Contents

■ TEST BY GENERATING A PERFECT SYMMETRIC ARRAY

```
clear; close all;
9888888888888888888
%This is for JONES radar
C = 299.792458;% Speed of light in Mega m/s
F0 = 31;\% 47.0 MHz
lambda0=C/F0;% meter
xy = [...
    0, 2;...
    0, -2.5;...
    -2, 0;...
    2.5, 0;...
    0, 0];
xpos = zeros(size(xy,1),1)*lambda0;
ypos = zeros(size(xy,1),1)*lambda0;
zpos = zeros(size(xy,1),1)*lambda0;
c0 = 2.99792e8;
%THIS IS FOR PANSY RADAR
%%%%%%%%%%%%%%%%%%%%%%%%
% f0 = 46.5e6;
% % f0 = 50e6;
% lambda0 = c0/f0;
% load('subarrypos101.txt')
% core = find(sqrt(sum(subarrypos101.^2,2)) <= 50);
% outlier = find(sqrt(sum(subarrypos101.^2,2)) > 50);
% subarrypos101(outlier,:) = [];
% subarrypos101(6,:) = subarrypos101(end,:);
subarrypos101(end,:) = [0,0];
%THIS IS FOR EISCAT 3D
% C = 299.792458;% Speed of light in Mega m/s
% F0 = 233;% 47.0 MHz
% lambda0=C/F0;% meter
% xy = subarrypos101./lambda0;
% xpos = zeros(size(xy,1),1)*lambda0;
% ypos = zeros(size(xy,1),1)*lambda0;
% zpos = zeros(size(xy,1),1)*lambda0;
%THIS is for MU RADAR
% C = 299.792458;% Speed of light in Mega m/s
% F0 = 46.5; % 46.5 MHz
% lambda0=C/F0;% meter
\% \ \log (\ '\ /\ me/danielk/IRF/IRF\_GITLAB/METEOR\_MUSIC\_MC\_ORB\_DISTRIBUTION/DEPLOY/xy\_muant 25.mat')
% [xpos,ypos,zpos] = muantennas();
% INDEX_OF_ASSYMETRIC = 1:4:21;
% INDEX_OF_SYMETRIC = 1:25;
% INDEX_OF_SYMETRIC(INDEX_OF_ASSYMETRIC) = [];
% xpos = xpos(INDEX_OF_SYMETRIC,:);
% ypos = ypos(INDEX_OF_SYMETRIC,:);
% zpos = zpos(INDEX_OF_SYMETRIC,:);
% xy = xy(INDEX_OF_SYMETRIC,:);
% THIS IS FOR JICAMARCA RADAR
```

TEST BY GENERATING A PERFECT SYMMETRIC ARRAY

```
% d = 3;
% % xy = [d,0;...
        -d,0;...
% %
% %
        0,d;...
% %
        0.-d:..
        d/sqrt(2),d/sqrt(2);...
왕 왕
        -d/sqrt(2),d/sqrt(2);...
% %
왕 왕
        d/sqrt(2),-d/sqrt(2);...
왕 왕
        -d/sqrt(2),-d/sqrt(2);...
% %
        0,0];
xy = [d*cosd(67.5), d*sind(67.5);...
%
      d*cosd(-67.5), d*sind(-67.5);...
      d*cosd(112.5),d*sind(112.5);...
%
      d*cosd(-112.5),d*sind(-112.5);...
%
      0,0];
% xpos = zeros(5,1);
% ypos = zeros(5,1);
% zpos = zeros(5,1);
xy = [d*cosd(67.5), d*sind(67.5);...
      d*cosd(112.5),d*sind(112.5);...
      0,-d;...
      0,0];
% xpos = (zeros(4,4) + repmat([0,0,-d/6,d/6],4,1) + repmat(xy(:,1),1,4))*lambda0;
% ypos = (zeros(4,4) + repmat([-d/6,d/6,0,0],4,1) + repmat(xy(:,2),1,4))*lambda0;
% zpos = zeros(4,4);
% MP_tol = 1e-3
% MP_tol = 3.5e-2
MP_{tol} = 1e-1
% MP_tol = 1.13e-1
gain_yagi = 7.24;
gain_yagi_base = 10^(gain_yagi/10);
%figure size configurations
axis_font_size = 18;
title_font_size = 16;
legend_font_size = 14;
xant = xpos/lambda0;
yant = ypos/lambda0;
Zn = size(zpos, 2); %subgroup size
\% DO NOT COUNT THE LAST GROUP AS IT IS DEFINED AS THE ORIGIN
Sn = size(zpos, 1); %sensorgroups
Sn = Sn - 1;
% Sn = 4;
r=zeros(3,Zn,Sn);
R=zeros(3,Sn);
R(:,:) = [xy(1:Sn,1).'; xy(1:Sn,2).'; (xy(1:Sn,1).')*0];
    r(:,:,i) = [xpos(i,:); ypos(i,:); zpos(i,:)]./lambda0;
end
rho=zeros(3,Zn,Sn);
for i=1:Sn
    rho(:,:,i) = r(:,:,i) - repmat(R(:,i),1,Zn);
k=@(th,ph) [cos(th).*sin(ph); sin(th).*sin(ph); cos(ph)];
%calculate the linear coefficients
```

```
K = zeros(Sn,1);
 for i=1:Sn
          Rni = norm(R(:,i));
          K(i) = R(1,i)^2/Rni + R(2,i)^2/Rni + R(3,i)^2/Rni;
end
% calculate the base numbers
n0 = floor(2*K);
% k's from 1 to 2n0+1
k_{ength} = 2*n0+1;
%calculate all the integer planes
r_{jk} = @(j,k) (n\theta(j)+1-k)/K(j);
 \begin{array}{lll} p \bar{\theta}_{-j} k & = & & & & & & & & \\ p \bar{\theta}_{-j} k & = & & & & & \\ p \bar{\theta}_{-j} k & & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & & & & \\ p \bar{\theta}_{-j} k & & 
 nvec_j = @(j) R(:,j)./norm(R(:,j));
I_j = @(j,x,y,z) R(1,j)*x + R(2,j)*y + R(3,j)*z;
%create all possible permutations from 3 first sets of planes
PERMS_n = prod(k_length(1:3));
PERMS J = zeros(PERMS n, 3);
cnt = 0;
 for i1=1:k_length(1)
          for i2=1:k_length(2)
                   for i3=1:k_length(3)
                            cnt = cnt + 1:
                            PERMS_J(cnt,:) = [i1,i2,i3];
                  end
         end
end
 fprintf('First intersection calculation: %i permutations of 3 planes\n',PERMS_n);
%3 first sets of planes
I = [1,2,3];
%start looping over all combinations
 pinv_norm = zeros(PERMS_n,1);
intersection_line = zeros(3,PERMS_n);
parfor j=1:PERMS_n
          %intersection matrix for planes J from groups I
          J = PERMS_J(j,:);
         W_matrix = zeros(length(I),3);
          b_vector = zeros(length(I),1);
          for i=1:length(I)
                   W_{matrix}(i,:) = nvec_j(I(i)).';
                   b\_vector(i) = -dot(nvec\_j(I(i)),p0\_jk(I(i),J(i)));
          % solution_check
         Moore Penrose solution check = W matrix*pinv( W matrix )*b vector - b vector;
         intersection_line(:,j) = pinv( W_matrix )*b_vector;
         pinv_norm(j) = norm(Moore_Penrose_solution_check);
intersection_line_norm = sqrt(sum(intersection_line.^2,1)).';
 intersections3_inds = find(pinv_norm < MP_tol & intersection_line_norm <= 2 & intersection_line_norm ~= 0);</pre>
intersections3_n = length(intersections3_inds)
intersections3 = PERMS_J(intersections3_inds,:);
 SURVIVORS = zeros(Sn-2,1);
SURVIVORS(1) = intersections3_n;
intersections_n = intersections3_n;
intersections_inds = intersections3_inds;
intersections = PERMS_J(intersections_inds,:);
intersections_integers = R.'*intersection_line(:,intersections_inds.');
%%%% STEP 2
 for ii=4:Sn
          PERMS_J_base = PERMS_J;
          %create all possible permutations from ii first sets of planes with only the
          %surviving set + all new
          %and recursibvly iterate
          PERMS_n = intersections_n*k_length(ii);
```

```
fprintf('Starting plane intersections for new sensor %i of %i with %i permutations on %i remaining solutions\n',ii,Sn,PERMS_n, intersections_n);
    PERMS J = zeros(PERMS n,ii);
    cnt = 0;
    for iperm=1:intersections n
        for i_add=1:k_length(ii)
            cnt = cnt + 1;
            PERMS_J(cnt,:) = [PERMS_J_base(intersections_inds(iperm),:), i_add];
    %4 first sets of planes
    I = [1:ii];
    %start looping over all combinations
    pinv_norm = zeros(PERMS_n,1);
    intersection_line = zeros(3,PERMS_n);
    parfor j=1:PERMS_n
        %intersection matrix for planes J from groups I
        J = PERMS_J(j,:);
        W_matrix = zeros(length(I),3);
        b_vector = zeros(length(I),1);
        for i=1:length(I)
            W_matrix(i,:) = nvec_j(I(i)).';
            b_{\text{vector}(i)} = -\text{dot}(\text{nvec}_{j}(I(i)), p0_{j}k(I(i),J(i)));
        % solution check
        Moore Penrose solution check = W matrix*pinv( W matrix )*b vector - b vector;
        intersection_line(:,j) = pinv( W_matrix )*b_vector;
        pinv_norm(j) = norm(Moore_Penrose_solution_check);
    end
    intersections_inds = find(pinv_norm < MP_tol);</pre>
    intersections_n = length(intersections_inds);
intersections = PERMS_J(intersections_inds,:);
    intersections_integers = R.'*intersection_line(:,intersections_inds.');
    SURVIVORS(ii-2) = intersections_n;
end
intersections__last = intersections_n
intersections_integers_complete = [zeros(1, size(intersections_integers, 2)); intersections_integers];
ambiguity_distances_INT_FORM_MAT = abs(intersections_integers_complete - round(intersections_integers_complete));
ambiguity_distances_INT_FORM_mean = mean(ambiguity_distances_INT_FORM_MAT,1)
ambiguity_distances_WAVE_FORM_MAT = exp(li*2*pi*intersections_integers_complete) - exp(li*2*pi*round(intersections_integers_complete));
ambiguity_distances_WAVE_FORM = sqrt(sum(ambiguity_distances_WAVE_FORM_MAT.*conj(ambiguity_distances_WAVE_FORM_MAT),1))
fign = 0;
el0 = 50;
az0 = 270;
k0x = sind(az0)*cosd(el0);
k0y = cosd(az0)*cosd(el0);
k0z = sind(el0):
k0 = [k0x: k0v: k0z1:
cutoff_ph_ang = 90*pi/180;
%find all s-lines that intersect with the cap by range check
cap\_intersections\_of\_slines = repmat([k0x; k0y], 1, length(intersections\_inds)) - intersection\_line(1:2, intersections\_inds);
{\tt cap\_intersections\_of\_slines = sqrt(sum(cap\_intersections\_of\_slines.^2,1));}
cap_intersections_of_slines = cap_intersections_of_slines <= sin(cutoff_ph_ang);</pre>
cap_intersections_of_slines = find(cap_intersections_of_slines);
%from knowing what lines intersect with cap, find all possible DOA ambigs
%that are part of this
s_sel = intersection_line(1:3,intersections_inds(cap_intersections_of_slines));
k_finds = zeros(size(s_sel));
k_{\text{finds}}(1:2,:) = \text{repmat}([k0x; k0y],1,size(s_sel,2)) - s_sel(1:2,:);
k_{finds}(3,:) = sqrt(1- k_{finds}(1,:).^2 - k_{finds}(2,:).^2);
SUBGROUP_signal=@(k) exp ( -1i*2*pi*(xy(:,1)*k(1) +...
                                        xy(:,2)*k(2));
ambiguity_distances_EXPLICIT = zeros(1,size(k_finds,2));
```

```
ambiguity_normal_EXPLICIT = zeros(size(xy,1),size(k_finds,2));
for i = 1:size(k_finds,2)
    ambiguity\_distances\_EXPLICIT(1,i) = norm(SUBGROUP\_signal(k0) - SUBGROUP\_signal(k\_finds(:,i)));
    ambiguity_normal_EXPLICIT(:,i) = (SUBGROUP_signal(k0) - SUBGROUP_signal(k_finds(:,i)))/ambiguity_distances_EXPLICIT(1,i);
ambiguity_distances_EXPLICIT
ambiguity_normal_EXPLICIT
tick_font_size = 18;
fign = fign + 1; figure(fign); clf;
% set( gcf, 'Color', 'White', 'Unit', 'Normalized', ...
      'Position', [0.1,0.1,0.8,0.6] );
% - Build title axes and title.
% axes( 'Position', [0, 0.95, 1, 0.05] );
% set( gca, 'Color', 'None', 'XColor', 'White', 'YColor', 'White' );
subplot(1,1,1);
h = plot(xy(:,1)*lambda0,xy(:,2)*lambda0, 'ob');
for i=1:(size(xy,1))
plot(xant(i,:)*lambda0, yant(i,:)*lambda0, '*', 'color', [.8,.6,.6]);
if size(yant,2) > 1
K = convhull(xant(i,:)*lambda0, yant(i,:)*lambda0);
plot(xant(i,K)*lambda0, yant(i,K)*lambda0, '-k');
end
end
hold off
% axis('equal')
lh = legend('Sensor position','Subgroup antennas');
xh = xlabel('x [m]');
yh = ylabel('y [m]');
th = title('MU-radar sensor configuration');
set([xh,yh,th,lh],'Interpreter','latex','fontsize',axis_font_size+8)
axis equal
ax = ancestor(h, 'axes');
ax.XAxis.FontSize = tick_font_size;
ax.YAxis.FontSize = tick_font_size;
fign = fign + 1; figure(fign); clf;
plot(xant', yant', '*', 'color', [.8, .6, .6]);
hold on
plot(xy(:,1),xy(:,2), 'ob');
hold off
axis('equal')
fign = fign + 1; figure(fign); clf;
% plot(xant', yant', '*');
plot(xy(:,1),xy(:,2), 'o');
% hold off
axis('equal')
fign = fign + 1; figure(fign); clf;
set( gcf, 'Color', 'White', 'Unit', 'Normalized', ...
     'Position', [0.1,0.1,0.8,0.6] );
% - Build title axes and title.
axes( 'Position', [0, 0.95, 1, 0.05] );
set( gca, 'Color', 'None', 'XColor', 'White', 'YColor', 'White');
subplot(1.1.1):
plot(3:Sn,SURVIVORS)
xh = xlabel('Number of sensors included');
yh = ylabel('Number of common plane intersections');
th = title('Intersection plane counts');
set([xh,yh,th],'Interpreter','latex','fontsize',axis_font_size)
fign = fign + 1; figure(fign); clf;
set( gcf, 'Color', 'White', 'Unit', 'Normalized', ...
     'Position', [0.1,0.1,0.8,0.6] );
% - Build title axes and title.
axes( 'Position', [0, 0.95, 1, 0.05] );
set( gca, 'Color', 'None', 'XColor', 'White', 'YColor', 'White' );
plot(intersection_line(1,intersections_inds),intersection_line(2,intersections_inds),'.b')
xh = xlabel('$s_{x}$ [1]');
yh = ylabel('$s_{y}$ [1]');
th = title('Intersection lines');
axis([-2,2,-2,2])
set([xh,yh,th],'Interpreter','latex','fontsize',axis_font_size)
% save('MUSIC_DOA_AMBIG_base.mat');
```

```
% inds_inside_circle = find(sqrt(sum(intersection_line(:,intersections_inds).^2,1)) < sind(18)*2.5)
fign = fign + 1; figure(fign); clf;
set( gcf, 'Color', 'White', 'Unit', 'Normalized', ...
   'Position', [0.1,0.1,0.8,0.6] );
\ensuremath{\$} - Build title axes and title.
axes( 'Position', [0, 0.95, 1, 0.05] );
set( gca, 'Color', 'None', 'XColor', 'White', 'YColor', 'White' );
subplot(1,1,1);
hold on
for S_ind = 1:length(intersections_inds)
     fprintf('Starting Sind %i of %i\n',S_ind,length(intersections_inds));
     % for S_ind = 3
    s_point = intersection_line(:,intersections_inds(S_ind));
    s_line = repmat(s_point,1,100);
    s_line(3,:) = linspace(-sqrt(4-dot(s_point,s_point)),sqrt(4-dot(s_point,s_point)),100);
           s_line(3,:)
                        = linspace(0,sqrt(4-dot(s_point,s_point)),line_resolution);
     if S ind == 25
        plot3(s_line(1,:),s_line(2,:),s_line(3,:),'.r')
   h = plot3(s_line(1,:),s_line(2,:),s_line(3,:),'.b');
    end
end
hold off
xh = xlabel('$s_{X}$ [1]');
yh = ylabel('$s_{Y}$ [1]');
zh = zlabel('$s_{z}$ [1]');
th = title('Solution set $\Omega$');
axis([-2,2,-2,2])
view([-143,53])
set([th],'Interpreter','latex','fontsize',axis_font_size+6)
set([xh,yh,zh],'Interpreter','latex','fontsize',axis_font_size+12)
ax = ancestor(h, 'axes');
ax.XAxis.FontSize = 19:
ax.YAxis.FontSize = 19:
ax.ZAxis.FontSize = 19;
fign = fign + 1; figure(fign); clf;
set( gcf, 'Color', 'White', 'Unit', 'Normalized', ...
     'Position', [0.1,0.1,0.8,0.6] );
  - Build title axes and title.
axes( 'Position', [0, 0.95, 1, 0.05] );
set( gca, 'Color', 'None', 'XColor', 'White', 'YColor', 'White' );
subplot(1,1,1);
for S_ind = 1:length(intersections_inds)
    s_point = intersection_line(:,intersections_inds(S_ind));
   plot(s_point(1),s_point(2),'.b')
   text(s point(1)+0.1,s point(2),sprintf('%.2f',ambiguity distances WAVE FORM(S ind)));
end
hold off
xh = xlabel('$s_{x}$ [1]');
yh = ylabel('$s_{y}$ [1]');
th = title('Solution set $\Omega$');
axis([-2,2,-2,2])
set([xh,yh,th],'Interpreter','latex','fontsize',axis_font_size)
\label{eq:circ_cutoff_ph_ang_x = sin(cutoff_ph_ang)*cos(linspace(0,2*pi,100));} \\
circ_cutoff_ph_ang_y = sin(cutoff_ph_ang)*sin(linspace(0,2*pi,100));
fign = fign + 1; figure(fign); clf;
                                        'Normalized', ...
set( gcf, 'Color', 'White', 'Unit',
     'Position', [0.1,0.1,0.8,0.6] );
\ensuremath{\$} - Build title axes and title.
axes( 'Position', [0, 0.95, 1, 0.05] );
set( gca, 'Color', 'None', 'XColor', 'White', 'YColor', 'White' );
subplot(1,2,1);
hold on
for S_ind = 1:length(intersections_inds)
     s_point = intersection_line(:,intersections_inds(S_ind));
   plot(s_point(1),s_point(2),'.b')
   text(s_point(1)+0.1,s_point(2),sprintf('%.2f',ambiguity_distances_WAVE_FORM(S_ind)));
plot(k0x,k0y,'or')
plot(circ_cutoff_ph_ang_x ,circ_cutoff_ph_ang_y,'-r')
hold off
xh = xlabel('$s_{x}$ [1]');
```

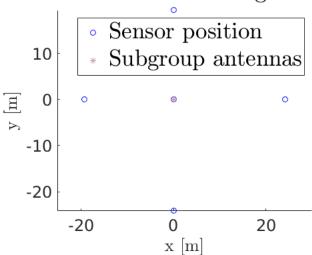
```
yh = ylabel('$s_{y}$ [1]');
th = title('Solution set $\Omega$');
axis([-2.2.-2.21)
set([xh,yh,th],'Interpreter','latex','fontsize',axis_font_size)
subplot(1,2,2);
hold on
plot(k0x,k0y,'or')
for i = 1:size(k_finds,2)
   plot(k_finds(1,i),k_finds(2,i),'.b')
   text(k\_finds(1,i)+0.1,k\_finds(2,i),sprintf('\%.2f',ambiguity\_distances\_EXPLICIT(i)));
hold off
xh = xlabel('$k_{x}$ [1]');
yh = ylabel('$k_{y}$ [1]');
th = title('Explicit ambiguities $\Omega(\mathbf{k})$');
axis([-1,1,-1,1])
set([xh,yh,th],'Interpreter','latex','fontsize',axis_font_size)
MP_tol =
    0.1000
First intersection calculation: 891 permutations of 3 planes
Starting parallel pool (parpool) using the 'local' profile ...
connected to 4 workers.
intersections3_n =
    34
Starting plane intersections for new sensor 4 of 4 with 374 permutations on 34 remaining solutions
intersections last =
    16
ambiguity_distances_INT_FORM_mean =
  Columns 1 through 7
    0.0000
              0.0450
                        0.0900
                                  0.0450
                                            0.0900
                                                      0.0000
                                                                0.0450
  Columns 8 through 14
    0.0450
              0.0450
                        0.0450
                                  0.0000
                                            0.0900
                                                      0.0450
                                                                0.0900
  Columns 15 through 16
    0.0450
              0.0000
ambiguity_distances_WAVE_FORM =
  Columns 1 through 7
    0.0000
              0.9837
                        1.3912
                                  0.9837
                                            1.3912
                                                      0.0000
                                                                0.9837
  Columns 8 through 14
    0.9837
              0.9837
                        0.9837
                                  0.0000
                                            1.3912
                                                      0.9837
                                                                1.3912
  Columns 15 through 16
    0.9837
              0.0000
ambiguity_distances_EXPLICIT =
    0.9837
             1.3912
                       0.9837
                                 0.9837
                                            0.9837
                                                      1.3912
ambiguity_normal_EXPLICIT =
  Columns 1 through 4
                      0.1373 - 0.4225i
   0.1941 - 0.5975i
                                         0.0000 - 0.0000i
                                                            0.0000 - 0.0000i
   0.2977 - 0.7188i
                      0.2105 - 0.5083i
                                         0.0000 + 0.0000i
                                                            0.0000 + 0.0000i
   0.0000 + 0.0000i
                      0.3816 - 0.2275i
                                         0.5396 - 0.3218i -0.6257 - 0.0569i
   0.0000 + 0.0000i
                      0.1517 - 0.5288i
                                         0.2146 - 0.7478i -0.6805 + 0.3771i
                                         0.0000 + 0.0000i
                      0.0000 + 0.0000i
   0.0000 + 0.0000i
                                                           0.0000 + 0.0000i
  Columns 5 through 6
   0.1941 + 0.5975i
                      0.1373 + 0.4225i
   0.2977 + 0.7188i
                      0.2105 + 0.5083i
   0.0000 + 0.0000i
                      0.3816 - 0.2275i
   0.0000 + 0.0000i
                      0.1517 - 0.5288i
```

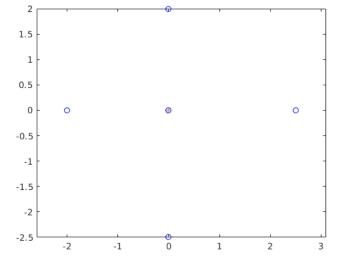
Starting Sind 16 of 16

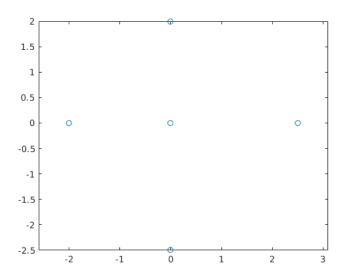
```
0.0000 + 0.0000i
                     0.0000 + 0.0000i
Starting Sind 1 of 16
Warning: Imaginary parts of complex X, Y, and/or Z arguments ignored
Starting Sind 2 of 16
Starting Sind 3 of 16
Starting Sind 4 of 16
Starting Sind 5 of 16
Starting Sind 6 of 16
Starting Sind 7 of 16
Starting Sind 8 of 16
Starting Sind 9 of 16
Starting Sind 10 of 16
Starting Sind 11 of 16
Starting Sind 12 of 16
Starting Sind 13 of 16
Starting Sind 14 of 16
Starting Sind 15 of 16
```

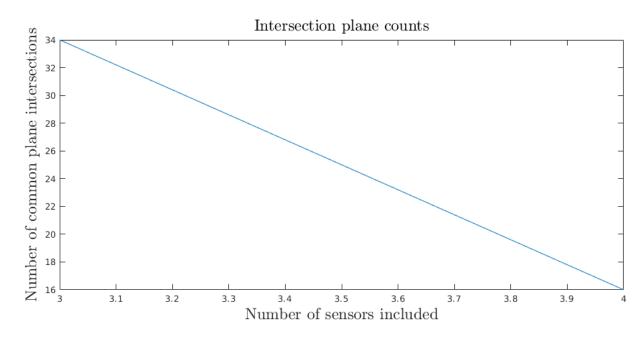
Warning: Imaginary parts of complex X, Y, and/or Z arguments ignored

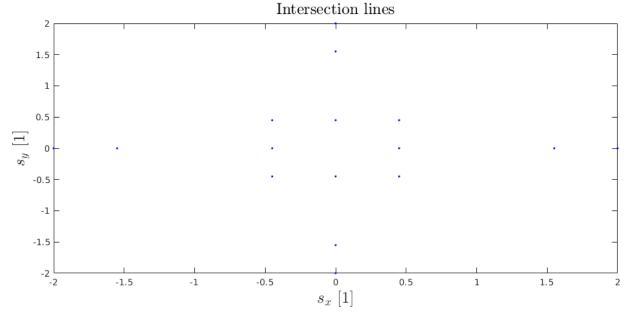
MU-radar sensor configuration



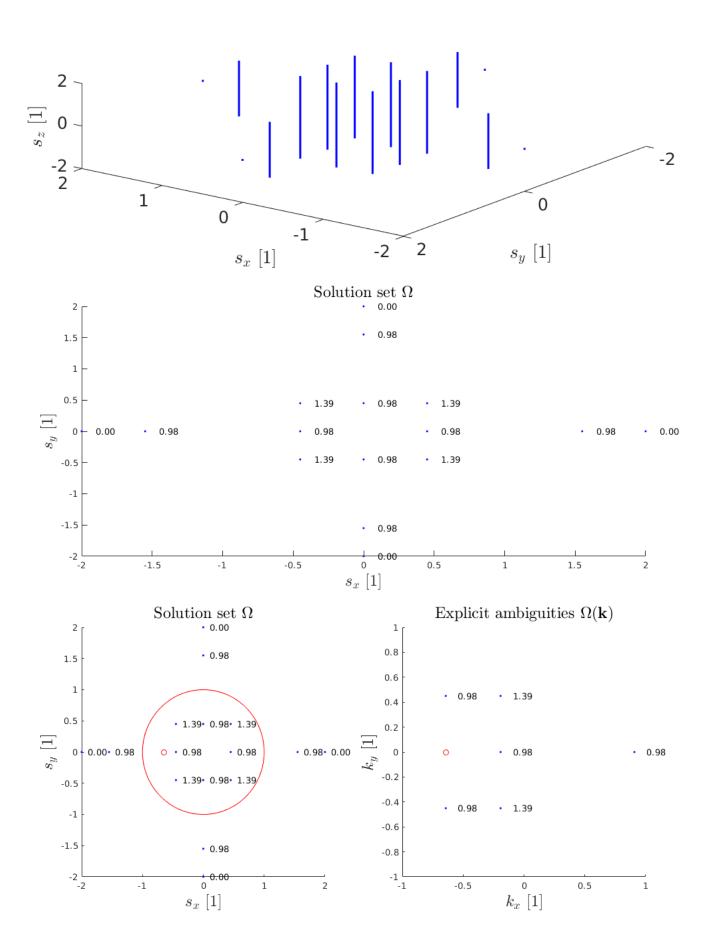








Solution set Ω



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