Long-term Portfolio Choice with the Presence of a Liquidity Friction

Summer School Market Microstructure 2021

UvA, Macro and International Economics

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Pension funds, endowment funds, etc. investing in asset classes that are illiquid even over large investment horizons...

- Hedge Fund lock-up periods;
- Private Equity, Infrastructure Funds exited usually once they mature;

- Examine portfolio allocation in a dynamic setting when part of the asset mix is illiquid;
- Examine how significant should illiqidity concerns be for long term investors;
- Quantify the cost of illiquidity in certainty equivalent terms;

Model

Model based on Ang, Papanikolaou, Westerfield (2014) as an extension to Merton (1971, 1973)

- Continuous time;
- Liquidity is exogenous and random: an illiquid asset cannot be traded for periods of random length;
- Quantify illiquidity through the average time one needs to wait between transactions: a Poisson Process governs tradability;
- The investor can consume out of liquid wealth only

- Investment choice:
  - Liquid Wealth W_t, consisting of a risk-free asset $(1-\theta_t)$ and a liquid risky asset (θ_t)
  - Illiquid wealth X_t consisting of a risky asset
  - Rebalancing between liquid and illiquid wealth through cash withdrawals dI_t
- Agent optimizes lifetime utility of consumption

$$V(W_t, X_t) = \sup_{\theta_s, dI_s, c_s} E_t \int_t^{\infty} \mathrm{e}^{-\beta(s-t)} u(C_s) ds$$

- Subject to the wealth constraints

$$dW_t/W_t = (r + (\mu_1 - r)\theta_t - c_t)dt + \theta_t \sigma_1 dZ_{1t} - dI_t/W_t$$
$$dX_t/X_t = \mu_2 dt + \sigma_2 \rho dZ_{1t} + \sigma_2 \sqrt{1-\rho^2} dZ_{2t} + dI_t/X_t$$

Using CRRA Utility $u(C_s) = \frac{C_s^{1-\gamma}}{1-\gamma}$ we can write

$$V(W_t, X_t) = (X_t + W_t)^{1-\gamma} H(\xi_t)$$

where $\xi_t = \frac{X_t}{X_t + W_t}$ and whenever liquidity is available, the investor reshuffles the portfolio such that

$$\xi^* = arg \max_{\xi} H(\xi)$$

- Discretize and solve numerically through Dynamic Programming techniques.
- ξ_t is both a stochastic state variable and a decision variable whenever liquidity is available.

$$\xi_t$$
$$H(\xi_t)$$
$$\theta_t = \theta_t(\xi_t, H(\xi_t))$$
$$c_t = c_t(\xi_t, H(\xi_t))$$
$$Q_t$$

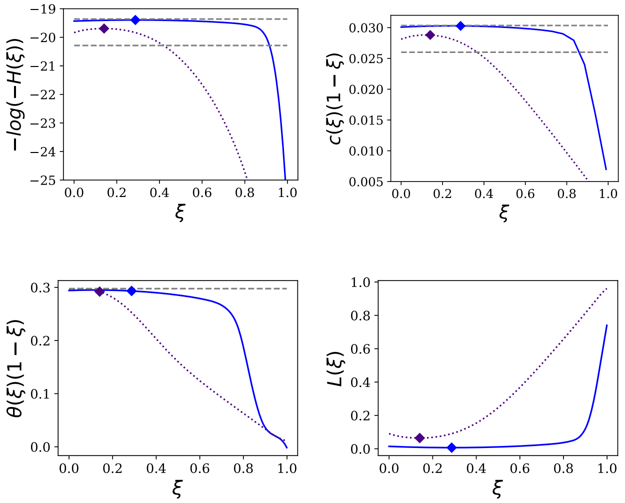
Trading of X is possible with prob. p

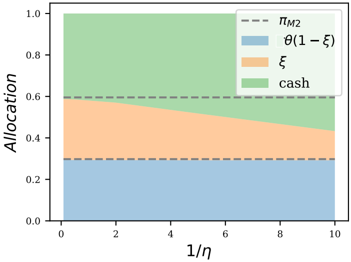
$$\xi_{t+1} = \xi^*$$
$$H(\xi^*) = H^*$$
$$dI_{t+1}(\xi^*)$$
$$Q_{t+1} = W_{t+1}(c_t, \theta_t) - dI_{t+1} + X_{t+1} + dI_{t+1}$$

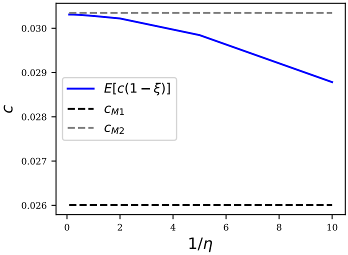
Trading of X not possible with prob. $1 - p$

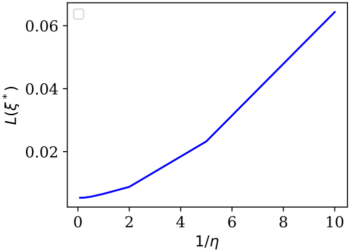
$$\xi_{t+1} = \frac{W_{t+1}(c_t, \theta_t)}{W_{t+1} + X_{t+1}}$$
$$H(\xi_{t+1})$$
$$dI_{t+1} = 0$$
$$Q_{t+1} = W_{t+1}(c_t, \theta_t) + X_{t+1}$$

Continuous trading (dashed lines) vs. one-year (solid curve) vs. ten-year friction (dotted curve)




1.0
0.8
0.6
0.4
0.2
0.0
Allocation
0
2
4
6
8
10
1/η
π_M2
θ(1 − ξ)
ξ
cash


0.030
0.029
0.028
0.027
0.026
c
E[c(1 − ξ)]
c_M1
c_M2
0
2
4
6
8
10
1/η


0.06
0.04
0.02
L(ξ*)
0
2
4
6
8
10
1/η

Asset Class	$1/\eta$	p	μ_2	σ_2	$\frac{\mu_2 - r}{\sigma_2}$	ρ
Public Equity	-		5.5	14.0	39.1	1.00
LT Gov. Bonds	-		2.5	12.8	4.0	(0.35)
Municipal Bonds	0.50	63.2%	3.5	5.1	67.9	(0.01)
HF - Multi-strategy	0.90	42.6%	4.3	7.5	56.4	0.08
HF - ED	1.01	39.0%	4.8	9.0	52.6	0.66
HF - Long Bias	0.90	42.6%	4.8	10.5	45.0	0.80
HF - Relative Value	0.68	52.1%	4.5	7.0	64.0	0.86
HF - Global Macro	0.59	57.1%	3.8	7.0	53.3	0.68
Private Equity	4.00	11.8%	7.3	21.0	34.4	0.82
Direct Real Estate	9.00	5.4%	5.3	10.8	48.7	0.30
Infrastructure	55.00	0.9%	6.3	11.8	53.0	0.30

	$CEC(\xi^*)$	$\pi_1;\ \theta^*(1-\xi^*)$	$\pi_2;\xi^*$	$E[\theta(1-\xi)]$	$E[\xi]$
Public Equity	2.5	29.8	-		
LT Gov. Bonds	2.7	35.8	18.9		
Municipal Bonds	3.2	29.7	72.3	29.4	73.2
HF - Multi-strategy	2.6	13.5	26.7	12.9	27.7
HF - ED	2.8	10.4	44.8	9.7	46.6
HF - Long Bias	2.6	13.5	26.7	12.9	27.7
HF - Relative Value	3.1	-	80.9	-	82.6
HF - Global Macro	2.6	17.6	34.9	17.4	35.5
Private Equity	2.6	15.5	9.6	15.5	12.2
Direct Real Estate	2.8	25.3	18.0	20.4	33.8
Infrastructure	2.6	28.3	4.7	13.3	55.3