Intergenerational Risk Sharing with Market Liquidity Risk

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- Intergenerational risk sharing (IRS) can happen institutionally through
  - PAYG/ Defined Benefit arrangements
  - Funded pension schemes with benefit/contributions indexing based on fund surpluses
  - Public debt and taxes
- Market liquidity as the ability to sell costlessly an asset
- Affects long-run investment decisions, e.g
  - Pension funds exploiting liquidity premia in their asset allocations;
  - Illiquid housing wealth locks up large portions of households' life-cycle investments;

- Main question
  - Can the risk of illiquidity be shared across generations?
  - What are benefits of intergenerationally sharing illiquidity?
- Goals
  - Provide a stylized framework for thinking about tradability in an inter-generational setting;
  - Evaluate the welfare losses resulting from illiquidity;
  - Quantify the welfare improvements from the introduction of risk-sharing transfers between coexisting generations;

Relate to the literature on:

- IRS: Gordon and Varian [1988], Shiller [1999], Ball and Mankiw [2007], Beetsma and Romp [2016], Lancia et al. [2020]
- Portfolio choice with risk sharing between generations: Merton [1981], Gollier [2008], Cui et al. [2011]
- Asset pricing and allocation with illiquidity and transaction costs: Acharya and Pedersen [2005], Brunnermeier and Pedersen [2009], Ang et al. [2014], Munk [2020]
- Policy implications when agents are liquidity constrained: Kaplan and Violante [2014]

- Liquid risk-free asset (R^f)
- Risky assets

$$\frac{P_t^i}{P_{t-1}^i} = R_t^i = \mu_i + \epsilon_t^i$$

- Illiquid risky asset

$$\tilde{R}_t^x = \frac{P_t^x(1 - l_t)}{P_{t-1}^x} = \mu_x - \mu_x l_t + \epsilon_t^x(1 - l_t) \tag{1}$$

- Illiquidity risk Appendix

$$l_t = \begin{cases} 0 & \text{with probab.} \quad p \\ \bar{l} & \text{with probab.} \quad 1 - p \end{cases} \tag{2}$$

- Two-period OLG small open economy with fixed young-age endowment Y and savings technology.
- Lifetime utility

$$u_y(C_{y,t}) + \beta \mathbb{E}_t u_o(C_{o,t+1})$$

- Social welfare evaluated *ex ante*:

$$V(\tau) = \mathbb{E}\left(\sum_{t=1}^{\infty} \delta^{t-1}\left(\frac{\beta}{\delta} u_o(C_{o,t}) + u_y(C_{y,t})\right)\right) \tag{3}$$

  with δ a policy-relevant discount factor

- Translates into utility equivalent risk-free consumption per period (CEC)
- Two types of IRS:
  1. Infinitely-lived planner can accumulate wealth buffers as tools of risk sharing between all future generations
  2. In a decentralized frameworks, we introduce a policymaker who sets transfers between coexisting generations

- Centralized planner solution
- Planner who takes over each generation's endowment and invests it optimally through time
- Risk can be spread through time accross multiple generations

$$V(W_t, X_t) = \max_{C_{y,t}, C_{o,t}, S_t, D_t^+, D_t^- \in \mathcal{A}} \left\{ \tilde{u}(C_{y,t}, C_{o,t}) + \delta \mathbb{E} V(W_{t+1}, X_{t+1}) \right\}$$

$$W_{t+1} = \left(W_t + Y - C_{y,t} - C_{o,t} - D_t^+ + D_t^-(1 - l_t)\right) R^f + S_t' r_{t+1}^s$$

$$X_{t+1} = \left(X_t + D_t^+ - D_t^-\right) R_{t+1}^x$$

with $\tilde{u}(C_{y,t}, C_{o,t}) = \frac{\beta}{\delta} u_o(C_{o,t}) + u_y(C_{y,t})$

- Subject to borrowing and pledgeability constraints

- Decenttralized solution: Individuals manage wealth over two periods framework subject to transfers

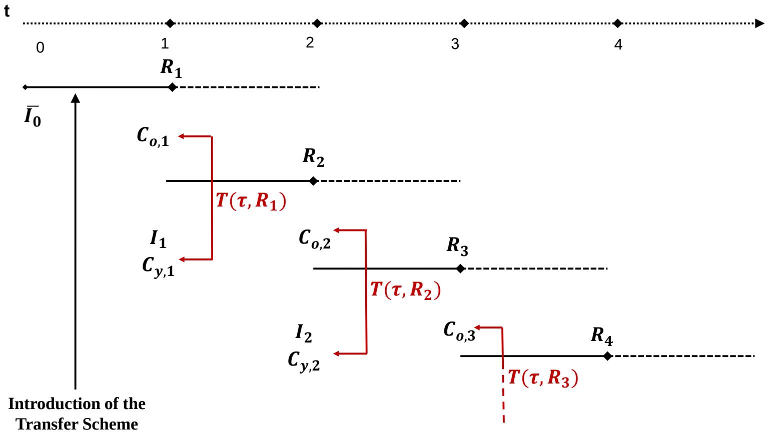
$$v = \max\{u_y(C_{y,t}) + \beta \mathbb{E}_t u_o(C_{o,t+1})\}$$

- subject to

$$\begin{aligned}
W_{t+1} &= (Y - C_{y,t} - D_t^+ - T_t)R^f + S_t' r_{t+1}^s \\
X_{t+1} &= D_t^+ R_{t+1}^x \\
C_{o,t+1} &= W_{t+1} + X_{t+1}(1 - l_{t+1}) + T_{t+1}
\end{aligned}$$

- Translates into the individuals' problem

$$\begin{aligned} & \max_{I_t} \left\{ u_y(C_{y,t}) + \beta \mathbb{E}_t u_o(C_{o,t+1}) \right\} \\ \text{s.t.} \quad & C_{y,t} = Y - I_t' \mathbb{1} - T_t \\ & C_{o,t+1} = I_t' R_{t+1} + T_{t+1} \end{aligned} \tag{4}$$



- IRS policy is not anticipated by the first old cohort when they took their savings decisions while still young

- No wealth buffers between generations
- Linear transfers

$$T(\tau, R_t) = Y\tau' \left(\mathbb{E}(R_t) - R_t\right) \tag{5}$$

- A policymaker controls the risk-sharing instruments τ
- Risk transfers can go both ways between the young and the old
- No systematic transfers

- Policymaker setting transfers in a decentralized economy:

$$V(\tau) = \mathbb{E}\frac{\beta}{\delta} u\left(\overline{C}_o(\tau, R_1)\right) + \frac{1}{1-\delta}\mathbb{E}v(\tau, R_t)$$

- First-order condition

$$\frac{\beta}{\delta}\mathbb{E}\left[u_o'(C_{o,1})\frac{\partial C_{o,1}}{\partial \tau_i}\right] + \frac{1}{1-\delta}\mathbb{E}\left[u_y'(C_{y,t})\frac{\partial C_{y,t}}{\partial \tau_i} + \beta u_o'(C_{o,t})\frac{\partial C_{o,t}}{\partial \tau_i}\right] = 0$$

where $C_{y,t}$ and $C_{o,t}$ are set optimally by the generations taking into account the transfer policy.

- Quadratic Utility
- Single asset with return $\tilde{R}^x_t$
- Combined shock $\tilde{\epsilon}^x_t = \mathbb{E}\left(\tilde{R}^x_t\right) - \tilde{R}^x_t$
- Savings are fixed and exogenous, $\overline{S}$;
- Consumption evolves as:

$$C_{y,t} = Y - \overline{S} + \tau \tilde{\epsilon}^x_t Y$$
$$C_{o,t+1} = \overline{S}\tilde{R}^x_{t+1} - \tau \tilde{\epsilon}^x_{t+1} Y$$

- The policymaker sets

$$\tau^* = \left(\frac{\beta}{\beta + \delta}\right)\frac{\overline{S}}{Y} \tag{6}$$

- Quadratic Utility; Single asset with return $\tilde{R}_t^x$
- Utility from retirement wealth only, savings S_t are endogenous

$$S_t = Y + \tau Y \tilde{\epsilon}_t^x$$
$$C_{o,t+1} = S_t \tilde{R}_{t+1}^x - \tau Y \tilde{\epsilon}_{t+1}^x$$

- The policymaker sets

$$\tau^* = \frac{1}{\delta \mathbb{E}\left((\tilde{R}_{t+1}^x)^2 \right) + 1} \tag{7}$$

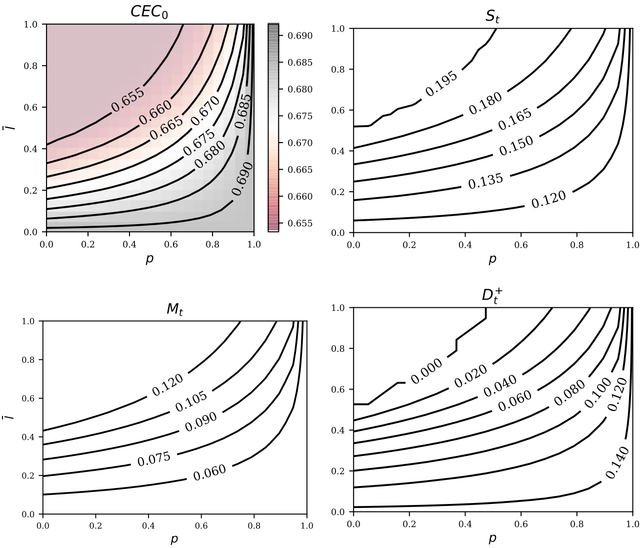
- Due to independence:
  $\mathbb{E}\left((\tilde{R}_{t+1}^x)^2 \right) = \mathbb{E}\left((R_{t+1}^x)^2 \right) \mathbb{E}\left((1 - l_t)^2 \right)$
- Then we have $\frac{\partial \tau^*}{\partial \sigma_t^x} < 0$, $\frac{\partial \tau^*}{\partial p} < 0$, $\frac{\partial \tau^*}{\partial \bar{l}} > 0$

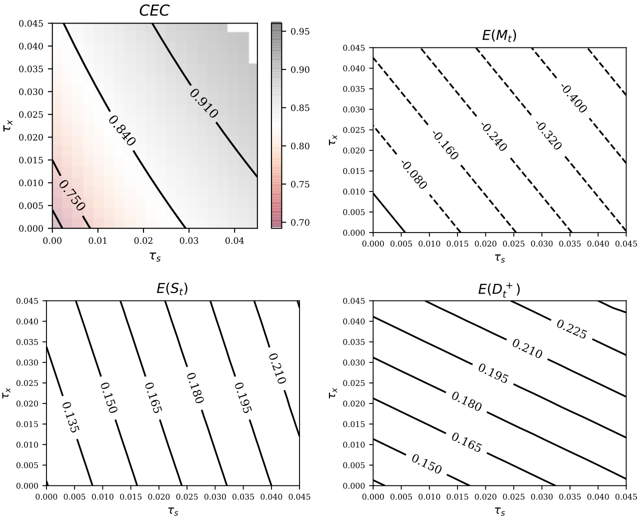
- CRRA Utility of young and old-age consumption with $\gamma = 5$
- Economy scaled with $Y = 1$
- Three assets, multivariate-normal log returns
- Probability to trade evaluated as $p = 1 - \mathrm{e}^{-\eta \Delta t}$ through the average waiting time to trade η.

	Holding Time	p	Weight	μ	σ	ρ
Liquid Risk-Free Asset						
- **Mid-Term Gov Bonds**			100%	**0.002**	-	
Liquid Risky Asset				-	-	1.000
- **Global Equity**			100%	**0.061**	**0.156**	
Illiquid Risky Asset				**0.049**	**0.120**	**0.586**
- Hedge Funds	1 - 2	0.92 - 0.99	16%	0.030	0.074	0.730
- Private Equity	4	0.71	23%	0.078	0.202	0.800
- Institutional Real Estate	8 - 10	0.39	39%	0.046	0.111	0.500
- Institutional Infrastructure	50 - 60	0.08	14%	0.047	0.105	0.550
- Private Loans	-	-	8%	0.017	0.045	0.150

- Calibration based on JPM Capital Market Assumptions, OECD pension fund data, Ang et al. [2014], Nadauld et al. [2019]
- Probability of costless trading :

$$p = P(N_{t+\Delta t} - N_t \geq 1) = 1 - P(N_{t+\Delta t} - N_t = 0) = 1 - \mathrm{e}^{-\eta \Delta t}$$


CEC_0
S_t
M_t
D_t^+
p
$\bar{l}$
0.655
0.660
0.665
0.670
0.675
0.680
0.685
0.690
0.195
0.180
0.165
0.150
0.135
0.120
0.105
0.090
0.075
0.060
0.000
0.020
0.040
0.080
0.100
0.140


CEC
0.910
0.840
0.750
τ_x
τ_s
0.95
0.90
0.85
0.80
0.75
0.70
$E(M_t)$
-0.400
-0.320
-0.240
-0.160
-0.080
$E(S_t)$
0.135
0.150
0.165
0.180
0.195
0.210
$E(D_t^+)$
0.225
0.210
0.195
0.180
0.165
0.150

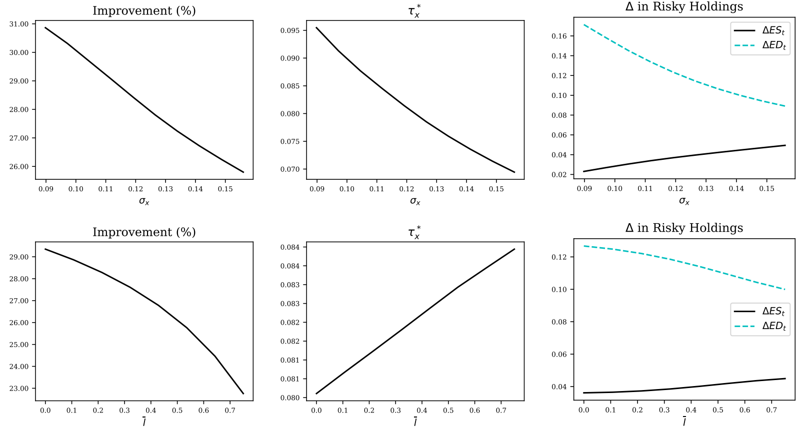
- Optimal IRS transfer scheme

	No Risk Sharing	Risk Sharing
$\mathbb{E}Cy$	0.694	1.008
$\mathbb{E}Co$	1.359	1.374
$\mathbb{E}M$	0.054	-0.446
$\mathbb{E}S$	0.119	0.224
$\mathbb{E}D^{+}$	0.133	0.214
τ_s^*		0.050
τ_x^*		0.021

The economy scale for a young-age endowment of 1. Investment numbers in amounts.

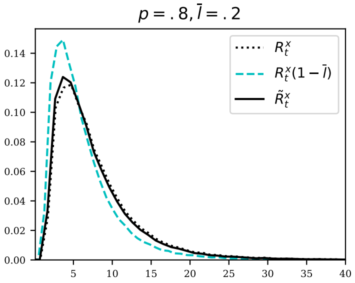
- Welfare improvement from IRS

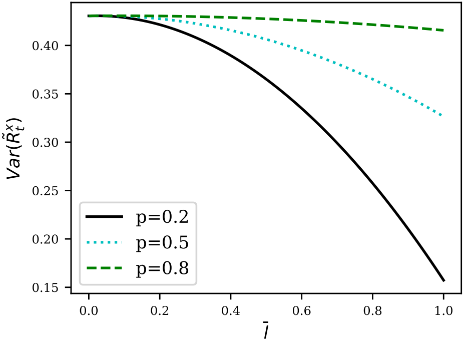
	No Risk Sharing	Policymaker	Planner
	With borrowing		
CEC	0.687	0.932	1.014
Improvement	-	36%	48%
	Without borrowing		
CEC	0.687	0.805	0.830
Improvement	-	17%	21%


Improvement (%)
31.00
30.00
29.00
28.00
27.00
26.00
0.09 0.10 0.11 0.12 0.13 0.14 0.15
σ_x
τ_x^*
0.095
0.090
0.085
0.080
0.075
0.070
0.09 0.10 0.11 0.12 0.13 0.14 0.15
σ_x
Δ in Risky Holdings
ΔES_t
ΔED_t
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.09 0.10 0.11 0.12 0.13 0.14 0.15
σ_x
Improvement (%)
29.00
28.00
27.00
26.00
25.00
24.00
23.00
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
$\bar{l}$
τ_x^*
0.084
0.084
0.083
0.083
0.082
0.082
0.081
0.081
0.080
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
$\bar{l}$
Δ in Risky Holdings
0.12
0.10
0.08
0.06
0.04
ΔES_t
ΔED_t
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
$\bar{l}$

To sum up

- Risk sharing between coexisting generations is welfare improving
  - It enhances individuals' capacity to bear risk and invest in risky and illiquid assets
  - If excessive, however, may destabilize the welfare of the young
- More illiquidity justifies higher levels of risk sharing
- Mode volatility justifies lower risk sharing


$p = .8, \bar{l} = .2$
$p = .5, \bar{l} = .5$
R_t^x
$R_t^x(1-\bar{l})$
$\tilde{R}_t^x$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.05
0.15
0.20
5
10
15
20
25
30
35
40


0.40
0.35
0.30
0.25
0.20
0.15
$Var(\tilde{R}_t^x)$
0.0
0.2
0.4
0.6
0.8
1.0
$\bar{l}$
p=0.2
p=0.5
p=0.8

back