1) $Y = a + bX$ each datapoint: $y_i = a + bx_i$

$$m(Y) = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$M(Y) = \frac{1}{N}\sum_{i=1}^{N} a + bx_i$$

$$N(M(Y)) = \sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i$$

$$N(M(Y)) = Na + b\sum_{i=1}^{N} x_i$$

$$M(Y) = \frac{N}{N}a + b\frac{1}{N}\sum_{i=1}^{N} x_i$$

$$M(Y) = a + bM(X)$$

$$M(a+bX) = a + bM(X)$$

2) $$\mathrm{cov}(x,x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(X)))$$

$$\mathrm{cov}(x,x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$$

$$\mathrm{cov}(x,x) = s^2$$

3) $\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$

$$\mathrm{cov}(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - (a+bM(Y))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(a+by_i - a - bM(Y))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(b(y_i - M(Y))$$

$$= b(\frac{1}{N}\sum_{i=1}^{N}(x_i - M(X))(y_i - M(Y))$$

$$= b\,\mathrm{cov}(X, Y)$$

4) $\mathrm{cov}(a+bX, a+bY)$

by our proof in #3: $\mathrm{cov}(a+bx, a+bY) = b\,\mathrm{cov}(a+bx, Y)$

$$b\,\mathrm{cov}(a+bx, Y) = b\,(b\,\mathrm{cov}(X, Y))$$

$$\mathrm{cov}(a+bx, a+by) = b^2\,\mathrm{cov}(X, Y)$$

5) Since $b>0$, order is maintained.

med$(a+bx)$ = all values are scaled by b, shifted by A, and then the median is taken

$a+b(\text{med}(x))$ = median is taken, and then all values are scaled/shifted.

Because $b>0$ keeps the order, the middle item (median) remains the middle item through transformations.

IQR: $IQR(X) = Q_3(X) - Q_1(X)$

$Y = a + bX$ where $b > 0$

$Q_1(Y) = a + bQ_1(X)$, $Q_3(Y) = a + bQ_3(X)$

$IQR(Y) = a + bQ_3(X) - a - bQ_1(X)$

$IQR(Y) = b(Q_3(X) - Q_1(X))$

$IQR(Y) = b(IQR(X)$

$IQR(a+bX) = b(IQR(X)$

Therefore, $IQR(a+bX) = a + b(IQR(X))$

is only true when $a = 0$.

6) X^2:

$X = \{0, 1, 2\}$

mean of $X^2 = \frac{0+1+2^2}{3} = 5/3$

mean of $X = \frac{1+2}{3} = 1$

$(m(X))^2 = 1 = 1$

$1 \neq 5/3$ by counterexample, $M(X^2) \neq (M(X))^2$

$\sqrt{X}$:

$x = \{0, 1, 4\}$

mean of $\sqrt{X} = \frac{0+1+2}{3} = 1$

mean of $X = 5/3$

$\sqrt{(m(x))} = \sqrt{5/3}$

$1 \neq \sqrt{5/3}$

by counterexample, $M(\sqrt{X}) \neq \sqrt{M(X)}$