Computational Applications in International Trade
Lecture 3: Endogenous Production Networks

Felix Tintelnot

University of Chicago

Open Source Economics Laboratory
August 7th, 2019

Motivation

- Few firms are directly participating in international trade
- Of course, firms can be indirectly exposed to international trade through the domestic network
- Want to take domestic firm-to-firm connections into account as well as how the domestic network responds to trade shocks

Motivation: Evidence from the Firm-to-Firm network in Belgium

- Three main data sources:
  - NBB B2B transaction database: Panel of VAT-id to VAT-id transactions among the universe of Belgian firms, over years 2002-2014 (Dhyne et al., 2015).
  - Annual accounts: total sales, labor cost, ownership of other VAT-ids, zip code, 4-digit NACE code, etc.
  - Int'l trade dataset: VAT-year-country-product(CN8) level.
- We aggregate VAT-ids into firms. ▸ Aggregation details
- Selected sample includes firms in the non-financial private sector with at least one full-time employee. ▸ Sampling ▸ Sectoral composition

Summary of descriptive results

1. Fact 1: Firms obtain a large share of their imports indirectly
2. Fact 2: By comparison, fewer firms are connected to the export market
3. Fact 3: Firms do not have to be large to obtain a large fractions of imports indirectly or to see most of their sales go to the foreign market
4. Fact 4 (older working paper version): Import supply and export demand shocks transmit through the production network

Direct and indirect importing

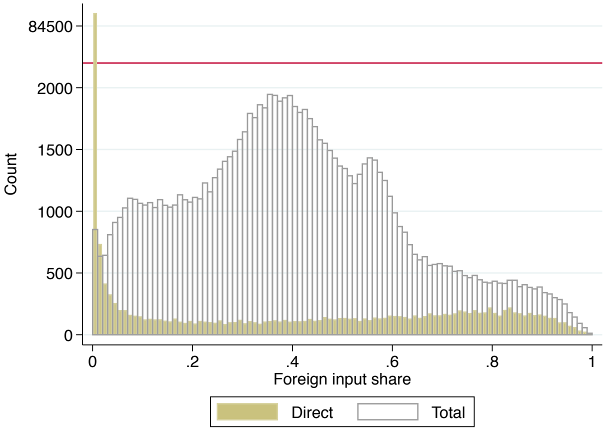
- Firm j's direct foreign input share:

$$s_{Fj} = \frac{\text{Imports}_j}{\text{Labor costs}_j + \text{Dom. purchases}_j + \text{Imports}_j}$$

- Firm j's total foreign input share:

$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j^D} s_{ij} \underbrace{\left[s_{Fi} + \sum_{k \in Z_i^D} s_{ki} \left(s_{Fk} + \cdots \right) \right]}_{s_{Fi}^{Total}}$$

Direct and indirect importing



99% of firms import directly or indirectly; Median: 39%; 80th percentile: 58%

▸ by sector ▸ excluding imports by wholesale and retail sectors

Approaches to endogenous production networks

1. Stochastic processes approach:
   - Jackson and Rogers (2007); Atalay, Hortacsu, Roberts, and Syverson (2011); Chaney (2014)
   - No firm optimization in network formation

2. Extreme value approach:
   - Oberfield (2018); Eaton, Kortum, and Kramarz (2018)
   - Constant returns to scale, continuum of firms, select single supplier for each task. Difficulties in matching empirical relationship between firm size and outsourcing.

3. Fixed cost approach:
   - Lim (2018); Tintelnot, Kikkawa, Mogstad, and Dhyne (2018); Taschereau-Dumouchel (2018)

Challenges

- Large fixed point problem: firms make choices based on cost / prices by other firms that themselves are equilibrium objects.
- Large discrete choice problem of which suppliers to add.
- Network effect of pair-wise choice: your choice to sell to firm a may affect you indirectly in your transaction with firm b if firm a is a supplier to firm b.
- Strategic interaction: a typical firm has 33 suppliers and 9 customers.

Overview: Lim (2018)

- Lim (2018) assumes that there are continuum of firms and that the seller makes the decision whether to offer the good to a supplier.
  - Seller can go independently through list of customer firms and decide whether it's worth incurring the fixed cost to sell
  - Everyone is measure zero, so stuff never comes back to you (almost surely)
  - No strategic interaction and no need to internalize the network effect

TKMD (2019)

- TKMD allow the buyer firm to decide from whom to purchase inputs. Assume that there are is an ordering of firms, that determines who can purchase from whom.
  - Solve large discrete choice problem via Jia (2008)'s algorithm
  - Assume no mark-ups in firm-to-firm trade (buyer has the full bargaining power), monopolistic competition in final goods trade
  - Acyclic network structure implies sold goods are not coming back to you or affect your profitability elsewhere

Note

- Both papers create an artificial economy that looks like the actual network in some (but not all) observed dimensions
- Cannot perfectly replicate all the observed connections between firms in the model

Lim (2018): Basic Environment

- Exogenous unit continuum of firms producing differentiated goods
- Firms heterogeneous over states $\chi \equiv (\phi, \delta)$
  - ϕ: **fundamental productivity** (labor input more productive)
  - δ: **fundamental quality** (household prefers product more)
  - exogenous distribution function G_χ and support $S_\chi \subset \mathbb{R}_+^2$

Lim (2018): Basic Environment

- Exogenous unit continuum of firms producing differentiated goods
- Firms heterogeneous over states $\chi \equiv (\phi, \delta)$
  - ϕ: **fundamental productivity** (labor input more productive)
  - δ: **fundamental quality** (household prefers product more)
  - exogenous distribution function G_χ and support $S_\chi \subset \mathbb{R}_+^2$
- Representative household supplies L units of labor inelastically, with preferences:

$$U = \left[\int_{S_\chi} \left[\delta x_H (\chi) \right]^{\frac{\sigma - 1}{\sigma}} dG_\chi (\chi) \right]^{\frac{\sigma}{\sigma - 1}}$$

- Conditional on prices, household demand $x_H (\chi)$ is greater for firms with higher δ

Lim (2018): Production Network

- Network specified by matching function m
  - $m\left(\chi,\chi^{'}\right)$ = probability that χ-firm buys from $\chi^{'}$-firm

- Production CES in labor and supplier inputs, given matching function:

$$X\left(\chi\right)=\left[\left[\phi l\left(\chi\right)\right]^{\frac{\sigma-1}{\sigma}}+\int_{S_\chi} m\left(\chi,\chi^{'}\right)\left[\alpha x\left(\chi,\chi^{'}\right)\right]^{\frac{\sigma-1}{\sigma}}dG_\chi\left(\chi^{'}\right)\right]^{\frac{\sigma}{\sigma-1}}$$

- Marginal cost:

$$\eta\left(\chi\right)=\left[\phi^{\sigma-1}+\alpha^{\sigma-1}\int_{S_\chi} m\left(\chi,\chi^{'}\right)\left[p\left(\chi,\chi^{'}\right)\right]^{1-\sigma}dG_\chi\left(\chi^{'}\right)\right]^{\frac{1}{1-\sigma}}$$

- Firms differ in labor productivity (not TFP); every firm has the same α; same elasticity of substitution as in utility function

Lim (2018): Market structure

- Market structure: monopolistic competition
- With continuum of sellers for each buyer, all firms charge constant CES markup, $\mu = \frac{\sigma}{\sigma - 1}$

Lim (2018): Production Network

- Firms characterized by endogenous variables:

$$\Phi(\chi) \equiv \eta(\chi)^{1-\sigma} \quad \text{(inverse marginal cost)}$$
$$\Delta(\chi) \equiv \frac{1}{Y_H / P_H^{1-\sigma}} X(\chi)\, \eta(\chi)^{\sigma} \quad \text{(demand shifter)}$$

- Depend on fundamental characteristics and characteristics of suppliers/customers through matching function

$$\Phi(\chi) = \phi^{\sigma-1} + \mu^{1-\sigma}\alpha^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi'\right) \Phi\left(\chi'\right) dF_\chi\left(\chi'\right)$$
$$\Delta(\chi) = \mu^{-\sigma}\delta^{\sigma-1} + \mu^{-\sigma}\alpha^{\sigma-1} \int_{S_\chi} m\left(\chi', \chi\right) \Delta\left(\chi'\right) dF_\chi\left(\chi'\right)$$

- Assume $\alpha < 1$. Decoupled contraction mappings in $\Phi(\cdot)$ and $\Delta(\cdot)$ $\Rightarrow$ easily solved

Lim (2018): Production Network

- These endogenous variables determine all variables of interest:

$$
\begin{aligned}
\text{firm revenue:} \quad & R\left(\chi\right) && \propto \Delta\left(\chi\right)\Phi\left(\chi\right) \\
\text{firm profit:} \quad & \Pi\left(\chi\right) && \propto \Delta\left(\chi\right)\Phi\left(\chi\right) \\
\text{firm-to-firm sales:} \quad & r\left(\chi,\chi^{'}\right) && \propto \Delta\left(\chi\right)\Phi\left(\chi^{'}\right) \\
\text{firm-to-firm profit:} \quad & \pi\left(\chi,\chi^{'}\right) && \propto \Delta\left(\chi\right)\Phi\left(\chi^{'}\right)
\end{aligned}
$$

- Unique market equilibrium for any matching function $m : S_\chi \times S_\chi \to [0,1]$

Lim (2018): Network Formation

- Seller pays stochastic relationship fixed cost: ξ units of labor to activate relationship
- Static variable profit earned by a $\chi^{'}$-firm from selling to χ-firm:

$$\pi\left(\chi,\chi^{'}\right)\propto\Delta\left(\chi\right)\Phi\left(\chi^{'}\right)$$

- **Acceptance function** - probability that a relationship is selected:

$$a\left(\chi,\chi^{'}\right)=\Pr\left[\pi\left(\chi,\chi^{'}\right)\geq\xi\right]=F_{\xi,t}\left[\pi\left(\chi,\chi^{'}\right)\right]$$

  where $F_{\xi,t}$ is the unconditional distribution of ξ
- Note: some relationships that buyers would like to form, do not get formed.

Lim (2018): Network Formation – Computation

- Given guess of $\{\Phi, \Delta\}$, compute π, then compute a and iterate
- Earlier version of paper included dynamic frictions:
  - $1 - \nu$ chance for each relationship to be activated/terminated
  - introduces option values into relationship formation
  - requires with more complicated dynamic algorithm
- Efficiency
  - Static market equilibrium is inefficient: double marginalization
  - End. equilibrium: additional sources of inefficiency. Do not form all relationships that (buyer) firms would like to form + network externality

Lim (2018): Application

- As a proof of concept, paper study the role of endogenous network adjustments in business cycles
- Note: decision problem is static, but imposes AR(1) on the log cost to form links and log productivity
- Suppose the network had been hold fixed - what would real output be?

Lim (2018): Data

- Compustat data
  - publicly-listed firms in the US
  - standard financial/accounting firm-level information
  - records of firms' major customers ($> 10\%$ of revenue)
  - panel data from 1979-2008, over 100,000 firm-year observations
  - firms account for just under 50% of US gross output in 2007
- Capital IQ data
  - both private and public firms
  - relationships recorded from multiple sources (publications, news reports)
  - select all firms in continental US with recorded relationship data and positive average revenue from 2003-2007
  - $\approx$9,000 firms, accounting for around 65% of US gross output in 2007

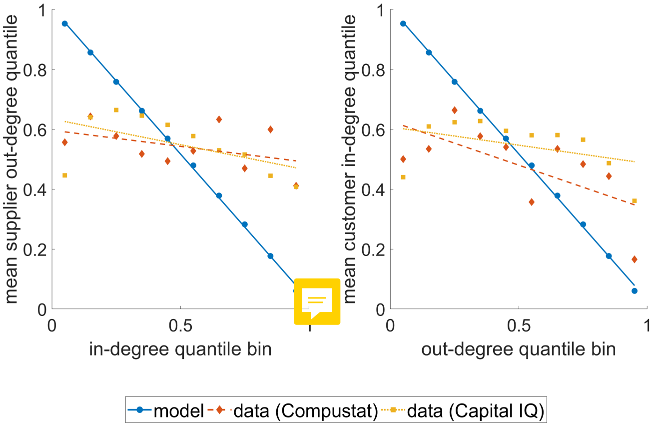
Lim (2018): Mapping model to data

- Ignore demand differences, $\delta = 1$
- AR(1) process for stochastic relationship cost to match persistence in links
- AR(1) process for productivity to match persistence of firm-level sales

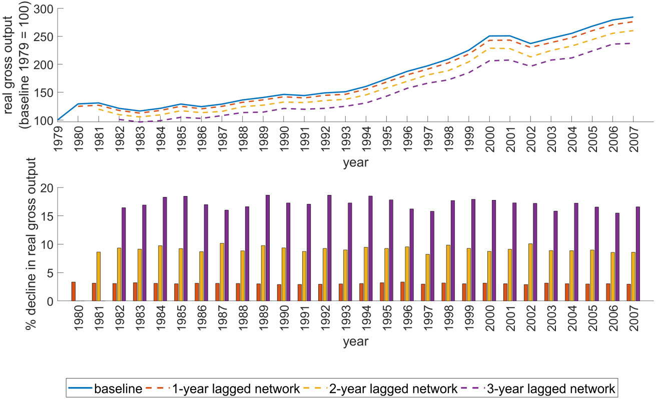
$$\overrightarrow{\phi}_t^{\sigma-1} = \frac{1}{\mu \Delta_{H,t}} \left[I - (\alpha/\mu)^{\sigma-1} M \right]^{-1} \left(\overrightarrow{R/\Delta_t} \right)$$

  - Value of general equilibrium term $\Delta_{H,t}$ is equivalent to a normalization of level of ϕ_t
  - choose arbitrary normalization in first period $t = 1$
  - calibrate level of ϕ_t for $t > 1$ to match US real GDP growth rates
- Not obvious how to match network characteristics (continuum of firms versus data)

Lim (2018): Model fit



Lim (2018): Counterfactual



- Restricting network adjustment always leads to lower output
- Endogenous network components of output are quantitatively large:
  - 4% at one-year horizons; 16% at three-year horizons
- No noticable differences between booms or busts

Huneeus (2018)

- Applies the Lim (2018) model to study the role of endogenous networks in the adjustment of Chile to the great recession (foreign demand and supply shocks)
- Uses the version of Lim's model with the Calvo fairy (firms can adjust certain links only with probablity ν)
- Assumes firm productivity is time-invariant

TKMD (2018)

- Use rich Belgian data to study the welfare effects of import price changes and their dependence on the domestic production network
- Comparison of:
  1. Fixed network: actual network versus roundabout production network
  2. Endogenous network versus Fixed Network
- Eaton and Kortum (2002), ACR (2012): Gains from trade are higher in models with intermediate goods than in models that abstract from intermediate inputs.
- Here: commonly observed firm-level and aggregate outcomes are all held fixed. Models may differ in the implied linkages between firms.

Exogenous network

Exogenous network

Model

Model environment

- Small open economy

Model environment

- Small open economy
- Fixed number of domestic firms

Model environment

- Small open economy
- Fixed number of domestic firms
- Firms sell to final consumers and to other firms

Model environment

- Small open economy
- Fixed number of domestic firms
- Firms sell to final consumers and to other firms
  - Monopolistic competition when selling to final consumers

Model environment

- Small open economy
- Fixed number of domestic firms
- Firms sell to final consumers and to other firms
  - Monopolistic competition when selling to final consumers
  - Bargaining over prices in any firm-to-firm trade (buyers have the full bargaining power)

Model environment

- Small open economy
- Fixed number of domestic firms
- Firms sell to final consumers and to other firms
  - Monopolistic competition when selling to final consumers
  - Bargaining over prices in any firm-to-firm trade (buyers have the full bargaining power)
  - Results extend to constant mark-ups in firm-to-firm trade

Model environment

- Small open economy
- Fixed number of domestic firms
- Firms sell to final consumers and to other firms
  - Monopolistic competition when selling to final consumers
  - Bargaining over prices in any firm-to-firm trade (buyers have the full bargaining power)
  - Results extend to constant mark-ups in firm-to-firm trade
- Here: Take as given firms' suppliers, customers, import and export participation

Consumers

- CES preferences (with $\sigma > 1$)
- Domestic demand:
$$q_{kH} = \beta_{kH}^{\sigma-1} \frac{p_{kH}^{-\sigma}}{P^{1-\sigma}} E$$
- Foreign demand:
$$q_{kF} = \beta_{kF}^{\sigma-1} \frac{p_{kF}^{-\sigma}}{P_F^{1-\sigma}} E_F,$$

Production Technology

- Firms produce single products
- Marginal cost of final good producer j:

$$c_j(Z_j) = \frac{1}{\phi_j} \left(\sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho} \right)^{1/(1-\rho)}$$

- Assume $\sigma > \rho > 1$

Firm-level cost changes due to foreign price change

- Given fixed linkages between firms, the change in firm j's unit cost, $\hat{c}_j \mid^{\hat{p}_{F\cdot}}$ due to an uniform change in foreign price, $\hat{p}_{F\cdot}$, is

$$\hat{c}_j \mid^{p_{F\cdot}} = \left(\left(1 - s_{Fj}^{Total}\right) \hat{w}^{1-\rho} + s_{Fj}^{Total} \hat{p}_{F\cdot}^{1-\rho} \right)^{1/(1-\rho)}$$

▸ Autarky ▸ Small changes

Firm-level cost changes due to foreign price change

- Given fixed linkages between firms, the change in firm j's unit cost, $\hat{c}_j \mid^{\hat{p}_{F\cdot}}$ due to an uniform change in foreign price, $\hat{p}_{F\cdot}$, is

$$\hat{c}_j \mid^{p_{F\cdot}} = \left(\left(1 - s_{Fj}^{Total}\right) \hat{w}^{1-\rho} + s_{Fj}^{Total} \hat{p}_{F\cdot}^{1-\rho} \right)^{1/(1-\rho)}$$

Autarky Small changes

- s_{Fj}^{Total} summarizes firm j's exposure to foreign inputs.

Change in real wage

- Given fixed linkages between firms, the change in the real wage, $\frac{\hat{w}}{\hat{P}}$, due to an uniform change in foreign price, $\hat{p}_{F\cdot}$, is:

$$\frac{\hat{w}}{\hat{P}} \, |^{p_{F\cdot}} = \hat{w} \left(\sum_i s_{iH} \left(\hat{c}_i \, |^{p_{F\cdot}} \right)^{1-\sigma} \right)^{\frac{1}{\sigma-1}}$$

$$= \left(\sum_i s_{iH} \left(\left(1 - s_{Fi}^{Total}\right) + s_{Fi}^{Total} \frac{\hat{p}_{F\cdot}^{1-\rho}}{\hat{w}^{1-\rho}} \right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}}$$

Autarky
Small changes

Implications for the gains from trade

- Effects of a change in the foreign price depends on:
  - Change in nominal wage
  - Firms' share in household demand, s_{jH}
  - Firms' total share of foreign inputs, s_{Fj}^{Total}

Implications for the gains from trade

- Effects of a change in the foreign price depends on:
  - Change in nominal wage
  - Firms' share in household demand, s_{jH}
  - Firms' total share of foreign inputs, s_{Fj}^{Total}
- These objects depend on the network structure

Implications for the gains from trade

- Effects of a change in the foreign price depends on:
  - Change in nominal wage
  - Firms' share in household demand, s_{jH}
  - Firms' total share of foreign inputs, s_{Fj}^{Total}

- These objects depend on the network structure

- Two economies with the same firm-level domestic sales, value added, exports, and imports, may have different gains from trade Example

Quantitative results

- Consider a 10 percent increase in the price of foreign goods
- Baseline: $\sigma = 4$, $\rho = 2$.
- Median firm's cost increases by 2.7%. Nominal wage falls by 2%.
  Cost changes histogram
- Real wage declines by 6%
- Calculation based on Hulten's theorem predicts a 8% decline in real wages (if no import content in exports) or 4.5% decline (if the same import content in exports and final demand). Hulten
- Firm-level data with roundabout production assumption leads to similar results for 10 percent increase in foreign price (for autarky, real wage changes 5-10 percentage points different) Table Histogram

Endogenous network

Endogenous network

Model

Model environment

- Build on model in previous section:
  - Same preferences, production function, and market power

Model environment

- Build on model in previous section:
  - Same preferences, production function, and market power
- Buyers choose which domestic suppliers to add $\rightarrow$ Endogenous formation of domestic firm-to-firm production network

Model environment

- Build on model in previous section:
  - Same preferences, production function, and market power
- Buyers choose which domestic suppliers to add $\rightarrow$ Endogenous formation of domestic firm-to-firm production network
- Firms also choose whether to import or export

Sourcing strategy and export decision

- Total profits given Z_j and I_{jF}

$$\pi_j(Z_j, I_{jF}) = \pi_j^{\textsf{\textit{var}}}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w - I_{jF} f_{jF} w$$

Sourcing strategy and export decision

- Total profits given Z_j and I_{jF}

$$\pi_j(Z_j, I_{jF}) = \pi_j^{\textsf{var}}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w - I_{jF} f_{jF} w$$

- Exogenously given set of eligible suppliers, $\mathbf{Z}_j$

Sourcing strategy and export decision

- Total profits given Z_j and I_{jF}

$$\pi_j(Z_j, I_{jF}) = \pi_j^{\textsf{var}}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w - I_{jF} f_{jF} w$$

- Exogenously given set of eligible suppliers, $\mathbf{Z}_j$
- Optimal sourcing strategy and on export participation:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j \text{ , } I_{jF} \in \{0, 1\}$$

Sourcing strategy and export decision

- Total profits given Z_j and I_{jF}

$$\pi_j(Z_j, I_{jF}) = \pi_j^{\mathit{var}}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w - I_{jF} f_{jF} w$$

- Exogenously given set of eligible suppliers, $\mathbf{Z}_j$
- Optimal sourcing strategy and on export participation:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j \ , \ I_{jF} \in \{0, 1\}$$

- Profit function is super-modular in elements of the sourcing strategy and in exporting

$$\begin{aligned} \pi_j^{\mathit{var}}(Z_j, I_{jF}) = & \frac{1}{\mu} \beta_{jD}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \frac{E}{P^{1-\sigma}} \\ & + I_{jF} \frac{1}{\mu} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} \end{aligned}$$

Formation of domestic production network

- Set of eligible suppliers, $\mathbf{Z}_j$ ▸ Picture
- Order firms in an exogenous sequence
  - Firm 1 can only hire labor
  - Firm 2 can hire labor and purchase inputs from Firm 1
  - Firm 3 can hire labor and purchase inputs from Firm 1 and 2
  - ...

Formation of domestic production network

- Set of eligible suppliers, $\mathbf{Z}_j$ ▸ Picture
- Order firms in an exogenous sequence
  - Firm 1 can only hire labor
  - Firm 2 can hire labor and purchase inputs from Firm 1
  - Firm 3 can hire labor and purchase inputs from Firm 1 and 2
  - ...
- $\mathbf{Z}_j$ has maximum 300 elements
  - Random draws of firms positioned earlier in the sequence

Formation of domestic production network

- Set of eligible suppliers, $\mathbf{Z}_j$ ▸ Picture
- Order firms in an exogenous sequence
  - Firm 1 can only hire labor
  - Firm 2 can hire labor and purchase inputs from Firm 1
  - Firm 3 can hire labor and purchase inputs from Firm 1 and 2
  - ...
- $\mathbf{Z}_j$ has maximum 300 elements
  - Random draws of firms positioned earlier in the sequence
- Sequentially solve the problem of the firms
  - Z_{k+1} does not affect firm k (other than through equilibrium objects w and A)

Equilibrium

- Similar to previous section: small open economy

Equilibrium

- Similar to previous section: small open economy
- Optimality of sourcing strategy and export decisions

Equilibrium

- Similar to previous section: small open economy
- Optimality of sourcing strategy and export decisions
- Fixed costs (paid in units of labor) enter the labor market clearing condition

Equilibrium

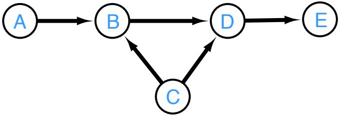
- Similar to previous section: small open economy
- Optimality of sourcing strategy and export decisions
- Fixed costs (paid in units of labor) enter the labor market clearing condition
- Trade balance condition is unchanged

Endogenous network

Shape of the network

Shape of network

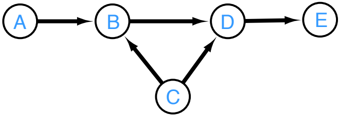
- Our theory of endogenous network formation by construction produces an acyclic directed network.



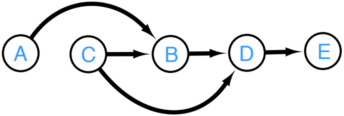
Directed acyclic network

Shape of network

- Our theory of endogenous network formation by construction produces an acyclic directed network.



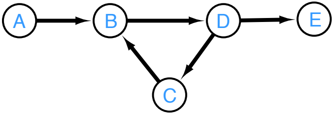
Directed acyclic network



Directed acyclic network, sorted

Shape of network

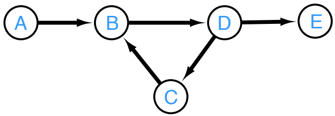
- Our theory of endogenous network formation by construction produces an acyclic directed network.



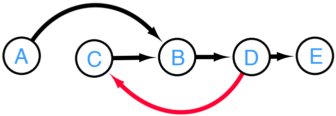
Directed cyclic network

Shape of network

- Our theory of endogenous network formation by construction produces an acyclic directed network.



Directed cyclic network



Directed cyclic network, sorted

Finding an ordering of firms

Finding an ordering of firms

- Let $\nu(i)$ be an ordering of firms that maps firms $\{i, j, k, \cdots\} \in \Theta$ into numbers from $\{1, \cdots, N\}$. We seek for the optimal $\nu(\cdot)$ that minimizes the following objective function.

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\}.$$

Finding an ordering of firms

- Let $\nu(i)$ be an ordering of firms that maps firms $\{i, j, k, \cdots\} \in \Theta$ into numbers from $\{1, \cdots, N\}$. We seek for the optimal $\nu(\cdot)$ that minimizes the following objective function.

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\}.$$

- We obtain an ordering that produces around 18% of edges or 23% of the value of transactions in the whole firm-to-firm network in 2012 violating the ordering.

Finding an ordering of firms

- Let $\nu(i)$ be an ordering of firms that maps firms $\{i, j, k, \cdots\} \in \Theta$ into numbers from $\{1, \cdots, N\}$. We seek for the optimal $\nu(\cdot)$ that minimizes the following objective function.

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\}.$$

- We obtain an ordering that produces around 18% of edges or 23% of the value of transactions in the whole firm-to-firm network in 2012 violating the ordering.
- Correlation between input-output table coefficients with 72 of full data and restricted data is 0.90 (unweighted ordering) / 0.97 (weighted ordering). ▸ Real wage changes with ordering data

Endogenous network

Empirical analysis

Estimation with endogenous domestic production network

1. Recover the productivity distribution of firms (scaled by some general equilibrium objects) from the identity

$$\frac{x_{iH}}{s_{Li}^{(\sigma-1)/(1-\rho)}} = \phi_i^{\sigma-1} \frac{\mu^{1-\sigma} w^{1-\sigma}}{P^{1-\sigma}} E.$$

Estimation with endogenous domestic production network

1. Recover the productivity distribution of firms (scaled by some general equilibrium objects) from the identity

$$\frac{x_{iH}}{s_{Li}^{(\sigma-1)/(1-\rho)}} = \phi_i^{\sigma-1}\frac{\mu^{1-\sigma}w^{1-\sigma}}{P^{1-\sigma}}E.$$

2. Estimate via simulated methods of moments distributional parameters of
   - Firm-pair specific shifter in the production function, α_{kj}, α_{Fj} details
   - Firm-specific foreign demand shifter, β_{jF}
   - Fixed cost to add domestic supplier, wf_{kj}
   - Fixed cost to export, wf_{jF}, and import wf_{Fj}
   - Assume log-normal distributions

Moments

- Domestic firm-to-firm sales, $\hat{m}_1(\delta)$:
  - Indegree
  - Outdegree
  - Labor share
  - Firm-to-Firm share
  - Calculate 25th, 50th, and 75th percentiles from data
- Imports and exports, $\hat{m}_2(\delta)$:
  - Share of importers and exporters
  - Direct and total foreign input share
  - Direct and total export share
  - Calculate 25th, 50th, and 75th percentiles from data
- Aggregate moments, $\hat{m}_3(\delta)$:
  - Ratio of agg. exports to agg. value added
  - $\sum_j s_{jH} s_{Fj}^{Total}$

Estimated parameters

$\hat{\Phi}^{\alpha_{\text{dom}}}_{scale}$	$\hat{\Phi}^{\alpha_F}_{scale}$	$\hat{\Phi}^{\beta_F}_{scale}$	$\hat{\Phi}^{\alpha,\beta}_{disp}$	$\hat{\Phi}^{f_{\text{dom}}}_{scale}$	$\hat{\Phi}^{f_{\text{imp}}}_{scale}$	$\hat{\Phi}^{f_{\text{exp}}}_{scale}$	$\hat{\Phi}^{f}_{disp}$
-4.42	-2.22	-2.01	2.34	-3.21	2.64	6.75	6.98

Model fit: targeted moments

	Data	Model
Number of dom. suppliers 25th percentile	19	17
Number of dom. suppliers 50th percentile	33	30
Number of dom. suppliers 75th percentile	55	49
Number of dom. buyers 25th percentile	2	2
Number of dom. buyers 50th percentile	9	13
Number of dom. buyers 75th percentile	34	46
Share of labor costs 25th percentile	0.17	0.12
Share of labor costs 50th percentile	0.34	0.25
Share of labor costs 75th percentile	0.54	0.49
Firm-to-Firm share 25th percentile	0.0002	0.0000
Firm-to-Firm share 50th percentile	0.0012	0.0003
Firm-to-Firm share 75th percentile	0.0053	0.0034

Model fit: targeted moments

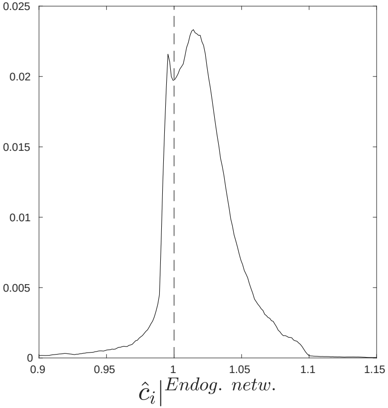
	Data	Model
Share of firms that export	0.12	0.10
Share of exports in total firm sales 25th percentile	0.01	0.02
Share of exports in total firm sales 50th percentile	0.10	0.14
Share of exports in total firm sales 75th percentile	0.62	0.50
Total export share 25th percentile	0.0008	0.0000
Total export share 50th percentile	0.0156	0.0328
Total export share 75th percentile	0.1005	0.2982
Share of firms that import	0.19	0.19
Share of imports in firm inputs 25th percentile	0.01	0.01
Share of imports in firm inputs 50th percentile	0.28	0.05
Share of imports in firm inputs 75th percentile	0.67	0.25
Total foreign input share 25th percentile	0.24	0.18
Total foreign input share 50th percentile	0.39	0.31
Total foreign input share 75th percentile	0.55	0.44
Ratio of aggregate exports to aggregate sales to domestic final demand	0.82	0.90
Import content of domestic final demand	0.58	0.65

Model fit: non-targeted moments

	Data	Model
Corr (Indegree Buyer, Outdegree Seller)	-0.05	-0.13
Corr (Sales Buyer, Sales Seller)	-0.02	0.01

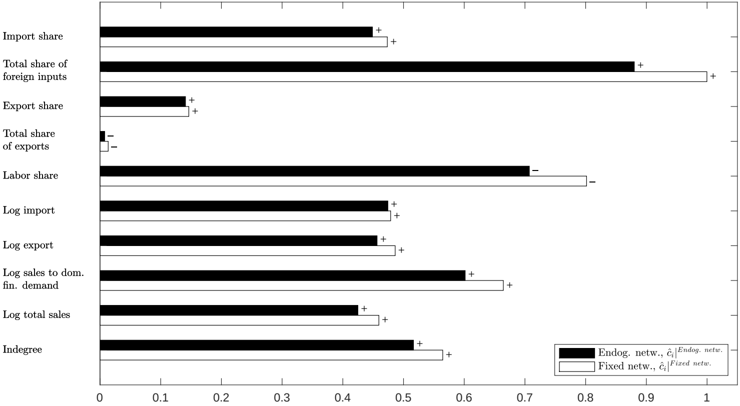
Counterfactual (10% increase in foreign price)

Figure 1: Density of cost changes under endogenous networks



Counterfactual (10% increase in foreign price)

Figure 2: Correlations with firm-level variables



Counterfactual

Table 1: Changes in real wage upon change in foreign price

	Endogenous Network	Fixed Network
Autarky ($\hat{p}_{F\cdot} \to \infty$)	0.4844	0.4157
50% decrease ($\hat{p}_{F\cdot} = 0.5$)	2.2779	2.0170

Counterfactual

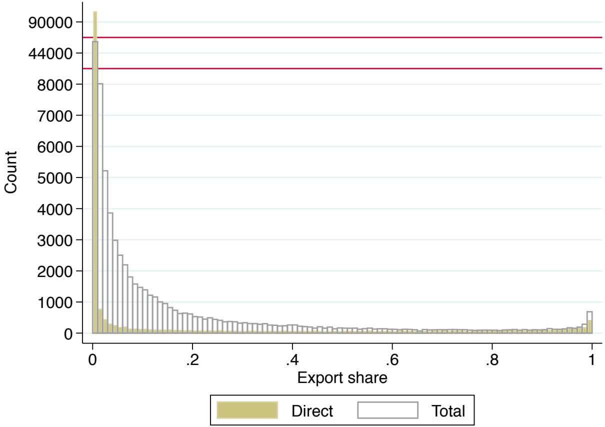
Table 1: Changes in real wage upon change in foreign price

	Endogenous Network	Fixed Network
Autarky ($\hat{p}_{F\cdot} \to \infty$)	0.4844	0.4157
50% decrease ($\hat{p}_{F\cdot} = 0.5$)	2.2779	2.0170
10% increase ($\hat{p}_{F\cdot} = 1.1$)	0.9388	0.9321
10% decrease ($\hat{p}_{F\cdot} = 0.9$)	1.0681	1.0881

Back-up

Direct and indirect exporting

$$r_{jF}^{Total} = r_{jF} + \sum_{k \in W_j} r_{jk} r_{kF}^{Total}$$



88% of firms export directly or indirectly; Median: 2%; 80th percentile: 14%

Export share by sector | Excluding exports wholesale and retail sectors

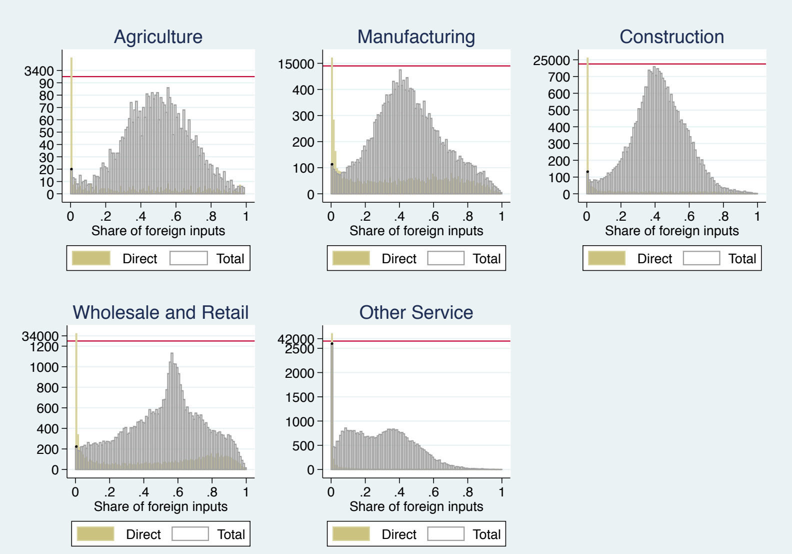
Quantiles of indegree and outdegree distributions

	25th	Median	75th
Indegree	19	33	55
Outdegree	2	9	34

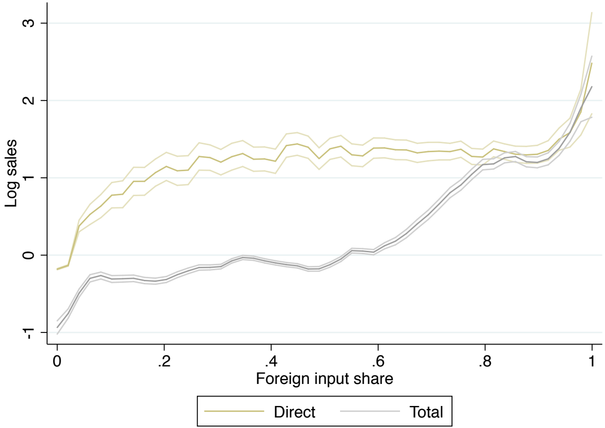
Size premium of direct and indirect linkages to ▸ Imports and ▸ Exports

▸ ICD and ICE by sector

Total and Direct share of foreign inputs by private sector firms in Belgium

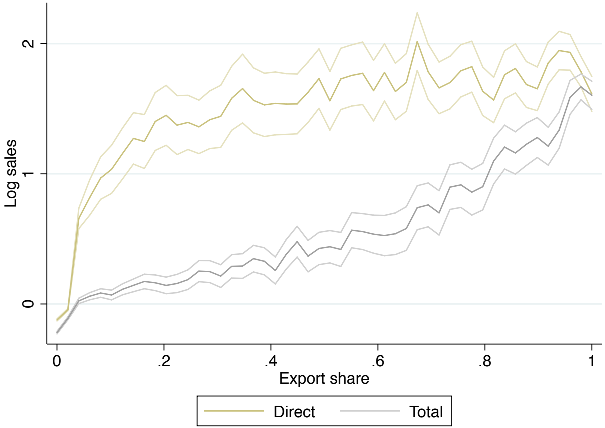


Size premium of direct and indirect linkages to imports



▸ Back

Size premium of direct and indirect linkages to exports



Back

Firm-level cost changes due to foreign price change

- Autarky ($p_{F\cdot} \to \infty$):

$$\hat{c}_j \mid^{p_{F\cdot} \to \infty} = \left(1 - s_{Fj}^{Total}\right)^{1/(1-\rho)} \hat{w}^{aut}$$

- Small percentage point change in the foreign price, $\frac{dp_{F\cdot}}{p_{F\cdot}}$:

$$\frac{dc_j}{c_j} = \left(1 - s_{Fj}^{Total}\right) \frac{dw}{w} + s_{Fj}^{Total} \frac{dp_{F\cdot}}{p_{F\cdot}}$$

Back

Change in real wage

- Autarky ($p_{F\cdot} \to \infty$):

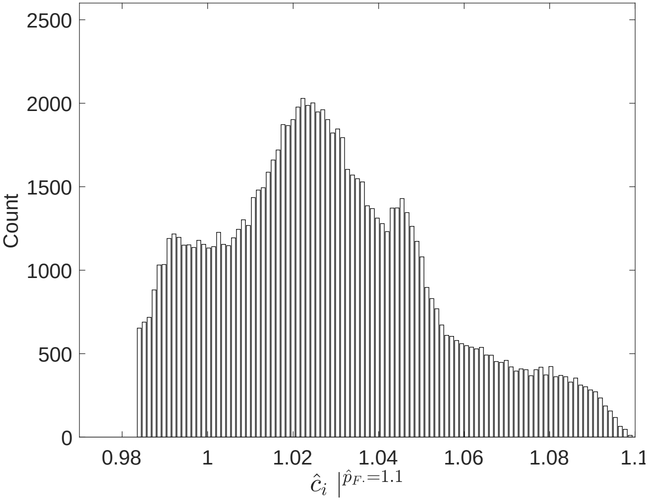
$$\frac{\hat{w}}{\hat{P}}\Big|^{\hat{p}_{F\cdot}\to\infty} = \left(\sum_i s_{iH}\left(1 - s_{Fj}^{Total}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{\sigma-1}}$$

- Small percentage point change in the foreign price, $\frac{dp_{F\cdot}}{p_{F\cdot}}$:

$$\frac{dw}{w} - \frac{dP}{P} = \frac{dw}{w} - \sum_j s_{jH}\frac{dc_j}{c_j}$$
$$= \left(\frac{dw}{w} - \frac{dp_{F\cdot}}{p_{F\cdot}}\right)\sum_j s_{jH} s_{Fj}^{Total}$$

Back

Firm-level cost changes



Back

Alternative modeling approaches

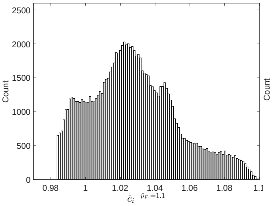
- Simple roundabout production economy:

$$c_j = \phi_i^{-1} \left(\alpha_{Dj}^{\rho-1} P^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

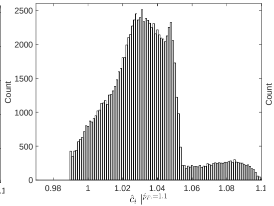
- Sectoral roundabout production economy:

$$c_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} \left(\prod_v P_v^{\gamma_{vu(j)}} \right)^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

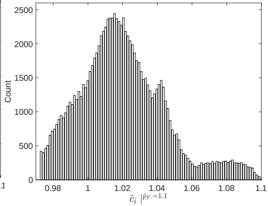
Firm-level cost changes



a: Baseline model



b: Simple roundabout



c: Sectoral roundabout

▸ Sensitivity ▸ Back

Comparison across models

a: Changes in real wage upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}}\big\rvert^{\hat{p}_{F\cdot}}$	0.940	0.941	0.931

b: Changes in real wage upon autarky

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}}\big\rvert^{\hat{p}_{F\cdot}\to\infty}$	0.558	0.596	0.441

Real income Sensitivity Back

Comparison with Hulten (1978)

- Real wage change under perfect competition

$$\underbrace{\frac{\mathrm{d}w}{w}-\frac{\mathrm{d}P}{P}}_{\substack{\text{Real wage change}\\ \text{under perfect competition}}} = \underbrace{\frac{\text{Imports}}{\text{VA}}}_{\text{Domar weight}}\left(\underbrace{\sum_j s_{jF}\left(\left(1-s_{Fj}^{Total}\right)\frac{\mathrm{d}w}{w}+s_{Fj}^{Total}\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right)}_{\substack{\text{Change in}\\ \text{export price}}}-\underbrace{\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}}_{\substack{\text{Change in}\\ \text{import price}}}\right)$$

$$= \frac{\text{Imports}}{\text{VA}}\left(\frac{\mathrm{d}w}{w}-\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right)\sum_j s_{jF}\left(1-s_{Fj}^{Total}\right)$$

- If no import content in exports: $\sum_j s_{jF}\left(1-s_{Fj}^{Total}\right)=1$.
- If same import content in exports and domestic consumption: $\sum_j s_{jF}\left(1-s_{Fj}^{Total}\right)=\frac{\text{VA}}{\text{VA+Exports}}$.

Back Baseline model

Sensitivity of firm-level cost change

a: Median $\hat{c}_i$

ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.035	1.038	1.030
1.5	4	1.017	1.025	1.008
1.5	6	1.009	1.019	0.998
2	2	1.046	1.047	1.041
2	4	1.027	1.032	1.018
2	6	1.017	1.024	1.006

b: 90th percentile $\hat{c}_i$

ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.069	1.058	1.059
1.5	4	1.061	1.047	1.045
1.5	6	1.057	1.042	1.038
2	2	1.075	1.064	1.065
2	4	1.065	1.052	1.051
2	6	1.060	1.046	1.043

Back

Changes in real income

a: Changes in real income upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}}\Big\rvert^{\hat{p}_{F\cdot}}$	0.883	0.878	0.887

b: Changes in real income upon autarky

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}}\Big\rvert^{\hat{p}_{F\cdot}\to\infty}$	0.374	0.390	0.291

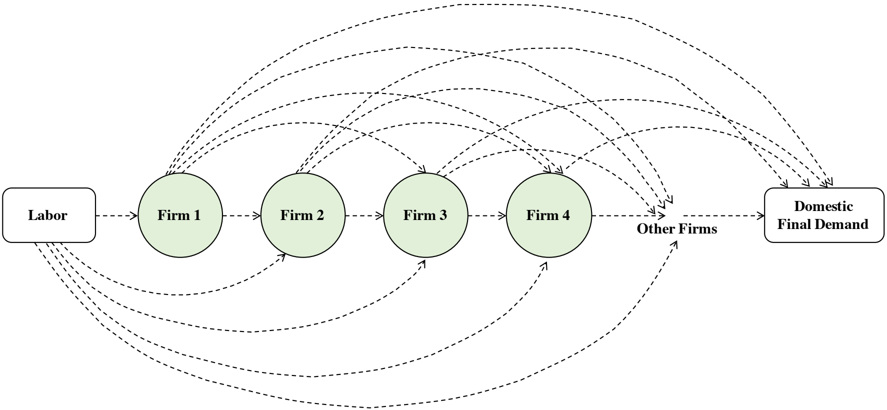
Back

Sensitivity of real wage change

ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	0.945	0.944	0.939
1.5	4	0.931	0.934	0.922
1.5	6	0.927	0.932	0.915
2	2	0.955	0.953	0.949
2	4	0.940	0.941	0.931
2	6	0.934	0.937	0.923

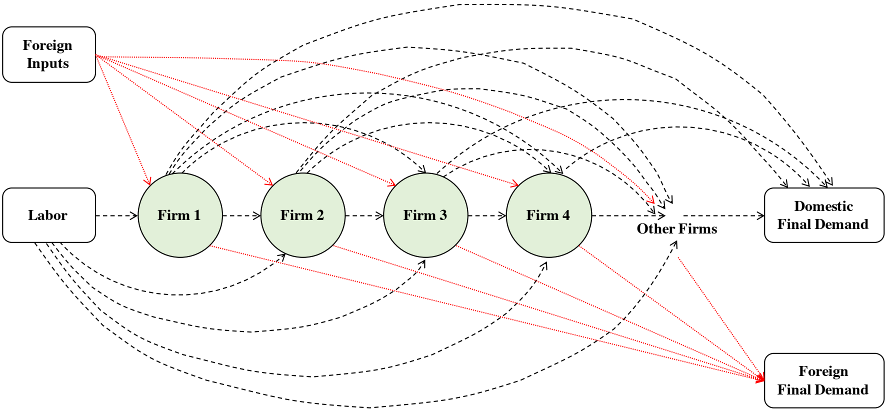
Back

Network formation



back

Network formation with trade



▸ back

Aggregating vats to firms

- We group all VAT-id into firms that are either
  - linked with more than 50% of ownership (ownership filings).
  - have common foreign parent firm (FDI filings).
- In 2012, 896K VAT-ids collapsed to 860K firms. Of those firms, 842K firms consisted of single VAT-ids. The number of VAT-ids for multiple VAT-id firms are as below.

	Mean	10%	25%	50%	75%	90%	max
Num. VAT-id	3	2	2	2	3	5	372

- The 18K firms with multiple VAT-ids account for $\sim 60\%$ of the total output.

Back

Sample of analysis

- We restrict the sample of analysis according to the criteria below:
  - Belgian firms in industries other than government and finance.
  - File positive sales, positive labor cost, and at least one FTE employment
  - Following De Loecker et al. (2014) we also restrict the sample to firms with tangible assets of more than 100 euro, positive total assets for at least one year throughout the period.

Year	GDP (Excl.	Output Gov. & Fin.)	Imports	Exports	Selected sample				
					Count	V.A.	Sales	Imports	Exports
2002	182	458	178	193	88,301	119	604	175	185
2007	230	593	254	267	95,941	152	782	277	265
2012	248	671	317	319	98,745	164	874	292	292

Notes: All numbers except for Count are denominated in billion Euro in current prices. Data for Belgian GDP, imports and exports are from Eurostat.

Back

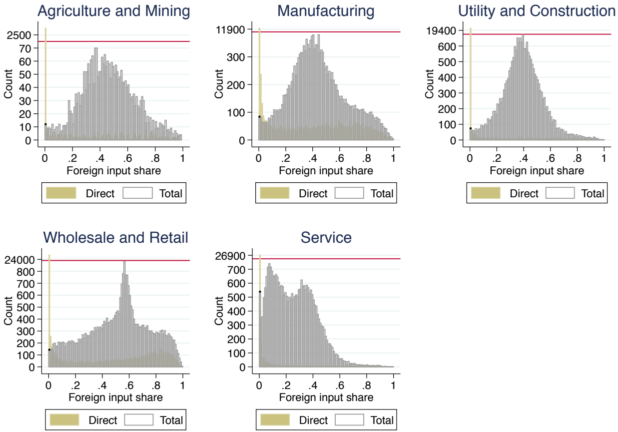
Sectoral composition

Table 5: Sectoral composition in 2012

Sector	Count	V.A.	Output	Imports	Exports
Agriculture and Mining	2,805	28.5	49.4	16.9	10.9
Manufacturing	16,577	138	272	146	193
Utility and Construction	20,421	23.3	77.0	27.8	17.5
Wholesale and Retail	31,117	87.8	241	84.1	53.4
Service	27,825	79.1	127	17.6	16.9
Total	98,745	356	874	292	292

Back

Direct and total foreign input share by firms' sector



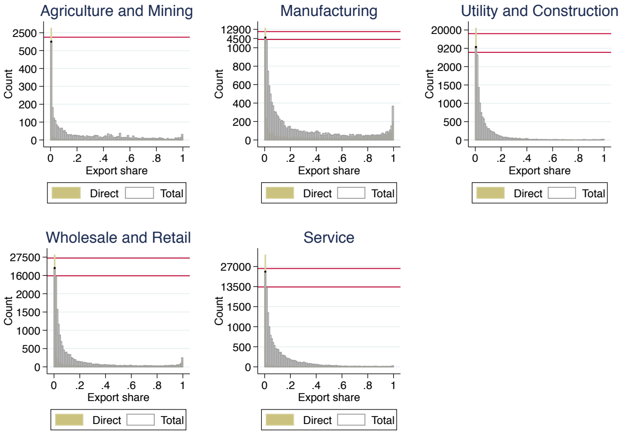
Back

Direct and total foreign input share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture and Mining	0.05	0.70	0	0.46	0.85	0.45
Manufacturing	0.12	0.59	0	0.44	0.75	0.42
Utility and Construction	0.02	0.31	0	0.39	0.59	0.39
Wholesale and Retail	0.12	0.43	0	0.52	0.75	0.55
Service	0.01	0.19	0	0.25	0.41	0.24
Total	0.07	0.45	0	0.40	0.68	0.39

Back

Direct and total export share by firms' sector



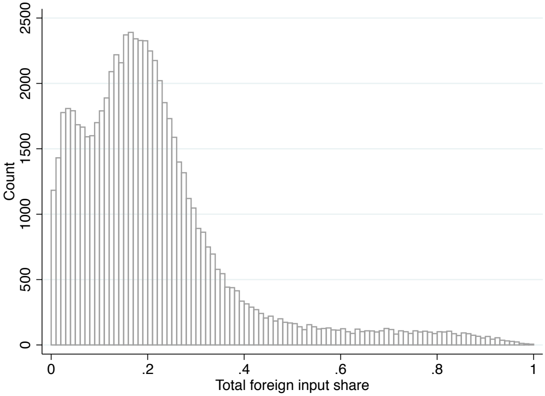
Back

Direct and total export share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture and Mining	0.05	0.22	0	0.24	0.30	0.11
Manufacturing	0.11	0.56	0	0.23	0.62	0.07
Utility and Construction	0.01	0.19	0	0.06	0.25	0.01
Wholesale and Retail	0.04	0.21	0	0.09	0.26	0.01
Service	0.01	0.13	0	0.07	0.23	0.01
Total	0.04	0.33	0	0.11	0.40	0.02

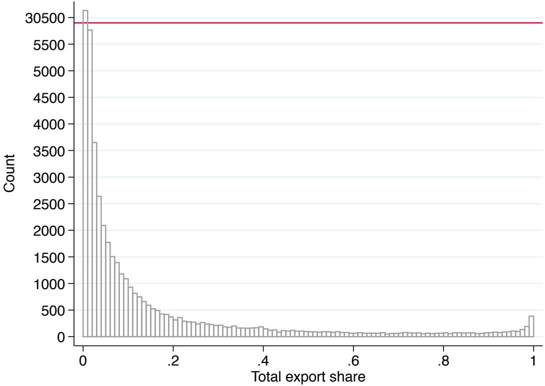
Back

Total foreign input share (excluding wholesale and retail sectors)



Back

Total export share (excluding wholesale and retail sectors)



Back

ICD and ICE across sectors

Sector	ICD	ICE
Agriculture and Mining	0.068	0.033
Manufacturing	0.158	0.522
Utility and Construction	0.059	0.052
Wholesale and Retail	0.237	0.155
Service	0.062	0.038
Total	0.584	0.800

Back

Gains from trade – example

- Consider two economies with the same level of GDP, gross production, and exports and imports
- Assume fixed network

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
Imports	100	0	0			
Exports	0	0	100			
Gross production	200	200	200			
Domestic sales	200	200	100			
Labor costs	50	100	50			
Domestic purchases	0	50	100			
Profits	50	50	50			
Firm-to-firm sales	$x_{12} = 50$	$x_{23} = 50$				
	$x_{13} = 50$					
Sales to households	100	150	100			

Gains from trade – example

- Consider two economies with the same level of GDP, gross production, and exports and imports
- Assume fixed network

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
Imports	100	0	0	100	0	0
Exports	0	0	100	0	0	100
Gross production	200	200	200	200	200	200
Domestic sales	200	200	100	200	200	100
Labor costs	50	100	50	50	100	50
Domestic purchases	0	50	100	0	50	100
Profits	50	50	50	50	50	50
Firm-to-firm sales	$x_{12} = 50$	$x_{23} = 50$			$x_{23} = 100$	$x_{32} = 50$
	$x_{13} = 50$					
Sales to households	100	150	100	200	100	50

Gains from trade – example (continued)

- Direct and total shares of foreign inputs and shares in household consumption:

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
s_{Fi}	2/3	0	0	2/3	0	0
s_{Fi}^{Total}	2/3	2/9	2/27	2/3	0	0
s_{iH}	2/7	2/7	3/7	4/7	1/7	2/7

Gains from trade – example (continued)

- Direct and total shares of foreign inputs and shares in household consumption:

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
s_{Fi}	2/3	0	0	2/3	0	0
s_{Fi}^{Total}	2/3	2/9	2/27	2/3	0	0
s_{iH}	2/7	2/7	3/7	4/7	1/7	2/7

- Plugging in $\frac{\hat{w}}{\hat{P}}|^{\hat{p}_{F\cdot}\to\infty}$ $(\sigma = 4,\ \rho = 2)$

$$\frac{\hat{w}}{\hat{P}}|^{\hat{p}_{F\cdot}\to\infty} = \left(\sum_i s_{iH} \left(1 - s_{Fi}^{Total}\right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}} = \begin{cases} 0.79 & \text{if economy 1} \\ 0.77 & \text{if economy 2} \end{cases}$$

Back

Log-changes in real wage, FOA

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$	-0.118	-0.112	-0.130
$\sum_j s_{jH} s_{Fj}^{Total}$	0.584	0.609	0.607
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right) \sum_j s_{jH} s_{Fj}^{Total}$	-0.069	-0.068	-0.079

Log-changes in real wage, FOA

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$	-0.118	-0.112	-0.130
$\sum_j s_{jH} s_{Fj}^{Total}$	0.584	0.609	0.607
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right) \sum_j s_{jH} s_{Fj}^{Total}$	-0.069	-0.068	-0.079

- $\frac{\text{Imports}}{\text{VA}} = 0.817$
- $\frac{\text{Imports}}{\text{VA+Exports}} = 0.451$

$\frac{\hat{w}}{\hat{P}}$ under acyclic network

Table 6: Change in real wage $\frac{\hat{w}}{\hat{P}}$ under acyclic network

ρ	σ	10% increase in p_F.		Autarky	
		$\frac{\hat{w}}{\hat{P}}$, Baseline	$\frac{\hat{w}}{\hat{P}}$, acyclic	$\frac{\hat{w}}{\hat{P}}$, Baseline	$\frac{\hat{w}}{\hat{P}}$, acyclic
1.5	2	0.945	0.942	0.251	0.223
1.5	4	0.931	0.927	0.437	0.408
1.5	6	0.927	0.922	0.541	0.519
2	2	0.955	0.952	0.391	0.388
2	4	0.940	0.936	0.558	0.530
2	6	0.934	0.929	0.634	0.609

Back

Firm-pair specific shifters

- Firms' outdegree distribution is more skewed that firms' indegree distributions
- Assume that the α_{kj} draw is a product of a supplier-k-specific random variables and an i.i.d. firm-pair-specific random variable
- The supplier-k-specific random variables are drawn from a Beta distribution with shape parameters 0.1 and 0.9

Back