Lecture 2: Benchmark Heterogeneous Firm Model and Overview of Solution Methods

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Motivating Facts: Doms and Dunne (1998)

Measurement

- Use Census data from LRD, 1972 - 1988
  - After 1988, stopped collecting data on sales and retirements

- Construct capital stock using perpetual inventory method
  - Focus on balanced panel

- Analyze the growth rate of capital for plant *i* at time *t*

$$GK_{it} = \frac{i_{it} - \delta k_{it-1}}{0.5 \times (k_{it-1} + k_{it})}$$

Plant-Level Investment is Lumpy Across Plants

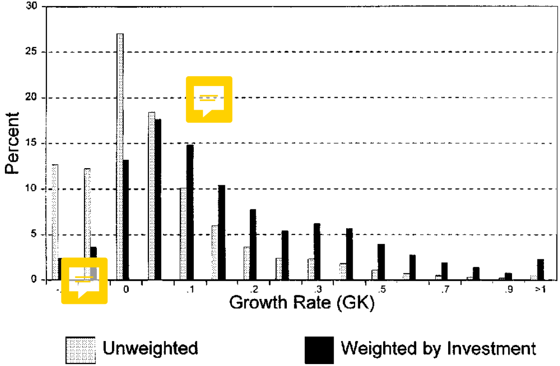
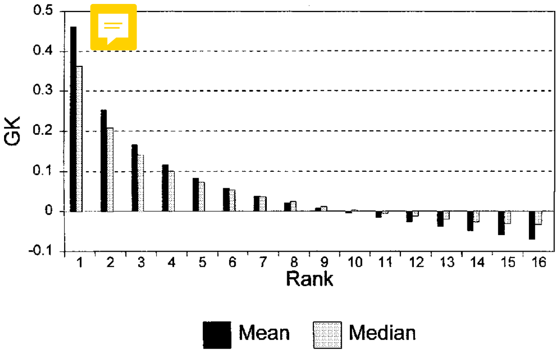


FIG. 1. Capital growth rate (*GK*) distributions: Unweighted and weighted by investment.

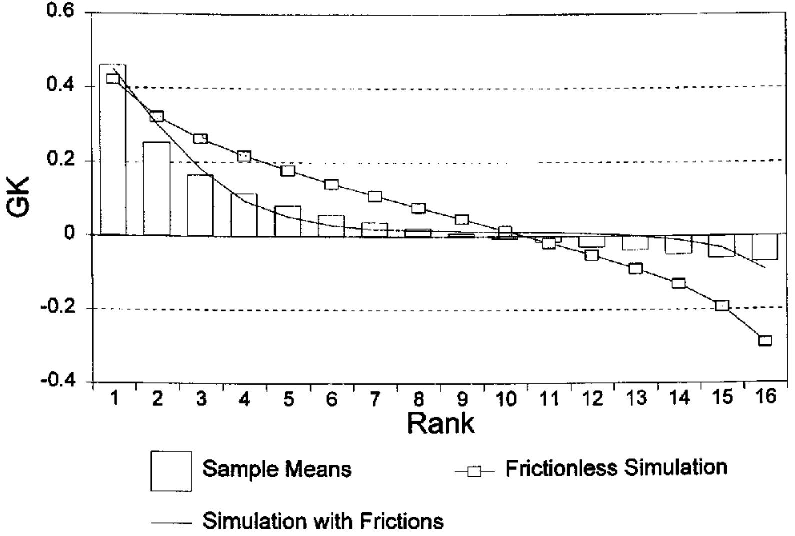
- 51.9% of plants increase capital $\leq$ 2.5%
- 11% of plants increase capital $\geq$ 20%

Plant-Level Investment is Lumpy Within Plants

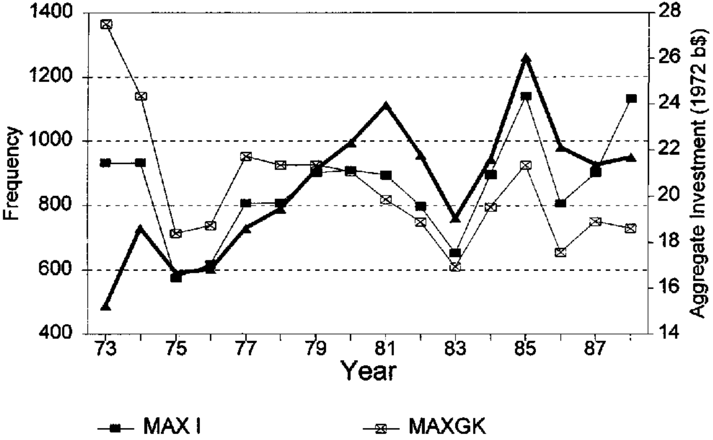


- Capital growth in largest investment episode nearly 50%
- In median investment episode approximately 0%

Plant-Level Investment is Lumpy Within Plants



Frequency of Spikes Correlated with Aggregate Investment



Benchmark Model: Khan and Thomas (2008)

Model Overview

Heterogeneous Firms

- Fixed mass
- Idiosyncratic + aggregate productivity shocks
- Fixed capital adjustment costs

Representative Household

- Owns firms
- Supplies labor
- Complete markets

Heterogeneous Firms

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}$, $\theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_\epsilon \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_\epsilon^2)$
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega_{t+1}^{z}$ where $\omega_{t+1}^{z} \sim N(0, \sigma_z^2)$

Firms accumulate capital according to $k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$

- If $\frac{i_{jt}}{k_{jt}} \notin [-a, a]$, pay fixed cost ξ_{jt} in units of labor
- Fixed cost $\xi_{jt} \sim U[0, \overline{\xi}]$

Firm Optimization Problem: Recursive Formulation

$$
\begin{aligned}
v(\varepsilon, k, \xi; \mathbf{s}) = \max_n\ & e^z e^{\varepsilon} k^{\theta} n^{\nu} - w(\mathbf{s})\, n \\
& + \max\left\{ v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s})\,\xi,\ v^N(\varepsilon, k; \mathbf{s}) \right\}
\end{aligned}
$$

Firm Optimization Problem: Recursive Formulation

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$$+ \max \left\{ v^{A}(\varepsilon, k; \mathbf{s}) - w(\mathbf{s})\, \xi, v^{N}(\varepsilon, k; \mathbf{s}) \right\}$$

$$v^{A}(\varepsilon, k; \mathbf{s}) = \max_{i \in \mathbb{R}} -i + \mathbb{E}\left[\Lambda(\mathbf{s}'; \mathbf{s})\, v(\varepsilon', k', \xi'; \mathbf{s}') \,|\, \varepsilon, k; \mathbf{s}\right]$$

$$v^{N}(\varepsilon, k; \mathbf{s}) = \max_{i \in [-ak, ak]} -i + \mathbb{E}\left[\Lambda(\mathbf{s}'; \mathbf{s})\, v(\varepsilon', k', \xi'; \mathbf{s}') \,|\, \varepsilon, k; \mathbf{s}\right]$$

Firm Optimization Problem: Recursive Formulation

$$v(\varepsilon, k, \xi; \mathbf{s}) = \max_n e^z e^\varepsilon k^\theta n^\nu - w(\mathbf{s})\, n$$
$$+ \max \left\{ v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s})\, \xi,\ v^N(\varepsilon, k; \mathbf{s}) \right\}$$

$$\widehat{v}(\varepsilon, k; \mathbf{s}) = \max_n e^z e^\varepsilon k^\theta n^\nu - w(\mathbf{s})\, n$$
$$+ \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{\overline{\xi}} \left(v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s}) \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{2} \right)$$
$$+ \left(1 - \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{\overline{\xi}} \right) v^N(\varepsilon, k; \mathbf{s})$$

Household

Representative household who owns all firms in the economy

$$\max_{C(\mathbf{s}),N(\mathbf{s})} \frac{C(\mathbf{s})^{1-\sigma}-1}{1-\sigma} - \chi\frac{N(\mathbf{s})^{1+\alpha}}{1+\alpha} \text{ such that}$$
$$C(\mathbf{s}) = w(\mathbf{s})N(\mathbf{s}) + \Pi(\mathbf{s})$$

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Complete markets implies that $\Lambda(\mathbf{s}';\mathbf{s}) = \beta\left(\frac{C(\mathbf{s}')}{C(\mathbf{s})}\right)^{-\sigma}$

- Firms maximize their market value
- Market value given by expected present value of dividends using stochastic discount factor
- With complete markets, SDF is household's intertemporal marginal rate of substitution

Defining Recursive Competitive Equilibrium

What is the aggregate state $\mathbf{s}$?

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Defining Recursive Competitive Equilibrium

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- Aggregate shock z
- Firm's individual states: productivity ε and capital k

$\implies$ need distribution of firms $g(\varepsilon, k)$

What is the law of motion for the **s**?

$$g'\left(\varepsilon', k'\right) = \int \left[\begin{array}{c} \mathbb{1}\left\{\rho_\varepsilon \varepsilon + \sigma_\varepsilon \omega'_\varepsilon = \varepsilon'\right\} \\ \times \int \mathbb{1}\left\{k'\left(\varepsilon, k, \xi; \mathbf{s}\right) = k'\right\} dG(\xi) \end{array} \right] \times p\left(\omega'_\varepsilon\right) g\left(\varepsilon, k\right) d\omega'_\varepsilon d\varepsilon dk$$

Recursive Competitive Equilibrium

A set of $\widehat{v}(\varepsilon, k; z, g)$, $C(z, g)$, $N(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ s.t.

1. **Firm optimization**: Taking $\Lambda(z'; z, g)$ and $w(z, g)$ as given, $\widehat{v}(\varepsilon, k; z, g)$ solves Bellman equation
2. **Household optimization**: $w(z, g)C(z, g)^{-\sigma} = \chi N(z, g)^{\alpha}$
3. **Market clearing**:

$$N(z, g) = \int n(\varepsilon, k; z, g) g(\varepsilon, k) d\varepsilon dk$$

$$C(z, g) = \int (y(\varepsilon, k, \xi; z, g) - i(\varepsilon, k, \xi; z, g)) dG(\xi) g(\varepsilon, k) d\varepsilon dk$$

$$\Lambda(z'; z, g) = \beta \left(\frac{C(z', g'(z, g))}{C(z, g)} \right)^{-\sigma}$$

4. **Consistency**: $g'(\varepsilon, k)$ satisfies law of motion for distribution

Model Parameterization

	Desc.	Value		Desc.	Value
β	Discount factor	.961	ρ_z	Aggregate TFP AR(1)	.859
σ	Utility curvature	1	σ_z	Aggregate TFP AR(1)	.014
α	Inverse Frisch	$\lim \alpha \to 0$	$\overline{\xi}$	Fixed cost	.0083
χ	Labor disutility	$\implies N^* = \frac{1}{3}$	a	No fixed cost region	.011
ν	Labor share	.64	ρ_ε	Idiosyncratic TFP AR(1)	.859
θ	Capital share	.256	σ_ε	Idiosyncratic TFP AR(1)	.022
δ	Capital depreciation	.085			

Overview of Computational Methods

Computing Equilibrium

- Key challenge: aggregate state g is infinite-dimensional

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- Two steps:
  1. Compute steady state without aggregate shocks $\Longrightarrow$ distribution constant at g^*
  2. Compute full model with aggregate shocks $\Longrightarrow$ distribution varies over time

Steady State Recursive Competitive Equilibrium

A set of $v^*(\varepsilon, k)$, C^*, N^*, w^*, and $g^*(\varepsilon, k)$ such that

1. **Firm optimization**: Taking w^* as given: $v^*(\varepsilon, k)$ solves Bellman equation
2. **Household optimization**: Taking w^* as given: $w^*(C^*)^{-\sigma} = \chi(N^*)^{\alpha}$
3. **Market clearing**:
$$N^* = \int n(\varepsilon, k) g(\varepsilon, k) d\varepsilon dk$$
$$C^* = \int (y(\varepsilon, k, \xi) - i(\varepsilon, k, \xi)) dG(\xi) g^*(\varepsilon, k) d\varepsilon dk$$
4. **Consistency**:
$g^*(\varepsilon, k)$ satisfies law of motion for distribution given g^*

Hopenhayn-Rogerson (1993) Algorithm

Start with guess of w^*

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Start with guess of w^*

- Solve firm optimization problem $\implies v^*(\varepsilon, k), n^*(\varepsilon, k), k'(\varepsilon, k, \xi)$
- Use $k'(\varepsilon, k, \xi)$ to compute stationary distribution $g^*(\varepsilon, k)$ by iterating on law of motion
- Compute implied labor demand $N^d = \int n^*(\varepsilon, k) g^*(\varepsilon, k) d\varepsilon dk$

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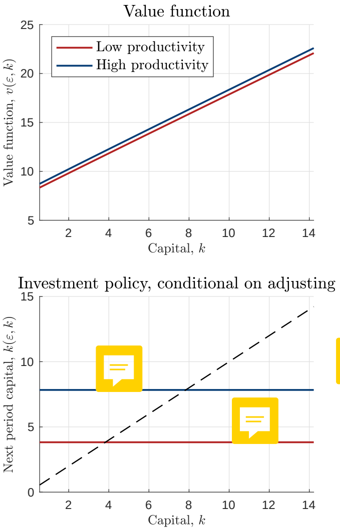
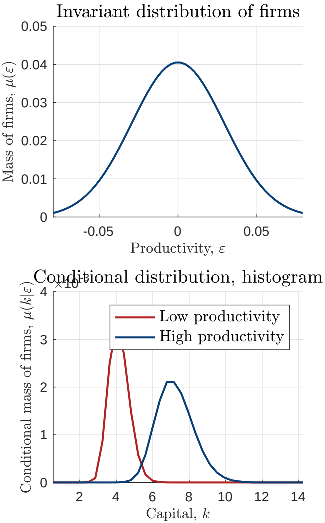
Update guess of w^* **based on** $N^d - N^s$

Iterate to convergence

Steady State Decisions



Steady State Distribution



Full Model with Aggregate Shocks

- Outside of steady state,
  - Distribution g varies over time $\implies$ how to approximate distribution?
  - Law of motion for g is complicated $\implies$ how to approximate law of motion?

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- Will provide overview of three approaches in the literature:
  1. **Krusell-Smith**: approximate distribution with moments (used in Khan and Thomas 2008)
  2. **Reiter methods**: perturbation w.r.t. the distribution (used in Winberry 2018; will discuss details on Friday if time)
  3. **MIT shocks**: perfect foresight transition paths (used in Ottonello-Winberry 2018; will discuss on Friday)

Krusell and Smith (1998)

- Typical approach to dealing with challenge: Krusell-Smith (1998)
  - Approximate distribution with moments, e.g. $g(\varepsilon, k) \approx \overline{K}$
  - Approximate law of motion with parametric form
    $\log \overline{K}' = \alpha_0 + \alpha_1 z + \alpha_2 \log \overline{K}$
  - Approximate prices with parametric form
    $\log C = \gamma_0 + \gamma_1 z + \gamma_2 \log \overline{K}$ and $\log w = \eta_0 + \eta_1 z + \eta_2 \log \overline{K}$

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- **Overview of the algorithm**:
  - Start with guess of α, γ, and η
    - Solve firm optimization $\implies \widehat{v}(\varepsilon, k; z, \overline{K}), n(\varepsilon, k; z, \overline{K}), k'(\varepsilon, k, \xi; z, \overline{K})$
    - Simulate large panel of firms using $n(\varepsilon, k; z, \overline{K}), k'(\varepsilon, k, \xi; z, \overline{K})$
    - Compute aggregate time series $z_t, \overline{K}_t, C_t, w_t$
  - Update α, γ, and η using OLS on simulated series
  - Iterate to convergence on α, γ, and η
- Assess accuracy using, e.g., R^2 of forecasting rules

Winberry (2018)

- Approximate distribution with parametric family:

$$g(\varepsilon, k) \cong g_0 \exp\{g_1^1 \left(\varepsilon - m_1^1\right) + g_1^2 \left(\log k - m_1^2\right) + \sum_{i=2}^{n_g} \sum_{j=0}^{i} g_i^j \left[\left(\varepsilon - m_1^1\right)^{i-j} \left(\log k - m_1^2\right)^j - m_i^j\right]\}$$

$\rightarrow$ Aggregate state approximated by $(z, g(\varepsilon, k)) \approx (z, \mathbf{m})$

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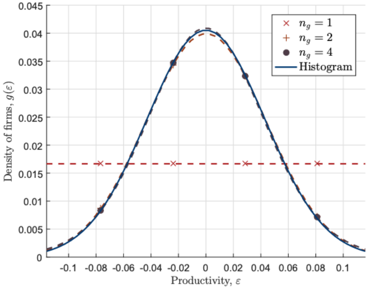
- Compute law of motion + prices directly by integration

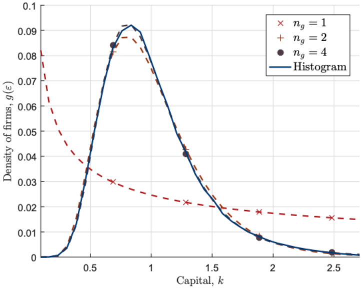
- Compute aggregate dynamics using perturbation methods
  - Solve for steady state in `Matlab`
  - Solve for aggregate dynamics using `Dynare`

Winberry (2018)

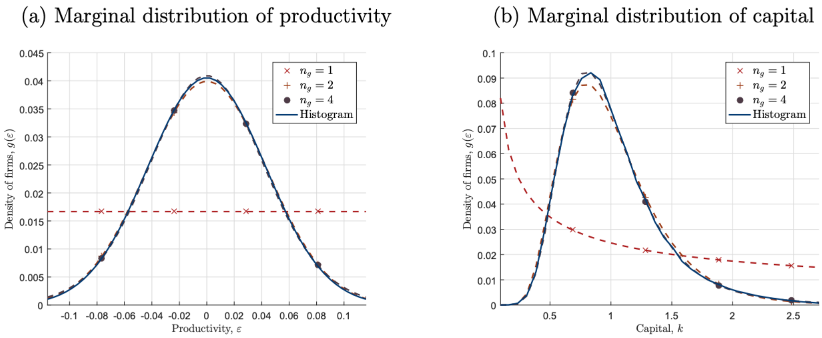
(a) Marginal distribution of productivity



(b) Marginal distribution of capital



Winberry (2018)



- Run time ≈ 20 - 40 seconds for accurate approximation
- Fast enough for likelihood-based estimation
- Codes at my `website`

MIT Shocks (aka Transition Paths)

- **MIT shock** = unexpected innovation Δz_0 at $t = 0$ + perfect foresign transition path back to steady state

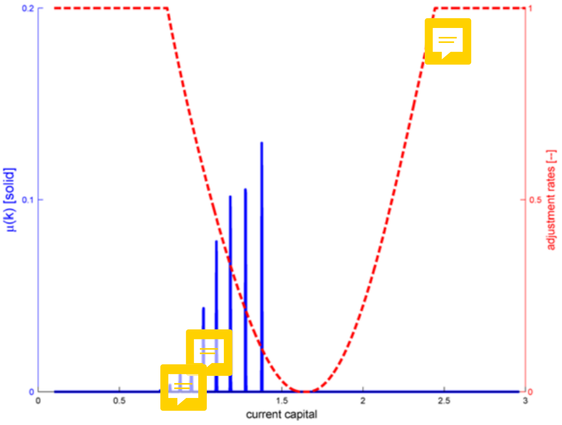
- In this model, characterized by $\{w_t\}_{t=0}^{\infty}$ and $\{C_t\}_{t=0}^{\infty}$
  1. Solve firm's problem by backward iteration
  2. Given $\{\widehat{v}_t(\varepsilon, k)\}_{t=0}^{\infty}$, simulate decisions to get $n_t^d(\varepsilon, k)$, $y_t(\varepsilon, k) - i_t(\varepsilon, k)$, and $g_t(\varepsilon, k) \implies$ get aggregates $N_t^d = \int n_t^d(\varepsilon, k) g_t(\varepsilon, k) d\varepsilon dk$ and $C_t^s = \int (y_t(\varepsilon, k) - i_t(\varepsilon, k)) g_t(\varepsilon, k) d\varepsilon dk$
  3. Equilibrium: $N_t^d = N_t^s$ and $C_t^s = C_t$

- Computational method: set $T =$ large enough and iterate over path of $\{w_t\}_{t=0}^{T}$ and $\{C_t\}_{t=0}^{T}$

- Converges to true IRF as $\Delta z_0 \to 0$

Results

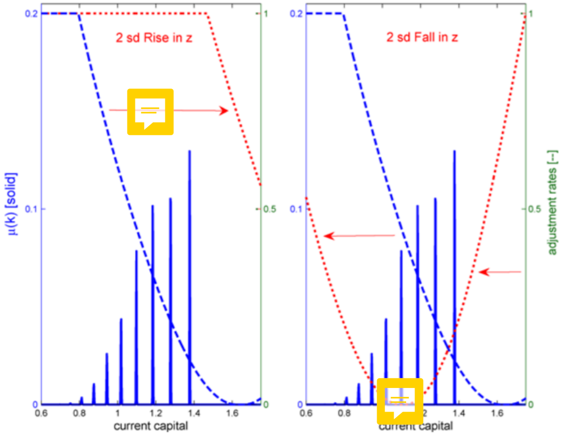
Complicated Impulse Responses with Fixed Prices



Distribution in model with no idiosyncratic productivity shocks

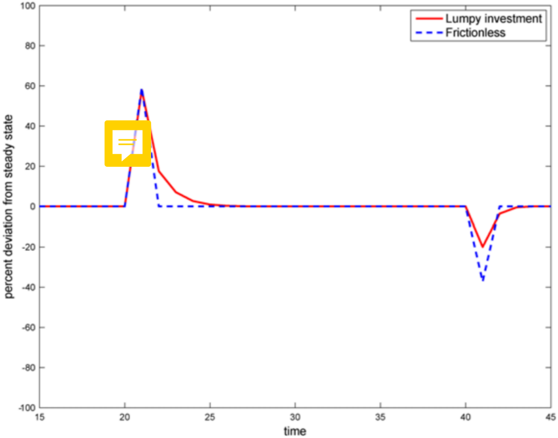
Investment decision characterized by adjustment hazard

Complicated Impulse Responses with Fixed Prices



Response of aggregate investment to shock depends on interaction of initial distribution and adjustment hazards

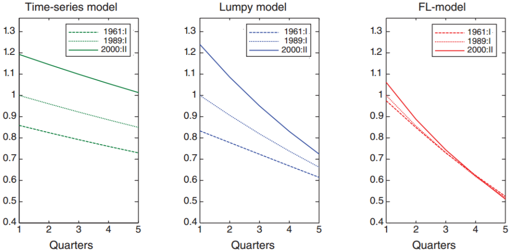
Implication: Sign Dependence



Aggregate investment more responsive to positive than negative shocks

Not true in frictionless model

Implication: State Dependence



From Bachmann, Caballero, and Engel (2013)

$$\frac{I_t}{K_t} = \sum_{j=1}^{p} \phi_j \frac{I_{t-j}}{K_{t-j}} + \sigma_t e_t$$

$$\sigma_t = \alpha_1 + \eta_1 \frac{1}{p} \sum_{j=1}^{p} \frac{I_{t-j}}{K_{t-j}}$$

Aggregate Nonlinearities with Fixed Prices

- Both of these are examples of nonlinear aggregate dynamics
  - Linear model has constant loading on aggregate shock

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- Both of these are examples of nonlinear aggregate dynamics
  - Linear model has constant loading on aggregate shock
- Some evidence in aggregate data
  - Sign and state dependence $\rightarrow$ distribution of $\frac{I_t}{K_t}$ positively skewed
  - State dependence $\rightarrow$ dynamics of $\frac{I_t}{K_t}$ feature conditional heteroskedasticity

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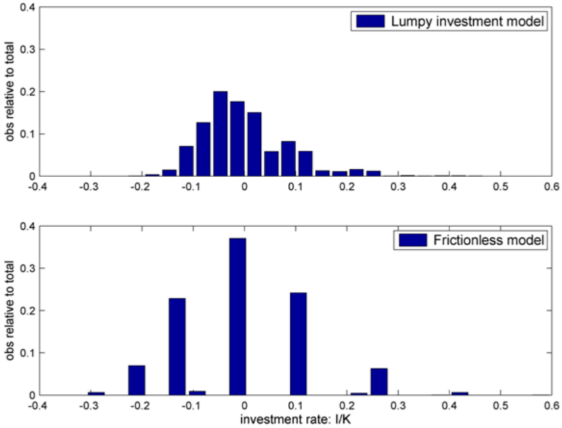
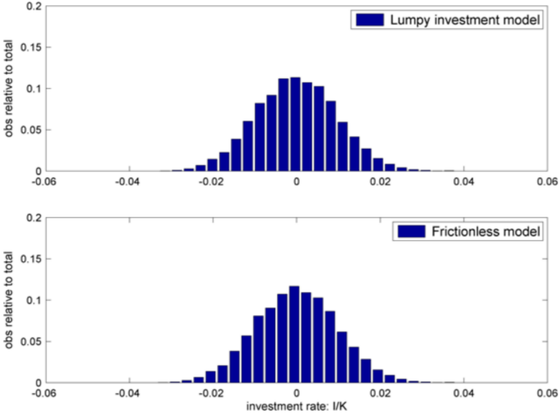
- My view: time series evidence is suggestive at best
  - Predictions are about extreme states, which are rare
  - But that is exactly when we care about these predictions!

    $\implies$ rely on cross-sectional data + carefully specified general equilibrium model

Distribution of Aggregate $\frac{I_t}{K_t}$ with Fixed Prices



Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium



Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium

TABLE III

ROLE OF NONCONVEXITIES IN AGGREGATE INVESTMENT RATE DYNAMICS

	Persistence	Standard Deviation	Skewness	Excess Kurtosis
Postwar U.S. data[a]	*0.695*	*0.008*	*0.008*	*−0.715*
A. Partial equilibrium models				
PE frictionless	−0.069	0.128	0.358	0.140
PE lumpy investment	0.210	0.085	1.121	2.313
B. General equilibrium models				
GE frictionless	0.659	0.010	0.048	0.048
GE lumpy investment	0.662	0.010	0.067	−0.074

[a] Data are annual private investment-to-capital ratio, 1954–2005, computed using Bureau of Economic Analysis tables.

Business Cycles Nearly Identical to Representative Firm

TABLE IV

AGGREGATE BUSINESS CYCLE MOMENTS

	Output	TFP[a]	Hours	Consump.	Invest.	Capital
A. Standard deviations relative to output[b]						
GE frictionless	(2.277)	0.602	0.645	0.429	3.562	0.494
GE lumpy	(2.264)	0.605	0.639	0.433	3.539	0.492
B. Contemporaneous correlations with output						
GE frictionless		1.000	0.955	0.895	0.976	0.034
GE lumpy		1.000	0.956	0.900	0.976	0.034

[a]Total factor productivity.

[b]The logarithm of each series is Hodrick–Prescott-filtered using a weight of 100. The output column of panel A reports percent standard deviations of output in parentheses.

Why Do the Nonlinearities Disappear?

General equilibrium price movements

- Time-varying elasticity comes from large movements in adjustment hazard
- Procyclical real interest rate and wage restrain those movements

$$1 + r_t = \frac{1}{\mathbb{E}_t[\Lambda_{t,t+1}]}$$

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Specification of adjustment costs

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General equilibrium price movements [Winberry (2018)]

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Specification of adjustment costs [Bachmann, Caballero, and Engel (2013)]

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Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas' calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities

Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas' calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities
- Argument based on decomposition between AC smoothing and PR smoothing
    - Frictionless partial equilibrium model excessively volatile
    - AC smoothing: dampening due to adjustment costs
    - PR smoothing: dampening due to price movements
- Measure AC smoothing in data and target in calibration $\rightarrow$ higher adjustment costs

Model

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{st}} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}, \theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_\epsilon \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_\epsilon^2)$
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega_{t+1}^{z}$ where $\omega_{t+1}^{z} \sim N(0, \sigma_z^2)$
- Sectoral productivity shock $\varepsilon_{st+1} = \rho_\epsilon \varepsilon_{st} + \omega_{st+1}^{\varepsilon}$ where $\omega_{st+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon_s}^2)$

Model

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{st}} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}$, $\theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_\epsilon \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_\epsilon^2)$
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- Sectoral productivity shock $\varepsilon_{st+1} = \rho_\epsilon \varepsilon_{st} + \omega_{st+1}^{\varepsilon}$ where $\omega_{st+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon_s}^2)$

Firms **accumulate capital** according to $k_{jt+1} = (1-\delta) k_{jt} + i_{jt}$

- If don't pay fixed cost, must undertake maintenance investment $\chi \times \delta k_{jt}$
- Otherwise, pay fixed cost ξ_{jt} in units of labor
- Fixed cost $\xi_{jt} \sim U[0, \overline{\xi}]$

Calibration

Set most parameters exogenously

Choose σ_z, $\bar{\xi}$, and χ to match degree of AC-smoothing

- Identify AC-smoothing using volatility of sectoral investment rates
  - Aggregated enough to capture interaction of distribution and hazards
  - Small enough to not generate price response

Calibration

Set most parameters exogenously

Choose σ_z, $\bar{\xi}$, and χ to match degree of AC-smoothing

- Identify AC-smoothing using volatility of sectoral investment rates
    - Aggregated enough to capture interaction of distribution and hazards
    - Small enough to not generate price response
- Targets:
    1. Volatility of aggregate investment rate
    2. Average volatility of sectoral investment rates
    3. Amount of conditional heteroskedasticity

AC vs. PR Smoothing Decomposition

TABLE 6—SMOOTHING DECOMPOSITION

	AC smoothing/total smoothing (in percent)		
Model	LB	UB	Average
Khan-Thomas-lumpy annual	0.0	16.1	8.0
Khan-Thomas-lumpy annual, our $\overline{\xi}$	8.1	59.2	33.7
Our model annual ($\chi = 0$), Khan and Thomas' $\overline{\xi}$	0.8	16.0	8.4
Our model annual ($\chi = 0$)	18.9	75.3	47.0
Our model annual ($\chi = 0.25$)	19.1	75.7	47.4
Our model annual ($\chi = 0.50$)	19.9	76.6	48.3
Our model quarterly ($\chi = 0$)	14.5	80.9	47.7
Our model quarterly ($\chi = 0.25$)	15.4	80.9	48.2
Our model quarterly ($\chi = 0.5$)	15.4	81.0	48.2

$$UB = \log\left[\sigma(\text{none})/\sigma(\text{AC})\right] / \log\left[\sigma(\text{none})/\sigma(\text{both})\right]$$

$$LB = 1 - \log\left[\sigma(\text{none})/\sigma(\text{PR})\right] / \log\left[\sigma(\text{none})/\sigma(\text{both})\right]$$

Calibrated Adjustment Costs

TABLE 4—THE ECONOMIC MAGNITUDE OF ADJUSTMENT COSTS—ANNUAL

Model	Adjustment costs/ unit's output (in percent) (1)	Adjustment costs/ unit's wage bill (in percent) (2)
This paper ($\chi = 0$)	38.9	60.9
This paper ($\chi = 0.25$)	12.7	19.8
This paper ($\chi = 0.50$)	3.6	5.6
Caballero-Engel (1999)	16.5	—
Cooper-Haltiwanger (2006)	22.9	—
Bloom (2009)	35.4	—
Khan-Thomas (2008)	0.5	0.8
Khan-Thomas (2008) "Huge Adj. Costs"	3.7	5.8

Notes: This table displays the average adjustment costs paid, conditional on adjustment, as a fraction of output (left column) and as a fraction of the wage bill (right column), for various models. Rows 4–6 are based on table IV in Bloom (2009). For Cooper and Haltiwanger (2006) and Bloom (2009) we report the sum of costs associated with two sources of lumpy adjustment: fixed adjustment costs and partial irreversibility. The remaining models only have fixed adjustment costs.

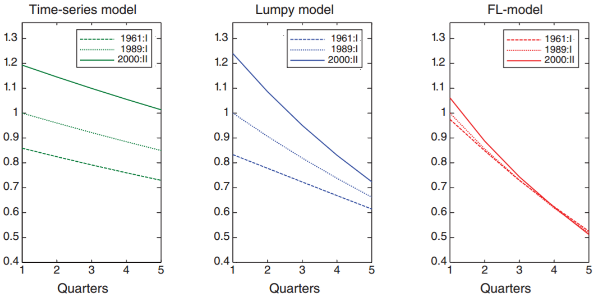
Aggregate Nonlinearities

TABLE 5—HETEROSCEDASTICITY RANGE

Model	$\log(\sigma_{95}/\sigma_5)$
Data	*0.3021*
This paper ($\chi = 0$)	0.1830
This paper ($\chi = 0.25$)	0.2173
This paper ($\chi = 0.50$)	0.2901
Quadratic adj. costs ($\chi = 0$)	0.0487
Quadratic adj. costs ($\chi = 0.25$)	0.0411
Quadratic adj. costs ($\chi = 0.50$)	0.0321
Frictionless	0.0539
Khan-Thomas (2008)	0.0468

Notes: This table displays heteroscedasticity range ($\log(\sigma_{95}/\sigma_5)$) for the data (row 1) and various model specifications that vary in terms of the maintenance parameter χ and the adjustment technology for capital: fixed adjustment costs (rows 2–4), quadratic adjustment costs (rows 5–7), a frictionless model, and the Khan-Thomas (2008) model. The adjustment costs for the models in rows 2–7 have been calibrated to match aggregate and sectoral investment rate volatilities.

Aggregate Nonlinearities



$$\frac{I_t}{K_t} = \sum_{j=1}^{p} \phi_j \frac{I_{t-j}}{K_{t-j}} + \sigma_t e_t$$

$$\sigma_t = \alpha_1 + \eta_1 \frac{1}{p} \sum_{j=1}^{p} \frac{I_{t-j}}{K_{t-j}}$$

Aggregate Nonlinearities

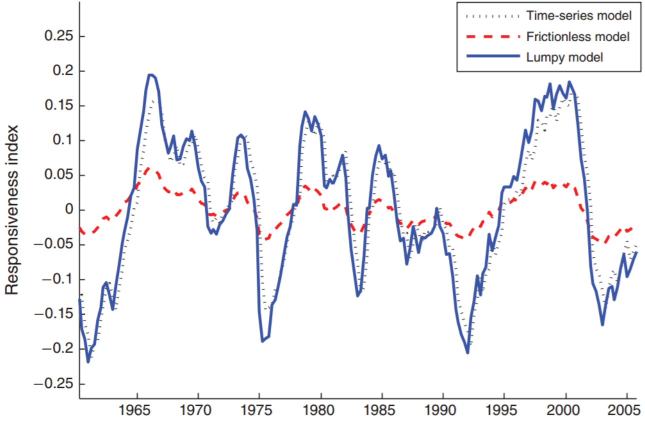


FIGURE 3. TIME PATHS OF THE RESPONSIVENESS INDEX

Aggregate Nonlinearities

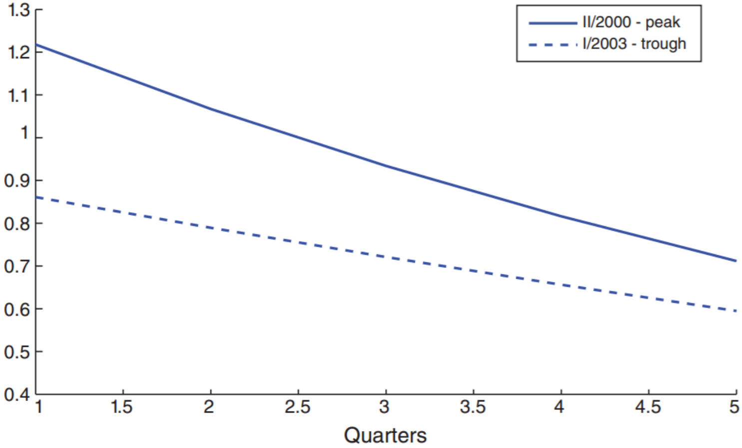


FIGURE 7. IMPULSE RESPONSES OF THE AGGREGATE INVESTMENT RATE IN THE 2000 BOOM-BUST CYCLE

Winberry (2018)

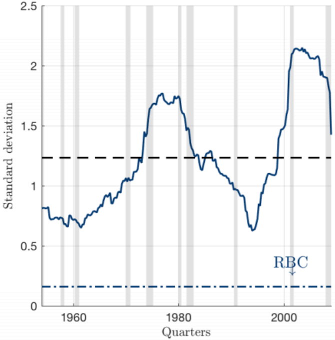
- Argues that procyclical interest rate inconsistent with data
- When consistent with data recover aggregate nonlinearities

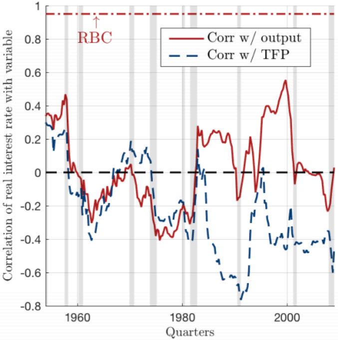
Winberry (2018)

- Argues that procyclical interest rate inconsistent with data
- When consistent with data recover aggregate nonlinearities

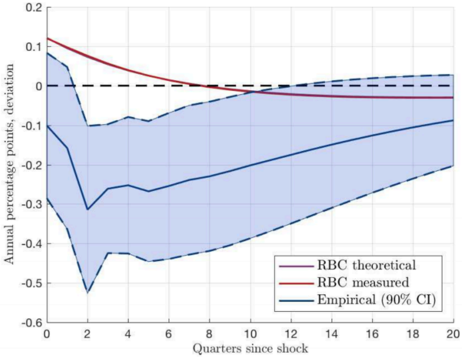
	$\boldsymbol{\sigma}(r_t)$	$\boldsymbol{\rho}(r_t, y_{t-1})$	$\boldsymbol{\rho}(r_t, y_t)$	$\boldsymbol{\rho}(r_t, y_{t+1})$
T-bill	2.18%	-0.08	-0.17	-0.251
AAA	2.34%	-0.29	-0.37	-0.40
BAA	2.43%	-0.32	-0.41	-0.45
Stock	24.7%	-0.24	-0.14	0.02
RBC	0.16%	0.61	0.97	0.74

Rolling Windows of r_t Dynamics





IRF of r_t to TFP Shock



Takeaways from this Lecture

- Specification of **benchmark heterogeneous firm model**
  - Individual vs. aggregate states
  - Role of the distribution in the aggregate state variable
  - Steady state: constant aggregates, but lots of churning at individual level
- Overview of how people **solve heterogeneous agent models**
- Response of aggregate investment to shocks **depends on distribution of firms**, which changes over the business cycle
  - Less responsive in recessions