilibrium model with heterogeneous households and firms, which features both a formalization decision by firms and a sector-of-purchase decision by households. The trade-offs in the firms' formalization decision give rise to critical implications aligned with the data: firms in the informal sector are smaller and less productive, and poorer households buy a larger share of their consumption bundle from the informal sector. We use the model to analyze the implications of different revenue-equivalent policy mixes for households across the wealth distribution. In particular, the government can use a proportional consumption tax that only raises revenue from the formal sector, and the inflation tax, which has a higher incidence in the informal sector. The use of these two instruments intends to underline the difficulty of relying on direct taxation and the trade-off between indirect taxation, which distorts the formal sector, and seigniorage. We study the degree of fiscal progressivity that can be achieved by countries with limited access to direct taxation, as well as the preferences for inflation of households with different incomes and wealth.

The model combines features from three different literatures: the household heterogeneity literature, the macro-development literature, and the literature on the welfare costs of inflation. The main source of household heterogeneity in our model is standard: households are subject to idiosyncratic labor productivity shocks and markets are incomplete. It is in the modeling of the formal and informal sectors that our approach is novel. Firstly, we need households in our model to consume a portion of their bundle from each sector. Moreover, this portion should vary with total expenditure. There are different possible explanations for these facts. Bachas, Gadenne, and Jensen (2023) consider four different explanation: life-cycle patterns, access to formal stores, non-homotheticity of preferences, and differences in the quality of goods in the two sectors. Their findings emphasize the role of the last two factors. Richer households spend more on goods sold in formal stores, possibly because they value higher quality varieties. They document that prices are 6.7% higher in formal than informal stores. Their findings guide our modeling decisions. We model a market for goods with different varieties and quality. Different varieties allows us to have households purchasing a part of their consumption bundle formally and another part informally. Differing quality allows us to introduce a mechanism by which the formal sector sells higher quality products that richer households are more interested in buying. After entry, firms can choose whether to formalize or operate informally. Formalization entails the payment of a fixed cost and will imply that the firm's sales are subject to consumption taxes. Firms that instead choose to stay informal face tax avoidance costs increasing in the size of a transaction. These assumptions deliver a positive relationship between firm size and formality and a negative relationship between the formality of a household's consumption bundle and its expenditure and wealth. Low productivity firms choose to remain informal because the fixed costs of formalization are more significant to them. Moreover, informal firms can produce lower quality goods at a lower cost because they are not subject to these fixed costs which must be covered by a markup. Nevertheless, informal firms cannot produce higher quality goods at a low cost because these goods involve large tax-avoidance costs. As a consequence, poorer households buy low quality goods which are cheaper in the informal sector, while richer households buy high quality goods in the formal sector. The third type of elements in our model relate to the means of payment used for purchases and the effects of inflation. For each good variety, households can purchase goods with cash or credit. Buying with credit is only possible for goods purchased in the formal sector and requires the payment of a financial services cost, in line with Dotsey and Ireland (1996).

For our quantitative analysis we calibrate the model to replicate the relevants features of the Peruvian economy such as the size of the informal sector and the degree of formality of the consumption bundle of households across the wealth distribution, measured by Bachas, Gadenne, and Jensen (2023). These ensure that in our model economy, the aggregate and heterogeneous exposure to changes in the inflation tax related to informality match the data. We use an estimation of the passthrough of the consumption tax into the price of formal goods as a target to discipline the elasticities of our model. Unfortunately, we are unable to get such a number for Peru but we use an estimate by Bachas, Gadenne, and Jensen (2023) resulting from a policy reform in 2014 in Mexico. In our calibrated model, seignorage revenues are in line with the data.

Using our calibrated model with baseline consumption tax rate of 18% and inflation rate of 4%, we evaluate changes in these two rates that are able to fund the same level of government expenditure. First, we find that under the benchmark policy, the tax system displays some progressivity: the effective tax rate of the top quintile is 9.1%, while this number is 5% for the bottom quintile. Increasing inflation to 10% allows the consumption tax rate to be reduced to 15.9%. This change in the policy mix reduces the effective tax rate of the top quintile by 0.1 p.p. while increasing the rate of households in the other four quintiles – the effective tax rate of the bottom quintile becomes 5.9%. When inflation is above 40%, the tax system becomes regressive, with poorer households facing a higher effective tax rate than richer households. In the other direction, decreasing inflation to 0%, requires an increase in the consumption tax rate by 2.2 p.p. and lowers the effective tax rate of the bottom quintile to 4.3% while increasing the rate for the top quintile by 0.1 p.p. We find substantial heterogeneity in the welfare costs of policy changes. Increasing inflation to 10% (while decreasing the consumption tax rate to 15.9%) has a welfare cost of 0.52% for the bottom quintile in consumption-equivalent unit, but only 0.19% for the top quintile. In turn, decreasing inflation to 0% (with an accompanying increase in the consumption tax of 2.2 p.p.) decreases welfare of the top quintile by 0.01% but increases it for the bottom quintile by 0.25%.

Related literature

We build on Bachas, Gadenne, and Jensen (2023), who document that there are large differences in the formal-informal composition of consumption across the income distribution in developing countries. More precisely, they harmonize consumption expenditure surveys for several developing countries, and categorize purchases as being formal or informal based on characteristics of the place of purchase. Using this classification, they show that the proportion of consumption purchased informally decreases steeply with income, which they term as a (negatively sloped) informality Engel Curve. Their results are not driven by household demographics nor location, but are consistent with formal retailers selling higher quality products, a view echoed by the literature (La Porta and Shleifer 2014).A key consequence of considering the *de facto* exemption of the informal sector from consumption taxes is that these taxes effectively become progressive. We develop a general equilibrium monetary model that can replicate these facts, and use it to study the trade-offs between inflation and indirect taxation.

We contribute to the body of work studying the public finance motive for inflation in economies with sizeable informal sectors. Nicolini (1998) finds an optimal nominal interest rate between 7 and 19% fort an economy like that of Peru, with an informal sector representing about 40% of GNP. Koreshkova (2006) undertakes a similar exercise but finds inflation rates substantially larger due to an explicit modeling of different production technologies for the formal and informal sectors. Koreshkova fings that high inflation rates observed across the developing world can potentially be rationalized by optimal policy when accounting for the size of the informal sector. More recently, Aruoba (2021) develops a model where money must be used in the informal sector and there are product market search frictions, while the formal sector is frictionless. The government has the ability to audit and punish the informal sector, but its capacity to do so depends on the quality of institutions. Aruoba (2021) characterizes the Ramsey problem and shows that measures of the quality of institutions can rationalize differences in inflation observed in the data. Our work reassesses this question in the light of the facts about heterogeneity recently documented in the empirical literature. In particular, we depart from the representative agent abstraction by incorporating idiosyncratic income risk and quality and sector-of-purchase decisions than can replicate the non-homothetic consumption patterns observed in the data.

An important assumption in our framework is that informal firms transact in cash to avoid government oversight. The connection between tax evasion and the use of cash has been explored in many theoretical papers. Caballé and Panadés (2004) develop a cash-in-advance model where there households choose how much to evade taxes at the risk of an audit. They analyze the impact of a higher inflation rate on evasion. Gomis-Porqueras, Peralta-Alva, and Waller (2014) use a monetary search model connecting the use of cash to tax evasion as a way of measuring the size of the informal sector based on data on the use of cash. Lahcen (2020) studies the impacts of inflation on unemployment in a model of an economy with a large informal sector with monetary and labor market search frictions. An application of such a model to a measurement exercise if found in Chodorow-Reich et al. (2019). The authors develop a model of an economy where cash is held for transactions and tax evasion and explore the implications of the large experience of demonetization in India in 2016.

Our work is related to the large body of work studying the welfare costs of long-run inflation in the context of the transaction motive for holding money (Dotsey and Ireland 1996; Erosa and Ventura 2002; Kurlat 2019; Lagos and Wright 2005; Lucas 2000). Erosa and Ventura (2002) examine inflation's distributional effects, showing that poorer households rely more on cash, and incorporate costly credit in a cash-in-advance model with heterogeneous agents. In their model, credit costs are fixed and thus are relatively cheaper for households that consume more, which generates significant heterogeneity on the welfare costs of inflation by wealth. Chiu and Molico (2010) and Cao et al. (2021) extend this analysis by introducing monetary search frictions and accounting for heterogeneity across age and birth cohorts, respectively. Our work proposes a different explanation for heterogeneous cash holdings that is particularly important for developing countries: differential demand for informal consumption. We show that this channel is sufficient to generate substantial heterogeneity both in cash holdings as well as preferences for different combinations of consumption taxes and the inflation across the wealth distribution.

More broadly, our paper contributes to the literature on how informality limits governments' capacity to raise revenue and shapes the distributional impacts of fiscal policy in emerging economies (Bachas, Jensen, and Gadenne 2024; Besley and Persson 2014). Existing research has examined this issue primarily through the lens of consumption taxes (Bachas, Gadenne, and Jensen 2023), labor income taxes and unemployment insurance (Cirelli 2022; Ndiaye et al. 2023), and social security (McKiernan 2021). In contrast, we focus on the combined effects of consumption taxes and the inflation tax. Our findings indicate that in economies with substantial informal sectors, redistribution achieved through any single tax instrument may be partially counterbalanced by adjustments in other tax instruments needed to maintain budgetary balance.

The paper proceeds as follows. Section 2 presents and characterizes the model, Section 3 discusses the calibration of the model and presents results, Section 4 concludes.

2 Model

Many of the model ingredients we use are common to Koreshkova (2006). The most obvious way our model is different is the introduction of heterogeneity on the household side. This not only entails introducing uninsurable productivity shocks, but also building a structure that delivers the essential non-homotheticities in the consumption bundle. An additional, more subtle way we deviate from Koreshkova (2006) is by making the informal sector unable to use credit. Koreshkova (2006) models the two sectors as equal in their payment systems. Consequently, the inflation tax affects the formal and informal sectors equally. We change this assumption to better match the higher importance of cash in the informal sector.

2.1 Setup

Environment

Time is infinite and discrete. The agents in the economy are the households, firms, and government.

There is a continuum of goods varieties indexed by $j \in [0, 1]$. Each variety can be produced by formal and informal firms. Varieties differ in two dimensions: the ease with which firms can hide their trade from the authorities and the cost consumers must incur in order to purchase this good with credit rather than cash. In particular, the government's monitoring ability is increasing in j, and the cost of using credit is decreasing in j. This means that varieties with a low j index are easier to hide from the government, but more costly to trade using credit, while the opposite holds for varieties with a high j index. Producers of a variety j can sell goods of different quality q. Labor is the only production input, and it can be hired in a competitive market. The markets for goods are also competitive: households and firms take the price of the good of variety j, sector s, with quality q, denoted by $p_{js}(q)$, as given. The numeraire is the wage for one unit of effective labor; all prices are measured in respect to it.

There is a unit measure of households that choose how much to work, consume, and save subject to uninsurable idiosyncratic shocks to their labor productivity. Households have preferences for variety: they want to consume at most a unit of each variety, and their utility depends on a Dixit-Stiglitz aggregator of the quality of the good they consume of each variety. To insure against income risk, households may save in cash or in interest-bearing risk-free assets.

Firms differ in productivity and must decide which variety to produce and whether to operate in the formal or the informal sector. Formality entails registration costs and taxation but allows firms to sell their goods with credit. In contrast, informal firms avoid registration costs and taxation but are subject to tax avoidance costs (this could represent probabilistic enforcement and fines by the government). Informal firms must rely exclusively on cash to sell their production, so households purchasing goods informally must do so with cash.

There is a financial services sector that allows households to pay for their purchases of goods with credit. If a household wants to purchase a good with cash, it must forego the interest rate it could have received by saving on the risk-free asset instead. When it instead uses credit, it can pay at the end of the period, after receiving repayment from the bonds purchased, so it does not need to forego the interest on these savings. To buy a specific variety with credit, the household must pay a cost $\gamma(j)$. Note that this is only possible if the household buys it from the formal sector.

The government's deterministic expenditure stream must be financed through consumption taxes on the formal sector and seigniorage. The government supplies money and government bonds.

For simplicity, we set the real interest rate exogenously and do not clear the market for risk-free assets. This can be rationalized by a supply of risk-free assets composed not only of government bonds but also of external debt issued elastically by foreigners at a fixed real interest rate. Because the stock of government debt is small relative to the total wealth in the economy we calibrate, we need an additional supply of risk-free assets. Adding capital and endogenizing the real interest rate in our model would not be quantitatively important for our results because with an ample supply of assets the policy interventions we analyze would have negligible effects on the real interest rate.

Households

A household starts period t with total wealth x_t and labor productivity ϵ_t. Labor productivity follows a Markov process with an unconditional mean equal to 1. Because we make the wage per efficiency unit of labor the numeraire, labor income (in these units) is $\epsilon_t l_t$, where l_t denotes labor hours. In each period, the household must decide how much of its wealth to hold in cash and the interest-bearing risk-free asset, how many hours to work, as well as its consumption bundle: the quality of the good it purchases of each variety, whether to buy it from the formal or the informal sector, and whether to use cash or credit whenever buying formal.

Households maximize expected discounted lifetime utility:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k u(q_{t+k}, l_{t+k}).$$

β is the discount factor. The period utility function, u, is strictly increasing and concave, and it is defined over a composite q which takes the shape of a Dixit-Stiglitz aggregator over varieties. Let q_{jf} denote the quality of the good of variety j from the formal sector (subscript f) that the household decides to purchase, and q_{jn} denote the quality of the good of variety j from the informal sector (subscript n) that the household decides to purchase, then:

$$q = \left(\int [\max\{q_{jf}, q_{jn}\}]^{1/\lambda} dj \right)^{\lambda}.$$

The maximum operator represents preferences in which formal and informal goods of a variety j are perfect substitutes and in which the household derives no utility from consuming two goods of the same variety. The degree of substitutability across varieties is determined by

$\lambda > 1$. We will restrict most of our analysis to the specific case of $\lambda \to \infty$ in which varieties are perfect complements. In that case, the household buys the exact same quantity of each variety, choosing solely whether to buy it from the formal or informal sector.

In period t, a household that ended the previous period with real wealth x_t chooses how much cash $m_t \geq 0$ and interest-bearing assets $b_t \geq 0$ (both in real terms) to hold, subject to the beginning of period budget constraint:

$$m_t + \frac{b_t}{1+i_t} = \frac{x_t}{1+\pi_t},$$

where π_t is the inflation rate that depreciates the value of the household's wealth from the previous period. i_t is the nominal interest rate paid on the interest-bearing asset.

In its portfolio decision, the household considers that it must carry enough cash to pay for its informal consumption and formal consumption of varieties for which it did not pay the financial services cost. This is described by the following constraint:

$$\int p_{jnt}(q_{jnt})dj + \int \mathbb{I}\{j \in J_M\}\, p_{jft}(q_{jnt})dj \leq m_t, \tag{CIA}$$

where $p_{jnt}(q_{jnt})$ is the price of a good of variety j with quality q_{jnt} purchased in the informal sector (trivially $p_{jnt}(0) = 0$), $p_{jft}(q_{jnt})$ is the price of a good of variety j with quality q_{jft} purchased in the formal sector and J_C is the set of varieties that, conditional on buying a variety from the formal sector, the household would prefer to purchase with cash. Because when the nominal interest rate is positive, the return of the risk-free asset strictly dominates that of cash, the household will only carry enough cash to finance the consumption it desires to pay with cash, so the constraint (CIA) will be binding.

The evolution of a household's assets from period t to period $t+1$ is given by:

$$x_{t+1} = m_t + b_t + \epsilon_t l_t - \int p_{jft}(q_{jft})\, dj - \int p_{jnt}(q_{jnt})\, dj.$$

We assume that formal good purchases are paid after the household receives payment on its government bond holdings, but before it receives its labor income. This implies the following borrowing constraint:

$$x_{t+1} \geq \epsilon_t l_t.$$

This means that buying with credit provides a way to avoid foregoing the nominal interest rate by holding cash, but it does not constitute a way of borrowing against the labor income paid at the end of the period when x_t is low.

Firms

On each period a measure of potential entrants may decide to participate in the economy for a single period, which involves paying an entry cost κ_e. Upon entry, firms draw a productivity $z \sim \Psi$ and must choose a variety j and a sector $s \in \{f, n\}$ in which to produce. Formalization entails the payment of a fixed cost κ_f (common across varieties) which subjects the firm to taxation but brings the possibility of selling goods with credit. Informal firms avoid these costs, but may only accept cash and face tax avoidance costs.

Firms operate according to a decreasing returns to scale production function that uses labor, l, as an input: $F(z,l) = zl^{\alpha}$ ($\alpha < 1$). Labor can be hired in a competitive labor market.

Given the firm's output, determined by the production function, each firm decides how many goods of each quality to produce. This transformation is linear: producing one good of quality q requires q units of output. This implies that the firm may produce $(s(q))_q$ units of each quality subject to the following capacity constraint:

$$\int qs(q)\, dq \leq F(z,l). \tag{CC}$$

Conditional on labor input l, the firm's revenue maximization problem consists on choosing how to allocate its capacity $F(z,l)$ into the production of goods of different quality:

$$\begin{aligned} R_{jf}(z,l) &= \max_{s(q)} \int \left(\frac{p_{jf}(q)}{1+\tau}\right) s(q)dq && \text{subject to (CC)},\\ R_{jn}(z,l) &= \max_{s(q)} \int [p_{jn}(q) - \chi_j(p_{jn}(q))]s(q)dq && \text{subject to (CC)}. \end{aligned}$$

Formal firms must pay a sales tax given by τ, whereas informal firms have to account for the tax avoidance costs embodied in function χ_j. We assume these costs are increasing in the size of the transaction, so that $\chi_j'(p_{jn}(q)) > 0$, and that the government can more easily monitor and detect trade of good varieties with a higher index, such that tax avoidance costs are increasing in j: $\chi_k(p) > \chi_j(p)$ for $k > j$.

Operating profits are defined as revenue net of labor costs, and a firm's sectoral choice is determined by operating profits net of fixed costs:

$$\begin{aligned} \Pi_{js}(z) &= \max_l R_{js}(z,l) - wl,\\ \Pi_j(z) &= \max\left\{\Pi_{jf}(z) - (\kappa_e + \kappa_f), \Pi_{jn}(z) - \kappa_e\right\}. \end{aligned}$$

Due to free entry, expected profits must equal zero:

$$\int \Pi_j(z)\, d\Psi(z) = 0 \tag{FEC}$$

The entry costs, formalization costs, and tax avoidance costs are paid to a firm that uses labor to deliver these services, with a linear technology. Because the wage is our numeraire, the total costs equal the amount of labor allocated to these non-productive uses.

Government

The government starts period t with liabilities issued in the previous period in the form of outstanding bond holdings B_{t-1} and money supply M_{t-1}. It collects the consumption taxes from the previous period T_{t-1}, issues new bonds and money, and must pay for expenditures G_t. Thus, the law of motion for government liabilities is as follows:

$$\frac{B_t}{1+i_t} + M_t + \frac{T_{t-1}}{1+\pi_t} - G_t = \frac{B_{t-1}+M_{t-1}}{1+\pi_t}.$$

Denoting a household index by $h \in [0,1]$, and q_{hjf} as the quality of the good of variety j that it purchased in the formal sector, taxes collected are given by:

$$T_{t-1} = \frac{\tau_{c,t-1}}{1+\tau_{c,t-1}} \int\int p_{jf,t-1}(q_{hjf,t-1})\, dj\, dh.$$

Government expenditures are purchases of goods from the formal sector:

$$G_t = \int\int p_{jft}(q) g_{jt}(q)\, dq\, dj,$$

where $g_{jt}(q)$ is the number of goods of variety j and quality q that the government purchases.

Market clearing

The markets in this economy are the money, risk-free asset, labor, and goods markets (one can think of each variety-sector-quality as a different market). As discussed

The money market clears if the money supply chosen by the government equals the money demanded by households. Letting $h \in [0,1]$ denote an household's index:

$$\int m_{ht}\, dh = M_t \tag{MMC}$$

Because we assume that there is an elastic foreign supply of risk-free assets at real interest rate r^*, the corresponding condition is given by:

$$\mathbb{E}_t \frac{1+i_t}{1+\pi_{t+1}} = 1 + r^* \tag{AMC}$$

Market clearing in the market for goods of variety j, sector s, and quality q requires that the total demand by households of this particular good equals the supply by firms. In order to write this mathematically, denote the firm index as $k \in [0, m^F]$, where m^F is the mass of firms that enter, and let $v_k \in [0,1]$ denote the firm's decision of which variety to produce, and $g_k \in \{f, n\}$ the decision of whether to be formal or informal. $s_k(q) \in \mathbb{R}_0^+$, which we have defined previously, is the amount of goods of quality q that firm k produces. Equilibrium in the formal sector, where both households and the government demand goods, is given by:

$$\int \mathbb{I}\{q_{hjft} = q\}\, dh + g_{jt}(q) = \int \mathbb{I}\{v_{kt} = j\}\mathbb{I}\{g_{kt} = f\}s_{kt}(q)\, dk,\ \ \forall j \in [0,1],\ q \in \mathbb{R}^{+}. \quad \text{(FGMC)}$$

In the informal sector, only households are on the demand side:

$$\int \mathbb{I}\{q_{hjnt} = q\}\, dh = \int \mathbb{I}\{v_{kt} = j\}\mathbb{I}\{g_{kt} = n\}s_{kt}(q)\, dk,\ \ \forall j \in [0,1],\ q \in \mathbb{R}^{+}. \quad \text{(NGMC)}$$

The labor market clears when total demand for labor equals supply. Labor demand includes the demand for labor by the two sectors and the amount of labor allocated to non-productive uses (entry costs, formalization costs, tax avoidance costs, and financial services costs). Given that households may hold external debt, consistency requires that such income be used in imports. We let those imports be in the form of labor. This results in the following labor market clearing condition:

$$\int l_{kt} dk + \kappa_e m_t^F + \int \kappa_f \mathbb{I}\{g_{kt} = f\} dk + \bar{\chi}_t + \bar{\gamma}_t = \int \epsilon_{ht} l_{ht} dh + \left(\int b_{ht}\, dh - B_t\right)\left[1 - \left(\frac{1}{1+i_t}\right)\right] \quad \text{(LMC)}$$

where $\bar{\chi}_t$ is the total amount of tax avoidance costs firms incur, $\bar{\gamma}_t$ is the total amount of financial services costs, and the last term on the right-hand side is the income received by households from lending to foreigners, which is then used to import labor.

Given policy decisions by the government, an equilibrium entails prices and policy decisions by households and firms such that these policy decisions are optimal given prices, and all markets clear.

The environment described above has a continuum of market clearing conditions. However, as shown in the next subsection, it has a structure that allows for a significant simplification of its characterization. In particular, two equilibrium objects will be sufficient to characterize the full equilibrium price schedules, $p_{jf}(q)$ and $p_{jn}(q)$, and goods market clearing will be characterized by a single condition for each sector. This holds because firms will be indifferent about how many goods of each quality to supply, so quality composition will be determined in equilibrium solely by the demand side. Moreover, it is unnecessary to specify the quality composition of government purchases.

2.2 Characterization

Price schedules and firm behavior

Proposition 1 shows that two equilibrium objects are sufficient statistics to characterize the equilibrium price schedules. We describe the intuition and leave formal proofs for the Appendix.

Proposition 1 *Equilibrium price schedules:*

1. *There exist (μ_n, μ_f), representing "revenue per unit of quality" in each sector, such that equilibrium price schedules $(p_{jf}(q), p_{jn}(q))_{j\in[0,1]}$ satisfy:*

(a) $p_{jf}(q) = \mu_f(1+\tau)q \ \forall j$,

(b) $p_{jn}(q) = \mu_n q + \chi_j\left(p_{jn}(q)\right) \ \forall j$.

2. *These prices make firms indifferent about the quality composition of their production.*

Variety and sector choice by firms:

3. *Producers are indifferent between producing any variety $j \in [0,1]$.*

4. *There exists a productivity threshold $\bar{z}^f$, common across varieties, that fully characterizes the formalization decision of firms. In particular, firms with productivity $z \geq \bar{z}^f$ will formalize and firms with productivity $z < \bar{z}^f$ will stay informal.*

To understand the intuition behind the results in Proposition 1, it is useful to focus on a firm producing variety j in sector f. Due to the linearity of the quality transformation, a firm can produce q_1 units of quality q_2 or q_2 units of quality q_1 with the same inputs. For the firm to be willing to sell the two different quality levels, she must be indifferent. This holds if the revenue per unit of quality is the same. That is, the revenue per unit of quality from selling q_1, given by $\frac{p_{jf}(q_1)}{(1+\tau)q_1}$, must equal $\frac{p_{jf}(q_2)}{(1+\tau)q_2}$, the equivalent from selling q_2. An analogous logic holds for informal producers and thus, the following equalities regarding variety j price schedules must hold:

$$\frac{p_{jf}(q)}{(1+\tau)q} = \mu_{jf}, \quad \frac{p_{jn}(q) - \chi_j(p_n(q))}{q_n} = \mu_{jn}, \quad \forall q_f, q_n. \tag{1}$$

Hence, the structure of the revenue maximization problem implies that revenue per unit of quality produced is independent of quality and given by μ_{js}, which is an equilibrium object that is taken as given by the firm. Our assumption of free entry into varieties implies that producers with productivity z must be indifferent between varieties (statement 3), hence μ_f and μ_n are the same across all varieties.

Proposition 1 implies that revenues in sector s are simply given by $R_s(z,l) = \mu_s F(z,l)$, and we arrive at a simple expression for operating profits:

$$\Pi_s(z) = \max_l \mu_s F(z,l) - wl.$$

Suppose $\mu_n > \mu_f$, then $\Pi_n(z) > \Pi_f(z)$. Given that formalization has an additional fixed cost, no firm would want to formalize under those circumstances and supply of formal goods would be 0. Therefore, in an equilibrium with formal and informal goods, it must be the case that $\mu_f > \mu_n$ and formalization implies that firms get access to a higher μ_s at a fixed cost κ_f. Only productive enough firms will do so, and this is determined by productivity threshold $\bar{z}^f$ given by the following condition: $\Pi_f(\bar{z}^f) - \Pi_n(\bar{z}^f) = \kappa_f$.

Household behavior

Similar to the firm's problem, we can simplify the household's problem significantly by incorporating the results from Proposition 1. The household's consumption bundle decision is summarized in Proposition 2.

Proposition 2 *Household consumption bundle and payment means decision*

1. *For each variety there exists a quality threshold $\bar{q}_j^f$ such that, for $q_j < \bar{q}_j^f$ the household will purchase in the informal sector, and for $q_j \geq \bar{q}_j^f$ the household will purchase in the formal sector. Threshold $\bar{q}_j^f$ is decreasing in j.*

2. *For each variety there exists a quality threshold $\bar{q}_j^M$ such that, for $q_j < \bar{q}_j^M$ it is cheaper for the household to buy the formal good with cash rather than credit, and for $q_j \geq \bar{q}_j^M$ credit is cheaper. Threshold $\bar{q}_j^M$ is decreasing in j.*

3. *The expenditure associated with purchasing a good of quality q and variety j is given by:*

$$e_j(q) = \begin{cases} p_{jn}(q)(1+i) & \text{if } q < \bar{q}_j^f, \\ \mu_f(1+\tau)q(1+i) & \text{if } q \geq \bar{q}_j^f \ \& \ q < \bar{q}_j^M \\ \mu_f(1+\tau)q + \gamma(j) & \text{if } q \geq \bar{q}_j^f \ \& \ q \geq \bar{q}_j^M. \end{cases}$$

4. *When varieties are perfect complements, the household consumes an equal amount of each variety. If the household wants to consume q, there exists a variety threshold $\bar{j}^f(q) \in [0,1]$ such that the household will buy informally all varieties $j < \bar{j}^f(q)$ and formally the rest[3]. $\bar{j}^f(q)$ is decreasing on total aggregate quality desired q.*

5. *If the household wants to consume q, there is a variety threshold $\bar{j}^M(q) \in [0,1]$ such that the household will buy with cash all varieties $j < \bar{j}^M(q)$ and will use credit for the remaining. $\bar{j}^M(q)$ is also decreasing on total aggregate quality desired q and $\bar{j}^M(q) \geq \bar{j}^f(q)$.*

6. *The household's total expenditure function can be written as:*

$$E(q) = \int_0^{\bar{j}^f(q)} (1+i)p_{jn}(q)\,dj + \int_{\bar{j}^f(q)}^{\bar{j}^M(q)} \mu_f(1+\tau)(1+i)q\,dj + \int_{\bar{j}^M(q)}^1 [\mu_f(1+\tau)q + \gamma(j)]\,dj$$

To understand Proposition 2, consider a household who wants to buy a good of variety j of quality q. The household can do so in the formal sector (with cash or credit) or the informal sector. Given that formal and informal goods are perfect substitutes, the decision of whether to purchase formally or informally is based on a comparison of the expenditure necessary to undertake this purchase. Buying it from the formal sector costs $p_{jf}(q) = \mu_f(1+\tau)q$. If the purchase is made with cash, the household foregoes interest on this amount, meaning that the total cost incurred is $\mu_f(1+\tau)q(1+i)$. Buying it with credit requires payment of $\gamma(j)$, meaning that this option involves an expenditure of $\mu_f(1+\tau)q+\gamma(j)$. Purchasing it from the informal sector has a direct cost given by $p_{jn}(q)$ plus the foregone interest from carrying cash, making the total expenditure from purchasing q informally equal to $p_{jn}(q)(1+i)$. The household compares these three expenditures to pick the cheaper purchasing option. The convexity of χ and conditions on μ_n and μ_f imposed by equilibrium explain why the choice is characterized

[3]This allows for the case in which $\bar{j}^f = 0$ and the household buys all its consumption bundle from the formal sector, or $\bar{j}^f = 1$ in which it buys everything from the informal sector.

by two thresholds, as explained in the first two statements of Proposition 2. We prove this in the Appendix.

Statements 4 and 5 of Proposition 2 imply that a household that consumes a higher q level (a wealthier household) will purchase a larger proportion of its consumption bundle from the formal sector and a will also buy more varieties with credit.

Finally, the proposition also implies that we can solve the problem of optimal shopping behavior separately from the intertemporal savings decision. We can construct the expenditure function for a choice of consuming q_t units, and use condition (CIA) (which holds with equality), to write the law of motion for household assets as:

$$x_{t+1} = \left(\frac{1+i_t}{1+\pi_t}\right) x_t + \epsilon_t l_t - E_t(q_t)$$

Goods market clearing

Finally, Proposition 3 transforms our continuum of market clearing conditions into two conditions.

Proposition 3 *Goods market clearing*

1. *Suppose μ_f and μ_n are such that, given the equilibrium price schedules as defined in Proposition 1 and household and firm decisions, the following two conditions hold:*

   (a)
   $$\int\int q_{hjf}\,dj\,dh + \frac{G}{\mu_f} = \int \mathbb{I}\{g_k = f\}F(z_k, l_k)\,dk$$

   (b)
   $$\int\int q_{hjn}\,dj\,dh = \int \mathbb{I}\{g_k = n\}F(z_k, l_k)\,dk$$

   Then, there is an equilibrium with such price schedules where all goods markets clear.

Through Propositions 1 to 3, our environment with a continuum of prices and a continuum of market clearing conditions simplifies to one in which there are two equilibrium "prices", μ_f and μ_n, and two "market clearing" conditions, one for each sector. Moreover, the quality composition of government demand does not need to be specified. To understand this, remember that according to Proposition 1 equilibrium price schedules were such that firms were indifferent about the quality composition of their supply. That means that this supply can be the one that makes the proportion of supply of a certain variety and quality be the same as the demand for it. Hence, the only two conditions that must be satisfied are that the aggregate quality demanded equals the aggregate quality supplied in each of the two sectors. This is what the two conditions in Proposition 3 require. A formal proof is provided in the Appendix.

2.3 Recursive Formulation

We now write the households' problem in recursive formulation and define a Stationary Equilibrium.

Households

The individual state variables for the household are the labor productivity ϵ and initial wealth x. The Bellman Equation of the household, following Proposition 2, can be written as:

$$V(\epsilon, x) = \max_{q \in [0, q^{max}], l \geq 0} u(q) + \beta \mathbb{E} V(\epsilon', x'),$$
$$st : x' = \left(\frac{1+i}{1+\pi}\right) x + \epsilon l - E(q),$$

where maximum consumption level q^{max} is a consequence of the borrowing constraint $x' \geq \epsilon l$ and is given by:

$$E(q^{max}) = \left(\frac{1+i}{1+\pi}\right) x$$

Given q, one can compute the threshold for the formal variety and the threshold for the variety at which the household stops buying with cash, $\bar{j}^f(q)$ and $\bar{j}^M(q)$ respectively, and use it to get the expenditure:

$$E(q) = \int_0^{\bar{j}^f(q)} (1+i) p_{jn}(q)\, dj + \int_{\bar{j}^f(q)}^{\bar{j}^M(q)} \mu_f (1+\tau)(1+i) q\, dj + \int_{\bar{j}^M(q)}^{1} [\mu_f (1+\tau) q + \gamma(j)]\, dj,$$

the demand for formal and informal goods, as well as money demand:

$$m = \int_0^{\bar{j}^f(q)} p_{jn}(q)\, dj + \int_{\bar{j}^f(q)}^{\bar{j}^M(q)} \mu_f (1+\tau) q\, dj.$$

Thus, having constructed $E(q)$ and used it in the Household Bellman Equation to find $q(\epsilon, x)$, one can compute $m(\epsilon, x)$, $b(\epsilon, x)$, $\{q_{jf}(\epsilon, x)\}_j$, and $\{q_{jn}(\epsilon, x)\}_j$.

Stationary Equilibrium

Definition 4 *A Stationary Equilibrium in this environment is given by:* **i)** *Household value function V and policy functions $(q, l, m, b, \{q_{jf}\}_j, \{q_{jn}\}_j)$;* **ii)** *firm policy functions for labor demand $(l_f(z), l_n(z))$ and formalization decision (characterized by the threshold $\bar{z}^f$);* **iii)** *a mass of firms m^F;* **iv)** *revenues per unit of quality in the two sectors, (μ_f, μ_n), and price schedules, $(p_{jf}(q), p_{jn}(q))_j$;* **v)** *inflation rate π;* **vi)** *government policy mix (B, M, i, τ, G, g_N), where B is the amount of real bonds outstanding, M is the amount of real money and g_N is the nominal growth rate of both of these liabilities; and* **vii)** *a household distribution across individual states $\Phi(\epsilon, x)$ and its law of motion $\mathcal{T}$, such that:*

1. *The household value function and policy functions satisfy the Bellman Equation for the*

household problem. Given a policy function for q, other policy functions can be found as described in Proposition 2.

2. *Firms of each variety are indifferent about the quality composition of supply. This holds if the price schedules are as described in Proposition 1.*
3. *The firms' policy decisions for labor demand and formalization are optimal.*
4. *The inflation rate equals the growth rate of nominal government liabilities,* $\pi = g_N$.
5. *The policy mix of the government satisfies the law of motion for government liabilities.*
6. *The free entry condition (FEC) is satisfied.*
7. *All markets clear. In accordance with Proposition 3 there is a single condition for each sector in the goods market.*

   (a) *Money market:*

   $$\int m(\epsilon, x)\, d\Phi(\epsilon, x) = M.$$

   (b) *The real interest rate is equal to the exogenously given rate:*

   $$\frac{1+i}{1+\pi} = 1 + r^*$$

   (c) *Formal sector goods market:*

   $$\int\int q_{jf}(\epsilon, x)\, dj\, d\Phi(\epsilon, x) + \frac{G}{\mu_f} = m^F \int_{\bar{z}^f}^{\bar{z}} F\left(z, l_f(z)\right)\, d\Psi(z)$$

   (d) *Informal sector goods market:*

   $$\int\int q_{jn}(\epsilon, x)\, dj\, d\Phi(\epsilon, x) = m^F \int_{\underline{z}}^{\bar{z}^f} F\left(z, l_n(z)\right)\, d\Psi(z)$$

   (e) *The labor market clears. By Walras' law, if all other markets clear, then the labor market clears.*
8. *The household distribution is the fixed point of its law of motion:*

   $$\Phi = \mathcal{T}(\Phi)$$
9. $\mathcal{T}$ *is compatible with household policy functions.*

The Appendix describes the algorithm for the computation of the Stationary Equilibrium.

3 Quantitative section

The model described above has two immediate implications that are in accordance with the data. First, formal firms are larger and more productive. Second, poorer households purchase a larger part of their consumption bundle from the informal sector. In addition, the size of the informal sector is endogenous and responds to changes in the household distribution and inflation. We now use the model as a laboratory to analyze the implications of different policy choices. We are interested in the problem of a government who must finance a certain level of expenditure and can do so using consumption taxes and seignorage revenues accruing from inflation.

In our definition of a Stationary Equilibrium we considered general government policies. A policy mix was described as including six variables: (B, M, i, τ, G, g_N). Due to the conditions imposed by equilibrium, the Government does not have six degrees of freedom: it must satisfy the law of motion for government liabilities, the choice of inflation pins down the nominal rate (to match the exogenous real rate), and the nominal interest rate determines the demand for money. This means that the government has three degrees of freedom. In the exercise we are interested in the total amount of government liabilities and government expenditures should be fixed, so that we are looking into the problem of a government with fixed fiscal obligations, considering how to finance them. Thus, we have:

$$\frac{B}{1+i} + M = \bar{D}, \quad G = \bar{G}$$

Notice that one should consider the total indebtedness of the government including both liabilities, not just government bonds. One way to think about it is that there is a central bank engaging in open market operations (buying government bonds with issued money). $\bar{D}$ is the total amount of public debt issued by the fiscal authority. Open market operations change the composition of outstanding liabilities between B and M, but not its total. Given that the government has three degrees of freedom, this leaves one additional variable to be specified. This is the policy dimension we want to analyze. We consider different choices of g_N, which directly pin down inflation, and let τ be determined to satisfy the government's obligations[4].

3.1 Calibration

We calibrate the model to match several data moments from the Peruvian economy. The length of the period in our model is important because with a cash-in-advance constraint the longer the period the lower money velocity and the higher the seignorage revenues from inflation. We choose our period to correspond to a quarter and check that this gives us an amount of seignorage revenues aligned with the data. Our strategy involves calibrating the model parameters for a benchmark policy mix, in which the consumption tax and inflation rate are set to match the average rates for the period going from 1995 to 2016. We compute the level of government expenditures that can be financed under this policy mix and fix this $\bar{G}$ in our quantitative exercise, where we evaluate different policy mixes that are able to finance it.

[4]Different choices of g_N will affect the nominal interest rate i and the composition of outstanding liabilities B and M according to households' demand for these assets.

3.1.1 Functional forms

We use a Constant Relative Risk Aversion utility function:

$$u(q,l) = \frac{q^{1-\sigma}-1}{1-\sigma} - \psi \frac{l^{1+\xi}}{1+\xi}.$$

The distribution of firm productivity $\Psi(z)$ is lognormal with two parameters, μ_z and σ_z, to be calibrated.

The tax avoidance cost function $\chi_j(p)$ takes the following functional form:

$$\chi_j(p) = \chi_{0,j}\, p^{\chi_1}, \quad \chi_{0,j} = \underline{\chi_0}^{1-j}\, \overline{\chi_0}^{j}, \quad \overline{\chi_0} > \underline{\chi_0}, \quad \chi_1 > 1.$$

For a given variety, the tax avoidance cost function is parameterized by two objects: $\chi_{0,j}$ and χ_1. The first one determines the scale of enforcement costs, while the second determines how they grow in the size of the transaction. Because tax avoidance is easier for varieties with low j, we parameterize $\chi_{0,j}$ with two parameters: $\underline{\chi_0}$ determines $\chi_{0,j}$ for variety $j = 0$ and $\overline{\chi_0}$ determines it for variety $j = 1$.

For the financial services cost we follow Dotsey and Ireland (1996), Erosa and Ventura (2002), and Koreshkova (2006):

$$\gamma(j) = \gamma_0 \left(\frac{1-j}{j}\right)^{\gamma_1}$$

3.1.2 Targeted moments

Preferences We set σ, which determines the degree of risk-aversion and is the inverse of the intertemporal elasticity of substitution, equal to 1 (log preferences). The discount factor β is set to 0.99. As discussed before, we make varieties perfect complements, which corresponds to setting the elasticity of substitution of varieties λ to $-\infty$. The inverse elasticity of labor supply is set to 2.5, in line with the range of estimates in the micro literature for the United States.

Idiosyncratic labor productivity shocks The process for idiosyncratic labor productivity is given by:

$$\ln(\epsilon_t) = \rho_\epsilon \ln(\epsilon_{t-1}) + \nu_t, \quad \nu_t \sim N\left(0, \sigma_\nu^2\right)$$

We discretize the process using the Tauchen method with eleven states. We use Peru's National Household Survey on Living Conditions and Poverty (ENAHO) to estimate the autocorrelation of log income across two years and the cross-sectional standard deviation of log income. We target these moments in our model.

Production and firm productivity distribution The degree of decreasing returns to scale, determined by α, affects the elasticity of supply in our two sectors. In particular, when α is close to 1, the model approximates constant returns to scale, so the supply is close to infinitely elastic. To discipline this elasticity in our model, we target the passthrough of a consumption tax increase to the price of formal goods. This depends on both the value for α, but also the value of parameters related to tax avoidance costs. We set α to 0.7, a value sufficiently below 1 to avoid elasticities that are too high, and in line with some of the literature on firm dynamics in developing economies, for example, Arellano, Bai, and Zhang (2012). Then, we use the other parameters to target the passthrough as discussed in more detail below.

The firm productivity distribution Ψ is log-normal with parameters μ_z and σ_z. We normalize average productivity to 1 and calibrate the dispersion to match the dispersion in the employment distribution. We use Peru's ENAHO to estimate percentiles of this distribution. This is a household survey which asks about the size of the firm in which the household member works. To estimate the number of firms of size X we divide the number of workers surveyed who report to work on a firm of that size by X itself. This corrects for the higher probability of sampling a worker of a larger firm (there are 80 workers working in a firm with 80 workers, but only one working in a firm with a single worker). We compare our firm size distribution with the one reported by the Ministry of Production for the share of micro, small, medium and large enterprises and find that it matches this well. Table 1 displays some of the percentiles of the distribution. We pick σ_z to get as close to this distribution as possible, though our log-normal distribution with a single parameter will evidently struggle to match this perfectly.

Size (employment)	**Relative size (relative to average firm)**	**Cumulative share**
2	0.47	0.499
3	0.70	0.723
5	1.16	0.901
10	2.33	0.967
25	5.82	0.990
50	11.65	0.996
100	23.29	0.998
250	58.23	0.999

Table 1
Firm size distribution in Peru

Tax avoidance and formalization costs Parameters $\underline{\chi_0}$, $\overline{\chi_0}$, χ_1 and κ_f are set jointly to target three moments: i) the level of informality in the Peruvian economy; ii) the slope of the Engel Curve of formality; and iii) the passthrough of an increase in the consumption tax to the price of formal goods. For *ii*) and *iii*) we rely on the work in Bachas, Gadenne, and Jensen (2023). The authors estimate the percentage of expenditure on formal stores for households with different expenditure level. They do this for several different economies, including Peru. They estimate that the bottom decile purchases about 75% of their bundle in the informal sector while the top decile purchases 35%. For a VAT reform in Mexico in 2014, they explore spatial variation to estimate a consumption tax passthrough of 75% in formal stores. We consider a similar change in the consumption tax in our model and use the computed passthrough into formal prices as a calibration target.

Financial services costs The financial services costs has two parameters, γ_0 and γ_1. We set γ_1 to 1.65, following Koreshkova (2006). **Aurazo2021** report that between 2015 and 2018, only about 40% of the Peruvian population had a bank account or debit / credit card. In turn, only 20% of payments are reported to be made digitally, with the remaining portion being made in cash. We set γ_0 to get 65% of transactions in cash, which is probably a lower bound.

Government policy We set parameters pertaining to government policy based on data for Peru in the period 1995 to 2016 collected from the World Bank. This is displayed in Table 2. The public-debt-to-GDP ratio is set to 33%, the real interest rate to 3.2% based on the real deposit rate, and our benchmark policy mix has a 18% consumption tax rate (corresponding to a 16% value added tax and a 2% municipal tax) and a 4% inflation rate. Table 2 also displays our calculation of seignorage revenues as a percentage of GDP using two different measures, the change in reserve money and the product of reserve money and inflation rate[5]. While we do not target this explicitly, we make sure that our choice of a period of a quarter allows this value to be in the right ballpark. Government revenues in Peru in the period analyzed were 19.38% of GDP. We do not target this directly because our model only has consumption and inflation taxes. Instead, we compute government revenues endogenously, given the consumption tax and inflation rates. As we will see, we get a lower value for government expenditures, which aligns with the idea that a part of these revenues would be funded by other taxes from which our model abstracts.

Variable	Value
Seigniorage	
$\Delta RM/GDP$	1.64%
(Inflation × RM)/GDP	0.55%
Real Interest Rates	
Deposit Rate	3.21%
Lending Rate	19.16%
CPI Inflation and Fiscal Indicators	
CPI Inflation	4.07%
Public Debt-to-GDP	33.29%
Government Revenues-to-GDP	19.38%

Note: Averages for the period 1995–2016.

Table 2
Macroeconomic outcomes for Peru

[5]These are the measures often used in the literature that empirically measures seignorage. See **Cukierman1992** or Aisen and Veiga (2008)

Summary Tables 3 and 4 summarize our calibration distinguishing between external and internally calibrated parameters. Due to data limitations, some of our targets are imperfect. In the next section, we discuss the features of our calibrated economy to argue that they are reasonable in the dimensions that are important for our quantitative exercise.

Table 3
Externally calibrated parameters

Parameter	Description	Value	Source
Households			
β	Discount factor	0.99	
σ	Inverse IES	1.0	
λ	Elasticity of substitution of varieties	$-\infty$	
ξ	Inverse elasticity of labor supply	2.5	
Production Technology			
α	Decreasing returns	0.7	Arellano et al. (2012)
Financial services costs			
γ_1	Elasticity of cost across varieties	1.65	Koreshkova (2006)
Government policy			
τ	Consumption tax	18%	VAT 16% + IPM 2%
π	Inflation	4%	CPI inflation
$\bar{D}$	Public debt		Debt-to-GDP ratio
Other			
t	Time period	3 months	

Table 4
Internally calibrated parameters

Parameter	Description	Value	Target
Households			
ρ_ϵ	Persistence labor productivity	0.911	autocorrelation of log income
σ_ν	Std dev of labor productivity shocks	0.487	st. dev. of log income
Production Technology			
σ_z	Firm productivity dispersion	0.513	Firm size dispersion
κ_f	Formalization cost	9.855	informal sector size
Tax avoidance costs			
$\underline{\chi_0}$	Tax avoidance cost scale $j=0$	0.105	informality engel curve
$\overline{\chi_0}$	Tax avoidance cost scale $j=1$	0.483	&
χ_1	Tax avoidance cost exponent	1.235	consumption tax passthrough
Financial services costs			
γ_0	Financial services cost scale	0.072	65% of transactions with cash
Government policy			
$\bar{G}$	Government spending	0.085	Revenues in model

3.2 Calibrated Results

Table 5 shows that the model does a good job at matching the targeted empirical moments. The model also replicates a series of (untargeted) relevant features of the Peruvian economy. Seignorage revenues add up to 0.9% of GDP, which amounts to approximately 10% of total government revenues. When applying the methodology of Aisen and Veiga (2008) to compute seigniorage using Peruvian data in the time window we study, we compute seigniorage values of 0.5% and 1.6% of GDP, depending on the methodology (see Table 2). Revenue from consumption (VAT) taxes are 8% in the model, while it is 7.6% in the data[6]. Combined, these results imply that the model accurately captures the relative importance of both tax instruments in the government's budget. With the consumption tax and seigniorage, the government can finance government expenditures totaling 8.5% of GDP (after payment of interest expenditures on its debt).

In the model, 97% of firms operate informally. Although a direct counterpart for this measure is not readily available, in Peru 94.2% of firms are microenterprises, and 99% are microenterprises plus small firms, which are predominantly informal (Ministerio de la Producción 2024). Furthermore, the model estimates that the informal sector contributes 50% of aggregate output, close to the 52.9% estimated by Elgin et al. (2021). Formalization costs are about three times the cost of entry, in the line with the findings of Ulyssea (2018) for Brazil, who finds formal entry costs twice as high as informal entry costs.

Table 5
Model fit

Moment	**Data**	**Model**
autocorrelation log earnings	0.91	0.903
std dev log earnings	1.6	1.643
size of the informal sector	0.5	0.476
slope informality engel curve	0.35	0.353
passthrough consumption tax	0.75	0.895
proportion of cash transaction	0.65	0.656

Figure 1 plots several variables pertaining to household consumption behavior along the distribution. The top left panel shows expenditure by decile. The bottom 10% of households consume less than a tenth of the top 10%. The top right panel shows the level of formality of the consumption bundle. The bottom decile spends close to 80% of its budget in the informal sector compared to 43% for the top decile. As a consequence of both this and the financial costs of credit, which are more negligible to households with a larger consumption, poorer households hold a much higher proportion of money relative to their expenditure, as shown in the bottom left panel. Notice that this ratio is generally higher than the percentage of expenditure in the informal sector, which means that households are still buying some formal

[6]This number is the average IGV revenue for years 1995-2016, as shown here.

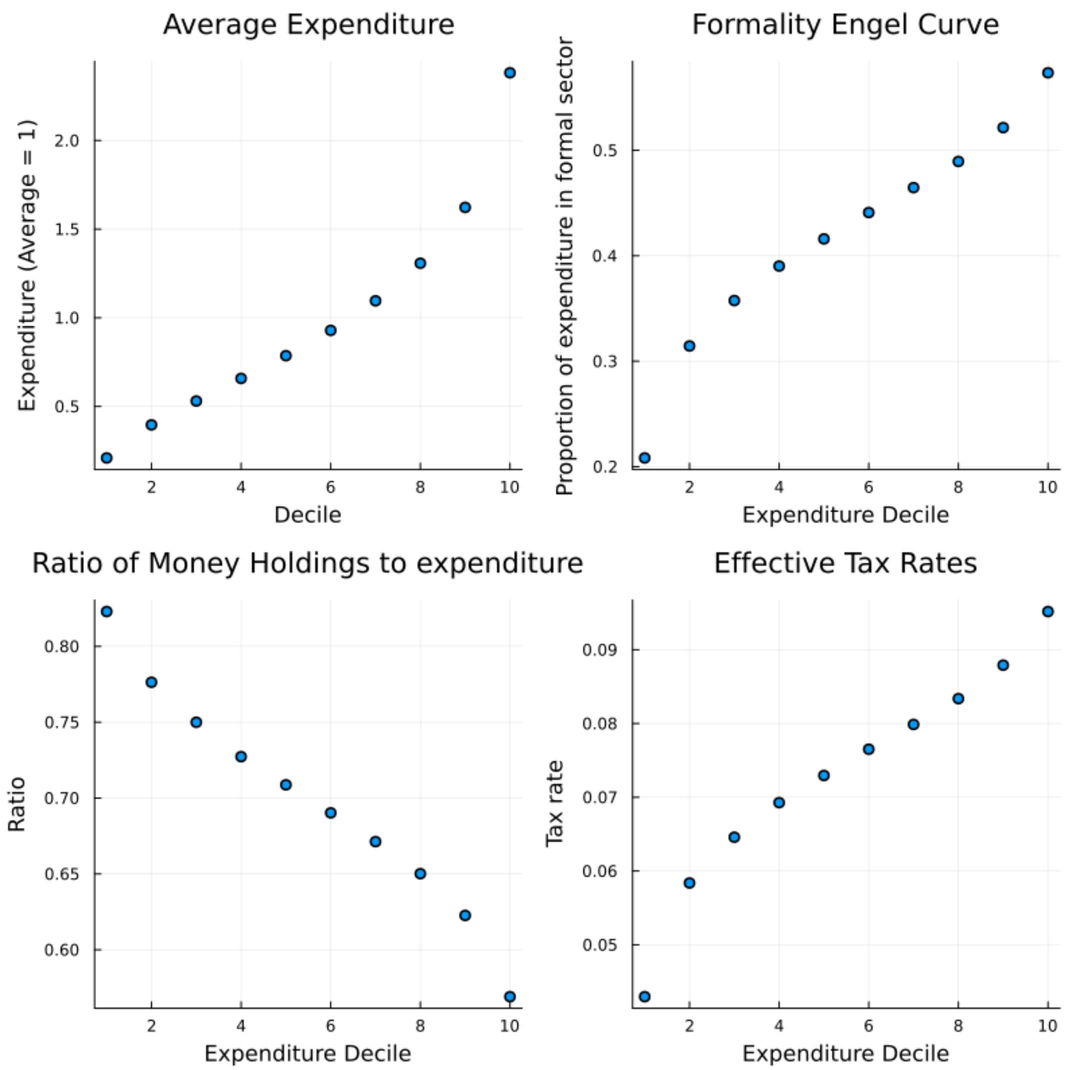

Figure 1
Household consumption bundle and taxation

goods with cash. The final plot, in the bottom right panel shows the effective tax rates as a percentage of expenditure. This sums a household's spending on consumption taxes and the foregone interest from holding money and computes it as a function of total expenditure. The first thing to notice is the level of effective tax rates. Even though the value added tax is 18%, households pay substantially less than that in taxes because of the larger share of their expenditure in the informal sector. The second thing one notices is the progressivity of the tax. As discussed in Bachas, Gadenne, and Jensen (2023), due to the positively sloped Engel Curve of formality, a proportional value added tax on the formal sector is a progressive tax. Contrarily, as suggested by the ratio of money holdings to expenditure, the inflation tax is regressive, taxing at a higher rate poorer households. In the benchmark calibration, with a consumption tax of 18% and inflation rate of 4%, the effect of the consumption tax dominates and the policy mix achieves a degree of progressivity.

3.3 Main Results

3.3.1 Policy mix and the size of the informal sector

In our quantitative exercise we vary inflation from 0 to 75% and find the consumption tax rate that balances the budget. Figure 2 displays some aggregate results. On the top left panel we see that as inflation is increased, the consumption tax rate can be decreased because of a substitution of consumption tax revenues for seignorage revenues, as displayed in the top right and bottom left panel. To decrease inflation from the benchmark policy of 4% to 0% requires an increase of the value added tax from 18% to 20.2%. In the other direction, an increase of inflation to 10% would allow the government to reduce the value added tax rate to 15.9%. The bottom right panel shows the effect of the policy mix on the degree of informality of the economy. As the inflation rate increases and tax rate on the consumption of formal goods decreases, households shift their consumption away from informal goods into formal goods. On the supply side, this increase in demand for formal goods pushes prices up, providing incentives for more firms to take the decision to formalize. However, the effects of increased inflation on the size of the informal sector are modest. The combined effect of increasing inflation from 4% to 10% and decreasing the value added tax from 18% to 15.9% is a decrease in the size of the informal sector by 1.8 percentage points.

3.3.2 Fiscal progressivity and inequality

Figure 3 shows how the policy mix affects different households along the wealth distribution. In the top right panel, we see that as the inflation rate is increased and the consumption tax is decreased, the effective tax rate of the poorest of households is going up faster than for other households. The top quintile experiences a decrease in its effective tax rate. Notice that around 40% inflation the system becomes regressive, effectively taxing poorer households at higher rates than richer ones. This is the point at which the inflation tax rate, appropriately computed, becomes larger than the consumption tax rate. The top right panel shows how this change in the progressivity of taxation affects consumption inequality. It plots the change in consumption quality for households in different quintiles of the distribution. By construction, the index equals 100 for each quintile when inflation is 0%; hence, what we see is the % change in the quality of consumption of households in each of the five quintiles. Households in the bottom quintile see the quality of their consumption decrease. An inflation of 20% increases consumption inequality by raising the quality of consumption of the top quintile by 0.7% and decreasing the bottom quintile's consumption quality by more than 1.5%. The bottom left panel displays money holdings as a ratio of expenditure. Even though for low inflation richer households also carry a large amount of money holdings (every household would purchase all of its goods with money at the Friedman Rule), they react to inflation more quickly due to the nature of the financial services cost – the financial services cost for a variety does not depend on the expenditure in that variety, which means that it is more worth paying for households with larger expenditures. Money holdings react strongly to inflation

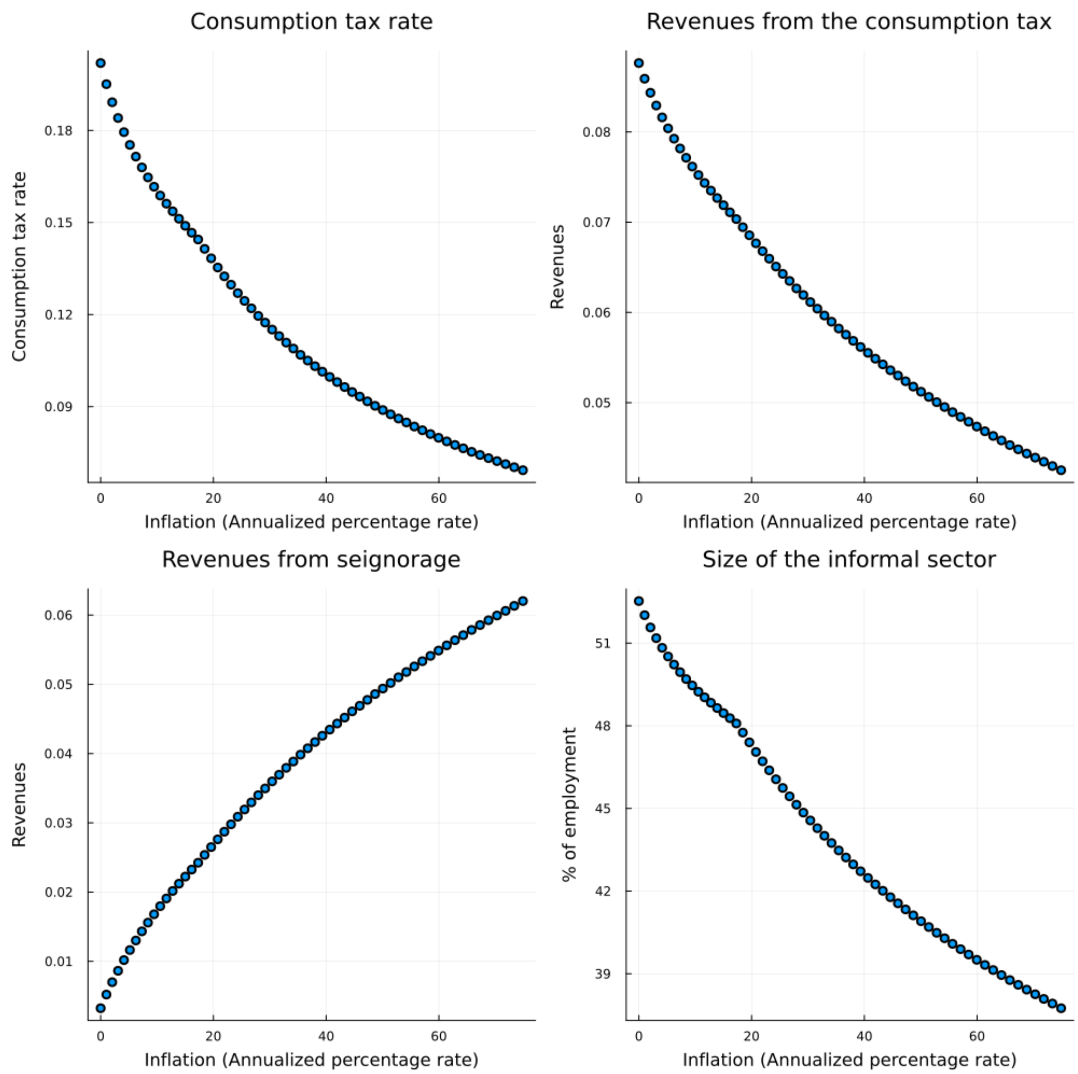

Figure 2

Effects of changing the policy mix on different aggregate variables

around 0 because they can be reduced through financial services. Once households stop using cash to buy formal goods, decreasing money holdings requires shifting consumption from the informal to the formal sector. The bottom right panel shows how every household increases its consumption of formal goods in response to the change towards a policy with more inflation and less consumption taxes.

3.3.3 Welfare

To analyze welfare, we compute the consumption-equivalent effect of moving from a certain policy mix to the benchmark policy. This tells us by how much the consumption quality of a household would have to increase in % permanently, for the household to be as well-off as with the policy change. This measure is similar in nature to compensating / equivalent variation measures commonly used to measures effects of policy. However, the fact that our model thinks of consumption in terms of the quality of the bundle of goods consumed makes it

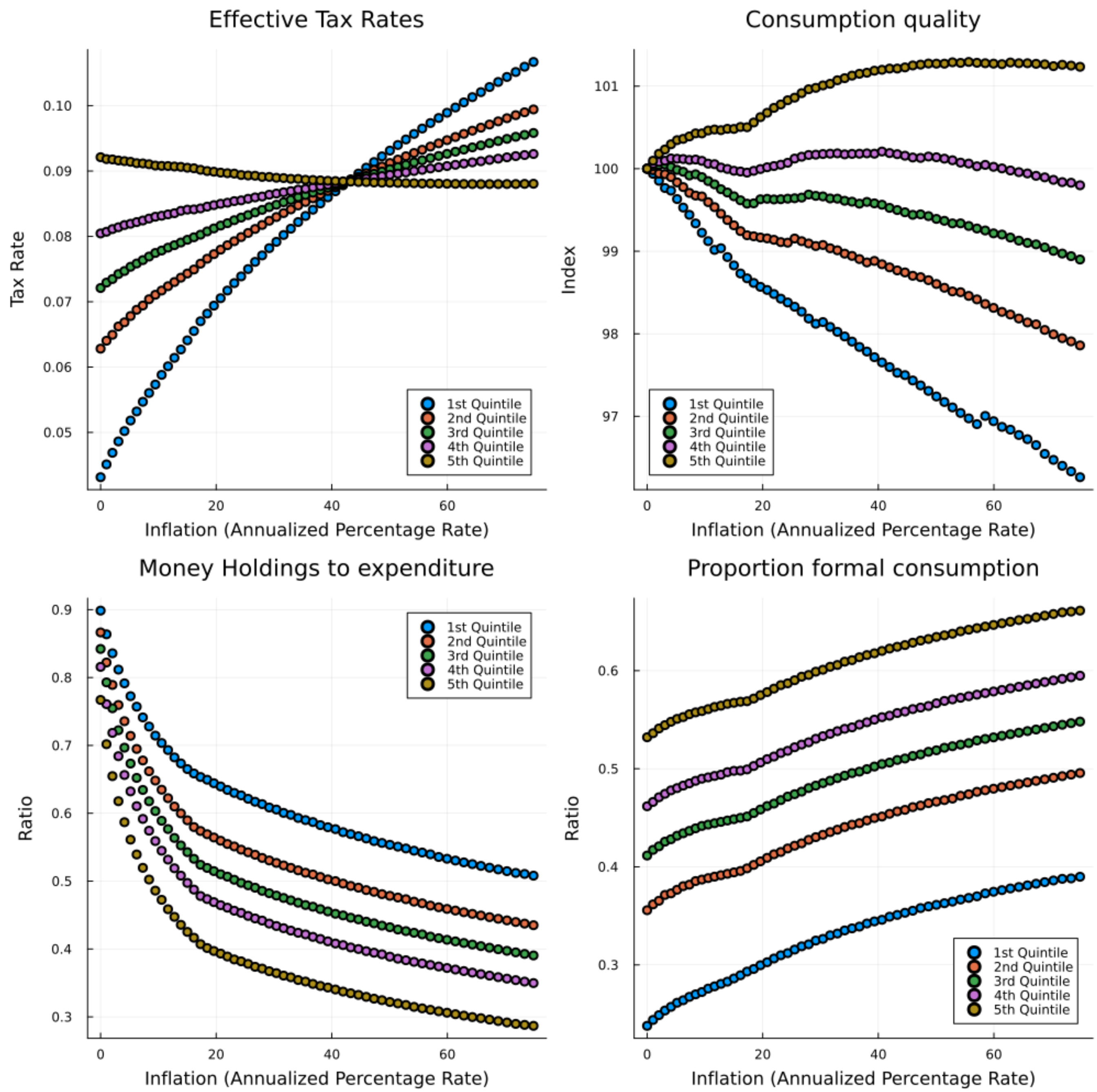

Figure 3
Effects of changing the policy mix on different households

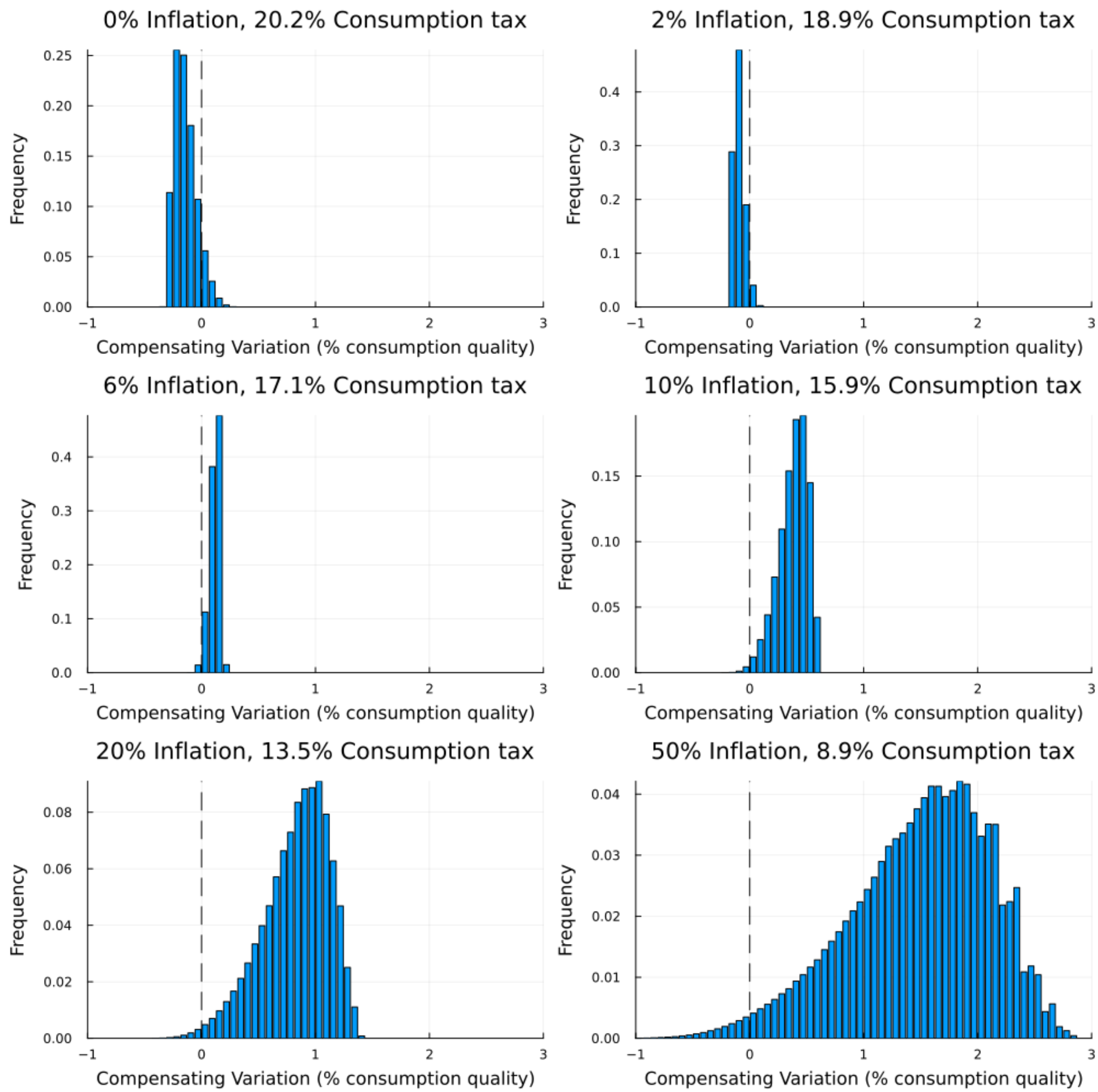

Figure 4
Distribution of compensating variation relative to the benchmark policy of 4% inflation, 18% consumption tax rate

less easily interpretable (given the linearity of prices in the formal sector, this measure would be equivalent to an increase in consumption expenditure for the formal sector, but this is not true for the informal sector). The distribution of consumption-equivalent welfare impacts is plotted in Figure 4.

About 90.7% of households prefer the policy mix with 0% inflation to the benchmark policy. Moving from the benchmark policy to 0% inflation, would benefit the bottom quintile by 0.25% consumption-equivalent units, and would hurt the top quintile by 0.01%. As inflation increases, the share of households who prefer it to the benchmark policy mix decreases and the welfare costs of poorer households hurt by the policy mix become larger.

3.4 Decomposing effects

The role of financial services costs

The scale of financial services costs plays an important part because it determines how costly inflation is on average, beyond the distributional effect between the formal and informal sectors. In our main results, financial services were high enough that inflation in the range of 10-20% was undesirable even for households at the top of the wealth distribution. When financial services costs are lower, this gives rise to more acute disagreement in preferences for policy, as households at the top are hurt less by inflation. Figure 5 illustrates this by plotting the results for welfare under a reduced value for the parameters determining the scale of financial services costs, γ_0. First, we see that the heterogeneity in welfare related to different policy mixes is a feature of the model which is largely independent of the magnitude of aggregate distortion produced by inflation, as pinned down by the scale of financial services costs. Under this calibration, there is still a large range of welfare costs of different policy mixes. Second, under lower financial services costs, the desirability of inflation can have a non-monotonic effect for households at the top. This is because increases in inflation at low rates affect richer households who are consuming a large part of their formal bundle with cash. Once inflation is high enough, households just buy formal goods with credit and informal with cash, and further increases in inflation hurt poorer households with a large informal share of consumption a lot more than they hurt richer households. Few households prefers inflation of 6% to the benchmark policy. However, close to half of the households prefer a 20% inflation to the benchmark policy.

4 Conclusion

Inflation has notable redistributional effects, especially in economies with large informal sectors, where the prevalence of cash use often goes hand-in-hand with tax evasion. This reliance on informality is particularly pronounced among poorer households, who make up a significant share of its consumers. In this paper, we develop a model to explore the implications of relying on consumption taxes versus inflation as a revenue source, accounting for the informal sector's unique characteristics.

Our findings highlight substantial differences in policy impacts across the wealth distribution, suggesting that distributional effects should be considered alongside average outcomes when evaluating tax policy. Importantly, acknowledging these distributional consequences reduces the rationale for relying on inflation as a tax tool in economies with high informality.

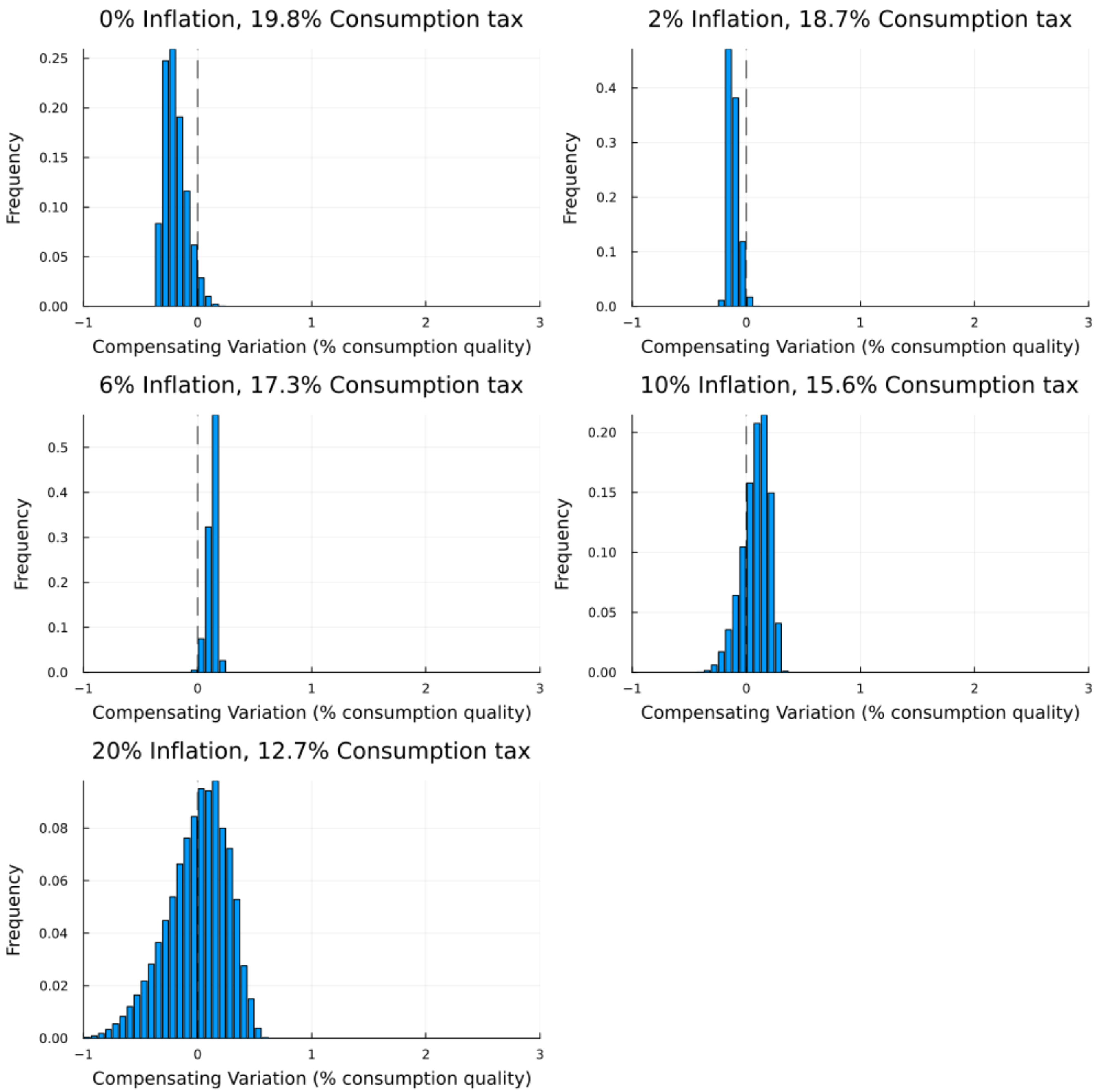

Figure 5
Distribution of compensating variation relative to the benchmark policy of 4% inflation, 18% consumption tax rate with lower financial services costs

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A Proofs

A.1 Equilibrium price schedules

We prove the characterization of the equilibrium price schedules described in the first three statements of Proposition 1.

The revenue maximization of a formal firm of variety j with productivity z and l units of labor is given by:

$$R_{jf}(z,l) = \max_{s(q)} \int \left(\frac{p_{jf}(q)}{1+\tau}\right) s(q)dq$$
$$s.t.: \quad \int qs(q)\, dq \leq F(z,l)$$

The first order condition of this problem with respect to $s(q)$ is:

$$\left(\frac{p_{jf}(q)}{1+\tau}\right) - \mu_{jf} q = 0,$$

where μ_{jf} is the Lagrangian multiplier on the constraint.

The revenue maximization of an informal firm of variety j with productivity z and l units of labor is given by:

$$\begin{aligned} R_{jn}(z,l) = & \max_{s(q)} \int \left[p_{jn}(q) - \chi_j\left(p_{jn}(q)\right)\right] s(q) dq \\ s.t.: & \quad \int q s(q)\, dq \leq F(z,l) \end{aligned}$$

The first order condition of this problem with respect to $s(q)$ is:

$$\left[p_{jn}(q) - \chi_j\left(p_{jn}(q)\right)\right] - \mu_{jn} q = 0,$$

where μ_{jn} is the Lagrangian multiplier on the constraint.

These two conditions correspond to equation (1).

Moreover, we can plug them back into our revenue expressions to write:

$$R_{jf}(z,l) = \mu_{jf} F(z,l), \quad R_{jn}(z,l) = \mu_{jn} F(z,l)$$

And profits for a firm producing variety j and in sector s with productivity z are given by:

$$\Pi_{js} = \max_{l} \mu_{js} F(z,l) - wl$$

Due to free entry across varieties, a firm with productivity z in sector s must be indifferent between all varieties j. This can only be the case if $\mu_{js} = \mu_s$.

Notice, further, that for any equilibrium where there is both formal and informal firms, it must be the case that $\mu_f > \mu_n$. If this is not true, then $\Pi_f(z) < \Pi_n(z), \forall z$ and no firm chooses to be formal.

A.2 Formalization decision

The proof that the formalization decision takes the form of a cutoff rule is a straightforward application of the intermediate value theorem. The firm wants to become formal iff

$$\Pi_f(z) - \kappa_f \geq \Pi_n(z).$$

Define the function $g(z)$ as:

$$g(z) \equiv \Pi_f(z) - \kappa_f - \Pi_n(z)$$

Clearly, g is continuous. We want to show that $g(z)$ is monotonically increasing in z. Then, if $g(\underline{z}) < 0$ and $g(\overline{z}) > 0$, by the intermediate value theorem, there is $\bar{z}^f \in (\underline{z}, \overline{z})$, such that $g(\bar{z}^f) = 0$ and above this cutoff $g(z)$ is positive and a firm wants to be formal, and below it $g(z)$ is negative and a firm wants to operate informally. If, $g(\underline{z}) > 0$, then $\bar{z}^f = \underline{z}$ and all firms formalize, while if $g(\overline{z}) < 0$, $\bar{z}^f = \overline{z}$ and all firms operate informally.

We now prove monotonicity of g, when $\mu_f > \mu_n$ and the production function $F(z, l)$ satisfies $F_z > 0$, $F_l > 0$, $F_{lz} > 0$, $F_{ll} < 0$ and $F(0, l) = 0$. The derivative of g is given by:

$$g'(z) = \frac{\partial \Pi_f}{\partial z} - \frac{\partial \Pi_n}{\partial z}$$

Thus,

$$g'(z) > 0 \Leftrightarrow \frac{\partial \Pi_f}{\partial z} - \frac{\partial \Pi_n}{\partial z} > 0 \Leftrightarrow \frac{\partial \Pi(z, \mu_f)}{\partial z} - \frac{\partial \Pi(z, \mu_n)}{\partial z} > 0 \Leftrightarrow \int_{\mu_n}^{\mu_f} \frac{\partial^2 \tilde{\Pi}(\tilde{\mu}, z)}{\partial z \partial \mu} . d\tilde{\mu} > 0,$$

where the last equivalence uses the fundamental theorem of calculus. It is then sufficient to prove that:

$$\frac{\partial^2 \tilde{\Pi}(\mu, z)}{\partial z \partial \mu} > 0, \quad \forall \mu > 0$$

The envelope theorem yields:

$$\frac{\partial \tilde{\Pi}(\mu, z)}{\partial \mu} = F\left(z, l^*(z, \mu)\right).$$

Taking the derivative of this with respect to z:

$$\frac{\partial^2 \tilde{\Pi}(\mu, z)}{\partial z \partial \mu} = F_z\left(z, l^*(z, \mu)\right) + F_l\left(z, l^*(z, \mu)\right) \frac{\partial l^*(z, \mu)}{\partial z}.$$

All the terms are positive, hence, this derivative is positive and g is monotonic and increasing. The first term is positive because we assumed $F_z > 0$. We also assumed $F_l > 0$. We now show that

$$\frac{\partial l^*(z, \mu)}{\partial z} > 0.$$

This can be shown through the implicit function theorem. The optimal l is characterized by the first order condition:

$$\mathcal{I}(z, \mu, l) \equiv \mu F_l(z, l) - w = 0.$$

The implicit function theorem tells us that:

$$\frac{\partial l^*(z, \mu)}{\partial z} = -\frac{\frac{\partial \mathcal{I}}{\partial z}}{\frac{\partial \mathcal{I}}{\partial l}} = -\frac{\mu F_{lz}(z, l)}{\mu F_{ll}(z, l)} = -\frac{F_{lz}(z, l)}{F_{ll}(z, l)} > 0$$

The inequality follows from $F_{zl} > 0$, $F_{ll} < 0$.

A.3 Household consumption bundle decision

Optimality of cutoff rule for informal consumption

First, we show that if χ_j is convex, the optimal choice of whether to buy a good of a certain quality in the formal or informal sector obeys a cutoff rule.

To show this, we start by analyzing the equilibrium price schedule in the informal sector. This is:

$$p_{jn}(q) = \mu_n q + \chi_j\left(p_{jn}(q)\right) \tag{IPS}$$

We start by showing that if $\chi_j' > 0$ and $\chi_j'' > 0$, the price schedule is also increasing and convex up to a maximum level $p_{jn}(\bar{q}_n)$. Above this quality there is no supply of informal goods.

Rewrite equation (IPS) as:

$$p - \chi_j\left(p\right) = \mu_n q$$

We want to derive a function $p(q)$ from this. Notice however that if $\lim_{p\to\infty} \chi_j'(p) > 1$, then the left hand side of the expression above has a maximum, at $\bar{p}_n$ such that:

$$\chi_j'(\bar{p}_n) = 1.$$

Above this price, the expression is decreasing in p. Hence, if we define $\bar{q}_n$ as

$$\bar{p}_n - \chi_j\left(\bar{p}_n\right) = \mu_n \bar{q}_n,$$

this is the highest quality for which there is a price that satisfies the equilibrium price schedule and the price schedule is only defined for $q < \bar{q}_n$.

Now we show that in this domain $p(q)$ is convex iff χ_j is convex.

The first derivative of the price schedule is:

$$\frac{\partial p}{\partial q} = \mu_n + \frac{\partial \chi_j}{\partial p}\frac{\partial p}{\partial q} \implies \frac{\partial p}{\partial q} = \frac{\mu_n}{\left(1 - \frac{\partial \chi_j}{\partial p}\right)^2}$$

The second derivative is:

$$\frac{\partial^2 p}{\partial q^2} = \frac{\mu_n \frac{\partial^2 \chi_j}{\partial p^2}\frac{\partial p}{\partial q}}{1 - \frac{\partial \chi_j}{\partial p}} = \frac{\mu_n^2 \frac{\partial^2 \chi_j}{\partial p^2}}{\left(1 - \frac{\partial \chi_j}{\partial p}\right)^3}$$

Thus, $p(q)$ will have the same first and second derivatives as χ_j.

Now we proceed to show that given $p_{jn}(q)$ is increasing and convex, the cutoff rule applies. As in the case of the formalization decision, this is an application of the intermediate value theorem. For a variety j, define function $h_j(q)$:

$$h_j(q) = p_{jn}(q)(1+i) - \min\{\mu_f(1+\tau)q(1+i), \mu_f(1+\tau)q + \gamma(j)\}$$

This is the difference between the expenditure of purchasing q units in the informal sector and in the formal sector (where the cost of purchasing in the formal sector is the minimum cost between buying it with cash and credit). Hence, given that the household just wants to purchase from the cheaper sector, it should buy formal when $h_j(q) \geq 0$. h is continuous.

We want to show that:

$$\exists \bar{q} : \begin{cases} h_j(q) < 0, & q \in (0, \bar{q}) \\ h_j(q) > 0, & q \in (\bar{q}, \bar{q}_n] \end{cases} \tag{2}$$

To do so we make two claims and show that they imply (2):

1. $h_j(0) = 0$
2. $h_j''(q) > 0, \quad \forall q$

It is trivial to see that $h(0) = 0$. The derivative of h_j is:

$$h_j'(q) = \begin{cases} \frac{\partial p_{jn}}{\partial q}(1+i) - \mu_f(1+\tau)(1+i), & \text{if } \mu_f(1+\tau)q(1+i) < \mu_f(1+\tau)q + \gamma(j) \\ \frac{\partial p_{jn}}{\partial q}(1+i) - \mu_f(1+\tau), & \text{otherwise} \end{cases}$$

and we have:

$$\frac{\partial p_{jn}}{\partial q} = \frac{\mu_n}{\left(1 - \frac{\partial \chi_j}{\partial p}\right)^2}.$$

Hence, the second derivative is:

$$h_j''(q) = \frac{\partial^2 p_{jn}}{\partial q^2}(1+i) = \frac{\mu_n^2 \frac{\partial^2 \chi_j}{\partial p^2}}{\left(1 - \frac{\partial \chi_j}{\partial p}\right)^3}(1+i)$$

Thus, given $\chi_j'' > 0$, we get $h_j''(q) > 0$. This means that the derivative of h_j is increasing. Thus, if $h_j'(0) > 0$, then $h_j(q) > 0, \forall q$ and $\bar{q} = 0$. That is, the household buys only from the formal sector. When $h_j'(0) < 0$, due to the monotonicity of the first derivative, there will be at most one quality, at which $h_j(q) = 0$ in the domain $q > 0$. This is the cutoff above which $h_j(q)$ is positive and below which it is negative.

Cutoff decreasing in j

Above we showed that for all varieties, $h_j(0) = 0$ and $h_j''(q) > 0, \forall q$, which proved the optimality of a cutoff rule. Now, we show that this cutoff, satisfying

$$h_j(\bar{q}_j) = 0,$$

is decreasing in j. First we show that for $k > j$, $h_k(q) > h_j(q)$:

$$h_k(q) - h_j(q) \geq [p_{kn}(q) - p_{jn}(q)](1+i)$$

where the inequality stems from $\gamma(k) < \gamma(j)$ because we assumed that the financial cost of varieties with higher j index is lower.

we have assumed that $\chi_k(p) > \chi_j(p)$ for $k > j$. We now show that this implies $p_{kn}(q) > p_{jn}(q)$ and thus $h_k(q) > h_j(q)$.

Notice, that for any two varieties j, k, the good of quality q satisfies:

$$p_{kn}(q) - \chi_k(p_{kn}(q)) = p_{jn}(q) - \chi_j(p_{jn}(q)) \implies p_{kn}(q) - p_{jn}(q) = \chi_k(p_{kn}(q)) - \chi_j(p_{jn}(q))$$

One can write the right-hand side expression as:

$$\chi_k(p_{kn}(q)) - \chi_k(p_{jn}(q)) + (\chi_k(p_{jn}(q)) - \chi_j(p_{jn}(q)))$$

Using the mean value theorem (relying on continuity and differentiability of χ_k) we have that there is p^* in the interval between the prices for variety k and j, such that:

$$\chi_k(p_{kn}(q)) - \chi_k(p_{jn}(q)) = \chi_k(p^*)(p_{kn}(q) - p_{jn}(q))$$

Then, putting all this together, we have:

$$p_{kn}(q) - p_{jn}(q) = \chi_k'(p^*)(p_{kn}(q) - p_{jn}(q)) + (\chi_k(p_{jn}(q)) - \chi_j(p_{jn}(q)))$$

Hence, we find:

$$p_{kn}(q) - p_{jn}(q) = \frac{(\chi_k(p_{jn}(q)) - \chi_j(p_{jn}(q)))}{1 - \chi_k'(p^*)}$$

So, if $\chi_k(p) > \chi_j(p)$, then $p_{kn}(q) > p_{jn}(q)$, and $h_k(q) > h_j(q)$. Thus, if $h_j(\bar{q}_j) = 0$, $h_k(\bar{q}_j) > 0$, and $\bar{q}_k$ such that $h_k(\bar{q}_k) = 0$ is lower that $\bar{q}_j$.

Cutoff rule for use of credit

This is straightforward. Define:

$$h_j(q) := \mu_f(1+\tau)q(1+i) - [\mu_f(1+\tau)q + \gamma(j)]$$

This is the difference between buying a formal variety j, quality q, with cash and credit. Notice that $h_j(0) = -\gamma(j) < 0$. Moreover, $h_j'(q) = \mu_f(1+\tau)i > 0$ Hence, there is a point $\bar{q}_j^M$ at which $h_j(\bar{q}_j^M) = 0$ and $h_j(q) > 0$ when $q > \bar{q}_j^M$ and, $h_j(q) < 0$ when $q < \bar{q}_j^M$.

Cutoff decreasing in j

This follows from $\gamma'(j) < 0$.

Threshold varieties

Given that $\bar{q}_j^f$ is decreasing in j, when the households wants consumption q, there is a single variety $\bar{j}^f(q)$ satisfying $q = \bar{q}_j^f$. Moreover, for $j < \bar{j}^f(q)$, then $q < \bar{q}_j^f$ so the household wants to buy informal, and for $j > \bar{j}^f(q)$, then $q > \bar{q}_j^f$ and the household prefers to buy formal. The same argument applies to the choice of credit, with the only modification that because households can only use credit for formal goods, if the variety that satisfies $q = \bar{q}_j^M$ is below $\bar{j}^f(q)$, then the actual threshold for the use of credit is $\bar{j}^M(q) = \bar{j}^f(q)$.

A.4 Goods market clearing

Work in progress

B Computation

Solving the household problem

1. Define a grid for aggregate quality: $\mathcal{G}^q = [q_1, q_2, ..., q_N]$.
2. For each aggregate quality q_k in the grid $\mathcal{G}^q$, find the threshold varieties, $\bar{j}^f(q_k)$ and $\bar{j}^M(q_k)$ and the total expenditure:

$$E(q_k) = \int_0^{\bar{j}^f(q_k)} (1+i)p_{jn}(q_k).dj + \int_{\bar{j}^f(q)}^{\bar{j}^M(q_k)} \mu_f(1+\tau)(1+i)q_k.dj + \int_{\bar{j}^M(q)}^{1} [\mu_f(1+\tau)q_k + \gamma(j)].dj$$

3. Interpolate the expenditure function computed in the grid $\mathcal{G}^q$.
4. Solve the dynamic programming problem of the household:

$$\begin{aligned} V(\epsilon, x) = \max_{q\in[0,q^{max}], l\geq 0} & u(q,l) + \beta \mathbb{E} V(\epsilon', x'), \\ st: x' &= \left(\frac{1+i}{1+\pi}\right) x + \epsilon l - E(q), \\ E(q^{max}) &= \left(\frac{1+i}{1+\pi}\right) x. \end{aligned}$$

Stationary Equilibrium

The government policy mix is given by (B, M, i, τ, G, g_N). In a Stationary Equilibrium g_N pins downs inflation directly, so we can consider instead inflation directly as part of the mix.

We can define total nominal liabilities as $D = B/(1+i) + M$ and consider policies in which the government does not pick B and M but instead accommodates demand for these.

We are left with (i, τ, G, π, D). The Government has three degree of freedom, so it can pick three of these variables and the two others will be endogenous (from i and π, at most one can be chosen, due to the exogenous real rate condition). The policies we consider, are D, τ, π for calibration, D, π, G for our main quantitative exercise, and D, π, τ for our additional quantitative exercise to decompose effects.

The general algorithm for computation is as follows:

1. Guess a value for the variables in the set (τ, μ_f, μ_n) which are endogenous.

2. Compute i to match the exogenous real rate.
3. Compute the productivity cutoff $\bar{z}^f$ for the formalization decision.
4. Compute the labor demand of firms $(l_f(z), l_n(z))$.
5. Compute price schedules $(p_{jf}(q), p_{jn}(q))_j$.
6. Solve the household problem, getting policy functions.
7. Find the stationary distribution of households.
8. Use the money market clearing condition to get aggregate money M.
9. Compute the aggregate bond supply using D and M and then compute tax revenues and use the law of motion of government assets to compute implied G.
10. Use the formal sector goods market condition to compute the mass of firms m^F.
11. Check the errors in the informal sector goods market clearing condition, free entry condition, and government expenditure if G is exogenous. If error is small, conclude. If not, update guess in step 1 and repeat.