We start with a compressible Naviere-Stokes equation with a given equation of state.

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} + \frac{\mu}{3\rho}\nabla(\nabla \cdot \vec{u}) + \frac{\mu}{\rho}\nabla^2 \vec{u} \tag{1}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{u}) \tag{2}$$

$$p = c^2 \rho \tag{3}$$

To simplify the equations we non-dimensionalize the equations using the following scaling:

$$\vec{x} = \vec{\tilde{x}} \cdot L \tag{4}$$

$$\vec{u} = \vec{\tilde{u}} \cdot U \tag{5}$$

$$t = \tilde{t}\frac{L}{U} \tag{6}$$

$$\rho = \tilde{\rho}\rho_0 \tag{7}$$

$$p = \tilde{\rho}\rho_0 U^2 \tag{8}$$

We apply this to our version of the Naviere-Stokes equation, and solve for the $\sim$ values, When this is done, we can remove the $\sim$ signs, and we get:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} + \frac{1}{3\rho R_e}\nabla(\nabla \cdot \vec{u}) + \frac{1}{\rho R_e}\nabla^2 \vec{u} \tag{9}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{u}) \tag{10}$$

$$p = \frac{\rho}{M^2} \tag{11}$$

Where we have the reynolds number

$$R_e = \frac{\rho_0 U L}{\mu}$$

and the mach number

$$M = \frac{U}{c}$$