Macroeconomic forecasting with Bayesian mixed-frequency dynamic factor models

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Abstract

This thesis studies the Bayesian estimation of mixed-frequency dynamic factor models and the application of such models in macroeconomic forecasting.

Its main contribution is to discuss the issues associated with an MA unit root in the commonly used specification of mixed-frequency dynamic factor models and to propose an alternative model that avoids these problems. The suggested alternative has the same forecast accuracy as the original specification and a more compact state space representation.

The thesis also explores the possibility to improve the forecast accuracy by imposing a sparse prior on the factor loadings. However, the results of an out-of-sample forecast experiment with real-time data show that the sparse prior does not make a difference for a typical nowcasting dataset with monthly macroeconomic variables.

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Contents

1. Introduction

In order to set appropriate policies, both central banks and ministries need as precise information on the current and future economic conditions as possible. This is why macroeconomic forecasting is a very active field of research and central banks have hired many researchers specializing in forecasting. However, predicting macroeconomic variables is not only an important but also a very challenging task. In the following, I provide a bird's eye view on the field, structured by the challenges that macroeconomic data poses to forecasters:

First of all, a large number of potential predictors is available to the economic forecaster. The FRED-MD database (McCracken and Ng, 2016), for example, features more than 100 series at monthly frequency. Since most time series only span a few decades, having roughly as many observations as potential predictors is not a very unlikely scenario. Standard econometric models such as linear regressions and vector autoregressive models are not suitable forecast models for such "wide" datasets because they overfit and produce imprecise forecasts (Stock and Watson, 2006). Even for a medium-sized dataset of, say, 20-30 predictors, flexible models are likely to perform worse than more restrictive models. Common ways to deal with the curse of dimensionality are variable selection, shrinkage (towards a simple benchmark model), dimension reduction and combining the forecasts of smaller models.

Moreover, predictive relationships are often not stable over time. While there is often in-sample evidence that the parameters in a model are not constant, it has been difficult to exploit this knowledge in forecast models that take these instabilities explicitly into account - for example by allowing for several regimes of economic dynamics. To see why, note that the number of parameters in a model with n different regimes is n times the number of parameters in the plain model without regime changes. An alternative to discrete breaks is to allow for small changes in the parameters in every time step. Moreover, instead of attempting to model the parameter instability explicitly, a forecaster could also use ad-hoc approaches such as a rolling estimation window or averaging across estimation windows. See Rossi (2013) for a survey on forecasting in the presence of instabilities.

Not only slope parameters may vary over time, also error variances might change. In that case allowing for stochastic volatility can improve forecast accuracy, especially for density forecasts (Clark, 2011).

Furthermore, some of the potential predictor variables are measured at a frequency that is different from the sampling frequency of the target variable: Most macroeconomic variables are measured annually, quarterly or monthly and financial data are typically available at a daily (or even higher) frequency. By aggregating the high-frequency data to a lower frequency, valuable information on the evolution of the target variable might get lost. The primary approaches to this problem are state space models and MIDAS regressions: In a state space model specified at some baseline frequency (e.g. monthly), we can include a variable measured with a lower frequency (e.g. quarterly) by writing it as a function of lags of its unobserved counterpart at the baseline frequency and inserting missing values whenever the variable is not observed.[1] A MIDAS regression is a linear regression of a low-frequency (target) variable on a high-frequency (predictor) variable and its lags, quite similar to a distributed lag model. If the mismatch between the two sampling frequencies is very large, one might want to include a large number of lags of the high-frequency variable. In this case, restricting the coefficients of the lag polynomial to follow a certain functional form might improve forecast accuracy. If the mismatch is not that large, such as in the case of a quarterly left-hand side and a monthly right-hand side variable, an unrestricted MIDAS model may be more appropriate. For a more extensive survey of mixed-frequency models, see Foroni and Marcellino (2013). Furthermore, Bai et al. (2013) provide insights on the theoretical relationship between the state space and the MIDAS approach to mixed-frequency data.

Foroni and Marcellino also discuss another challenge of forecasting macroeconomic time series, namely the so-called “ragged edge” of the data: Macroeconomic time series are typically not published immediately after the end of the reference period but with a delay of several days or weeks. Consequently, a suitable forecast model should be able to deal with missing observations for some time series at the end of the sample.

For variables like GDP that are measured at a lower frequency than its potential predictors, not only forecasting its value in future time periods but also predicting its value in the current time period is an important task. Such predictions are referred to as nowcasts. If the variable has a long publication lag, even “backcasts” (e.g. predictions of GDP in the past quarter) might be of interest. Bańbura et al. (2013) provide an overview over the field of nowcasting. Naturally, how to deal with different sampling frequencies and the “ragged edge” of the data matters even more for nowcasting than for (long-term) forecasting.

Finally, the target variable and/or predictor variables may be revised. This has important implications for the design of forecast experiments: In order to compare the performance of competing models in a realistic way, we should use a so-called real-time dataset that allows us to reconstruct the data that was available at the forecast origin. Moreover, if the target variable is revised, it is not clear which estimate of the target variable to use as the “actual” values when evaluating the forecasts - there are good arguments for both early estimates and the latest available vintage. See Croushore (2011) for a more detailed discussion of the role of revisions in economic forecasting.

Dynamic factor models have emerged as one of the most popular family of forecast models because they provide a coherent framework to deal with many of the challenges

[1] The exact functional form of the temporal aggregation depends on whether the variable with lower sampling frequency is a stock or a flow variable and on what transformations have been applied to it. See section 2.1 for a more detailed treatment for the case of a log-differenced flow variable such as GDP growth.

in macroeconomic forecasting described above: They are suitable models for forecasting with many predictors by design because they model the common dynamics of variables with a much smaller number of unobserved variables, so-called factors. See Stock and Watson (2002a) for an early contribution that focuses on (long-term) forecasting based on a large dataset. Since dynamic factor models can be cast into state space form, they also provide an elegant way for dealing with missing values that may arise due to publication lags or the inclusion of variables with a lower sampling frequency. This feature of dynamic factor models is especially appreciated in the nowcasting literature (Bańbura et al., 2013) and in the closely related literature on constructing coincident business cycle indexes (Mariano and Murasawa, 2003).

The two main estimation approaches are principal components and maximum likelihood. Principal components-based estimation does not require strict parametric assumptions on the distribution of error terms and is therefore a more robust estimation procedure than maximum likelihood. However, if the error terms are indeed normally distributed, maximum likelihood estimation is more efficient. Furthermore, the EM-algorithm for maximum likelihood estimation of state space models can easily deal with missing values. This is why maximum likelihood estimation as studied in Doz et al. (2012) and Banbura and Modugno (2014) is the state-of-the-art estimation technique for dynamic factor models that are used in macroeconomic forecasting and nowcasting (for example, see Bok et al., 2017, for a description of model of the Federal Reserve Bank of New York for nowcasting real GDP growth). Stock and Watson (2011) provide a more detailed review of estimation methods for dynamic factor models.

Bayesian estimation of (mixed-frequency) dynamic factor models is not widespread in economic forecasting although, being also a likelihood-based method, it is as convenient as maximum likelihood estimation for dealing with missing values. The few exceptions in the literature are Luciani and Ricci (2014), D'Agostino et al. (2016), Marcellino et al. (2016) and Antolin-Diaz et al. (2017).

In this thesis, I study macroeconomic forecasting with Bayesian mixed-frequency dynamic factor models. As dealing with mixed-frequency data matters most for short forecast horizons, my focus is on short-term forecasts, including nowcasts and backcasts. I contribute to the literature in the following ways:

First, I describe an issue with the Bayesian estimation of mixed-frequency dynamic factor models that arises due to a MA unit root in the model and that has not been addressed in the previous literature. Moreover, I suggest an alternative model specification for which this issue does not appear and demonstrate that the generated forecasts are as accurate as for the specification that has been used previously.

Second, I consider the use of a sparse prior on the factor loadings as an alternative to heuristic data-based variable selection. Although sparse factor models have been extensively studied by Kaufmann and Schumacher (2013, 2017), Fruhwirth-Schnatter and Lopes (2018) and others, there has not been a lot of research on their applications in macroeconomic forecasting yet.

The thesis is structured as follows: Chapter 2 discusses two simple mixed-frequency dynamic factor models and possible extensions. Chapter 3 is concerned with the estimation of these models, both for a normal prior and a sparse prior on the factor loadings. In chapter 4, I compare the forecast accuracy of different model specifications in an out-of-sample forecast experiment with real US GDP growth as the target variable. Chapter 5 concludes.

2. Dynamic factor models and mixed-frequency data

This chapter is solely on the specification of dynamic factor models, with a special focus on models that are commonly used for forecasting GDP growth. Bayesian estimation of these models is discussed in the following chapter.

In the first section, I describe a simple model that is the common basis of the more general models proposed in Luciani and Ricci (2014), D'Agostino et al. (2016), Marcellino et al. (2016) and Antolin-Diaz et al. (2017). Then, I suggest an alternative basis specification and argue why it is preferable. The last section briefly covers possible extensions of the basic models.

2.1. A simple dynamic factor model for forecasting GDP growth (MF-DFM 1)

The use of dynamic factor models for forecasting GDP growth can be motivated as follows: After they have been transformed to stationarity, autoregressive models

$$y_t = \mu + u_t \tag{2.1}$$
$$u_t = \psi_1 u_{t-1} + \cdots + \psi_p u_{t-q} + w_t \tag{2.2}$$

approximate the dynamics of many macroeconomic variables well and are commonly used as univariate forecast models. However, they do not take into account that there is co-movement between many variables, especially between variables related to real economic activity. A simple way to model the common dynamics is to include an additional term in equation 2.1 that is proportional to an unobserved variable f_t, a so-called factor:

$$y_{i,t} = \mu_i + \lambda_i f_t + u_{i,t} \qquad \text{for } i = 1:n \tag{2.3}$$
$$u_{i,t} = \psi_{i,1} u_{i,t-1} + \cdots + \psi_{i,q_i} u_{i,t-q_i} + w_{i,t} \quad \text{for } i = 1:n. \tag{2.4}$$

The factor is typically assumed to follow AR dynamics itself:

$$f_t = \phi_1 f_{t-1} + \cdots + \phi_p f_{t-p} + v_t. \tag{2.5}$$

In the equations above, i is an index for the n variables included in the factor model, the parameters $\lambda_{1:n}$ are referred to as factor loadings and the innovations $v_t, w_{1,t}, \dots w_{n,t}$ are assumed to be i.i.d. and to follow a Normal distribution with mean zero and variance $\text{diag}(\sigma_v^2, \sigma_{w,1}^2, \dots \sigma_{w,n}^2)$. In order to make sure that the observed variables $y_{i,t}$ are stationary, the autoregressive coefficients $\phi_{1:p}, \psi_{1,1:q_1}, \ldots \psi_{n,1:q_n}$ need to be such that f_t and $u_{1,t}, \ldots u_{n,t}$ are stationary.

The model in equations 2.3 - 2.5 is referred to as a dynamic factor model.[1] Using matrix notation, we can rewrite the model as

$$y_t = \mu + \lambda f_t + u_t \tag{2.6}$$
$$f_t = \phi_1 f_{t-1} + \cdots + \phi_p f_{t-p} + v_t \tag{2.7}$$
$$u_t = \Psi_1 u_{t-1} + \cdots + \Psi_q u_{t-q} + w_t \tag{2.8}$$

where $q = \max_i(q_i)$, y_t, u_t, μ and λ are column vectors of length n and the $n \times n$ matrices $\Psi_1, \ldots \Psi_q$ are diagonal.

As macroeconomic variables such as industrial production, employment series and sales are released monthly with a publication lag of the order of several days to weeks, information on the state of the economy (and therefore also on GDP growth) in the current quarter becomes available long before the first GDP estimate is released. A one-factor model can extract this information on the business cycle in a parsimonious way and is therefore especially useful for short-term GDP forecasting. Moreover, simple dynamic factor models as in equation 2.3-2.5 are also used to construct an coincident index of economic activity (Mariano and Murasawa, 2003).

However, the model discussed above is only able to deal with variables measured at one frequency. If we want to include quarterly GDP growth y_t into the model that is specified at a monthly frequency, we need to write it in terms of unobserved monthly GDP growth y_t^*

$$y_t \approx \frac{1}{3}y_t^* + \frac{2}{3}y_{t-1}^* + y_{t-2}^* + \frac{2}{3}y_{t-3}^* + \frac{1}{3}y_{t-4}^* \tag{2.9}$$

where quarterly GDP growth y_t is assigned to the last month of a quarter. Equation 2.9 first appeared in Mariano and Murasawa (2003) but as their derivation does not make clear that it can be seen as an approximation and because it has sometimes been misunderstood, I provide an alternative derivation in appendix A.

In the following, let us assume that real GDP growth is the only quarterly variable in the model and that it belongs to the index $i = 1$. Then, by applying equation 2.9 to the first row in the measurement equation, we obtain a simple mixed-frequency dynamic factor model (MF-DFM 1)

$$\begin{aligned} y_{1,t} = 3\mu_1 + \lambda_1 &\left(\frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4}\right) + \\ &\frac{1}{3}u_{1,t} + \frac{2}{3}u_{1,t-1} + u_{t-2} + \frac{2}{3}u_{1,t-3} + \frac{1}{3}u_{1,t-4} && (2.10) \\ y_{i,t} = \mu_i + \lambda_i f_t + u_{i,t} & \qquad \text{for } i = 2:n && (2.11) \\ f_t = \phi_1 f_{t-1} + \cdots + \phi_p f_{t-p} + v_t & && (2.12) \\ u_{i,t} = \psi_{i,1} u_{i,t-1} + \cdots + \psi_{i,q_i} u_{i,t-q_i} + w_{i,t} & \quad \text{for } i = 1:n. && (2.13) \end{aligned}$$

[1] Some authors, such as Bai and Wang (2015) would rather refer to such as model as a static factor model as there are not lags of the factor in equation 2.3. I will, however, stick to the terminology of e.g. Bańbura et al. (2013) and Marcellino et al. (2016).

where I have replaced the $\approx$ sign with a $=$ for convenience.

Note that there will be missing values for the following reasons: First, GDP growth in the last three months is only measured every third month (in March, June, September and December). Second, due to asynchronous data releases, observations of some variables are missing at the end of the sample. In terms of notation, we can deal with missing observations by redefining the vector y_t to contain only variables $y_{i,t}$ that are observed. As we will see in the following chapter on estimation, methods developed for state space models can easily deal with a measurement equation that is different for each t.

So far, I have ignored that the factor loadings λ and the factor f_t are not identified because $\lambda^{(1)} f_t^{(1)} = \lambda^{(2)} f_t^{(2)}$ for any $\lambda^{(1)} = c\lambda^{(2)}$ and $f_t^{(1)} = \frac{1}{c} f_t^{(2)}$ with $c \in \mathbb{R}$. The most common solutions for this problem are to either fix one of the factor loadings or to fix the variance of v_t and the signs of factor loadings. I set $\lambda_1 = 1$ such that the factor has the same sign and scale as monthly GDP.

2.2. An alternative dynamic factor model (MF-DFM 2)

The roots of the MA lag polynomial $w(L) = \frac{1}{3} + \frac{2}{3}L^1 + L^2 + \frac{2}{3}L^3 + \frac{1}{3}L^4$ in equation 2.9 are $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ which means that they all lie on the unit circle and $w(L)$ is not invertible. This is not a problem for the factor f_t because it is identified as soon as there are at least two monthly variables $y_{i,t}$ with nonzero loadings. However, due to the non-invertibility, we cannot recover the unobserved series $u_{1,t}$ from $w(L)u_{1,t}$. (Also note that information on $w(L)u_{1,t}$ is only available in every third month.) Consequently, modeling the idiosyncratic component of GDP as an unobserved monthly AR(q_1) process is unlikely to contribute to the forecast accuracy of the model. Furthermore, it is not clear what the non-invertibility of $w(L)$ means for inference about the parameters $\psi_{1,1}, \dots \psi_{1,q_1}$, $\sigma^2_{w,i}$ of the AR(q_1) process: What does the asymptotic distribution of the maximum likelihood estimator look like? How is the posterior distribution in the Bayesian estimation framework affected?[2]

In the following chapter, I try to answer the last question for the case $q_1 = 1$ and find that a sample size of several hundred observations (that is typical of forecasting with monthly data) is not sufficient to narrow down the range of probable values for ψ_1 and $\sigma^2_{w,1}$ to a small region in the parameter space. Moreover, the autocorrelation of the Markov chain for ψ_1 and $\sigma^2_{w,1}$ is unusually high. See figure 3.1 for an illustration of this problem.

As far as I know, these issues related to the MA unit root have not been addressed in the nowcasting literature. In particular, none of the four articles on forecasting with Bayesian mixed-frequency dynamic factor models mentions the problem. To some

[2] There is some literature on maximum likelihood estimation of the parameter θ in an MA(1) model $1 - \theta L$ if θ is either 1 or close to 1 (for example, see Plosser and Schwert, 1977; Davis and Dunsmuir, 1996). The theoretical results presented in these articles do however not help for our problem since we are rather concerned with estimating AR parameters in an ARMA(q_1, 4) if the roots of the known MA lag polynomial are on the unit circle.

degree, this is understandable: Despite the MA unit root, the mixed-frequency dynamic factor model from the previous section generates good short-term forecasts. Nonetheless, we should try to find an alternative specification for which these issues do not occur. As a starting point, consider the following idea from Mariano and Murasawa (2010):

> Many, if not all, will agree that if we observe real gross domestic product (GDP) promptly on a monthly basis, then we do not need a coincident index; that is, a coincident index is a proxy for monthly real GDP.

Put differently, in the one-factor model in equations 2.3-2.5 with real variables, the factor should be viewed as monthly real GDP (up to an additive constant) since real GDP is the most comprehensive measure of the state of an economy. Consequently, the only reason to allow quarterly GDP growth to deviate from $3\mu_1+w(L)f_t$ is measurement errors. According to Aruoba et al. (2016, section 3), GDP measurement errors should be modeled as i.i.d. in growth rates because the U.S. Bureau of Economic Analysis attempts to provide "best change" forecasts. Following this argument, we end up with the alternative model (MF-DFM 2)

$$\begin{aligned} y_{1,t} &= 3\mu_1 + \frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4} + e_t && (2.14)\\ y_{i,t} &= \mu_i + \lambda_i f_t + u_{i,t} \qquad \text{for } i = 2:n && (2.15)\\ f_t &= \phi_1 f_{t-1} + \cdots + \phi_p f_{t-p} + v_t && (2.16)\\ u_{i,t} &= \psi_{i,1}u_{i,t-1} + \cdots + \psi_{i,q_i}u_{i,t-q_i} + w_{i,t} \quad \text{for } i = 2:n && (2.17) \end{aligned}$$

where $e_t \sim \text{Normal}(0, \sigma_e^2)$ is uncorrelated with v_t and $w_{2:n,t}$.

Note that the way to avoid the problems with the MA unit root described above only works for GDP (and GDI, the income-side measure of output): For other quarterly variables, we cannot convincingly argue that their unobserved quarterly counterpart is equal to the factor.

2.3. Possible extensions

Both the basic model MF-DFM 1 and the alternative MF-DFM 2 suggested in this thesis can be generalized.

Stochastic volatility

In order to improve density forecasts, Marcellino et al. (2016) allow for time-variation in the innovation variances σ_v^2 and $\sigma_{w,1:n}^2$

$$\begin{aligned} \log\sigma_{v,t} &= \log\sigma_{v,t} + \nu_{v,t} && (2.18)\\ \log\sigma_{w,i,t} &= \log\sigma_{w,i,t} + \nu_{w,i,t} \qquad \text{for } i = 1:n && (2.19) \end{aligned}$$

where $\nu_{v,t} \sim \text{Normal}(0, \omega_v^2)$ and $\nu_{w,i,t} \sim \text{Normal}(0, \omega_{w,i}^2)$ are mutually uncorrelated. If we use stochastic volatility in MF-DFM 2, we should also model the variance of the measurement error σ_e^2 as time-varying.

Time-varying mean

Antolin-Diaz et al. (2017) use stochastic volatility and replace the constant mean μ_1 of real GDP growth with a time-varying mean

$$\mu_{1,t} = \mu_{1,t-1} + \zeta_t \tag{2.20}$$

where $\zeta_t \sim \text{Normal}(0, \tau^2)$. Although their primary goal is to describe the decline of real GDP growth in advanced economies, they also conduct an out-of-sample forecast experiment with US data and find that the time-varying mean improves the accuracy of nowcasts especially in the post-recession sample.

"Dynamic heterogeneity"

Luciani and Ricci (2014) and D'Agostino et al. (2016) allow for lags of the factor in equation 2.6

$$\begin{aligned} y_t &= \mu + \lambda(L) f_t + u_t &(2.21)\\ &= \mu + \lambda_0 f_t + \lambda_1 f_{t-1} + \cdots + \lambda_r f_{t-s} + u_t &(2.22)\end{aligned}$$

such that the corresponding mixed-frequency model with real GDP growth at $i = 1$ reads

$$\begin{aligned} y_{1,t} &= 3\mu_1 + w(L)\lambda_1(L) f_t + w(L) u_t &(2.23)\\ y_{i,t} &= \mu_i + \lambda_i(L) f_t + u_{i,t} \qquad \text{for } i = 2:n. &(2.24)\end{aligned}$$

Luciani and Ricci (2014) and D'Agostino et al. (2016) argue that this generalization is crucial to model the "dynamic heterogeneity" of the variables properly: As the dynamics of some variables do not coincide with the business cycle but either lag behind it or provide early signals, choosing $r = 0$ may be too restrictive.

More than one factor

Of course, factor models may have more than one factor:

$$\begin{aligned} y_t &= \mu + \Lambda f_t + u_t &(2.25)\\ f_t &= \Phi_1 f_{t-1} + \cdots + \Phi_p f_{t-p} + v_t &(2.26)\end{aligned}$$

Here, f_t is a $k \times 1$ vector of factors and Λ is an $n \times k$ matrix of factor loadings. Also, the autoregressive model for f_t has been replaced by a vector autoregressive model with coefficients matrices $\Phi_1, \ldots \Phi_p$. In the model above, factors and factor loadings are identified up to rotations, which means that we need k^2 restrictions on Λ and/or f_t. See Bai and Wang (2015) for an in-depth discussions of the identification of dynamic factor models. By using the Mariano and Murasawa (2003) relation in each row that corresponds to a quarterly variable, we can modify the model such that it can also accommodate quarterly variables. In short-term forecasting and nowcasting, however, one factor is often found to be sufficient.

Out of the four articles on macroeconomic forecasting with Bayesian mixed-frequency dynamic factor models, only Marcellino et al. (2016) consider a model with more than one factor. Because their variable selection algorithm is designed for a single factor, it is no surprise that they report the factor loadings for the second factor to be zero.

3. Bayesian estimation of mixed-frequency dynamic factor models

In this chapter, we consider the Bayesian estimation of the models discussed in chapter 2. I start with a general introduction to Bayesian estimation of state space models and apply these concepts first to MF-DFM 2, our preferred model. Then, I continue with MF-DFM 1 and illustrate the problems that occur due to the MA unit root. Finally, I consider variable selection implemented via a sparse prior on the factor loadings.

As Bayesian estimation of dynamic factor models with one sampling frequency has been extensively studied in e.g. Bai and Wang (2015), I focus on what is different when working with mixed-frequency data.

Throughout this section, I assume that the data $y_1, \dots, y_T$ has been demeaned. Therefore I set μ in both MF-DFM 1 and 2 to zero.

3.1. Bayesian estimation of state space models

A linear and Gaussian state space model has the form

$$x_t = Ax_{t-1} + R_\epsilon \epsilon_t \text{ for } t = 2 : T \tag{3.1}$$

$$y_t = Bx_t + R_\eta \eta_t \tag{3.2}$$

where x_t is an $m \times 1$ vector of unobserved states, y_t is an $n \times 1$ vector of observations, $\epsilon_t \sim \text{Normal}(0, \Sigma_\epsilon)$ are the disturbances in the transition equation and $\eta_t \sim \text{Normal}(0, \Sigma_\eta)$ are the disturbances in the measurement equation. ϵ_t and η_t are assumed to be uncorrelated with each other. The initial state is drawn from a Normal distribution as well: $x_1 \sim \text{Normal}(\mu_1, \Sigma_1)$. A is an $m \times m$ matrix, B is an $n \times m$ matrix and the matrices R_ϵ and R_η are selection matrices. (In the simple case that there is a disturbance term for every row of the transition and the measurement equation, these matrices are simply identity matrices.)

If there are missing values in y_t, we filter out these missing values such that y_t has the dimensions $n_t \times 1$, $n_t \leq n$ and do the same with the corresponding rows in the measurement equation

$$y_t = S_t B x_t + S_t R_\eta \eta_t \tag{3.3}$$

$$= B_t x_t + R_{\eta,t} \eta_t \tag{3.4}$$

where S_t is a suitable selection $n_t \times n$ matrix.

In many applications, some of the elements in A, B, Σ_ϵ and Σ_η are known. Therefore it is helpful to think of these matrices as functions of a vector of unknown parameters θ. The likelihood of a state space model with parameters θ has the form

$$p(x_{1:T}, y_{1:T}|\theta) = p_\theta(x_1) \prod_{t=2:T} p_\theta(x_t|x_{t-1}) \prod_{t=1:T} p_\theta(y_t|x_t). \tag{3.5}$$

By multiplying the likelihood with the prior distribution of the parameters, we obtain the posterior distribution of the states and the parameters given the data $y_{1:T}$:

$$p(x_{1:T}, \theta|y_{1:T}) \propto p(x_{1:T}, y_{1:T}|\theta)p(\theta). \tag{3.6}$$

The general recipe for sampling from this posterior distribution is a Gibbs sampler (Carter and Kohn, 1994):

- initialize the parameters of the model $\theta^{(0)}$
- for $s = 1 : S$
  - draw the states $x_{1:T}^{(s)}$ given the parameters $\theta^{(s-1)}$ and the data $y_{1:T}$
  - draw the parameters $\theta^{(s)}$ given the states $x_{1:T}^{(s)}$ and the data $y_{1:T}$
  - store the sampled states $x_{1:T}^{(s)}$ and parameters $\theta^{(s)}$

There are several approaches to drawing the states $x_{1:T}$ for a given state space model and the observations $y_{1:T}$. I decide to use the simulation smoother proposed in Durbin and Koopman (2002, section 2.4) because of its computational efficiency:

- simulate states $x_{1:T}^+$ and observations $y_{1:T}^+$ from the given state space model
- compute $x_{1:T}^* = \mathrm{E}(x_{1:T}|y_{1:T}^*)$ where $y_{1:T}^* = y_{1:T} - y_{1:T}^+$ with the fast state smoothing algorithm
- $x_{1:T}^* + x_{1:T}^+$ is a draw from the distribution of states given the observations $y_{1:T}$

The fast state smoothing algorithm can be found in Durbin and Koopman (2002, section 4.6.2). It is faster than the standard Kalman smoother because it only computes $\mathrm{E}(x_{1:T}|y_{1:T})$ and not the variances $\mathrm{Var}(x_1|y_{1:T}), \ldots, \mathrm{Var}(x_T|y_{1:T})$.

3.2. Bayesian estimation of MF-DFM 2

State space representation

In order to apply the general recipe for Bayesian inference in state space models to MF-DFM 2, we first need to find its state space representation. One possibility is to define the vector of states as

$$x_t = [f_t, \ldots f_{t-\max(4,p-1)}, u_{2,t}, \ldots u_{2,t-q_2+1}, \ldots u_{n,t}, \ldots u_{n,t-q_n+1}]' \tag{3.7}$$

and construct the matrices A, B, R_ϵ, Σ_ϵ, R_η and Σ_η such that the state space model is equivalent to MF-DFM 2. [1]

[1] This representation is almost the one chosen by Marcellino et al. (2016), except that they use MF-DFM 1 as their basic model and therefore also $u_{1,t}$ and it lags need to be included in x_t.

This naive approach has the disadvantage that the state vector has the length $m = \max(5, p) + \sum_{i=2:n} q_i$ and hence grows with n, the number of variables. For example, in a dynamic factor model with $n = 20$ variables, $p \leq 5$ and $q_2 = \cdots = q_n = 1$ the resulting state vector has length 24.

An alternative approach is to apply the lag polynomials $\psi_i(L) = 1 - \psi_{i,1}L - \ldots \psi_{i,q_i}L^{q_i}$, $i = 2 : n$ to the rows 2 to n of the measurement equation (Luciani and Ricci, 2014; D'Agostino et al., 2016; Antolin-Diaz et al., 2017). The resulting model is

$$y_{1,t} = \frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4} + e_t \tag{3.8}$$

$$\psi_i(L)y_{i,t} = \lambda_i\psi_i(L)f_t + w_{i,t} \quad \text{for } i = 2 : n \tag{3.9}$$

$$f_t = \phi_1 f_{t-1} + \cdots + \phi_p f_{t-p} + v_t \tag{3.10}$$

for $t = \max_{i=2:n}(q_i) + 1 : T$ and can be represented as a state space model with a state vector that contains only the factor and its lags:

$$x_t = [f_t, \ldots f_{t-r}]' \tag{3.11}$$

where r is the maximum of $\{4, p-1, q_2, \ldots q_n\}$. Hence, in a dynamic factor model with an arbitrary number of variables n, the corresponding state vector has length 5 (if $p \leq 5$ and $q_i \leq 4$ for all $i = 2 : n$). Since the computational speed of algorithms for state space models decreases with the number of variables n and the number of states m, we will work with the transformed state space model in the following. In appendix B, more information on the state space representation of the transformed MF-DFM 2 is given.

Note that in equations 3.8 - 3.10, we discard the first $q = \max_{i=2:n}(q_i)$ values of the quarterly variable[2] and use the first q observations of the monthly variables only for the transformation.

Moreover, we have so far ignored how to initialize x_t at $t = q + 1$. Clearly, $\mu_1 = [0, \ldots, 0]'$, but how do we choose Σ_1? A natural choice is to use the stationary distribution of f_t. Then, however, the posterior distribution of ϕ given σ_v^2 and the states $x_{q+1:T}$ would not have a closed form. In order to keep things simple, I choose diagonal covariance matrix Σ_1 that does not depend on ϕ or σ_v^2. The same choice is made by Bai and Wang (2015), Antolin-Diaz et al. (2017) and many other articles that do not address this issue explicitly.

Prior distribution

I choose the prior distribution such that the posterior is invariant under rescaling the variables. For the disturbance variances σ_v^2, σ_e^2, $\sigma_{w,2}^2, \ldots \sigma_{w,n}^2$, I therefore take a Jeffreys prior $p(\sigma^2) \propto \frac{1}{\sigma}$. For the autoregressive parameters $\phi_{1:p}, \psi_{2,1:q_2}, \ldots \psi_{n,1:q_n}$, any prior that is zero if the roots of the corresponding lag polynomials are outside the unit circle may be chosen. For example, if the lag length is 1, a uniform prior on $(-1, 1)$ is a reasonable choice. For larger lag lengths, a uniform prior restricted to the stationary

[2] If e.g. $t = 1$ is the first month of a quarter and $q \leq 2$, then these values are missing values anyway.

parameter area could be chosen, but in order to account for the increasing number of parameters, Luciani and Ricci (2014) and D'Agostino et al. (2016) instead choose a prior with decreasing variance for larger lags

$$\phi_{1:p} \sim \text{Normal}\left(0, \text{diag}\left(c, \frac{c}{2^2}, \dots \frac{c}{p^2}\right)\right) I_{\phi(L) \text{ stationary}}. \tag{3.12}$$

where $c = 0.2$ in D'Agostino et al. (2016) and $I_{\phi(L) \text{ stationary}}$ indicates that the Normal distribution has been truncated such that it is zero for any $\phi(L)$ with roots on or outside the unit circle.

Finally, for $\lambda_{2:n}$, a flat prior $p(\lambda_i) \propto 1$ would be a reasonable choice as well. Because we will put a spike-and-slab prior on the factor loadings in the next section, I prefer to choose a hierarchical Gaussian prior that corresponds to the "slab" for better comparability. In order to retain the invariance when changing the scale of $y_{i,t}$, I do not put the prior on λ_i directly but rather on $\tilde{\lambda}_i = \frac{\lambda_i}{\sigma_{w,i}}$

$$p(\tilde{\lambda}_i|\tau^2) \propto \frac{1}{\tau} \exp\left(-\frac{\tilde{\lambda}_i^2}{2\tau^2}\right) \tag{3.13}$$

$$p(\tau^2) \propto \frac{1}{\tau} \tag{3.14}$$

where τ^2 is the variance of the prior on $\tilde{\lambda}_i, i = 2 : n$.

Assuming a priori independence of the parameters, we end up with

$$p(\theta) = p(\tau^2) \prod_{i=2:n} p(\tilde{\lambda}_i|\tau^2) \cdot p(\phi_{1:p}) \cdot \prod_{i=2:n} p(\psi_{i,1:q_i}) \cdot \frac{1}{\sigma_e} \cdot \frac{1}{\sigma_v} \cdot \prod_{i=2:n} \frac{1}{\sigma_{w,i}}. \tag{3.15}$$

Sampling from the posterior

We already know that we can draw the states given a general state space model with the simulation smoothing algorithm, and hence the first step in the Gibbs sampler from section 3.1 is covered. What remains is to sample the parameters given the states. It turns out that this task boils down to the repeated application of a Bayesian linear regression model

$$y = X\beta + u \text{ with } u_i \sim \text{Normal}(0, \sigma^2) \tag{3.16}$$

with the prior

$$p(\beta, \sigma^2) \propto p_{\text{Normal}}(\beta|\underline{\beta}, \underline{\Sigma}) \cdot \frac{1}{\sigma}. \tag{3.17}$$

A Gibbs sampler for sampling from the posterior of this model is

$$\beta|y, \sigma^2 \sim \text{Normal}(\overline{\beta}, \overline{\Sigma}) \tag{3.18}$$

$$\tag{3.19}$$

with

$$\overline{\Sigma}^{-1} = \frac{X'X}{\sigma^2} + \underline{\Sigma}^{-1} \tag{3.20}$$

$$\overline{\beta} = \overline{\Sigma}\left(\frac{X'y}{\sigma^2} + \underline{\Sigma}^{-1}\underline{\beta}\right) \tag{3.21}$$

and

$$\sigma^{-2}|y, \beta \sim \text{Gamma}(\overline{a}, \overline{b}) \tag{3.22}$$

with $\overline{a} = \frac{n}{2}$ and $\overline{b} = \frac{2}{u'u}$, $u = y - X\beta$. For a flat prior, we can simple take $\underline{\Sigma}^{-1} = 0$.

As a start, the Gibbs steps for the parameters ϕ and σ_v^2 in the mixed-frequency dynamic factor model are the steps described above with y being the factors and X lags of the factors. The only difference is that the prior for ϕ is truncated such that the roots of the lag polynomial are inside the unit circle and therefore we have to discard all draws for which this does not hold.

Similarly, sampling $\tilde{\lambda}_i$ corresponds to a Bayesian linear regression of $\psi_i(L)y_{i,t}/\sigma_{w,i}$ on $\psi_i(L)f_t$ with the prior $\tilde{\lambda}_i \sim \text{Normal}(0, \tau^2)$ for the known error variance 1. The Gibbs step for $\sigma_{w,i}^2$ is also the same as in a Bayesian linear regression, with the elements of u being $\psi_i(L)y_{i,t} - \lambda_i\psi_i(L)f_t$. The same holds for σ_e^2 with the errors $y_{1,t} - w(L)f_t$.

The step for ψ_i is essentially the step for the slope parameters in a linear regression with a known error variance $\sigma_{w,i}^2$, this time with $y_{i,t} - \lambda_i f_t$ as the dependent variable and lags of this term as the explanatory variable. Again, we discard draws that do not correspond to a stationary process.

The final step is to sample τ^{-2} given everything else from a Gamma distribution with $\overline{a} = \frac{n-1}{2}$ and $\overline{b} = \frac{2}{\sum_{i=2:n}\lambda_i^2/\sigma_{w,i}^2}$.

Forecasts are then simply generated by simulating the model based on the MCMC draws from the parameters.

3.3. Bayesian estimation of MF-DFM 1

Again, we apply the transformation $\psi_i(L)$ to rows $i = 2 : n$ of the measurement equation. Since we cannot apply it to the first row due to the missing values in the first and the second month of each quarter, the state vector in the state space representation of MF-DFM 1

$$x_t = [f_t, \ldots f_{t-r}, u_{1,t}, \ldots, u_{1,t-s}]' \tag{3.23}$$

with $r = \max(4, p-1, q_2, \ldots q_n)$ and $s = \max(4, q_1-1)$ is larger than the one for MF-DFM 2. Appendix B provides more detailed information on the state space representation of MF-DFM 1.

The considerations on how to choose a prior $p(\theta)$ and how to sample from the posterior are similar as for MF-DFM 2. The only difference is that we are now working with the

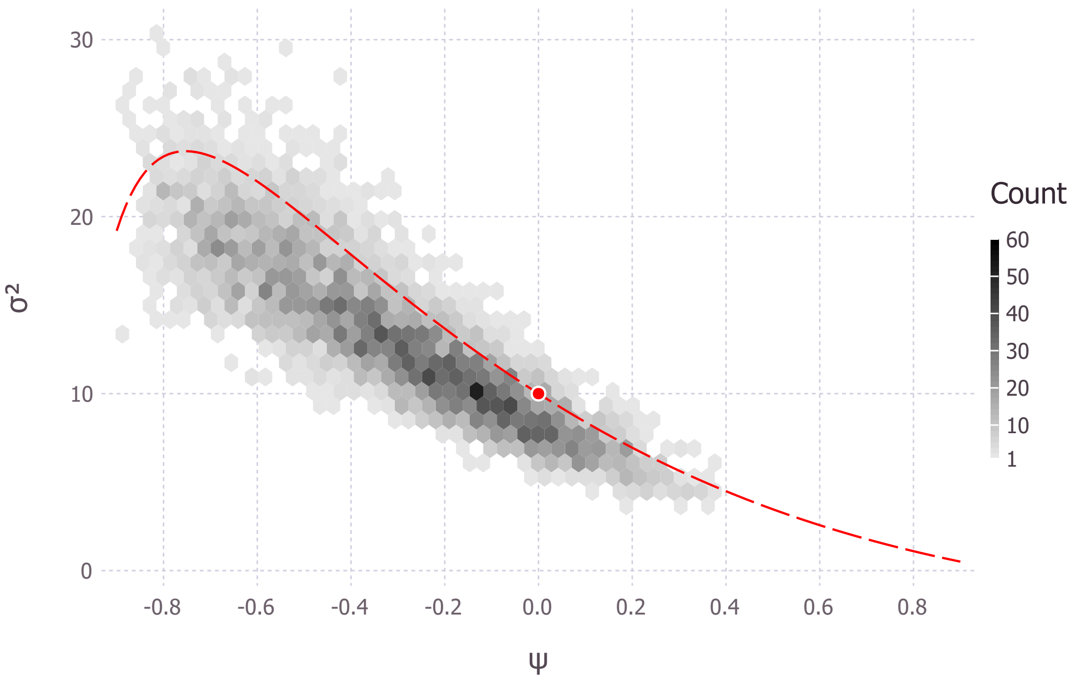

Figure 3.1.: Hexbinplot for 5000 MCMC draws from $p(\psi_1, \sigma^2_{w,1}|y_{1:T})$
We simulate $T = 300$ observations y_t from a MF-DFM 1 with $p = q_1 = \cdots = q_n = 1$, $\phi = 0.9$, $\sigma^2_v = 1$, $\psi_1 = \cdots = \psi_n = 0$, $\sigma^2_{w,1} = \cdots = \sigma^2_{w,n} = 10$ and $\lambda_2 = \cdots = \lambda_n = 1$. Then, we run the Gibbs sampler for estimating the MF-DFM 1 and retain $S = 5000$ samples after a burn-in phase of $S_0 = 5000$ samples. The red line marks all parameter combinations that correspond to the same variance of $w(L)u_{1,t}$ as the true parameters $\psi_1 = 0$, $\sigma^2_{w,1} = 10$ (denoted by a red circle).

parameters $\psi_{1,1:q_1}$ and $\sigma^2_{w,1}$ instead of σ^2_e. The steps for $\psi_{1,1:q_1}$ and $\sigma^2_{w,1}$ are essentially the same as for $\phi_{1:p}$ and σ^2_v as we can simply use the information in the sampled state vectors $x_{1:T}$ for a Bayesian linear regression of $u_{i,t}$ on its lags.

In order to find out how the presence of the MA unit root affects the posterior distribution of ψ_1 and $\sigma^2_{w,1}$, I simulate data from a MF-DFM 1 and plot the output of the Gibbs sampler for ψ_1 and $\sigma^2_{w,1}$ in figure 3.1. The figure shows that 300 months of data (which corresponds to 100 observations of the quarterly variable $y_{1,t}$) are not enough to pin down ψ_1 and $\sigma^2_{w,1}$ to a small region in the parameter space. This is because the covariance of $w(L)u_{1,t}$ with $w(L)u_{1,t-3}, w(L)u_{1,t-6}, \ldots$ does not provide enough information on ψ_1 to distinguish values close to zero from negative values. Hence it is primarily the variance of $w(L)u_{1,t}$ that determines which areas in the parameter space have a non-negligible posterior probability to contain $(\psi_1, \sigma^2_{w,1})$.

3.4. Bayesian estimation of sparse dynamic factor models

If there is some reason to believe that some variables in the model do not co-move with the factor at all, it may be possible to increase the forecast accuracy by imposing a sparse prior on the factor loadings. Some researchers use heuristic variable selection procedures before the actual estimation step to address this issue (Marcellino et al., 2016) but Bayesian variable selection in the estimation step is in my opinion the more elegant solution: For example, we get averaging over all probable variable sets for free.

Despite the recent interest in the Bayesian estimation of sparse factor models (Kaufmann and Schumacher, 2013, 2017; Fruhwirth-Schnatter and Lopes, 2018), I am not aware of any published article or working paper that uses a sparse prior in a standard nowcasting setting with monthly macroeconomic variables as predictors: Zhou et al. (2014) use a factor model with dynamic sparsity and apply it to a forecast problem in finance; Thorsrud (2016a,b) apply the same model in nowcasting with daily data extracted from newspaper articles. Therefore it seems worthwhile to explore the usefulness of sparse dynamic factor models in macroeconomic forecasting.

I will use the approach of Kaufmann and Schumacher (2013) in the following. The spike-and-slab prior on the factor loadings is

$$\tilde{\lambda}_i|(\omega_i = 0) = 0 \tag{3.24}$$

$$\tilde{\lambda}_i|(\omega_i = 1) \sim \text{Normal}(0, \tau^2) \tag{3.25}$$

$$\omega_i|\rho \sim \text{Binomial}(\rho) \tag{3.26}$$

$$\rho \sim \text{Beta}(a, b) \tag{3.27}$$

$$\tau^{-2} \sim \frac{1}{\tau} \tag{3.28}$$

By setting $\rho = 1$, we recover the hierarchical Gaussian prior from the previous sections. For sampling from the posterior, first calculate the posterior odds for $\omega_i = 1$:

$$\frac{P(\omega_i = 1|\cdot)}{P(\omega_i = 0|\cdot)} = \frac{p(y_i|\omega_i = 1)}{p(y_i|\omega_i = 0)} \cdot \frac{P(\omega_i = 1)}{P(\omega_i = 0)} \tag{3.29}$$

$$= \sqrt{\frac{\sigma^2_{\tilde{\lambda}_i}}{\tau^2}} \exp\left(\frac{\mu^2_{\tilde{\lambda}_i}}{2\sigma^2_{\tilde{\lambda}_i}}\right) \cdot \frac{\rho}{1-\rho}. \tag{3.30}$$

where $\sigma^2_{\tilde{\lambda}_i}$ and $\mu_{\tilde{\lambda}_i}$ are the posterior mean and variance if $\rho = 1$ and $|\cdot$ denotes that we condition on all other model parameters in this step. Then, we sample ω_i according these posterior odds. If $\omega_i = 0$, we set $\lambda_i = 0$; if $\omega_i = 1$, then $\lambda_i \sim \text{Normal}(\mu_{\tilde{\lambda}_i}, \sigma^2_{\tilde{\lambda}_i})$. Finally, we have

$$\tau^{-2} \sim \text{Gamma}\left(\frac{\sum_{i=2:n}\omega_i}{2}, \frac{2}{\sum_{i=2:n}\tilde{\lambda}_i^{\,2}}\right) \tag{3.31}$$

$$\rho \sim \text{Beta}\left((a + \sum_{i=2:n}\omega_i, b + n - 1 - \sum_{i=2:n}\omega_i\right). \tag{3.32}$$

While I was already working on this part of the thesis, I found out that Philipp Hauber and Christian Schumacher also use a sparse factor model for nowcasting GDP with monthly predictors in an ongoing research project. Their main motivation for imposing sparsity is that they want to work with large international datasets, such as a combined dataset of US and euro area data, which is likely to contain many irrelevant variables for forecasting either US or euro area GDP growth. Hauber and Schumacher choose a multiplicative Gamma shrinkage prior. In contrast to the spike-and-slab prior that I employ, this prior does not induce exact zeros in the model. For more detailed information on Hauber's and Schumacher's methodological choices and on their preliminary results, see their poster from the 2017 Deutsche Bundesbank workshop.[3]

[3] https://www.bundesbank.de/Redaktion/EN/Downloads/Bundesbank/Research_Centre/Conferences/2017/2017_09_08_frankfurt_p2_5_poster.pdf?__blob=publicationFile

4. Forecasting US GDP growth with mixed-frequency dynamic factor models

In order to assess the performance of mixed-frequency dynamic factor models in a realistic setting, I conduct an out-of-sample forecast experiment with real GDP growth as the target variable. All four articles on forecasting with Bayesian mixed-frequency dynamic factor models contain a similar empirical application: Luciani and Ricci (2014) generate nowcasts of Norwegian GDP growth, Marcellino et al. (2016) forecast euro area GDP growth and both D'Agostino et al. (2016) and Antolin-Diaz et al. (2017) use their models in order to nowcast US GDP growth. Because ALFRED, the real-time database for US data, seems more mature than its counterpart for the euro area[1], I will work with US data.

The first part of this chapter is concerned with the design of a suitable forecast experiment while the second part presents and discusses the results.

4.1. Description of the forecast experiment

An out-of-sample experiment with real-time data is conducted as follows: At the first forecast origin, the model is estimated with the data that was available at this time. The estimated model is then used to forecast the target variable. This procedure is repeated for all forecast origins.

Designing a forecast experiment therefore means to determine a set of forecast origins and the associated evaluation sample, to select a set of potential predictors and to set the start of the estimation sample. Some of these decisions are intertwined - for example, we can only include predictors for which real-time data is available at all forecast origins.

I proceed as follows: I set January 1985 as the beginning of the estimation sample because it is roughly the start of the so-called "Great Moderation". Extending the estimation sample is likely to decrease the overall forecast performance because pre-1985 data is more volatile and the economic dynamics between variables prior to 1985 may differ a lot from today.[2] As the evaluation sample, I choose the past 15 years, 2003 Q1 - 2017 Q4, such that there are 60 GDP values that we attempt to forecast. This means that the evaluation sample consists of three periods: the years between the early 2000s recession and the Great Recession, the Great Recession and the post-recession period.

[1] The Statistical Data Warehouse of the ECB refers to its real-time database as "experimental data".

[2] As documented in Stock and Watson (2002b), it was already clear in the early 2000s that the economic dynamics before the mid-eighties were a lot different from the period afterwards. Therefore, these considerations do not contradict the intended out-of-sample character of the forecast experiment.

For selecting potential predictors, there are many different approaches in the literature on short-term forecasting and nowcasting: Luciani and Ricci (2014) select 14 real and survey indicators that appear either in the Bloomberg calendar for Norway or on the news section of Norway's statical agency and its central bank. In this way, they attempt to select variables that are considered to be most informative by financial markets and policy makers. D'Agostino et al. (2016) include four series that are traditionally used for constructing a business cycle index and add the Purchasing Manager's Index because of its timely release. Marcellino et al. (2016) use a large dataset with 40 variables as a starting point and use a heuristic statistical procedure to select a small subset of variables. They end up with only nine indicators. Although they do not explicitly mention which estimation sample they apply their variable selection algorithm to, I suspect that it has been the one available at the first forecast origin because everything else would violate the out-of-sample character of the forecast experiment. Since Marcellino et al. (2016) do not use their variable selection procedure at every forecast origin, they may overlook variables that do not co-move with the business cycle in the first estimation sample but later.

I take an approach similar to Bańbura et al. (2013) and Antolin-Diaz et al. (2017) and use a medium-sized dataset of 19 indicators plus my target variable, real GDP growth. 10 of these variables are macroeconomic variables that are related to real economic activity, such as industrial production, employment, sales and residential construction. I also include 3 indexes constructed from surveys because of their timeliness. The remaining 6 variables are important financial variables and inflation series. See table 4.1 for more information.

Ideally, we would like to update our forecast/ nowcast every time new data is published or data is revised. However, this is not feasible within a reasonable time period with limited computation power. Hence I take a simpler approach and update forecasts only a few times per month. In table 4.2, all data releases in January 2017 are listed. In January 2017 and in many other months, the releases can be split into three groups: one right after the beginning of the month, one towards the middle and one towards the end. This is way I choose the 10th, the 20th and the last day of each month as forecast origins. Predictions are thought to be made at the end of these days, after all the data was published. In each forecast origin, I predict the next two values for real GDP growth that are not available yet. So, dependent on the forecast origin, we make either a backcast and a nowcast or a nowcast and a forecast. In total, there are 18 forecast origins for each value of GDP in the evaluation sample, ranging from the last day of the first month in the previous quarter (-1/1/30) to the 20th day of the first month in the following quarter (1/1/20). The first seven forecast origins correspond to forecasts in a strict sense (-1/1/30 to -1/3/30), the following eight forecast origins belong to nowcasts (0/1/10 to 0/3/20) and the remaining three dates are for doing backcasts (0/3/30, 1/1/10 and 1/1/20).[3]

[3] Although the 30th day of a month is not always its last day, I use the number 30 to refer to it in the short-hand notation Q/M/D for forecast origins. (Q is the quarter relative to the reference quarter, M is the month within this quarter and D is the day.)

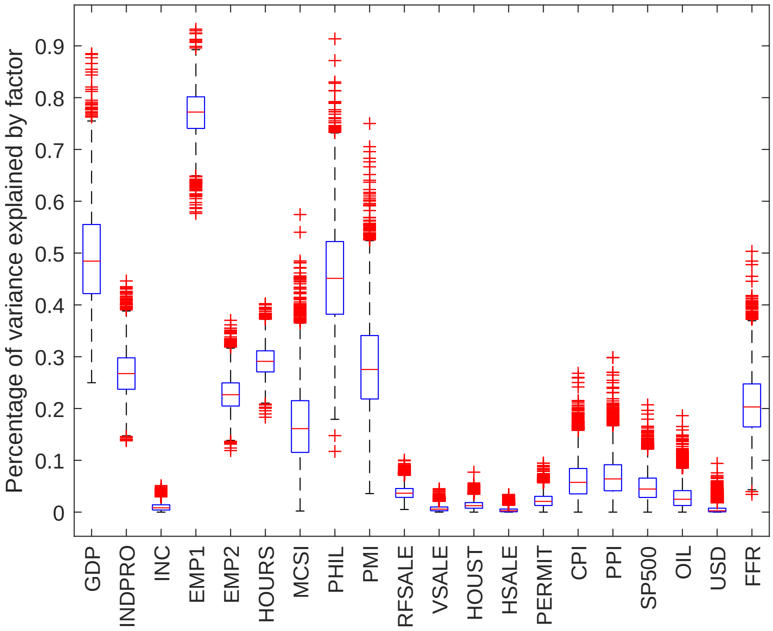

Figure 4.1.: Percentage of the variance that is explained by the co-movement with the factor (2018-01-20 vintage)

4.2. Discussion of the results

We conduct the forecast experiment with MF-DFM 2 with both a Gaussian and a sparse prior on the factor loadings. For simplicity, I have set all lag lengths $p, q_2, \ldots q_n$ to 1. I choose a prior that is uniform on $[0, 1]$ for the sparsity coefficient ρ. In order to confirm the presumption that the forecasts generated by MF-DFM 2 barely differ from the ones generated by MF-DFM 1, I also conduct the experiment with MF-DFM 1 ($q_1 = 1$) with a Gaussian prior. As the main benchmark, I use an AR(1) model. Since there are 550 forecasts origins in total, I use $S_0 = 5000$ burn-in samples and $S = 5000$ samples although longer MCMC chains would be preferable for models as complex as mixed-frequency dynamic factor models.[4]

Before I report the results of the out-of-sample forecast experiment, I provide some in-sample results that may help to gain a better understanding of the model.

In-sample results

Figure 4.1 is a boxplot for the fraction of variance in each variable that is explained by the factor. In my opinion, this quantity is a more comprehensible measure of the co-movement of variables with the business cycle than the alternatives: λ_i is not invariant

[4] Conducting the forecast experiment for one mixed-frequency dynamic factor model takes about one day on a computer with a Intel Core i7-6500U processor.

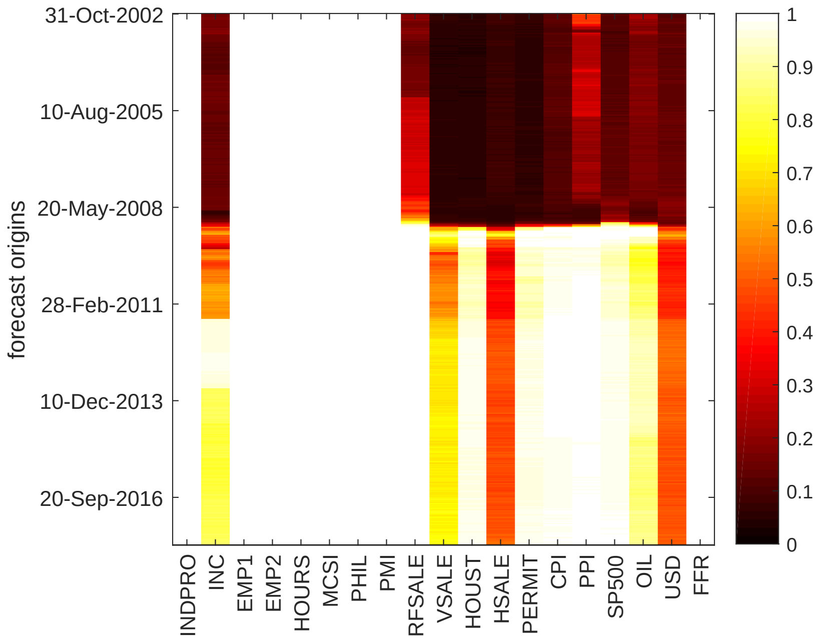

Figure 4.2.: Inclusion probabilities $\text{Prob}(\lambda_i \neq 0)$ for all estimation samples

under rescaling; $\lambda_i/\sigma_{w,i}$ is invariant but since ψ_i differs, a high $\lambda_i/\sigma_{w,i}$ does not necessarily mean that the factor explains much of the dynamics in $y_{i,t}$. Unsurprisingly, many of the variables that are directly related with real economic activity have a particularly strong co-movement with the factor.

We can clearly see that a large fraction of the variability of real GDP, industrial production, employment variables, survey indicators and the federal funds rate is explained by the factor. For financial variables, inflation, sales and residential construction, the factor is not important. Surprisingly, it does also not play a role for real disposable income as well.

As expected, the in-sample results for MF-DFM 1 with a Gaussian prior on the factor loadings are very similar to MF-DFM 2. Also, the marginal posterior distribution of ψ_1 and $\sigma^2_{w,i}$ looks almost as the one for simulated data that has been depicted in figure 3.1.

The in-sample results for MF-DFM 2 with a sparse prior on the factor loadings are also similar. The pattern of sparsity that is induced by the prior at each forecast origin can be studied in figure 4.2. In the pre-recession period, about a half of the variables is set to zero very frequently. After the Great Recession, however, almost all the factor loadings are included in the model. This means that during an recession as extreme as the Great Recessions variables that are normally not influenced by the business cycle, tend to co-move with the factor.

Out-of-sample results

In order to evaluate the results of the out-of-sample forecast experiment, I use both the squared error and the continuous ranked probability score as criteria. The tables in appendix C provide an extensive overview of the results, for different choices what the "actual GDP values" are in the presence of revisions. In the following, I will work with the third GDP estimate that is typically released 3 months after the reference quarter.

The tables in appendix C show that the three variants of the mixed-frequency dynamic factor model that I have estimated yield barely different results, for all 18 forecast origins. In the case of MF-DFM 1 and MF-DFM 2 with a Normal prior, this does not come as a surprise: Since the monthly idiosyncratic component provides little added value due to the MA unit root, replacing it with quarterly measurement errors does not change a lot. It is however disappointing that the sparse prior is not able to improve forecast accuracy. As the differences in forecast accuracy between the three models are so small, I will only consider the MF-DFM 2 Normal now.

In figure 4.3 and 4.4, the mean values of these criteria are depicted for all 18 forecast origins (-1/1/30 to 1/1/20). We can see that, according to both criteria, our forecasts get more accurate as new data comes in: While the dynamic factor model forecasts are as good as the AR(1) benchmark for predictions 1-2 months before the start of the quarter, the nowcasts and backcasts of the dynamic factor model are clearly better. Note that the AR(1) forecasts suddenly get better at the (0/1/30) forecast origin because GDP of the past quarter is released.

Figure 4.5 provides a closer look for predictions generated at the end of a quarter (0/3/30): The MF-DFM 2 with the Normal prior clearly dominates the AR(1) benchmark during the Great Recession; both in the pre- and post-recession sample it is much less clear if the dynamic factor model is better. Again, results for the MF-DFM 2 with the sparse prior and the MF-DFM 1 with the Normal prior are very similar and are therefore not reported here.

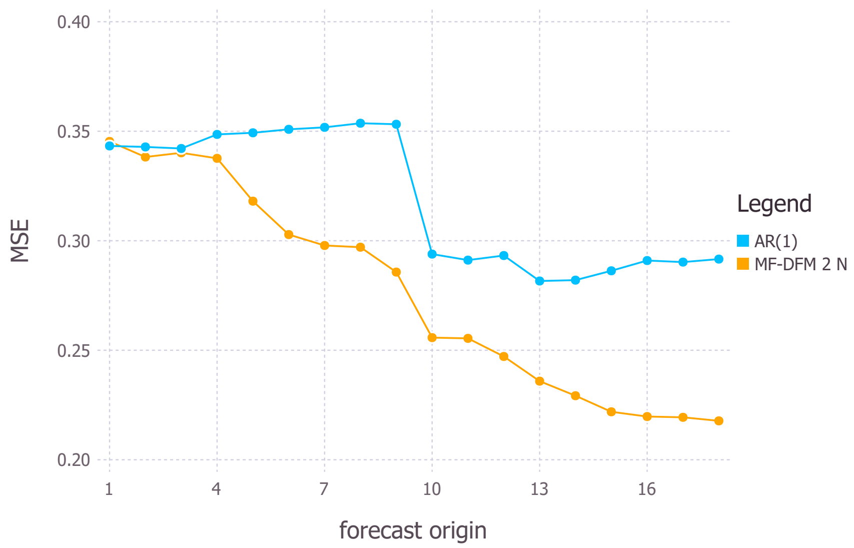

Figure 4.3.: Mean squared errors for the AR(1) forecasts and the forecasts of the MF-DFM 2 with a Normal prior for the different forecast horizons

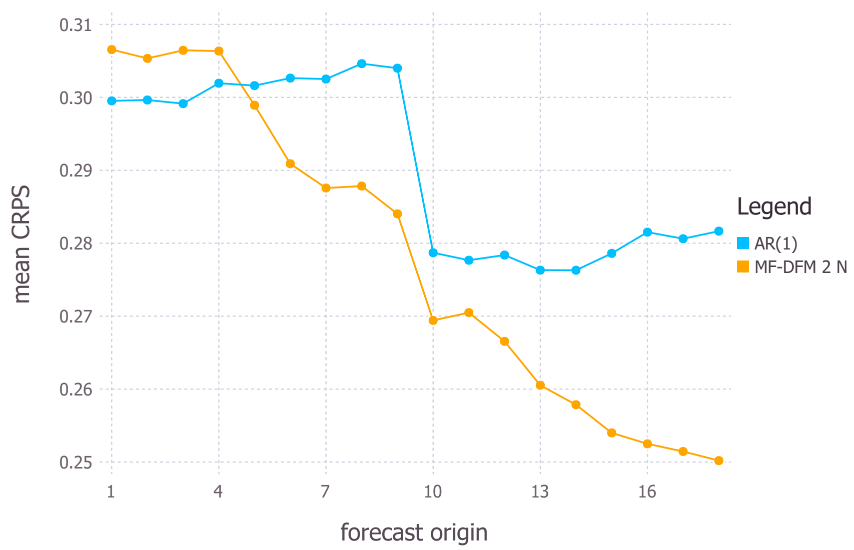

Figure 4.4.: Mean CRPS score for the AR(1) forecasts and the forecasts of the MF-DFM 2 with a Normal prior for the different forecast horizons

Table 4.1.: Macroeconomic and financial variables in the real-time dataset

ID	Description	Source	Source ID	Freq.	SA	Transf.
GDP	Real gross domestic product	ALFRED	GDPC1	Q	yes	$100 \cdot \log \Delta$
INDPRO	Industrial production index	ALFRED	INDPRO	M	yes	$100 \cdot \log \Delta$
INC	Real disposable personal income	ALFRED	DSPIC96	M	yes	$100 \cdot \log \Delta$
EMP1	All employees: Total nonfarm payrolls	ALFRED	PAYEMS	M	yes	$100 \cdot \log \Delta$
EMP2	Civilian Employment Level	ALFRED	CE16OV	M	yes	$100 \cdot \log \Delta$
HOURS	Index of aggregate weekly hours	ALFRED	AWHI	M	yes	$100 \cdot \log \Delta$
RFSALE	Real retail and food services sales	ALFRED	RRSFS + RSALES	M	yes	$100 \cdot \log \Delta$
VSALE	Light weights vehicle sales	ALFRED	ALTSALES	M	yes	$100 \cdot \log \Delta$
HSALE	New one family houses sold	ALFRED	HSN1F	M	yes	$100 \cdot \log \Delta$
HOUST	Housing starts	ALFRED	HOUST	M	yes	$100 \cdot \log \Delta$
PERMIT	Building permits	ALFRED	PERMIT	M	yes	$100 \cdot \log \Delta$
CPI	Consumer price index	ALFRED	CPIAUCSL	M	yes	$100 \cdot \log \Delta$
PPI	Producer price index	ALFRED	PPIACO	M	no	$100 \cdot \log \Delta$
OIL	Spot crude oil price: WTI	ALFRED	WTISPLC	M	no	$100 \cdot \log \Delta$
SP500	S&P 500 stock index	Yahoo Finance	-	M	no	$100 \cdot \log \Delta$
USD	Trade weighted US Dollar index: Broad	ALFRED	TWEXBMTH	M	no	$100 \cdot \log \Delta$
FFR	Effective federal funds rate	ALFRED	FEDFUNDS	M	no	Δ
MCSI	Consumer sentiment index	ALFRED	UMCSENT	M	no	Δ_{12}
PMI	Purchasing managers' index	Quandl	ISM/MAN_PMI	M	yes	-
PHIL	Philadelphia manufacturing index	ALFRED	GACDFSA066MSFRBPHI	M	yes	-

Table 4.2.: Data releases and forecast origins in January 2017

Day of month	Release	IDs	Time period
3	Purchasing managers' index	PMI	Dec 2016
5	Light weight vehicle sales	VS	Dec 2016
6	Employment situation	EMP1, EMP2, HOURS	Dec 2016
10	Backcast for Q4 2016 and nowcast for Q1 2017		
13	Producer price index	PPI	Dec 2016
18	Consumer price index	CPI	Dec 2016
18	Real retail and food services sales	RFSALE	Dec 2016
18	Industrial production	IP	Dec 2016
19	New residential construction	HOUST, PERMIT	Dec 2016
19	Philadelphia manufacturing index	PHIL	Jan 2017
20	Backcast for Q4 2016 and nowcast for Q1 2017		
26	New one family houses sold	HSALE	Dec 2016
27	GDP advance estimate	RGDP	Q4 2016
27	Consumer sentiment index	MSCI	Jan 2017
30	Personal income and outlays	INC	Dec 2016
31	Financial variables	SP500, OIL, USD, FFR	Jan 2017
31	Nowcast for Q1 2017 and forecast for Q2 2017		

For simplicity, revisions are omitted which means that only new releases are shown. The timing of the releases in other months is similar. In the second and third month of a quarter, a second estimate and third of GDP in the previous quarter are published - typically also towards the end of the month, such as the first estimate.

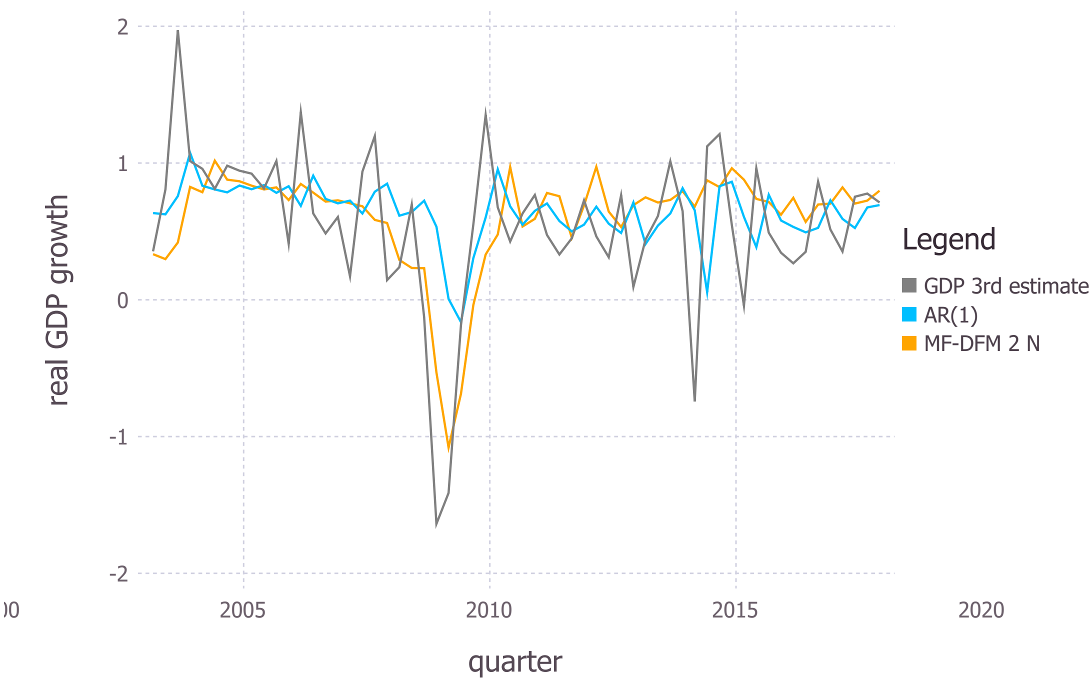

Figure 4.5.: Mean AR(1) forecasts, mean MF-DFM 2 Normal forecasts and third GDP growth estimate during the evaluation sample

5. Conclusion

First of all, the empirical results in the previous section confirm that mixed-frequency dynamic factor models are indeed very suitable for GDP nowcasting. Depending on the forecast horizon and which GDP estimates are seen as the “actual values”, the gains in terms of mean squared error with respect to the AR(1) benchmark up to 34% in the evaluation sample 2003 Q1 - 2017 Q4. The advantage over univariate forecast models are especially clear during the Great Recession.

The main contribution of this thesis to the literature is an alternative way to model the idiosyncratic component of quarterly GDP growth in a dynamic factor model specified at a monthly baseline frequency: By viewing the factor as a proxy for monthly GDP growth, we can replace the unobservable monthly AR process with i.i.d. measurement errors at a quarterly frequency. Due to this change, we can avoid the issues associated with the MA unit root and work with a more compact state space representation, without making any sacrifices in forecast accuracy or theoretical adequacy.

As this thesis is only focused on the Bayesian estimation of mixed-frequency dynamic factor models, I did not study what problems might occur when using other estimation methods in the presence of the MA unit root. It seems however natural that similar problems appear in maximum likelihood estimation. In particular, there may be problems with the likelihood being almost flat in a large, oddly shaped region in the $(\psi_1, \sigma^2_{w,1})$-parameter space. Moreover, asymptotic standard errors for these parameters will probably not provide an accurate characterization of the uncertainty associated with the point estimates.

My attempt to improve the accuracy of forecast by imposing a sparse prior on the factor loadings did not succeed: Although the sparse factor model found evidence for sparsity, especially in the pre-recession period, the overall forecast performance of the sparse model was basically the same as for the model with a Normal prior. These results are broadly in line with the preliminary results by Philipp Hauber and Christian Schumacher for an international dataset. Consequently, sparsity in factor models for nowcasting GDP growth do not seem to have any added value. Future research should instead try to apply sparse factor models to other variables such as inflation, especially since (dynamic) sparsity has already been shown to improve the forecast accuracy of other forecast models for inflation (Rockova and McAlinn, 2018).

The most promising way to improve the quality of our forecasts is to take instabilities of model parameters more seriously: As I have already explained in the introduction, both slope parameters and error variances of a model estimated with macroeconomic data are unlikely to stay the same in a model estimated with macroeconomic data. If the instabilities are large, taking them into account can improve forecast accuracy. In particular, introducing a time-varying mean as in Antolin-Diaz et al. (2017) would remove the upwards bias of our forecasts in the post-recession sample. Stochastic volatility might improve the density forecasts. Whether allowing for the time-variation in the factor loadings helps, is much less clear.

Moreover, allowing for "dynamic heterogeneity" in the factor model by including lags of the factor in the measurement equation may also help, especially as Luciani and Ricci (2014) and Marcellino et al. (2016) stress that survey variables are typically more in sync with year-on-year GDP growth than with quarter-on-quarter GDP growth.

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A. An alternative derivation of eqn. 2 in Mariano and Murasawa (2003)

Let Y_t^q be a flow variable that is measured at a quarterly frequency (e.g. real GDP) and assigned to the third month of each quarter. Y_t^m denotes the unobserved monthly counterpart such that

$$Y_t^q = Y_t^m + Y_{t-1}^m + Y_{t-2}^m \tag{A.1}$$

holds. We want to show that the log-difference of the quarterly variable $y_t^q = \log Y_t^q - \log Y_{t-3}^q$ is approximated by

$$y_t^q \approx \frac{1}{3}y_t^m + \frac{2}{3}y_{t-1}^m + y_{t-2}^m + \frac{2}{3}y_{t-3}^m + \frac{1}{3}y_{t-4}^m \tag{A.2}$$

for small $y_t^m = \log Y_t^m - \log Y_{t-1}^m$.

As a starting point, I use

$$y_t^q = \log Y_t^q - \log Y_{t-3}^q = \log(1 + \frac{Y_t^q - Y_{t-3}^q}{Y_{t-3}^q}) \approx \frac{Y_t^q - Y_{t-3}^q}{Y_{t-3}^q}, \tag{A.3}$$

replace Y_t^q by the sum $Y_t^m + Y_{t-1}^m + Y_{t-2}^m$ of monthly values and divide both the numerator and the denominator by Y_{t-5}^m

$$y_t^q \approx \frac{Y_t^m/Y_{t-5}^m + Y_{t-1}^m/Y_{t-5}^m + Y_{t-2}^m/Y_{t-5}^m - Y_{t-3}^m/Y_{t-5}^m - Y_{t-4}^m/Y_{t-5}^m - 1}{Y_{t-3}^m/Y_{t-5}^m + Y_{t-4}^m/Y_{t-5}^m + 1}. \tag{A.4}$$

The fractions Y_t^m/Y_{t-5}^m, $Y_{t-1}^m/Y_{t-5}^m, \ldots$ can be written in terms of growth rates by repeatedly applying $Y_t^m \approx Y_{t-1}^m(1 + y_t^m)$:

$$\begin{aligned}
Y_{t-4}^m/Y_{t-5}^m &\approx (1 + y_{t-4}^m) && \text{(A.5)}\\
Y_{t-3}^m/Y_{t-5}^m &\approx (1 + y_{t-4}^m)(1 + y_{t-3}^m) && \text{(A.6)}\\
&\approx 1 + y_{t-4}^m + y_{t-3}^m && \text{(A.7)}\\
&\vdots && \text{(A.8)}\\
Y_t^m/Y_{t-5}^m &\approx (1 + y_{t-4}^m)(1 + y_{t-3}^m)(1 + y_{t-2}^m)(1 + y_{t-1}^m)(1 + y_t^m) && \text{(A.9)}\\
&\approx 1 + y_{t-4}^m + y_{t-3}^m + y_{t-2}^m + y_{t-1}^m + y_t^m && \text{(A.10)}
\end{aligned}$$

Since we assume that monthly growth rates are small, it is ok to discard products of growth rates. Putting everything together yields

$$y_t^q \approx \frac{y_t^m + 2y_{t-1}^m + 3y_{t-2}^m + 2y_{t-3}^m + y_{t-4}^m}{3 + 2y_{t-4}^m + y_{t-3}^m} \tag{A.11}$$

$$\approx \frac{1}{3}y_t^m + \frac{2}{3}y_{t-1}^m + y_{t-2}^m + \frac{2}{3}y_{t-3}^m + \frac{1}{3}y_{t-4}^m. \tag{A.12}$$

The derivation above is quite tedious compared to the motivation in Mariano and Murasawa (2003). However, it shows more clearly that the relation is basically an approximation of quarter-on-quarter growth of a flow variable in terms of monthly growth rates.

There has been some confusion about equation A.2 in the literature: For example, both Bańbura et al. (2013) and Luciani and Ricci (2014) use the wrong relation

$$y_t^q \approx y_t^m + 2y_{t-1}^m + 3y_{t-2}^m + 2y_{t-3}^m + y_{t-4}^m \tag{A.13}$$

which means that the hidden variable y_t^m is their models is one third of the actual monthly GDP growth. However, using this wrong approximation in a forecast model does not affect the forecast accuracy because all model parameters can simply adjust to the different scale of the unobserved variable. Moreover, Luciani and Ricci (2014) claim in their quick derivation of the Muriano-Murasawa relation that $\log Y_t^q \approx \log Y_t^m + \log Y_{t-1}^m + \log Y_{t-2}^m$ holds which is simply not true.

B. State-space representation of MF-DFM 1 and 2

B.1. State space representation of MF-DFM 1

For simplicity, I assume $p = 1$ and $q_2 = \cdots = q_n = 1$ in the following. The generalization to other lag lengths is trivial.

$$\underset{5\times 1}{x_t} = [f_t, \dots f_{t-4}]' \tag{B.1}$$

$$\underset{5\times 5}{A} = \begin{pmatrix} \phi & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \underset{n\times 5}{B} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} \\ \lambda_2 & -\psi_2\lambda_2 & 0 & 0 & 0 \\ \vdots & & & & \vdots \\ \lambda_n & -\psi_n\lambda_n & 0 & 0 & 0 \end{pmatrix} \tag{B.2}$$

$$\underset{1\times 1}{\Sigma_\epsilon} = \sigma_v^2 \qquad \underset{5\times 1}{R_\epsilon} = [1, 0, 0, 0, 0]' \tag{B.3}$$

$$\underset{n\times n}{\Sigma_\eta} = \mathrm{diag}(\sigma_e^2, \sigma_{w,2}^2, \dots \sigma_{w,n}^2) \qquad \underset{n\times n}{R_\eta} = I_{n\times n} \tag{B.4}$$

B.2. State space representation of MF-DFM 2

For simplicity, I assume $p = 1$ and $q_1 = q_2 = \cdots = q_n = 1$ in the following. The generalization to other lag lengths is trivial.

$$\underset{10\times 1}{x_t} = [f_t, \dots f_{t-4}, u_{1,t}, \dots u_{1,t-4}]' \tag{B.5}$$

$$\underset{10\times 10}{A} = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \text{ with } \underset{5\times 5}{A_1} = \begin{pmatrix} \phi & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \underset{5\times 5}{A_2} = \begin{pmatrix} \psi_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{B.6}$$

$$\underset{n\times 10}{B} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} \\ \lambda_2 & -\psi_2\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_n & -\psi_n\lambda_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{B.7}$$

$$\underset{2\times 2}{\Sigma_\epsilon} = \mathrm{diag}(\sigma_v^2, \sigma_{w,1}^2) \tag{B.8}$$

$$\underset{n-1\times n-1}{\Sigma_\eta} = \mathrm{diag}(\sigma_{w,2}^2, \dots \sigma_{w,n}^2) \tag{B.9}$$

$$\underset{10\times 2}{R_\epsilon} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \underset{n\times n-1}{R_\eta} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \tag{B.10}$$

C. Additional results

In the tables C.1 - C.4, the forecast accuracy is evaluated for all 18 forecast horizons, using the mean squared error (MSE) as criterion. The first row of each table contains the MSE for the AR(1) benchmark forecasts; the other rows state the MSE of the mixed-frequency models over the benchmark MSE. Each of the four tables uses different GDP estimates for evaluation the forecasts: first estimates, second estimates, third estimates and the 2018-08-30 vintage.

The tables C.5 - C.8 use the continuous ranked probability score (CRPS) as the criterion and are structured in the same way.

Table C.1.: MSE for the AR(1) model and relative MSE for the MF-DFMs (first GDP estimate)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) MSE	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.21	0.21	0.21	0.20	0.20	0.21	0.21	0.21	0.21
MF-DFM 2 N	1.06	1.04	1.05	1.01	0.93	0.86	0.83	0.83	0.80	0.91	0.93	0.89	0.86	0.84	0.79	0.77	0.77	0.76
MF-DFM 2 S	1.07	1.06	1.04	1.02	0.93	0.87	0.84	0.84	0.80	0.92	0.93	0.90	0.88	0.86	0.80	0.78	0.79	0.77
MF-DFM 1 N	1.04	1.03	1.02	1.01	0.92	0.87	0.83	0.85	0.80	0.92	0.94	0.90	0.88	0.87	0.81	0.80	0.80	0.78

Table C.2.: MSE for the AR(1) model and relative MSE of the MF-DFMs (second GDP estimate)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) MSE	0.33	0.33	0.33	0.33	0.33	0.34	0.34	0.34	0.34	0.28	0.27	0.28	0.27	0.27	0.27	0.27	0.27	0.28
MF-DFM 2 N	1.03	1.01	1.01	0.99	0.93	0.87	0.85	0.85	0.82	0.89	0.91	0.88	0.86	0.84	0.80	0.78	0.77	0.76
MF-DFM 2 S	1.03	1.02	1.01	1.00	0.92	0.88	0.86	0.86	0.83	0.91	0.92	0.89	0.89	0.85	0.81	0.79	0.79	0.77
MF-DFM 1 N	1.02	1.01	1.00	0.99	0.93	0.88	0.86	0.87	0.83	0.91	0.93	0.90	0.89	0.87	0.82	0.81	0.81	0.78

Table C.3.: MSE for the AR(1) model and relative MSE of the MF-DFMs (third GDP estimate)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) MSE	0.34	0.34	0.34	0.35	0.35	0.35	0.35	0.35	0.35	0.29	0.29	0.29	0.28	0.28	0.29	0.29	0.29	0.29
MF-DFM 2 N	1.01	0.99	0.99	0.97	0.91	0.86	0.85	0.84	0.81	0.87	0.88	0.84	0.84	0.81	0.78	0.76	0.76	0.75
MF-DFM 2 S	1.01	1.00	0.99	0.97	0.91	0.87	0.85	0.84	0.82	0.89	0.88	0.85	0.86	0.82	0.79	0.76	0.77	0.75
MF-DFM 1 N	1.00	0.99	0.98	0.97	0.92	0.88	0.86	0.86	0.83	0.89	0.9	0.87	0.86	0.85	0.80	0.79	0.80	0.77

Table C.4.: MSE for the AR(1) model and relative MSE of the MF-DFMs (2018-06-30 vintage)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) MSE	0.41	0.41	0.41	0.41	0.41	0.42	0.42	0.42	0.42	0.36	0.36	0.36	0.35	0.35	0.35	0.36	0.36	0.36
MF-DFM 2 N	0.92	0.91	0.91	0.91	0.86	0.82	0.80	0.77	0.74	0.76	0.75	0.73	0.72	0.68	0.65	0.64	0.65	0.65
MF-DFM 2 S	0.93	0.92	0.91	0.91	0.86	0.82	0.80	0.77	0.75	0.77	0.76	0.74	0.73	0.69	0.66	0.65	0.67	0.66
MF-DFM 1 N	0.92	0.91	0.90	0.92	0.87	0.84	0.81	0.79	0.76	0.78	0.77	0.75	0.73	0.71	0.67	0.67	0.69	0.67

Table C.5.: Mean CRPS for the AR(1) model and relative mean CRPS for the MF-DFMs (first GDP estimate)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) CRPS	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.24	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25
MF-DFM 2 N	1.04	1.04	1.05	1.04	1.01	0.97	0.96	0.94	0.93	0.98	0.98	0.97	0.94	0.93	0.9	0.89	0.89	0.88
MF-DFM 2 S	1.04	1.05	1.04	1.04	1.00	0.97	0.96	0.95	0.93	0.98	0.99	0.97	0.95	0.94	0.91	0.90	0.90	0.89
MF-DFM 1 N	1.03	1.04	1.04	1.04	1.01	0.98	0.96	0.96	0.94	0.99	0.99	0.98	0.95	0.95	0.92	0.91	0.91	0.89

Table C.6.: Mean CRPS for the AR(1) model and relative mean CRPS of the MF-DFMs (second GDP estimate)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) CRPS	0.29	0.29	0.29	0.30	0.30	0.30	0.30	0.30	0.30	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.28
MF-DFM 2 N	1.03	1.03	1.04	1.03	1.00	0.97	0.96	0.96	0.95	0.98	1.00	0.98	0.96	0.95	0.93	0.91	0.91	0.90
MF-DFM 2 S	1.04	1.04	1.03	1.03	1.00	0.97	0.96	0.96	0.95	0.99	1.00	0.99	0.97	0.96	0.94	0.92	0.92	0.91
MF-DFM 1 N	1.03	1.03	1.03	1.04	1.01	0.98	0.96	0.98	0.96	1.00	1.01	0.99	0.98	0.97	0.94	0.94	0.94	0.92

Table C.7.: Mean CRPS for the AR(1) model and relative mean CRPS of the MF-DFMs (third GDP estimate)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) CRPS	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
MF-DFM 2 N	1.02	1.02	1.02	1.01	0.99	0.96	0.95	0.94	0.93	0.97	0.97	0.96	0.94	0.93	0.91	0.90	0.90	0.89
MF-DFM 2 S	1.03	1.02	1.02	1.02	0.99	0.96	0.95	0.95	0.94	0.97	0.98	0.97	0.95	0.94	0.92	0.90	0.90	0.89
MF-DFM 1 N	1.03	1.02	1.02	1.02	1.00	0.98	0.96	0.97	0.95	0.99	0.99	0.98	0.96	0.96	0.93	0.92	0.93	0.91

Table C.8.: Mean CRPS for the AR(1) model and relative mean CRPS of the MF-DFMs (2018-06-30 vintage)

Q	-1							0									+1	
M/D	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20	1/30	2/10	2/20	2/30	3/10	3/20	3/30	1/10	1/20
AR(1) CRPS	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.30	0.30	0.30	0.30	0.30	0.30	0.31	0.31	0.31
MF-DFM 2 N	1.01	1.01	1.01	1.01	0.99	0.96	0.94	0.92	0.91	0.92	0.92	0.91	0.89	0.88	0.86	0.85	0.86	0.86
MF-DFM 2 S	1.02	1.02	1.01	1.01	0.99	0.96	0.94	0.92	0.91	0.93	0.93	0.92	0.9	0.88	0.86	0.85	0.87	0.86
MF-DFM 1 N	1.01	1.01	1.00	1.01	0.99	0.97	0.95	0.94	0.92	0.93	0.93	0.92	0.9	0.89	0.87	0.87	0.89	0.87

D. Code manual

The majority of the code is written by myself. The only exceptions are:

- The `download_data.py` file that is a modified version of the `fredapi` library.
- I make use of the `gamrnd` function from the Matlab Statistics & Machine Learning Toolbox.

My results can be replicated as follows:

- run `download_data.py` with Master-Thesis-Code as the working directory
- install the MFDFM toolbox by running `install.m` (this adds the relevant folders to the Matlab PATH variable)
- if you want to change the setup of the forecast experiment, modify the file `change_setup.m` and run it (optional)
- run `run_experiments_mfdfm.m` and `run_experiments_arp.m` with `forecast_experiments` as the working directory
- run the matlab and julia files in the folders `plots_and_tables_matlab` and `plots_julia` in order to generate the plots and tables in the thesis
- uninstall the MFDFM toolbox by running `uninstall.m` (optional)

Erklärung

Ich erkläre, dass ich meine Masterarbeit „Macroeconomic forecasting with Bayesian mixed-frequency dynamic factor models“ selbstständig und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe und dass ich alle Stellen, die ich wörtlich oder sinngemäß aus Veröffentlichungen entnommen habe, als solche kenntlich gemacht habe. Die Arbeit hat bisher in gleicher oder ähnlicher Form oder auszugsweise noch keiner Prüfungsbehörde vorgelegen. Ich versichere, dass die eingereichte schriftliche Fassung der auf dem beigefügten Medium gespeicherten Fassung entspricht.

Amsterdam, 10. September 2018 Daniel Schmidt