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[CM0481] SOFTWARE PERFORMANCE AND SCALABILITY

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# IMDb: Performance and Scalability Analysis

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# Abstract

This report presents a comprehensive account of the implementation and performance testing process conducted on a web application that emulates the IMDb website<sup>1</sup>.

The report begins with an overview of the system implementation, in which the technologies utilized are outlined. Subsequently, a theoretical analysis of the queueing system network is presented, in which the implemented system is characterised. The load test conducted on the system will be subjected to a thorough examination, including an analysis of its theoretical and empirical findings.

In conclusion, a potential architectural solution is evaluated for its potential to enhance the scalability of the tested system.

It should be noted that the implementation of the system, along with all automated programs and scripts referenced in this report, is fully documented and accessible on the GitHub repository<sup>2</sup>.

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<sup>1</sup><https://www.imdb.com/>

<sup>2</sup><https://github.com/danieljaderpellattiero/unive-imdb>

# Chapter 1

## System implementation

The project entails the development of a three-tiered web application, comprising a database, a RESTful API and a website. The following sections will present an analysis of each of these components, with a focus on the technologies and design choices implemented.

### 1.1 Database – MongoDB

MongoDB is a popular NoSQL document-oriented database that is designed to store and manage large volumes of structured and unstructured data.

The database was selected as the optimal choice for managing the IMDb Non-Commercial Datasets<sup>1</sup> due to its adoption of a schemaless data modelling, which enables the management of any non-normalised records or fields within the data. Furthermore, the usage of JSON format for storing the records allows for enhanced efficiency in interactions with the backend.

#### 1.1.1 Data analysis

Each dataset, comprising a UTF-8-encoded tab-separated values (TSV) file, was subjected to analysis using the Python Pandas library. The primary steps of the data analysis process were as follows:

1. The replacement of missing values (denoted by ‘\N’) with default values of a type consistent with the column domain to which they belong.
2. The removal of records that lack some data fields.
3. The normalisation of fields containing arrays of elements.
4. The potential replacement of table indexes.

Further modifications were implemented to the tables, however, as these affect the database schema, they will be addressed subsequently.

Once the preliminary phase of preparing the datasets was complete, it was decided that the files should be exported in the JSON format, allowing them to be imported into the database. Additionally, the files were exported also in the Parquet format, enabling

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<sup>1</sup><https://developer.imdb.com/non-commercial-datasets/>

them to be uploaded to the GitHub repository via the GitHub Large File Storage (LFS) facility<sup>2</sup>.

### 1.1.2 Collections' schema

MongoDB represents objects using BSON (Binary JSON) types, which are binary-encoded serialisations of documents that adhere to the JSON format. As each table in MongoDB is translated into the concept of a document collection, the diagrams of the respective collections imported from the datasets obtained post data analysis are presented below. Furthermore, supplementary notes on the refactoring of the tables are provided.

Table 1.1: '*title.basics*' collection schema

Field	BSON type	Index	Index type
_id	String	yes	unique, ascending
titleType	String	no	-
name	String	no	-
nameEng	String	no	-
isAdult	Boolean	no	-
genres	Array[String]	no	-
startYear	32-bit integer	no	-
endYear	32-bit integer	no	-
runtime	32-bit integer	no	-
rating	Double	no	-
votes	32-bit integer	no	-

*N.d.R.*: A collection was created ad hoc for records with the field '*titleType*' equal to '*tvEpisode*'.

Table 1.2: '*title.akas*' collection schema

Field	BSON type	Index	Index type
_id	ObjectId	yes	unique, ascending
titleId	String	no	-
ordering	32-bit integer	no	-
region	String	no	-
name	String	no	-
nameLower	String	yes	ascending

*N.d.R.*: '*language*', '*types*', '*attributes*', '*isOriginalTitle*' fields dropped.

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<sup>2</sup><https://git-lfs.com/>

Table 1.3: ‘*title.crew*’ collection schema

Field	BSON type	Index	Index type
_id	String	yes	unique, ascending
writers	Array[String]	no	-
directors	Array[String]	no	-

Table 1.4: ‘*title.episodes*’ collection schema

Field	BSON type	Index	Index type
_id	String	yes	unique, ascending
titleId	String	yes	ascending
name	String	no	-
nameEng	String	no	-
season	32-bit integer	no	-
episode	32-bit integer	no	-
isAdult	Boolean	no	-
genres	Array[String]	no	-
startYear	32-bit integer	no	-
endYear	32-bit integer	no	-
runtime	32-bit integer	no	-
rating	Double	no	-
votes	32-bit integer	no	-

Table 1.5: ‘*name.basics*’ collection schema

Field	BSON type	Index	Index type
_id	String	yes	unique, ascending
fullName	String	no	-
birth	32-bit integer	no	-
death	32-bit integer	no	-
professions	Array[String]	no	-

*N.d.R.*: ‘*knownForTitles*’ field dropped.



Table 1.6: ‘*title.principals*’ collection schema

Field	BSON type	Index	Index type
<code>id</code>	ObjectId	yes	unique, ascending
<code>titleId</code>	String	yes	ascending
<code>ordering</code>	32-bit integer	no	-
<code>personId</code>	String	no	-
<code>job</code>	String	no	-
<code>characters</code>	Array[String]	no	-

*N.d.R.*: ‘*category*’ field used to fill empty ‘*job*’ fields; then dropped.

### 1.1.3 Schema indexes and bulk data insert

As can be seen from the tables above, in addition to the standard unique indexes, it was decided to add additional ones to optimise the queries requested by the backend. Given the significant use of wildcard searches based on regular prefix expressions, it was decided to implement ascending indexes on several ‘*String*’ type fields to drastically reduce search times without having to resort to a text index, which would take up much more space due to the tokenisation and stemming of the fields.

In order to perform a bulk insert of the approximately 42.5 million records contained in  $\sim 1.6$  GB of JSON files, it was decided to write a script in Python that would take advantage of a multithreaded execution of the `mongoimport` command-line tool provided by MongoDB. The overall script is responsible for generating the database’s collections, populating them and finally creating indexes.

#### 1.1.4 Technical details

The database was configured as a locally managed instance of MongoDB, thus avoiding the utilisation of the cloud version of the service (Atlas). The service’s local deployment facilitated the monitoring and control of its resources, as well as the measurement of its performance. These factors collectively contributed to the generation of more reliable results during the load tests that were conducted.

Technical insights into the environment are presented below.

- MongoDB 7.0.11 (Community Server service)
- MongoDB Compass 1.43.3 (database UI)
- MongoDB Shell 2.2.9 (database CLI)

## 1.2 RESTful API – Express

The web application’s backend was developed using the Express framework, a Node.js web application framework that was identified as the most widely utilised in the “State of JavaScript 2023” survey<sup>3</sup>.

<sup>3</sup>[https://2023.stateofjs.com/en-US/other-tools/#backend\\_frameworks](https://2023.stateofjs.com/en-US/other-tools/#backend_frameworks)

A variety of endpoints were incorporated into the API, enabling a constrained yet still functional utilisation of the web interface. The user-accessible API calls are enumerated below, along with a concise description of each.

1. `localhost:3000/search/preview/:title` – Returns the top 4 most voted titles that match the search query prefix.
2. `localhost:3000/search/:title` – Returns the most voted titles that match the search query prefix; the results are paginated in groups of 8.
3. `localhost:3000/search/episodes/:title` – Returns the episodes of a specific title that matches the `'titleId'` parameter.
4. `localhost:3000/title/:id` – Returns the details of a specific title that matches the `'_id'` parameter.
5. `localhost:3000/episode/:id` – Returns the details of a specific episode that matches the `'_id'` parameter.

The official MongoDB driver for Node.js (Typescript) `mongodb@6.8` is responsible for handling queries from the backend to the database. We deliberately decided not to utilise any form of Object-Relational/Document Mapping (e.g. `Mongoose` or `Prisma`) due to the suitability of the JSON record format to the application domain model and potential performance overheads.

Aggregation pipelines are used to facilitate the execution of queries to the database. These pipelines allow the definition of queries through the use of a sequential list of stages, thereby simplifying the grouping and sorting of data and providing control over the execution times of the individual stages within the pipeline.

## 1.3 Website – Vue and Tailwind CSS

The web interface of the system was realised using Vue.js “The Progressive JavaScript Framework”, a well-known framework used for building SPAs (Single Page Applications) renowned for its component-based architecture and reactive data binding system. The aesthetic component was addressed through the utilisation of Tailwind CSS, a utility-first CSS framework that can be suitably integrated with Vue. A bundle of the website, optimised for deployment on a static hosting service, was created at the end of the development process using Vite<sup>4</sup>. This design choice ensures that the frontend is efficiently packaged into static files, which can be served directly to users without requiring server-side processing. Consequently, during load testing with JMeter, the focus has been exclusively on the backend API endpoints, as the frontend’s static nature does not impose additional load on the server.

Thumbnails of the web interface are available on GitHub<sup>5</sup>.

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<sup>4</sup>A modern front-end build tool that facilitates optimized production builds through the utilization of ES hot module replacement (HMR).

<sup>5</sup><https://github.com/danieljaderpellattiero/unive-imdb/tree/frontend/thumbnails>

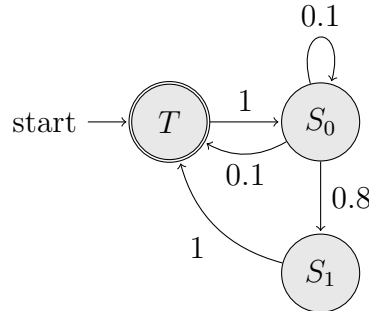
# Chapter 2

## Test case identification

### 2.1 High-level test case design

This brief chapter aims to present the test case designed to perform the load test of the system. In order to illustrate the user behaviour modelled by the test, we define a trivial finite-state automata with reference to the endpoints implemented in the RESTful API.

It should be noted that, although this aspect will be discussed in greater detail in the following chapters, the two endpoints that will be used in the load test result in two different classes of jobs, which must consequently be analysed individually.



The above automata has to be interpreted in the following manner:

- The user initiates a title search by name match, regardless of the customer language locale. (`/search/:title`)
  - In 80% of cases, the search continues with a precise request for the title information. (`/title/:id or /episode/:id` – node  $S_1$ )
  - In 20% of cases, either the user terminates the search process pre-emptively, without finding the title, thus entering a thinking state – (node  $T$ ) or it continues the search by hitting the same endpoint though the use of pagination. (`/search/:title?page=n` – node  $S_0$ )
- Upon receipt of the searched title information, the user is satisfied and returns to a thinking state before repeating a new search.

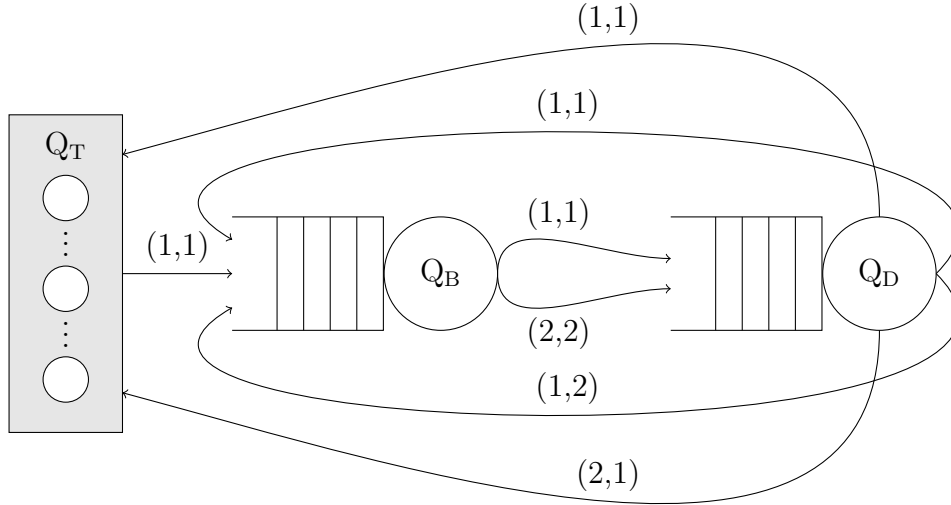
# Chapter 3

## Queueing network model

### 3.1 Introduction

This chapter presents a comprehensive theoretical analysis of the implemented model. The following illustration depicts the system components involved in the load test in the context of queuing theory.

For the purposes of this illustration, let  $Q_T$  represent the thinking station,  $Q_B$  represent the backend, and  $Q_D$  represent the database.



It is possible to derive several characteristics of the system under consideration from the illustration above.

The system is characterised by a closed and interactive nature, free of stability conditions, and a constant number of users that dictate the nature of the workload. A further aspect that emerges relates to the multiclass nature of the jobs within the system, as indicated by the labels placed at the station entry and exit arcs.

Subsequently, an in-depth examination will be conducted on the distinct service time/demand and probabilistic routing for the two classes of jobs.

In conclusion, the network under analysis can be classified as a BCMP, exhibiting a closed topology and utilising “Processor Sharing” (PS) as its scheduling discipline.

Prior to examining the nature of the queues that characterise our network and their associated scheduling disciplines, we proceed with the formalisation and resolution of the network's traffic equations. These equations will then be essential in producing the theoretical analysis of the system.

## 3.2 Traffic equations

The system's traffic equations are presented below.

$$\begin{bmatrix} p_{T1B1} \\ p_{B1D1} \\ p_{B2D2} \\ p_{D1B1} \\ p_{D1B2} \\ p_{D1T1} \\ p_{D2T1} \end{bmatrix} = \begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 0.1 \\ 0.8 \\ 0.1 \\ 1.0 \end{pmatrix} : \begin{cases} e_{B1} = e_{T1} + (p_{D1B1} \times e_{D1}) \\ e_{B2} = p_{D1B2} \times e_{D1} \\ e_{D1} = e_{B1} \\ e_{D2} = e_{B2} \\ e_{T1} = e_{D2} + (p_{D1T1} \times e_{D1}) \end{cases} \Rightarrow \begin{cases} e_{B1} = 1.1111 \times e_{T1} \\ e_{B2} = 0.8889 \times e_{T1} \\ e_{D1} = 1.1111 \times e_{T1} \\ e_{D2} = 0.8889 \times e_{T1} \end{cases}$$

The system of equations above demonstrates that our routing is irreducible, which implies that any queue can be accessed from any other queue within the network. From an analytical standpoint, our system is under-determined, indicating the existence of an infinite number of solutions. The non-trivial solutions can be derived from the non-zero multiplicative coefficients.

Moreover, it is essential to highlight that within this context, where the system under consideration is an interactive queueing network, it is imperative to impose an additional constraint on the previous system, specifically  $e_{T1} = 1$ , given that this is the thinking station  $Q_T$ .

## 3.3 Queueing network components

This section will provide a brief overview of the components of our network, with reference to Kendall's notation.

### 3.3.1 Thinking station

The thinking station can be represented by a queueing system described with the "G/D/ $\infty$ /IS" notation. The distribution of inter-arrival times for jobs at the thinking station is unknown, but the distribution of service times is known, as the jobs within it remain in service (*thinking state*) for a fixed constant time, known as thinking time. By definition, the thinking station is equipped with an infinite number of servers, indeed an undefined number of jobs can access it. Consequently, the service discipline of this system is of the "Delay center" or "Infinite Servers" type, as all the jobs remain in an idle state in parallel.

### 3.3.2 Backend station

With regard to the JavaScript runtime environment Node.js, despite its reputation as a "purely single-threaded" execution environment due to its Event Loop, it is in fact capable of offloading I/O blocking operations to separate threads or the operating system, thus

allowing the main thread to continue processing other tasks. The library that permits the dispatching of blocking operations to a thread pool is called `libuv` and it belongs to the Chrome V8 Engine. Furthermore, Node.js has also implemented the `worker_threads` module, which allows for further enhancements of performance by the utilisation of multi-core processors for CPU-bound operations.

In light of the aforementioned considerations, it can be concluded that the RESTful API functions as a “G/G/1/PS” queueing system. This is due to the fact that no manual instantiation of multiple worker threads was performed for the API, and thus a unique main thread is responsible for handling all incoming requests through a “Processor Sharing” scheduling discipline.

### 3.3.3 Database station

With regard to MongoDB, an examination of the “Production Notes” section reveals that its WiredTiger storage engine is inherently multithreaded. In particular, the total number of active threads (i.e. concurrent operations) relative to the number of available CPUs can impact performance as follows:

- The throughput increases in direct proportion to the number of concurrent active operations, up to the overall number of CPUs.
- The throughput decreases as the number of concurrent active operations exceeds the number of CPUs by a certain threshold amount.

The optimal number of concurrent active operations for a given application can be determined through the measurement of throughput.

In order to ascertain the exactness of the information provided in the documentation, an empirical test was conducted to monitor the thread pool instantiated by the MongoDB service in both idle and stressed states.

The results obtained from the Linux command-line utility `htop` are presented below.

In conclusion, the database can be identified as a “G/G/#C/PS” queueing system, where “#C” represents the number of cores. The inter-arrival times and service times are both unknown, however it can be stated that the database leverages all CPU cores with a “Processor Sharing” service discipline.

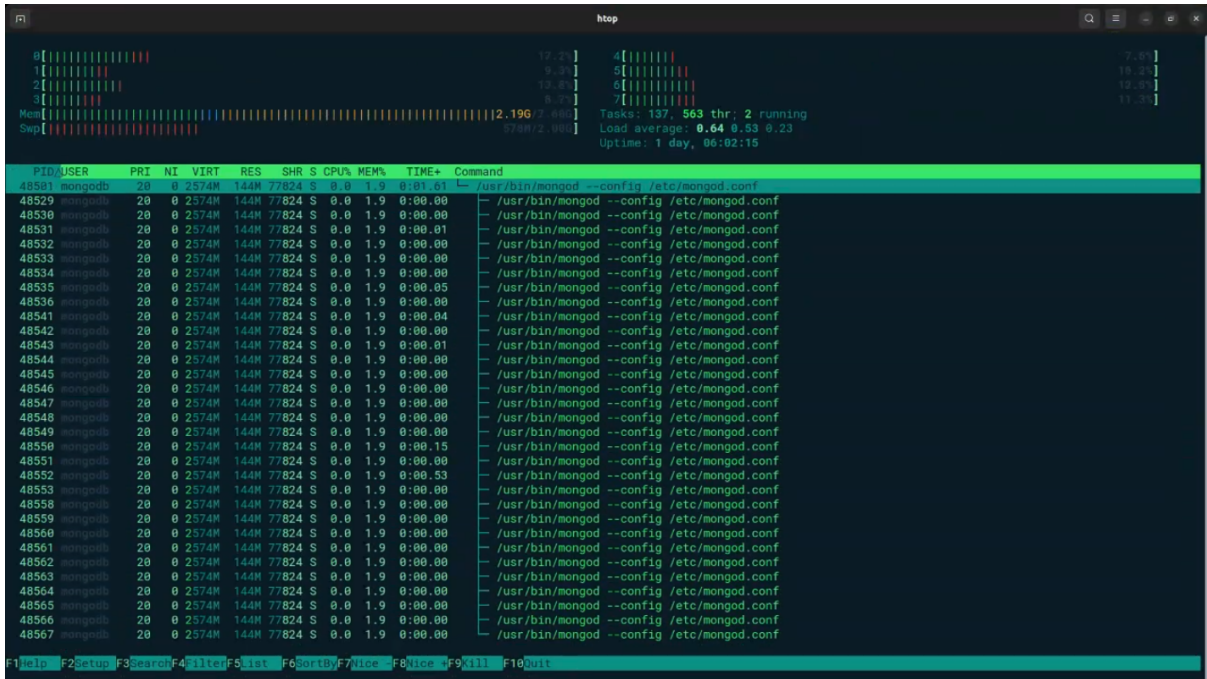


Figure 3.1: MongoDB thread pool in idle state

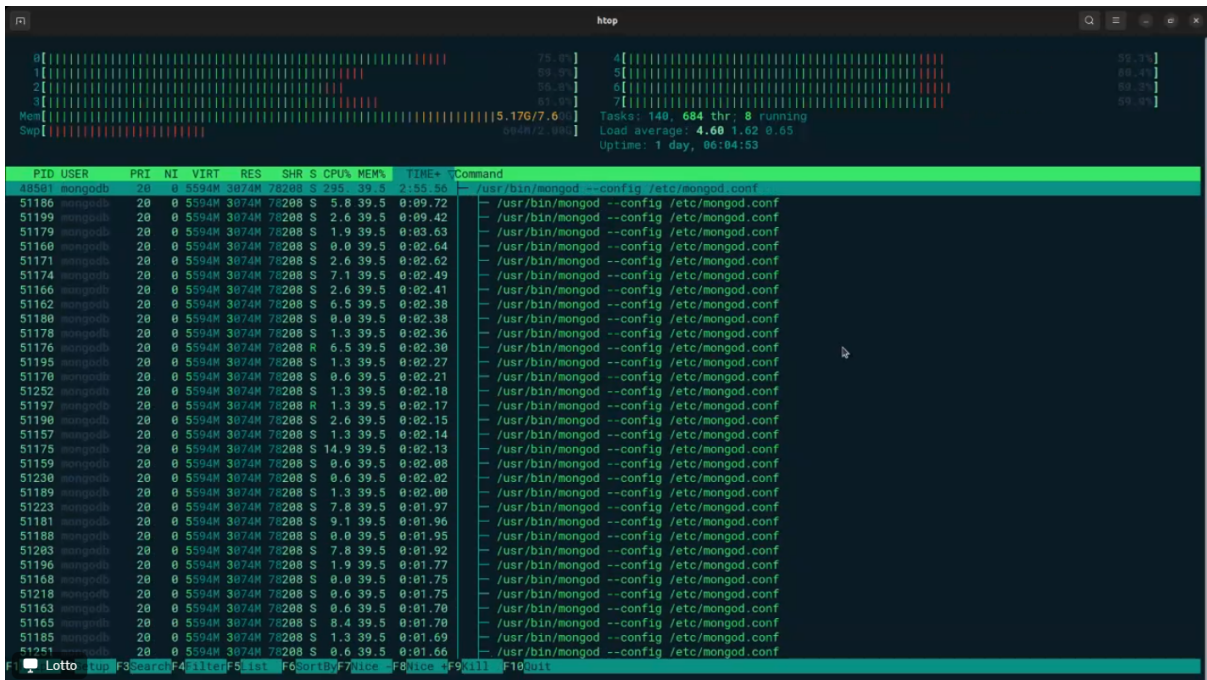


Figure 3.2: MongoDB thread pool under load conditions

# Chapter 4

## Q.N. theoretical analysis

To fulfil the requirements of the assignment, it is necessary to undertake a comprehensive theoretical analysis of the queueing network.

In order to apply the theoretical notions learned during the course to the estimation of the degree of scalability of the system, the optimal number of users and the identification of the bottleneck of the system, it is first necessary to measure the service times for the queueing network components with regard to the two classes of jobs previously defined.

Although the queries to the endpoint `‘/search/:title?page=n’` may vary depending on the parameterization of the URL, we assume that they belong to the same job class. This assumption will be subsequently confirmed by the empirical results.

### 4.1 Service times measurements

#### 4.1.1 Overview

Given that we are operating in a “Processor Sharing” regime, it is possible for us to exploit the fact that, in the event that there is only one job within the system, its service time is in fact equivalent to the response time. In light of these considerations, we employed JMeter, a load testing software that will be discussed in greater detail later on, to perform two tests (one for each class of jobs) with the objective of measuring the various service times of our stations.

Both tests comprise a user (a thread) executing a request to the RESTful API at a constant interval of 10 milliseconds. A total of 10,000 requests are made by the user in both cases, a quantity predetermined by the size of the query set imposed by the assignment.

Details on the tests carried out are provided in the following sections.

#### 4.1.2 Test query set

A Python script was employed to perform a random sampling with reinsertion of 10,000 movie titles based on our MongoDB collections.

It was assumed that the probability of a movie being selected in the extraction is proportional to the number of votes it received. In order to obtain the most comprehensive query set possible, it was decided to include all movies, all episodes, and all the various linguistic declinations of these in the extraction. The entire process was carried out using the `pandas.DataFrame.sample` function.



### 4.1.3 RESTful API Profiling

To obtain the most precise measurement of service times for the two classes of jobs, two different versions of the backend were implemented. One was employed for load testing and the other was used for the current employment; the latter version contained mechanisms to perform profiling on its execution.

Indeed, within the API, it was necessary to distinguish between the processing times of code in Node.js and those of database aggregation pipelines. To perform profiling on the Express execution, it was decided to utilise the `PerformanceObserver` Node interface, which forms part of the Performance Timing API. Indeed, this interface enabled the effective measurement of execution times without the addition of overheads through the utilisation of the `Marks`, namely high-resolution timestamps.

Conversely, regarding MongoDB profiling, the situation was more intricate. In order to facilitate the process, we proceeded to activate the `system.profile` collection within our database. This is a *capped collection*<sup>1</sup> that is created by default by the MongoDB Profiler for each database and hidden from the user. Its purpose is to measure precise metrics regarding CRUD operations and other administrative and/or configuration commands executed on a running instance of MongoDB. Consequently, the utilisation of the profiler enabled the accurate determination of the execution times inherent to the aggregation pipelines.

Upon completion of the whole process, all metrics evaluated by the API execution are transcribed asynchronously, thus maintaining optimal performance, by `Pino`.

`Pino.js` is a fast and lightweight Node.js logging library: it is a robust and scalable solution due to its comprehensive asynchronous logging of JSON-formatted files that provides customizable log levels and serializers while maintaining low processing costs.

### 4.1.4 Empirical service times

Following the execution of the two tests with JMeter, the service times of our system components with respect to the job classes identified in our network are as follows:

$$\mu_{B1}^{-1} = 0.001\,69\,\text{s} \quad \mu_{D1}^{-1} = 0.002\,58\,\text{s} \quad \mu_{B2}^{-1} = 0.001\,71\,\text{s} \quad \mu_{D2}^{-1} = 0.001\,46\,\text{s}$$

It should be noted that, although MongoDB employs a scheduling discipline of the “Processor Sharing” type, it is not a single-server system analogous to the M/G/1/PS queues analysed during the course. Consequently, a slight modification was required to the formula for calculating its service times:

$$\mu_{Dx}^{-1} = \frac{\overline{R}}{\text{number of CPUs}} \quad \text{where } x \in \{1, 2\}$$

## 4.2 Bottleneck identification

Thus far, we have calculated the service times of each station per job class and the relative visit ratios ( $\overline{V}_i$ ). These  $\overline{V}_i$  are derived from the solutions of the traffic equations and indicate the expected number of visits to each station with respect to each visit made to the reference station ( $Q_T$ ).

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<sup>1</sup>A fixed-size collection that performs document insertion and retrieval based on the order in which they were added, exhibiting behaviour analogous to that of circular buffers.

$$\bar{V}_{B1} = 1.111\ 11 \quad \bar{V}_{D1} = 1.111\ 11 \quad \bar{V}_{B2} = 0.888\ 89 \quad \bar{V}_{D2} = 0.888\ 89 \quad \bar{V}_T = 1.000\ 00$$

The data enables the calculation of the service demands ( $\bar{D}_i$ ), which represent the quantity of service requested by a user from each station for each visit to the reference station. This is achieved through the application of the following formula:

$$\bar{D}_i = \bar{V}_i \times \mu_i^{-1} \quad \text{where} \quad i \in \{B1, D1, B2, D2\} \quad (4.1)$$

$$\bar{D}_{B1} = 0.001\ 88 \quad \bar{D}_{D1} = 0.002\ 86 \quad \bar{D}_{B2} = 0.001\ 52 \quad \bar{D}_{D2} = 0.001\ 30$$

The results indicate that the database is the bottleneck station. A further calculation of the utilisation of each station can be made using the Bottleneck Law:

$$\rho_i = X_1 \times \bar{D}_i \quad \text{where} \quad i \in \{B1, D1, B2, D2\} \quad (4.2)$$

A graph is provided below, which illustrates the varying levels of station utilisation over the number of users.

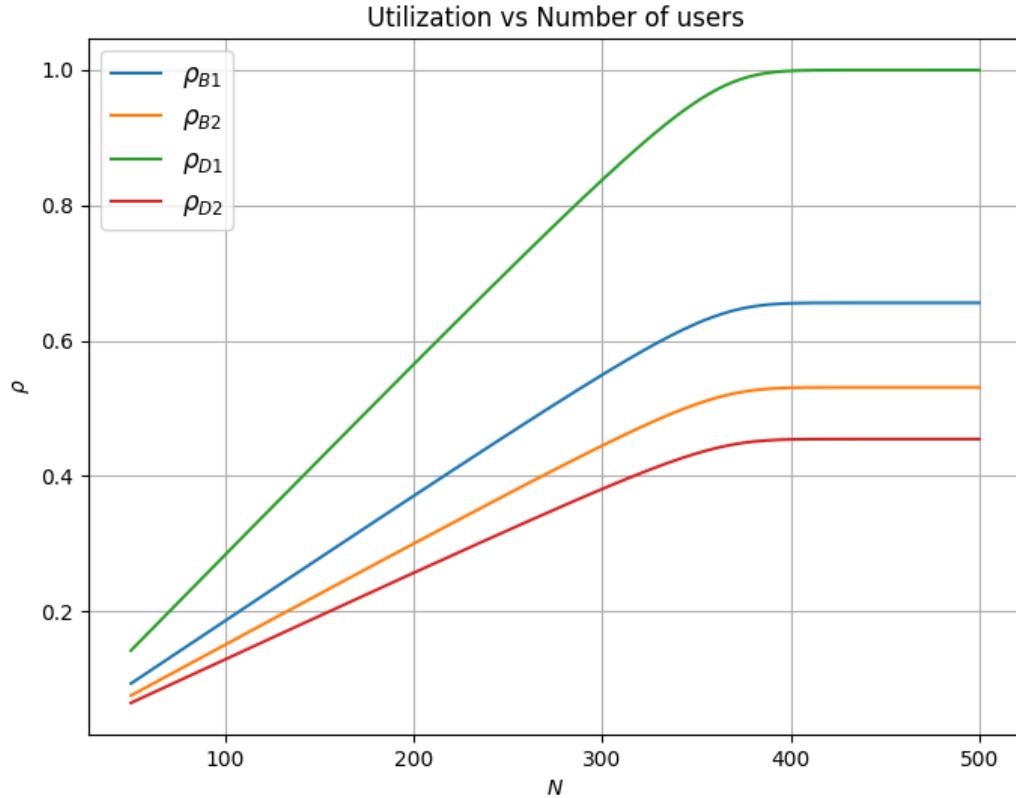


Figure 4.1: Stations utilisation over the number of users

It can be concluded that the database is the component with the highest utilisation rate.

### 4.3 Asymptotic bounds

In the next section, we will perform a mean value analysis to compute the expected performance indices of our system. This will be done through Little's Law and the recursive procedure on which this technique is based, without the need to compute the normalization constant  $G_K$  required by Gordon Newell's Theorem.

Prior to conducting the previously announced analysis, a preliminary calculation is conducted using a Python script to determine the upper and lower bounds of response time ( $\bar{R}$ ) and throughput ( $X$ ) in relation to the number of customers in the network.

This is done according to the following formulas:

$$\bar{R} \geq \max(\bar{D}, N\bar{D}_b - \bar{Z}) \quad (4.3)$$

$$X \leq \min\left(\frac{N}{\bar{D} + \bar{Z}}, \frac{1}{\bar{D}_b}\right) \quad \text{where} \quad (4.4)$$

$$\bar{Z} \quad \text{exp. thinking time,} \quad \bar{D}_b \quad \text{bottleneck service demand,} \quad \bar{D} = \sum_{j=2}^K \bar{D}_j$$

The parametric values of the calculated limits are provided below:

$$R \geq \max(0.00756, N \times 0.00286 - 1)$$

$$X \leq \min\left(\frac{N}{0.00756 + 1}, \frac{1}{0.00286}\right)$$

### 4.4 Mean Value Analysis

#### 4.4.1 JMVA emulation setup

In order to estimate the performance indices of our queueing network, we employed the "Java Modelling Tools (JMT)" suite of applications developed by the "Politecnico di Milano". In particular, we utilised the JMVA ("Mean Value Analysis and Approximate solution algorithms for queueing network models") application.

The subsequent steps are those required to prepare the emulation within the software according to our queueing network configuration.

1. The number of customers (50) and the number of classes of jobs were entered. As can be seen, only one class was indicated, as only class 1 jobs were output from the thinking station.

**Classes characteristics**  
Number, customized name, type of classes and number of customers (closed class) or arrival rate (open class). Add classes one by one or define total number at once. Higher number means higher priority

Number: 1

#	Name	Type	Priority	No. of Customers	Arrival Rate (λ)
1	Class1	closed	0	50	

< Back Next > Solve Exit

Figure 4.2: JMVA setup – classes of jobs

2. The stations in the queueing network are entered according to their respective load type. The thinking station is designated as a “delay type”, whereas other stations are classified as “load-independent”, as their service time remains consistent regardless of the number of customers.

**Stations characteristics**  
Number, customized name and type of stations. Add stations one by one or define the total number at once. Load Dependent stations necessarily require the use of MVA.

Number: 5

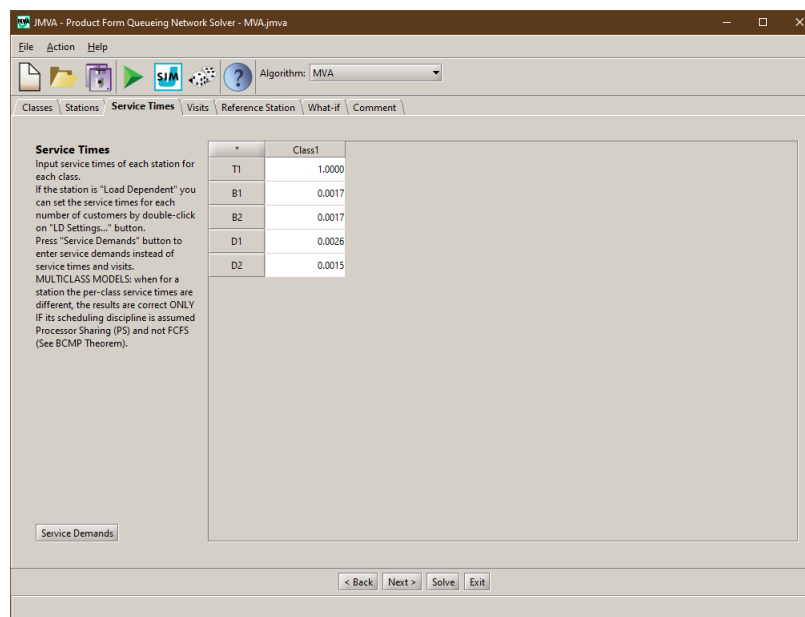
#	Name	Type
1	T1	Delay (Infinite Server)
2	B1	Load Independent
3	B2	Load Independent
4	D1	Load Independent
5	D2	Load Independent

< Back Next > Solve Exit

Click or drag to select stations; to edit data double-click and start typing. Right-click for a list of available operations

Figure 4.3: JMVA setup – stations

3. The service times previously calculated for each station are entered.



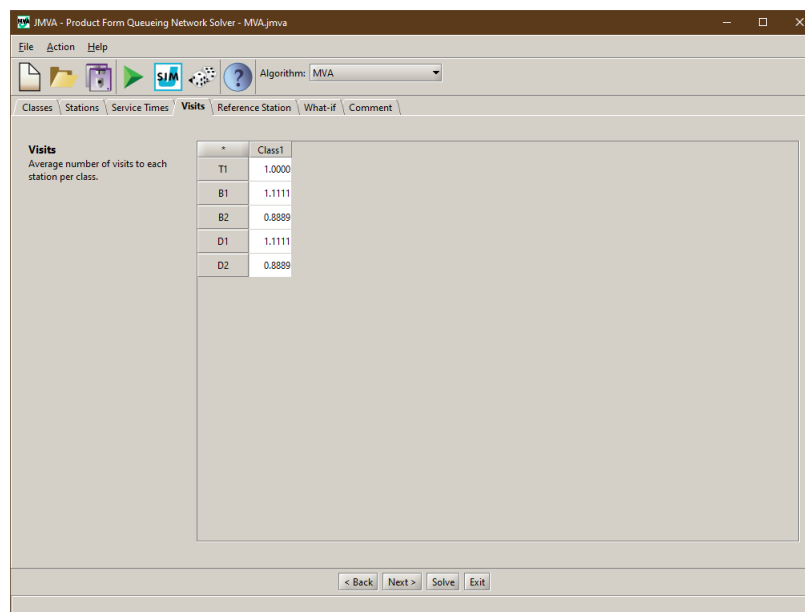
The screenshot shows the 'Service Times' tab in the JMVA software. The 'Algorithm' dropdown is set to 'MVA'. The 'Service Times' section contains instructions: 'Input service times of each station for each class. If the station is "Load Dependent" you can set the service times for each number of customers by double-click on "LD Settings..." button. Press "Service Demands" button to enter service demands instead of service times and visits. MULTICLASS MODELS: when for a station the per-class service times are different, the results are correct ONLY IF its scheduling discipline is assumed Processor Sharing (PS) and not FCFS (See BCMP Theorem).' Below the text is a table with two columns: '\*' and 'Class1'. The table contains the following data:

*	Class1
T1	1.0000
B1	0.0017
B2	0.0017
D1	0.0026
D2	0.0015

At the bottom of the window, there are buttons for '< Back', 'Next >', 'Solve', and 'Exit'.

Figure 4.4: JMVA setup – service times

4. The relative visit ratios previously calculated for each station are entered.



The screenshot shows the 'Visits' tab in the JMVA software. The 'Algorithm' dropdown is set to 'MVA'. The 'Visits' section contains instructions: 'Average number of visits to each station per class.' Below the text is a table with two columns: '\*' and 'Class1'. The table contains the following data:

*	Class1
T1	1.0000
B1	1.1111
B2	0.8889
D1	1.1111
D2	0.8889

At the bottom of the window, there are buttons for '< Back', 'Next >', 'Solve', and 'Exit'.

Figure 4.5: JMVA setup – relative visit ratios

5. The reference station of the queueing network is indicated.

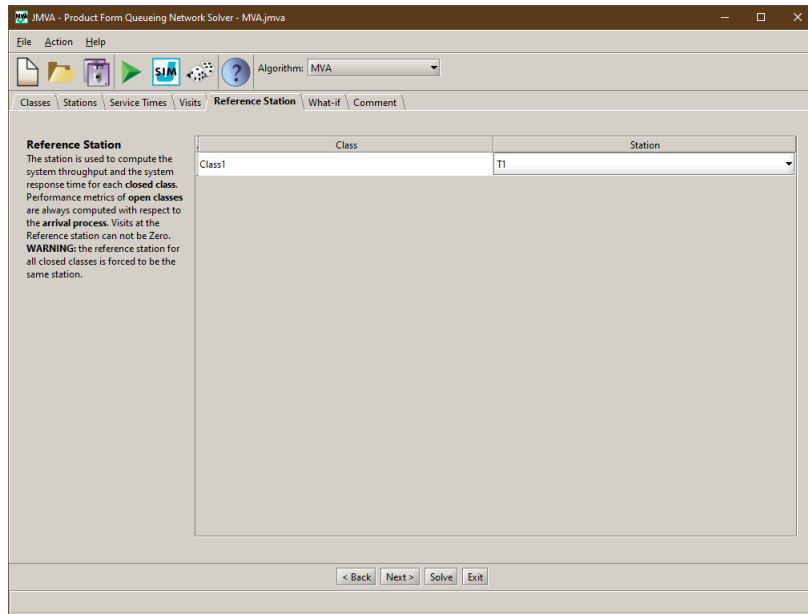


Figure 4.6: JMVA setup – reference station

6. The initial number of users is indicated as 50, with a subsequent increment of up to 500 users per increment of 1 user.

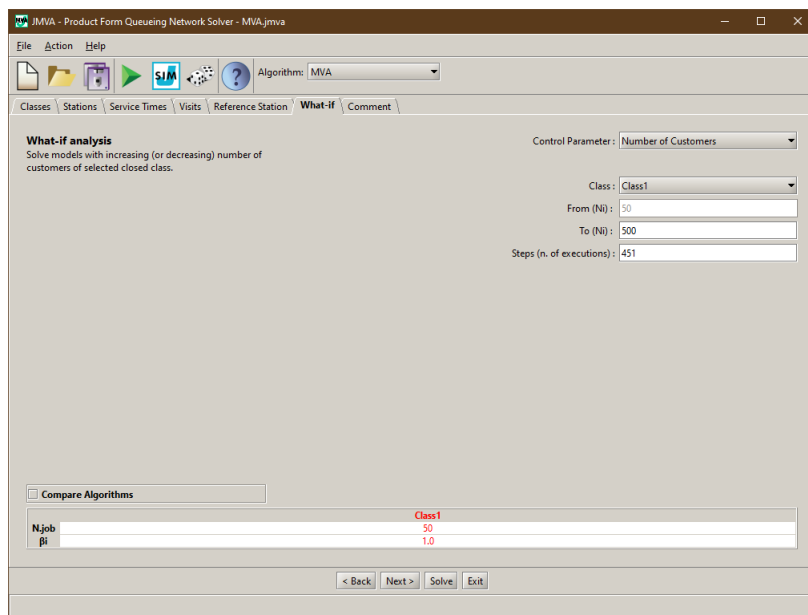


Figure 4.7: JMVA setup – emulation settings

#### 4.4.2 JMVA emulation results

The graphs generated by the emulation of JMVA, in conjunction with the previously calculated theoretical bounds, are presented below for the reader's convenience. As can be observed in the graphs, the optimal number of users has already been determined; this will be calculated in the following section.

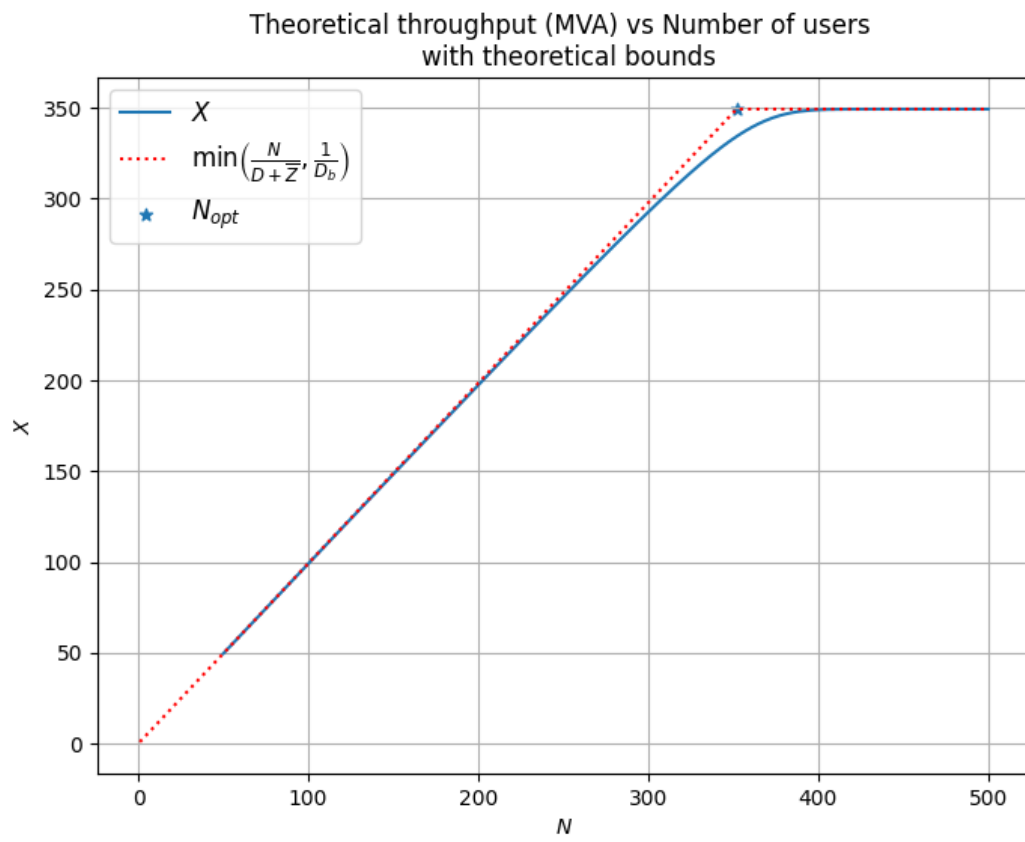


Figure 4.8: MVA – Throughput over the number of users

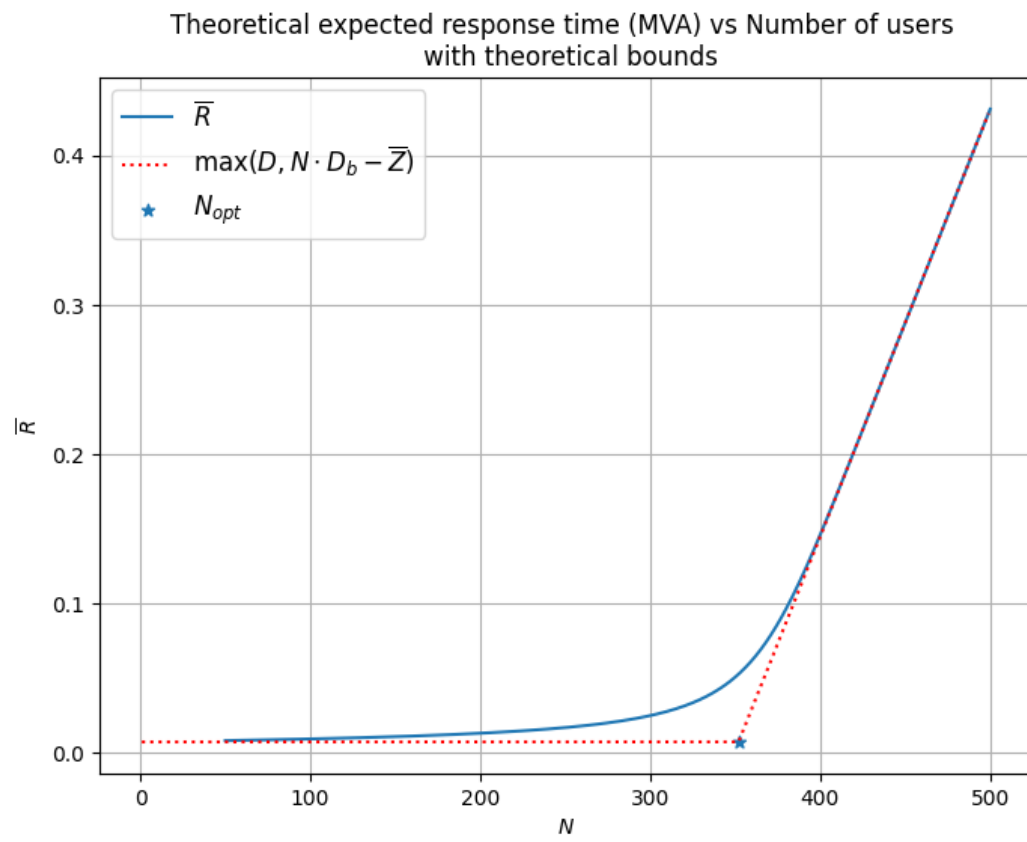


Figure 4.9: MVA – Response time over the number of users



## 4.5 Optimal – theoretical – number of users

In conclusion to the theoretical analysis of the queuing network, we proceed with the estimation of the optimal number of users. This can be obtained from the intersection of the previously calculated asymptotes for throughput and expected response time and with the following formula:

$$N_{opt} = \frac{\overline{D} + \overline{Z}}{\overline{D}_b} = 351.986\,51 \quad (4.5)$$

The outcome is fully reflected in the graphs illustrated above.