Time-Consistent Risk Sharing under Ambiguity

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Abstract

This paper examines optimal intergenerational risk sharing in an overlapping generations economy where agents face ambiguity about future savings volatility. Using smooth ambiguity preferences, I model how uncertainty about risk affects welfare and optimal sharing under intergenerational transfers. In a decentralized market setting, a policymaker implements a binding transfer policy under the veil of ignorance, accounting for both risk and model uncertainty in a time-consistent manner. In the context of retirement savings, I show that ambiguity (model uncertainty) about risk reduces the effectiveness of transfers by distorting precautionary behavior and by inducing more conservative policy responses to underlying shocks. (Deep) uncertainty, for example associated with climate change, can constrain fiscal insurance mechanisms and can complicate the design of intergenerational policy.

JEL codes: D81, E21, H55, G11, D91

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1 Introduction

There are well-known hurdles in a free market economy to sharing risk between generations that are subject to different economic prosperity over their lifetimes (Merton, 1981; Gordon and Varian, 1988; Shiller, 1999; Campbell and Nosbusch, 2007; Gollier, 2008; Cui et al., 2011; Lancia et al., 2024). Due to the natural physical limitation of a finite lifetime, individuals cannot directly participate in risk that materializes before or after they become economically active. This creates a classical incomplete market inefficiency and a policy intervention that sets contingent transfers between young and old generations can improve social welfare by widening the risk-bearing pool in the economy and thus increasing its capacity to bear risk.

The current intergenerational insurance models, however, are constructed under the assumption that the parameters governing future risks are known with certainty. This assumption becomes increasingly tenuous given the long-run horizons that such models are designed to address. The deep uncertainty surrounding long-term economic and environmental risks, as those posed for example by climate change, challenges the central premise expected utility theory of risk under known probabilities. The climate economics literature has already emphasized the role of model uncertainty in designing optimal climate policies (Millner et al., 2011; Heal and Millner, 2014; Millner et al., 2013; Barnett et al., 2020; Olijslagers and van Wijnbergen, 2024; Xepapadeas, 2024). This angle, however, is still missing from the literature on integenerational risk sharing.

This paper contributes to the literature by embedding unresolvable ambiguity – that is, uncertainty which cannot be reduced through learning – into a setting of intergenerational transfers. Using smooth ambiguity preferences (Klibanoff et al., 2005, 2009), I demonstrate that such ambiguity can be incorporated in a tractable manner and has material implications for optimal policy design. Specifically, I show that ambiguity induces a precautionary motive in the policymaker, who seeks to avoid destabilizing a generation's welfare through overly aggressive risk-sharing transfers. The ambiguity-adjusted policy that results still continues to facilitate intergenerational insurance, but does so with an attenuated sensitivity to uncertain shocks, thereby enhancing robustness in the face of uncertainty about future risk. In an environment where the level of future asset volatility is ambiguous, a more cautious transfer rule mitigates the compounding of misestimated risk over time. The optimal policy continues to insure across generations, but does so in a way that is robust to uncertainty about the distribution of future risks.[1]

I develop a stylized framework with two overlapping generations (OLG) in which, in the context of retirement savings, the young and the old share financial risk. Wealth shocks arise from the returns of risky assets in the savings portfolio of individuals used to fund retirement consumption. Shocks each period occur before the current young have accessed the capital markets, and before they have made any investment decisions. The young start with labor endowment that is not affected by the current shock while the old bear financial risk on their savings. In a fully decentralized market economy, the young are making consumption, savings, and allocation decisions that optimize their lifetime utility, while the old consume from the accumulated retirement wealth.

This arrangement leaves room for an institutional designer to intervene and enforce transfers between the young and the old, which are contingent on the accumulated return of the old generation's savings portfolio.[2] The transfers are designed from an ex-ante point of view and welfare in the economy is evaluated before any shocks materialize. The transfers are linear in the realized return on the individuals' retirement portfolio and act as partial insurance on retirement wealth, covered by the young. Once aware

[1]There are nuances in the decision theory literature between (Knightian, or deep) uncertainty and ambiguity. Knightian uncertainty typically refers to situations where the probability distribution of possible outcomes is unknown or incalculable. In this context, decision-makers cannot assign precise probabilities to different events because they lack the necessary information (e.g. lack of data, historical precedents) or because the future is genuinely unpredictable. Ambiguity, on the other hand, relates to situations where the information available is incomplete or vague, making it difficult to assess the likelihood of various outcomes. In the presence of ambiguity, decision-makers may have some information, but it is not clear how to interpret or use that information to make predictions about the future. Ambiguity arises when there is a lack of clarity or consensus on the relevant probabilities or when there are multiple possible interpretations of the available information. In this paper, I abstract from these nuances and use the two concepts interchangeably.

[2]I abstract from the particularities of the institutions through which IRS occurs. In reality, the contingent transfers between young and old, as modeled here, could be the result of several arrangements. Beetsma and Romp (2016) provide an overview of the institutional frameworks allowing intergenerational risk sharing; Gollier (2008); Bovenberg and Mehlkopf (2014); Cui et al. (2011); Chen et al. (2023) discuss in particular risk sharing between generations in collective pension plans; (Chen et al., 2016) look at counter-cyclical adjustments in the tax code in combination with adjustments to the public debt, through the pay-as-you-go pension system.

of the policy implementation, utility-optimizing individuals adjust their savings mix by factoring in the regulated transfer policy, giving rise also to indirect welfare effects.

First, I show that under known risk parameters risk-sharing induces two opposing effects. On one hand, a policy that engages the young in a shock that otherwise affects only the old widens the pool of people who can participate in that shock and increases the risk-bearing capacity of the economy. This is the well-known pooling effect of risk sharing (Merton, 1981; Gordon and Varian, 1988; Shiller, 1999). At the same time, transfers import additional risk in the youth's labor endowment, thus extending the horizon over which individuals will bear risk. Cumulatively, the latter effect also leads to more risk in their old age. This is the compounding effect, also alluded to in Gordon and Varian (1988). The larger the asset variance, the more the second effect dominates, and thus the lower optimal risk sharing needs to be. Similarly, the lower the asset variance, the more the first effect dominates, and the higher the optimal risk sharing should be.

Second, I show that this pooling-compounding trade-off gives rise to a precautionary motive for the policymaker when risk parameters are uncertain. Faced with the possibility that realized shocks could lead to disproportionately large compounding costs, the policymaker optimally adopts a more conservative approach to risk sharing. The intergenerational transfers are thus designed to be less sensitive to observed shocks, thereby mitigating the risk of excessive exposure for the young that could amplify to their old-age consumption. This cautious stance reflects a desire to hedge against adverse scenarios without fully committing to the worst-case outlook. In a smooth ambiguity framework, the policymaker does not follow a strict minimax rule, i.e., does not optimize solely against the worst possible outcome, but instead chooses a policy that balances across a range of plausible models Gilboa and Schmeidler (1989). This results in a risk sharing policy that lies at a prudent distance from the most pessimistic scenario, reflecting ambiguity aversion while still allowing for some responsiveness to observed shocks.

Ambiguity has become especially relevant in the context of climate risk, where deep uncertainty surrounds the physical and socioeconomic consequences of climate change and the potential feedback loops. As Heal and Millner (2014); Millner et al. (2013) argue, the probabilistic foundations of expected utility theory may be inadequate for guiding policy in this domain. Several contributions, including Millner et al. (2011); Olijslagers and van Wijnbergen (2024); Xepapadeas (2024), apply multiple-prior within an ambiguity framework to climate models, showing that ambiguity aversion can significantly raise the social value of abatement and adaptation efforts. This paper relates, keeping, however, the focus on intergenerational insurance, without the need to model climate dynamics explicitly.

The paper continues as follows: Section 2 outlines a stylized model of risk sharing over time and illustrates how ambiguity affects the optimal policy; Section 3 provides the full quantitative model of intergenerational welfare optimization; Section 4 discusses the model parametrization and the quantitative results.

2 Stylized Look at Risk Transfers between Generations

Intergenerational risk-sharing policies allow agents to participate early in their lifetime in lotteries that otherwise materialize in old age. To illustrate that, consider a simple two-period overlapping-generations economy in a partial equilibrium setting, in which an idiosyncratic shock $\epsilon_{t+1} \sim \text{i.i.d.}(0, \sigma^2)$ hits the return to savings in period $t+1$. To share this risk across cohorts, a linear transfer scheme is introduced between period zero and one: a policymaker chooses a parameter τ ex ante, and in each period $t+1$ for $t \geq 1$ the transfer from the young to the old is defined by $T_{t+1} = -\tau\, \epsilon_{t+1}$, so that a fraction τ of the realized shock is reallocated between the current old and the young.

2.1 Additive Shocks

Assume that shocks are simply added to young-age savings without any dynamic compounding. This is consistent with a simplifying assumption that the savings of the young do not react in any way to the risk sharing policy, so any shocks are fully absorbed by the consumption of the young. In reality, one can think of hand-to-mouth type consumers.

Then, suppose that with risk sharing the young's consumption in period t is

$$C_{y,t}(\tau) \;=\; 1 \;-\; \overline{S} \;+\; \tau\,\epsilon_t,$$

where $\overline{S}$ is a fixed savings amount, and that upon reaching old age they consume

$$C_{o,t+1}(\tau) = \overline{S}\,R_{t+1} - \tau\,\epsilon_{t+1} = \overline{S}\,\mu + (1-\tau)\,\epsilon_{t+1}.$$

Assume now, for the sake of the example, that a planner minimizes the sum of variances of young and old-age consumption, such that

$$\tau^* \;=\; \arg\min_{\tau}\Big\{\mathbb{Var}(C_{y,t}(\tau)) + \omega\mathbb{Var}(C_{o,t+1}(\tau))\Big\}$$

where ω is a policy weight on the relative value of the consumption of the old vs that of the young. As a result we get $\mathbb{Var}(C_{y,t}) = \tau^2\sigma^2, \quad \mathbb{Var}(C_{o,t+1}) = (1-\tau)^2\sigma^2$. Since there is no compounding term, the optimal risk sharing parameter is independent of the asset risk, and we get

$$\tau^* = \frac{\omega}{1+\omega}$$

In other words, risk sharing becomes dependent only on the relative value of the young vs. old age consumption in the policymaker's optimization problem.

In this setting, pooling across many generations eliminates risk from the point of view of single generation: if generation t bears only $1/n$ of each shock ϵ_{t-i} for $i = 0, \ldots, n$, then aggregating over time

$$\mathbb{Var}\Big(\frac{1}{n}\sum_{i=0}^{n}\epsilon_{t-i}\Big) = \frac{1}{n}\,\sigma^2 \;\xrightarrow{n\to\infty}\; 0.$$

This reflects the classic pooling effect: as more generations share the burden of shocks, the variance borne by each individual generation diminishes. However, this intuition relies on an additive treatment of shocks, which overlooks how returns compound over time.

2.2 The Effect of Risk Compounding

Now, in order to examine how intergenerational risk sharing induces the compounding of lotteries and interact negatively with the size of risk sharing, assume that a cohort born at $t \geq 0$ saves an amount S_t as young out of its (unit) young-age endowment, and the gross return on savings is $R_{t+1} = \mu + \epsilon_{t+1}$, then its old-age consumption is

$$C_{o,t+1} \;=\; S_t R_{t+1} + T_{t+1} \;=\; S_t(\mu + \epsilon_{t+1}) - \tau\, \epsilon_{t+1}.$$

In equilibrium, each young generation chooses S_t, anticipating the transfer scheme, apart from the generation born in period $t = 0$ which does not anticipate the introduction of the risk-sharing mechanism. In particular for $t \geq 1$ the preceding shock enters their wealth so that effectively $S_t = 1 + \tau\epsilon_t$. We now examine how the parameter τ affects the variability of old-age consumption, decomposing the effects of risk pooling versus risk compounding.

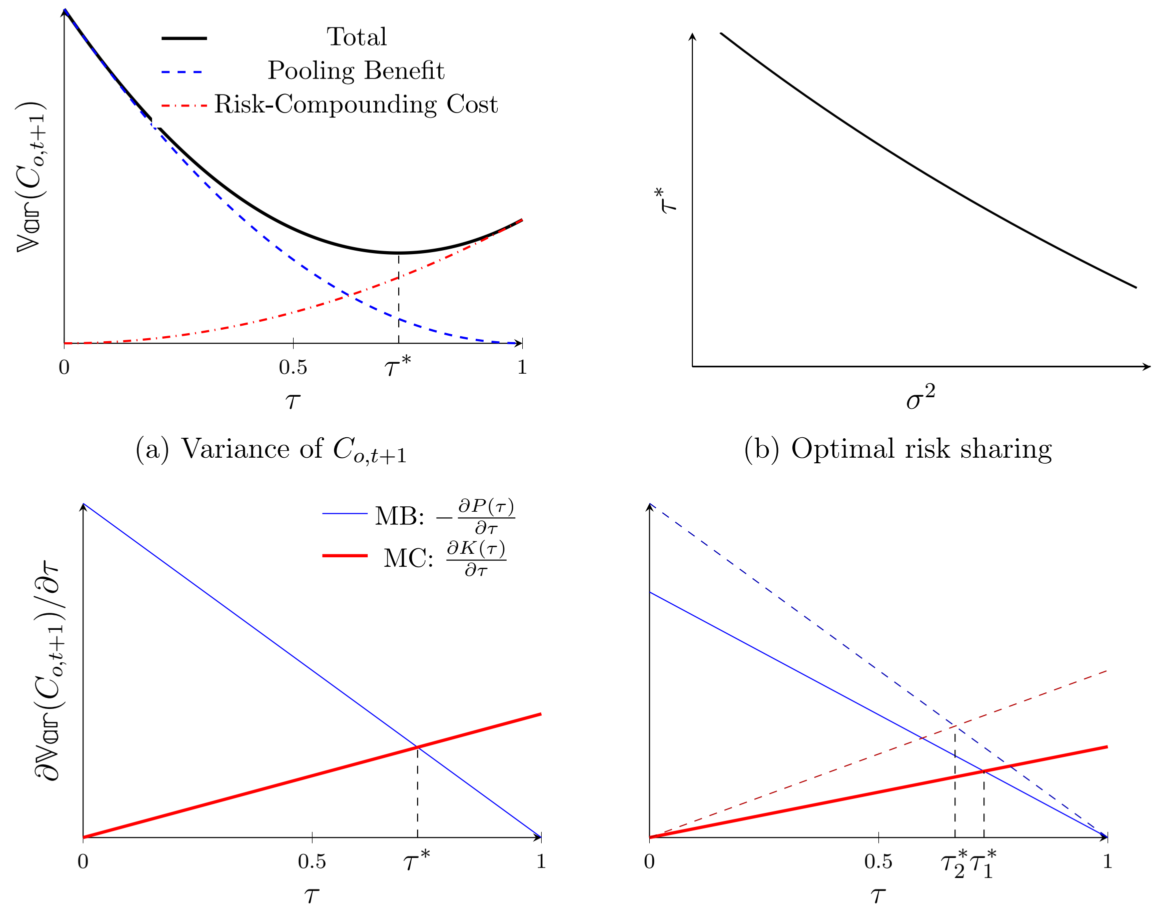
The generation born at $t = 0$ enters old age at $t = 1$ without having been exposed to any youth shocks or transfers. For this first generation, $S_0 = 1$ and their old-age consumption is simply $C_{o,1} = (\mu + \epsilon_1) - \tau\epsilon_1 = \mu + (1 - \tau)\epsilon_1$. Hence the variance of their old-age consumption is

$$\mathbb{Var}(C_{o,1}) = (1 - \tau)^2\sigma^2.$$

This expression shows the pure *risk-pooling benefit* of transfers: the old at $t = 1$ bear only a fraction $1 - \tau$ of the shock ϵ_1, with the rest effectively passed to the newborn young generation. In consequence, their consumption variance is reduced by a factor $(1 - \tau)^2$, and a policymaker focused solely on minimizing $\mathbb{Var}(C_{o,1})$ would set $\tau = 1$ to eliminate the shock entirely from the consumption of the old. In this case, the shock is fully transferred to the young from the new cohort. In short, for generation zero the transfer scheme unambiguously shrinks old-age risk by broadening the risk pool.

By contrast, consider a cohort born after the introduction of the policy at $t \geq 1$. It participates in risk sharing in both periods of its life. In their youth, they receive a transfer $T_t = -\tau\epsilon_t$ (coupled with a reduction in their initial endowment if the shock is

Figure 1: Variance Decomposition and Optimal Risk Sharing Parameter



(a) Variance of $C_{o,t+1}$

(b) Optimal risk sharing

(c) Marginal Pooling Benefit vs. Marginal Compounding Cost

(d) MC and MB shift with higher σ^2

Notes: Panel (a) displays the total variance of old-age consumption and its decomposition into a total pooling benefit and a risk-compounding cost, with the optimal τ^* again marked where the total variance is minimized. Panel (b) shows how optimal risk sharing varies with the risk. Panel (c) shows marginal cost and marginal benefit curves associated with intergenerational risk sharing for two levels of risk. The intersection points define the optimal sharing parameters. Panel (d) shows how the optimal risk-sharing shifts with an increase in the saving's variance.

negative) and therefore save $S_t = 1 + \tau\epsilon_t$. In old age, they consume

$$C_{o,t+1} = (1 + \tau\epsilon_t)(\mu + \epsilon_{t+1}) - \tau\epsilon_{t+1} = \mu + (1 - \tau)\epsilon_{t+1} + \tau\mu\,\epsilon_t + \tau\,\epsilon_t\epsilon_{t+1}.$$

Crucially, the policymaker evaluates the variance of old-age consumption from an ex-ante perspective — before any shocks are realized. This means that both shocks, ϵ_t and ϵ_{t+1} are treated as independent and the variance is computed unconditionally. As ϵ_t and ϵ_{t+1}

are independent with mean zero, the variance of $C_{o,t+1}$ is the sum of the variances of each term in this decomposition. In particular, we have that: $\mathbb{Cov}(\epsilon_t, \epsilon_t\epsilon_{t+1}) = \mathbb{E}(\epsilon_t^2\epsilon_{t+1}) + \mathbb{E}(\epsilon_t)\mathbb{E}(\epsilon_t\epsilon_{t+1}) \overset{i.i.d.}{=} \mathbb{E}(\epsilon_t^2)\mathbb{E}(\epsilon_{t+1}) = 0$; and $\mathbb{Var}(\epsilon_t\epsilon_{t+1}) = \mathbb{E}(\epsilon_t^2\epsilon_{t+1}^2) \overset{i.i.d.}{=} \mathbb{E}(\epsilon_t^2)\mathbb{E}(\epsilon_{t+1}^2) = \sigma^4$. Then, one obtains the unconditional variance of the old-age consumption of a generation born after the introduction of the risk-sharing arrangement as:

$$\mathbb{Var}(C_{o,t+1}) = (1-\tau)^2\sigma^2 \; + \; \tau^2 \left(\mu^2\sigma^2 \; + \; \sigma^4\right). \tag{1}$$

The first term, $P(\tau) = (1-\tau)^2\sigma^2$, again represents the *pooling benefit* on the period $t+1$ shock: by transferring a share τ of that shock to the young, the old generation directly reduces its exposure by a factor $(1-\tau)^2$. Now, there is also a second term: $K(\tau) = \tau^2\left(\mu^2\sigma^2 + \sigma^4\right)$. It represents the cost of compounding risk over time and arises because the uncertainty ϵ_t, introduced into the endowment of the same cohort when it is young, is subsequently reinvested and interacts with the same generation's next-period return. Specifically, the term $\tau^2\mu^2\sigma^2$ reflects the variance contribution from the interaction of the youth shock ϵ_t with the mean return of the savings asset, while $\tau^2\sigma^4$ captures the variance from the interaction between the youth and old-age shocks themselves. Intuitively, the transfer scheme injects volatility into young-age savings via $-\tau\epsilon_t$, which amplifies it in the future period. Overall, $K(\tau)$ implies that intergenerational transfers can also amplify shocks over time rather than dilute them.

The optimal intergenerational scheme is set by trading off these two opposing forces. As shown in Panel (a) of Figure 1, the total variance of old-age consumption is minimized at the point where these two effects exactly offset. The simple rule is to keep increasing τ as long as the marginal benefit (the reduction in variance from pooling equal to $-P'(\tau)$) outweighs the marginal cost (the increase in variance from compounding, which is $K'(\tau)$). This is illustrated in panel (c) where the marginal benefits of increasing τ offset the marginal costs exactly. Formally, setting the derivative of $\mathbb{Var}(C_{o,t+1})$ with respect to τ

to zero yields the first-order condition, which provides the optimal sharing parameter:

$$-P'(\tau) = K'(\tau) \tag{2}$$

$$\implies \tau^* = \frac{1}{1+\mu^2+\sigma^2} \tag{3}$$

Note the inverse relationship between the asset volatility and the optimal risk sharing: greater return volatility calls for intergenerational risk sharing arrangements that are less sensitive to the realization of the asset shock. The intuition behind this result is illustrated in Panel (d) of Figure 1, which shows how both curves shift with an increase in σ^2. Higher volatility steepens the marginal cost curve and it also steepens the marginal benefit curve but by less, pulling the optimal risk-sharing parameter leftward to τ_2^*. Thus, while pooling reduces risk at first, aggressive transfers eventually exacerbate lifetime volatility. In this stylized setup, greater asset volatility dampens the gains from intergenerational insurance, as compounding dominates pooling. The optimal policy τ^* internalizes this trade-off.

In effect, the argument here hinges on the fact that returns compound multiplicatively. When shocks affect savings that are reinvested over time, the variance of an n-period compounded return becomes:

$$\mathbb{Var}(\epsilon_t \epsilon_{t-1} \cdots \epsilon_{t-n+1}) = \left(\sigma^2\right)^n,$$

assuming i.i.d. shocks with zero mean and variance σ^2. This insight underpins the so-called "fallacy of time diversification" (Samuelson, 1963), and what Gordon and Varian (1988) describe as the compounding of lotteries in the context of intergenerational risk-sharing. Contrary to the intuition that risk averages out over time, when agents save and reinvest across periods, shocks introduced early in life are amplified exponentially with the investment horizon. This introduces a compounding cost that must be accounted for in an intergenerational transfer policy trade-off.

2.3 Ambiguity About Risk

Now, consider the situation in which the true value of the return variance is not known with certainty. Assume instead that agents hold a uniform (uninformative) prior over σ^2, such that

$$\pi(\sigma^2) = \begin{cases} \frac{1}{\overline{\sigma}^2 - \underline{\sigma}^2}, & \text{if } \sigma^2 \in [\underline{\sigma}^2, \overline{\sigma}^2] \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

For the sake of tractability, assume that the policymaker is ambiguity-neutral, implying that the agent is aggregating linearly her risk preferences over the distribution of σ^2. Later on, in Section 3 we will introduce a concave aggregator $\phi(.)$ corresponding to an ambiguity aversion in the sense of Klibanoff et al. (2005). Even if the agent has neutral preferences towards ambiguity, however, we will see that this will still affect the optimal policy. In Equation (1) we saw that the variance of old-age consumption is a nonlinear function of the uncertainty parameter σ^2. As a result, it is not surprising that $\mathbb{E}_{\sigma^2}\left[\mathbb{Var}(C_{o,t+1})\right] \neq \mathbb{Var}(C_{o,t+1})\big|_{\sigma^2=\mathbb{E}[\sigma^2]}$.

Formally, now the policymaker's objective becomes:

$$\min_{\tau} \mathbb{E}_{\sigma^2}[\mathbb{Var}(C_{o,t+1})] = \left[(1-\tau)^2 + \tau^2\mu^2\right] \cdot \mathbb{E}_{\sigma^2}[\sigma^2] + \tau^2 \cdot \mathbb{E}_{\sigma^2}[\sigma^4].$$

For a uniform prior, the first and second moments of σ^2 are:

$$\mathbb{E}_{\sigma^2}[\sigma^2] = \frac{\underline{\sigma}^2 + \overline{\sigma}^2}{2}, \qquad \mathbb{E}_{\sigma^2}[\sigma^4] = \frac{1}{3}\left(\overline{\sigma}^4 + \overline{\sigma}^2\underline{\sigma}^2 + \underline{\sigma}^4\right).$$

This then yields:

$$\tau^* = \frac{\mathbb{E}[\sigma^2]}{\mathbb{E}[\sigma^2](1+\mu^2) + \mathbb{E}[\sigma^4]} \tag{5}$$

Compared to the baseline expression $\tau^* = \frac{1}{1+\mu^2+\sigma^2}$, ambiguity leads to a more cautious intergenerational risk-sharing policy. Because $\mathbb{E}[\sigma^4]$ grows faster than $\mathbb{E}[\sigma^2]$ as the upper bound of the support increases, the denominator of τ^* grows faster than the numerator, and thus τ^* falls. This reflects a form of precaution: the policymaker internalizes the possibility of extreme volatility realizations and limits the extent of shock transmission to future generations.

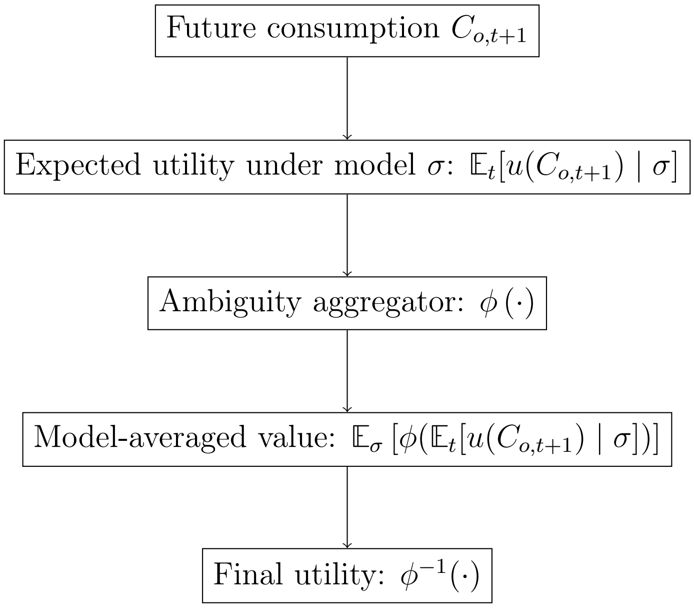
It is worth emphasizing that the decline in τ^* with greater return uncertainty does not imply a retreat from intergenerational insurance. Instead, it reflects the need to moderate the sensitivity of the transfer rule to avoid amplifying risk under model uncertainty. Since here the policymaker evaluates transfers under a double veil of ignorance – before any shocks are realized, and under ambiguity about the volatility itself —- the policy must account for the possibility of miscalibrated risk. A lower τ thus means that while the structure of long-term intergenerational insurance is preserved, its responsiveness to uncertain shocks is deliberately restrained. This improves robustness by containing the compounding of uncertainty across cohorts in the face of ambiguous risk.

3 Full Model with Ambiguity

Consider a discrete-time overlapping generations (OLG) economy indexed by $t \in \{0, 1, 2, \dots\}$. Each generation lives for two periods: young in t, old in $t+1$. The population is stationary, and each generation receives a fixed endowment when young. Time is infinite, and there is no aggregate population growth or technological progress.

The economy features uncertainty about the true volatility of risky asset returns. In particular, agents do not know the exact value of σ, the volatility parameter of risky returns, but hold a smooth ambiguity belief: they consider all values $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ to be possible, with a uniform prior. Formally, the uncertainty set is $\Sigma = [\underline{\sigma}, \bar{\sigma}]$, and the prior $\pi(\sigma)$ is uniform on Σ.

Figure 2: Smooth Ambiguity Utility Aggregation



3.1 Decentralized Lifetime Savings-Consumption Problem under Transfers

Each generation t derives lifetime utility from consumption when young and old:

$$U_t = u(C_{y,t}) + \beta \cdot \tilde{u}(C_{o,t+1}),$$

where $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$ is a CRRA utility function with risk aversion parameter $\theta > 0$, and $\tilde{u}(C)$ is the ambiguity-adjusted utility from old-age consumption.

To capture ambiguity aversion, we adopt smooth ambiguity preferences (Klibanoff et al., 2005), where agents evaluate uncertain consumption outcomes using a two-stage expectation: first over risk, and then over models. Specifically, utility from ambiguous consumption C is given by:

$$\tilde{u}(C) = \phi^{-1}\left(\int_{\underline{\sigma}}^{\bar{\sigma}} \phi\left(\mathbb{E}_t\left[u(C) \mid \sigma\right]\right) \pi(\sigma)\, d\sigma\right), \tag{6}$$

where $\phi(\cdot)$ is a strictly increasing and concave ambiguity aggregator, and $\mathbb{E}_t[\cdot \mid \sigma]$ denotes the expectation over future consumption conditional on given volatility σ and conditional on observing risk that materializes up to time t. Figure 2 illustrates this onion-like layer structure of smooth ambiguity preferences.

This structure ensures that each generation internalizes both risk and model uncertainty when evaluating retirement consumption, while utility from youth consumption $C_{y,t}$ remains deterministic from the agent's perspective at the time of decision.

Asset Market and Portfolio Risk. Young agents allocate their savings S_t between a risk-free and a risky asset. Let $\alpha_t \in [0,1]$ denote the share invested in the risky asset. The gross return on the portfolio is given by:

$$R_{p,t+1}(\sigma) = (1-\alpha_t)R_f + \alpha_t R_t(\sigma), \tag{7}$$

where $R_f = \exp(rn)$ is the gross risk-free return over holding period n, and the risky return follows:

$$R_t(\sigma) = \exp\left(\mu n + \sigma\sqrt{n}\epsilon_t\right), \quad \epsilon_t \sim \mathcal{N}(0,1). \tag{8}$$

As a result, the future retirement consumption is stochastic both because of ϵ_t (risk) and because of the unknown σ (ambiguity).

Transfer policy. Between periods zero and one, a transfer policy T_t between the old and the young is introduced to dampen the inter-generational variability of consumption of the old. The policy is not anticipated before its introduction.[3] All future cohorts are obliged to participate without a walk-out option.

The transfers are designed to be counter-cyclical with respect to the consumption of the old, effectively smoothing idiosyncratic returns across cohorts. This means that when the accumulated return on the portfolio of the old is larger than expected, they give up part of it to the young; and when the return is lower than expected, the old are (partially) compensated by the young. As the policymaker does not have a redistributive objective but purely acts with the mandate to smooth out risks over time, the transfers

[3]A more granular multi-period specification can incorporate anticipation effects. In a two-period setting, however, where each period represents 30 years the implementation of an unanticipated policy seems more realistic.

are designed to be neutral in expectation. In order to keep the transfers also symmetric in all moments of the distribution, they are constructed terms of the logarithmic distance between the realized and the expected return risky asset return.[4] The transfers are thus zero for period $t = 0$, and for periods $t > 0$ they are given by

$$T(\tau, R_t) = \tau\,(n \cdot \mu - \ln R_t) \tag{9}$$

where τ is a vector of parameters standing for the portion of the risk in each asset that is transferred from the old to the young.

Note that, when a policymaker sets the risk-sharing parameters τ, this affects optimal consumption in two ways: first, through the transfers that are directly dependent on the policy parameters, and second, through the adjustment that individuals make on their savings and asset allocations mix in anticipation of the policy. This will then guide the marginal effect on the individuals' utility from changing the policy parameters. As policy anticipation effects have been ruled out, the generation born in period zero will not factor in the possibility of a transfer in its optimal investment-consumption decision, but still will get to participate in the risk-sharing scheme once it is old.

Decentralized Household Problem. Each young agent then solves:

$$\begin{aligned} v(\tau, R_t) = \max_{C_{y,t},\alpha_t} \quad & u(C_{y,t}) + \beta \cdot \tilde{u}(C_{o,t+1}) && (10)\\ \text{s.t.} \quad & C_{y,t} + S_t = Y - T(\tau, R_t), && (11)\\ & C_{o,t+1} = R_{p,t+1}(\sigma) S_t + T(\tau, R_{t+1}), && (12) \end{aligned}$$

where Y is the endowment and T_t denotes the net transfer between generations.

[4]While log-linear transfers are, to some extent, restrictive, this does capture first-order effects and significantly simplifies the consequent optimization problems. Non-linear transfers, which will further increase welfare, are possible, but the added insight relative to the added modeling complexity is likely to be low.

The first-order condition with respect to savings S_t is given by:

$$u'(C_{y,t}) = \beta \cdot \left(\phi^{-1}\right)'(\cdot) \cdot \mathbb{E}_\sigma \Big[\phi' \left(\mathbb{E}_t \left[u(C_{o,t+1}) \mid \sigma \right] \right) \cdot \mathbb{E}_t \left[u'(C_{o,t+1}) \cdot R_{p,t+1}(\sigma) \mid \sigma \right] \Big] \qquad (13)$$

where the derivative of the outer ϕ^{-1} function reflects how marginal utility under ambiguity differs from a standard expected utility setting.

The first-order condition with respect to the portfolio share of risky assets α_t is:

$$\mathbb{E}_\sigma \Big[\phi' \left(\mathbb{E}_t \left[u(C_{o,t+1}) \mid \sigma \right] \right) \cdot \mathbb{E}_t \left[u'(C_{o,t+1}) \cdot S_t \left(R_t(\sigma) - R_f \right) \mid \sigma \right] \Big] = 0 \qquad (14)$$

These Euler conditions generalize the standard ones by weighting model-specific marginal utility expectations using the ambiguity aggregator ϕ'. Intuitively, when ambiguity aversion is present (i.e., $\phi'' < 0$), the agent behaves as though placing greater weight on models with lower expected utility, thereby dampening risky investment and savings relative to the expected utility case.[5]

For generations born at $t \geq 1$ the problem (10) (or specifically the two first-order conditions) implicitly defines the optimal allocation and savings-consumption decisions of each cohort. As the only thing that differs across generations is the shock that hits their young-age endowment, through the transfers, we can write implicitly the optimal

[5]Note that when the ambiguity aggregator is linear, i.e., $\phi(x) = x$ (agents are ambiguity-neutral in the sense of Klibanoff et al. (2005)), the utility collapses to standard expected utility, the preference functional reduces to a standard expected utility over models. In this case, the smooth ambiguity preferences collapse to: $u(C) = \mathbb{E}_{\sigma \in \Sigma} \left[\mathbb{E}_t^\sigma \left[\tilde{u}(C) \mid \sigma \right] \right]$, which is a double expectation over risk (inner) and model uncertainty (outer). In that case, the Euler equations simplify to:

$$u'(C_{y,t}) = \beta \cdot \mathbb{E}_\sigma \mathbb{E}_t \left[u'(C_{o,t+1}) \cdot R_{p,t+1}(\sigma) \mid \sigma \right],$$
$$\mathbb{E}_\sigma \mathbb{E}_t \left[u'(C_{o,t+1}) \cdot S_t \left(R_t(\sigma) - R_f \right) \mid \sigma \right] = 0.$$

However, even in this ambiguity-neutral case, the agent is still integrating over a set of models $\sigma \in \Sigma$. As long as the utility function or the return process is nonlinear in σ—which is typically the case—the resulting behavior *does not coincide* with that of an agent facing a known volatility. In other words, even ambiguity-neutral agents exhibit behavior that differs from pure risk-averse agents due to the second-order effects of parameter uncertainty. If agents are certain about σ, the expectations over σ collapse and one recovers the baseline Euler equations from the standard CRRA portfolio choice problem.

household solutions as

$$\begin{aligned} \alpha_t^* &= \alpha(\tau, R_t); \quad S_t^* = S(\tau, R_t) \\ C_{y,t}^* &= C_y(\tau, R_t); \quad C_{o,t+1}^* = C_o(\tau, R_t, R_{t+1}) \end{aligned} \tag{15}$$

Generation zero, on the other hand, is surprised by the introduction of the policy, so it does not factor the risk transfers into its investment decision. As a result, we write

$$\begin{aligned} \alpha_0^* &= \overline{\alpha}; \quad S_0^* = \overline{S} \\ C_{y,0}^* &= \overline{C}; \quad C_{o,t+1}^* = C_{o,1}(\tau, R_{t+1}) \end{aligned} \tag{16}$$

3.2 Intrergenerational Welfare and Optimal Transfers

Now, we can formalize the welfare analysis under which the policymaker sets the optimal policy. Assume that welfare in this economy is quantified *ex-ante* through the unconditional expectation with respect to all generations' lifetime utilities in all possible states of the world, over all future time periods, by taking into account also the ambiguity with respect to σ. The resulting social welfare is the discounted sum of the weighted expected utilities of all future young and old generations:

$$V = \sum_{t=1}^{\infty} \delta^{t-1} \left(\frac{\beta}{\delta} \tilde{u} \left(C_{o,t} \right) + \tilde{u} \left(C_{y,t+1} \right) \right) \tag{17}$$

where, $\tilde{u}(.)$ is the smooth-ambiguity utility function defined in Equation (6); $\delta < 1$ is the policymaker's discount factor that governs intertemporal trade-offs between generations, and $\frac{\beta}{\delta}$ keeps the relative social weights between young and old utility fixed between time periods.

The policymaker maximizes expected welfare, accounting for the fact that each generation chooses optimal savings and portfolio allocation in response to the policy τ. It is important to note that, while each agent observes their own youth consumption $C_{y,t}$ at the time of decision-making and treats it as deterministic, the policymaker chooses the transfer policy τ *ex ante*, before any realization of consumption shocks. As a result, from the planner's perspective, both $C_{y,t}$ and $C_{o,t+1}$ are stochastic and subject to model uncertainty. As a result of this distinction in the welfare function of the policymaker the ambiguity aversion applies uniformly to all components of individual utility, including youth consumption. The policymaker then sets τ in period zero once and for all future generations, based on information up to that period, and with model uncertainty in mind.

All generations born after $t > 0$ are the same, and the one born in period one is different only in the fact that it does not participate in the transfer policy in its youth. Then, given that welfare is evaluated through an unconditional expectation at period zero, the expression of Equation 17 can be simplified, and we can write it as a function of the risk-transfer parameter τ:

$$V(\tau) = \frac{\beta}{\delta}\left(\tilde{u}\left(C_{o,1}(\tau, R_1)\right) + \frac{1}{1-\delta}\tilde{u}\left(C_y(\tau, R_t)\right) + \frac{\beta}{1-\delta}\tilde{u}(C_o(\tau, R_{t+1}))\right) \tag{18}$$

where $C_y(\tau)$ and $C_o(\tau)$ denote the stationary consumption levels for young and old for the cohorts born after $t \geq 1$ in line with the household optimization setting of (10).

This implies that the policymaker needs to balance the utility of the old generation present immediately after the scheme is implemented with young age and old age utilities of future cohorts, weighted appropriately through the discount factors of the policymaker and the individuals. To reduce the notation overload going forward, we write $C_{o,1}$,$C_{y,t}$ and $C_{o,t}$ while keeping in mind that each of these satisfies the household lifecycle investment problem (10).

As all consumption terms are assumed to satisfy the individual optimality conditions, relying on the Envelope Theorem for the individuals' optimal consumption sensitivity to τ, we can write:

$$\frac{\beta}{\delta}\left[\tilde{u}'(C_{o,1})\frac{\partial C_{o,1}}{\partial \tau_i}\right] + \frac{1}{1-\delta}\left[\tilde{u}'(C_{y,t})\frac{\partial C_{y,t}}{\partial \tau_i} + \beta\tilde{u}'(C_{o,t})\frac{\partial C_{o,t}}{\partial \tau_i}\right] = 0 \tag{19}$$

4 Parametrization and Results

To account for attitudes towards risk, assume that agents have a CARA type utility with γ a coefficient of risk aversion, and a logarithmic ambiguity aggregator such that in aggregate we have:

$$\tilde{u}(C) = -\exp\left(\int \log\left(\mathbb{E}_t\left[-\exp(-\gamma C) \mid \sigma\right]\right)\pi(\sigma)d\sigma\right)$$

Then the certainty equivalent consumption (CEC) associated with one-off ambiguous consumption in the future is

$$CEC = -\frac{1}{\gamma}\log(-\tilde{u}(C)) \tag{20}$$

And in the case of the cumulative welfare function defined in (17), we have the CEC:

$$CEC_V(\tau) = u^{-1}\left(\frac{1-\delta}{\beta+\delta}\delta V(\tau)\right) \tag{21}$$

as this is the level of consumption that solves the risk-free equivalent of (17), such that $V = \sum_i^\infty \delta^{t+1}\left(\frac{\beta}{\delta}u(CEC) + u(CEC)\right)$.

This can be interpreted as the level constant lifetime consumption that, if repeated each period, would deliver the same lifetime utility across generations as the given policy.

Parameter choice: The model is parameterized to reflect plausible long-term economic conditions and intergenerational structures. The generation length is set to 30 years, capturing roughly a typical working-to-retirement transition over a life cycle. The subjective discount rate of individual agents is calibrated as $\beta = \exp(-0.03\cdot 30)$, implying a 3% annual rate of time preference. The planner's social discount rate is set lower, at $\delta = \exp(-0.01 \cdot 30)$, reflecting a more patient social objective function and the desire to weigh future generations more heavily in the welfare aggregation. This can also be seen as a policymaker who believes that free-market welfare is not optimally distributed over time.

Preferences exhibit CARA risk aversion with a coefficient $\gamma = 10$, consistent with values used in the long-term asset pricing and social insurance literature to capture strong aversion to consumption fluctuations. The return process on the risky asset assumes an expected annual excess return $\mu = 6\%$ and a risk-free rate of $r = 3\%$, consistent with stylized facts from historical equity returns. Risk enters through a log-normal return structure with volatility σ, which is ambiguous ex ante. The agent perceives volatility as lying within a bounded interval $[\sigma_{\min}, \sigma_{\max}]$, where $\sigma_{\min} = 0.10$ and $\sigma_{\max} \in \{0.25, 0.30, 0.35\}$ in different experimental runs.

This range of $\sigma_{\max}$ allows us to evaluate how increasing model uncertainty about risk affects portfolio choice, welfare, and the optimal degree of intergenerational risk sharing. The volatility bounds are chosen to reflect uncertainty over macro-financial regimes that agents may face over the course of retirement and that planners must anticipate when setting transfer policies.

4.1 Results

Figure 3 illustrates the effects of increasing risk ambiguity on a range of model outcomes across the transfer grid $\tau \in [0, 0.25]$, where τ governs the extent of intergenerational risk sharing. The case of autarky, where no intergenerational pooling occurs, corresponds to $\tau = 0$. This provides a useful benchmark across all panels.

The charts compare results for three levels of perceived model ambiguity, captured by the upper bound of the volatility interval $\bar{\sigma} \in \{0.25, 0.30, 0.35\}$. The solid black lines depict the baseline case ($\bar{\sigma} = 0.25$), while dashed blue and dash-dotted green lines correspond to moderate and high ambiguity. Vertical dotted lines mark the level of τ that maximizes cumulative social welfare (in certainty-equivalent terms) for each ambiguity level.

Panel (a) shows the risky asset share of generation t. As expected, the share of wealth invested in the risky asset increases sharply with the degree of risk sharing. This is consistent with prior findings Campbell and Nosbusch (2007); Ball and Mankiw (2007); Gollier (2008), which document the insurance and incentive effects of intergenerational transfers on portfolio choice. The strength of this relationship is preserved across all ambiguity levels, although higher ambiguity slightly dampens the risk-taking response.

Panel (b) plots the certainty-equivalent consumption (CEC) of the initial old generation, o_1, who benefit from the policy but do not participate in its cost-sharing side. Their welfare is strictly increasing in τ, and largely insensitive to changes in ambiguity. This reflects their unique position: they receive transfers from the young but do not face portfolio risk or ambiguity in their decision problem. Only at high levels of τ does ambiguity modestly reduce their gain.

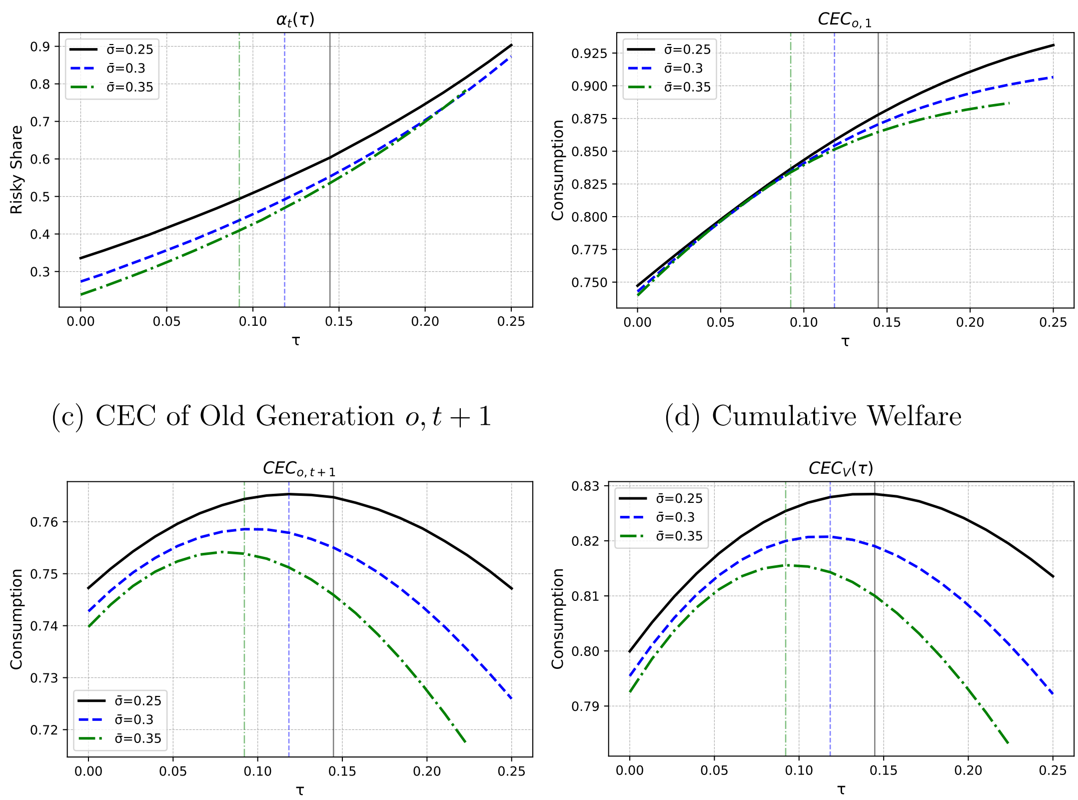
Panel (c) focuses on old generations born after the reform, o_{t+1}, who are fully exposed to the effects of pooling. Their certainty-equivalent welfare is significantly more sensitive to ambiguity. While they initially benefit from risk sharing, at higher τ values—especially under high ambiguity—their welfare begins to decline. In fact, the green curve for $\bar{\sigma} = 0.35$ falls below the autarky level when $\tau > 0.2$, highlighting the potential downside of over-extending intergenerational insurance in ambiguous environments.

Panel (d) displays total social welfare in certainty-equivalent terms. As predicted by the stylized model in Section 2, the optimal level of intergenerational sharing declines with increasing ambiguity. The vertical lines show that the welfare-maximizing value of τ falls from approximately 0.145 under low ambiguity to 0.118 and 0.092 for the intermediate and high ambiguity cases, respectively. This reinforces the core insight that ambiguity aversion acts as a constraint on intergenerational redistribution, particularly when future risk environments are poorly understood.

Additionally, social welfare is non-monotonic in τ: excessive pooling leads to a decline in aggregate utility, even relative to autarky. This underscores the possibility of overshooting the optimal level of intergenerational insurance. A similar reversal is observed in the welfare of future old generations in Panel (c), who are adversely affected when ambiguity-adjusted downside risks dominate the expected benefits of transfers.

Taken together, these results confirm that while intergenerational risk sharing is welfare-enhancing, its optimality is conditional on the level of model uncertainty. Policymakers thus have to account for both risk and ambiguity aversion when designing redistributive transfer systems.

Figure 3: Effects of Risk Ambiguity on Intergenerational Outcomes



(a) Risky Asset Share, Generation t (b) CEC of Old Generation $o, 1$

(c) CEC of Old Generation $o, t+1$ (d) Cumulative Welfare

Note. Each panel shows outcomes across different values of the transfer parameter τ for three levels of perceived risk ambiguity, captured by the upper bound of volatility $\bar{\sigma} \in \{0.25, 0.30, 0.35\}$. Solid black lines represent the baseline ($\bar{\sigma} = 0.25$), dashed blue lines show the intermediate ambiguity ($\bar{\sigma} = 0.30$), and dash-dot green lines represent high ambiguity ($\bar{\sigma} = 0.35$). Vertical dotted lines mark the τ value that maximizes cumulative welfare in certainty-equivalent terms.

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