AMC 10 Extra Practice

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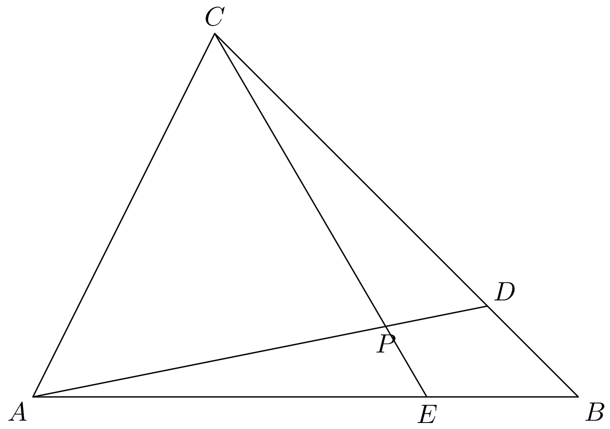
§1 Problem Set 1

§1.1 Problems

1. (1963 AHSME #39) In $\triangle ABC$ lines CE and AD are drawn so that $\dfrac{CD}{DB} = \dfrac{3}{1}$ and $\dfrac{AE}{EB} = \dfrac{3}{2}$. Let $r = \dfrac{CP}{PE}$ where P is the intersection point of CE and AD. Then r equals:



(A) 3 **(B)** $\dfrac{3}{2}$ **(C)** 4 **(D)** 5 **(E)** $\dfrac{5}{2}$

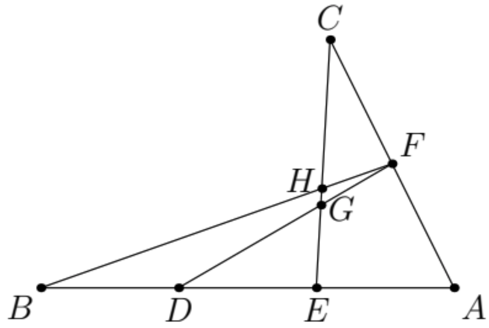
2. $\triangle ABC$ is a triangle with $\angle B = 90°$, $BC = 3$, and $AB = 4$. Point D is on AC such that $AD = 1$, and point E is the midpoint of AB. Join D and E, and extend DE to meet CB extended at F. Find BF.

3. (2011 AIME II #4) In triangle ABC, $AB = \frac{20}{11}AC$. The angle bisector of $\angle A$ intersects BC at point D, and point M is the midpoint of AD. Let P be the point of the intersection of AC and BM. The ratio of CP to PA can be expressed in the form $\dfrac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

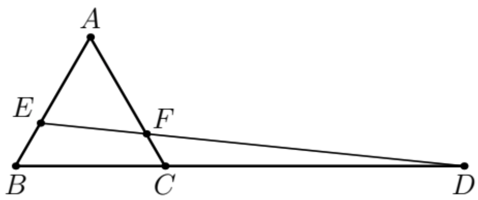
4. Let $\triangle ABC$ have $AC = 6$ and $BC = 3$. Point E is such that $CE = 1$ and $AE = 5$. Construct point F on segment BC such that $\angle AEB = \angle AFB$. EF and AB are extended to meet at D. If $\frac{[AEF]}{[CFD]} = \frac{m}{n}$ where m and n are positive integers, find $m+n$ (note: $[ABC]$ denotes the area of $\triangle ABC$).

5. Triangle ABC has $AB = 2007$ and $AC = 2015$. The incircle ω of the triangle is tangent to AC and AB at E and F respectively, and P is the intersection point of EF and BC. Suppose B is the midpoint of $\overline{CP}$. Compute the length of BC.

6. Given noncollinear points A, B, C, segment $\overline{AB}$ is trisected by points D and E, and F is the midpoint of segment AC. DF and BF intersect CE at G and H respectively. If triangle DEG has area 18, compute the area of triangle FGH.



7. (Purple Comet 2014) The diagram below shows equilateral $\triangle ABC$ with side length 2. Point D lies on ray BC so that $CD = 4$. Points E and F lie on AB and AC, respectively, so that E, F, and D are collinear, and the area of $\triangle AEF$ is half of the area of $\triangle ABC$. Then $\frac{AE}{AF} = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + 2n$.

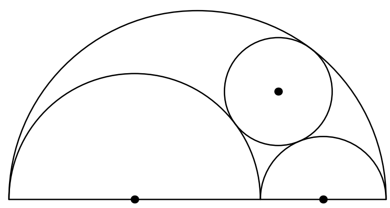


8. (1989 AIME #15) Point P is inside $\triangle ABC$. Line segments APD, BPE, and CPF are drawn with D on BC, E on AC, and F on AB (see the figure below). Given that $AP = 6$, $BP = 9$, $PD = 6$, $PE = 3$, and $CF = 20$, find the area of $\triangle ABC$.

9. (2002 AMC 12B #23) In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side $\overline{BC}$ and the median from A to $\overline{BC}$ have the same length. What is BC?

(A) $\frac{1+\sqrt{2}}{2}$ (B) $\frac{1+\sqrt{3}}{2}$ (C) $\sqrt{2}$ (D) $\frac{3}{2}$ (E) $\sqrt{3}$

10. (2017 AMC 12A #16) In the figure below, semicircles with centers at A and B and with radii 2 and 1, respectively, are drawn in the interior of, and sharing bases with, a semicircle with diameter JK. The two smaller semicircles are externally tangent to each other and internally tangent to the largest semicircle. A circle centered at P is drawn externally tangent to the two smaller semicircles and internally tangent to the largest semicircle. What is the radius of the circle centered at P?



(A) $\frac{3}{4}$ **(B)** $\frac{6}{7}$ **(C)** $\frac{1}{2}\sqrt{3}$ **(D)** $\frac{5}{8}\sqrt{2}$ **(E)** $\frac{11}{12}$

11. (2002 AMC 12A #23) In triangle ABC , side AC and the perpendicular bisector of BC meet in point D, and BD bisects $\angle ABC$. If $AD = 9$ and $DC = 7$, what is the area of triangle ABD?

    (A) 14 (B) 21 (C) 28 (D) $14\sqrt{5}$ (E) $28\sqrt{5}$

12. (2013 AMC 12B #19) In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

    (A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

ANSWERS ARE ON THE NEXT PAGE

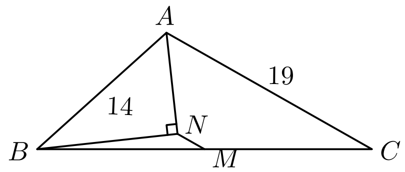
§1.2 Answers

1. D
2. 1
3. 51
4. 17
5. 24
6. $\frac{9}{5}$
7. 26
8. 108
9. C
10. B
11. D
12. B

§2 Problem Set 2

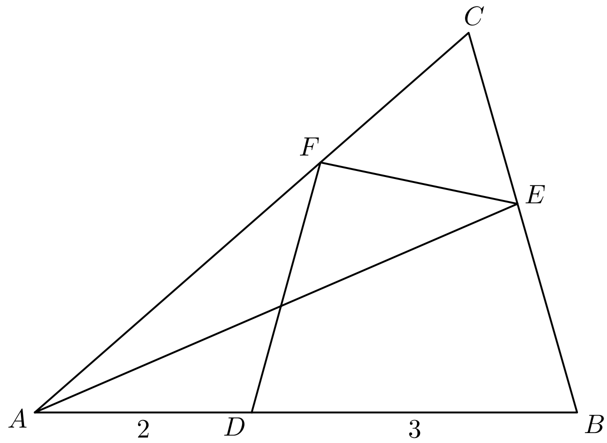
§2.1 Problems

1. (1981 AHSME #19) In $\triangle ABC$, M is the midpoint of side BC, AN bisects $\angle BAC$, and $BN \perp AN$. If sides AB and AC have lengths 14 and 19, respectively, then find MN.



2. Point D is chosen on minor arc AC of the circumcircle of equilateral triangle $\triangle ABC$ with side length 2 such that the perimeter of quadrilateral $ABCD$ is 7. Find the length of BD.

3. (1983 AHSME #28) Triangle $\triangle ABC$ in the figure has area 10. Points D, E and F, all distinct from A, B and C, are on sides AB, BC and CA respectively, and $AD = 2, DB = 3$. If triangle $\triangle ABE$ and quadrilateral $DBEF$ have equal areas, then that area is



(A) 4 **(B)** 5 **(C)** 6 **(D)** $\frac{5}{3}\sqrt{10}$ **(E)** not uniquely determined

4. (2011 AMC 12A #13) Triangle ABC has side-lengths $AB = 12, BC = 24$, and $AC = 18$. The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at N. What is the perimeter of $\triangle AMN$?

(A) 27 **(B)** 30 **(C)** 33 **(D)** 36 **(E)** 42

5. A hexagon with sides of lengths 2, 2, 7, 7, 11, and 11 is inscribed in a circle. Find the diameter of the circle.

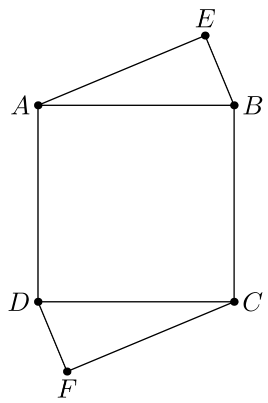
6. (1991 AIME #14) A hexagon is inscribed in a circle. Five of the sides have length 81 and the sixth, denoted by $\overline{AB}$, has length 31. Find the sum of the lengths of the three diagonals that can be drawn from A.

7. (2018 AMC 12A #20) Triangle ABC is an isosceles right triangle with $AB = AC = 3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and

$\overline{AB}$, respectively, so that $AI > AE$ and $AIME$ is a cyclic quadrilateral. Given that triangle EMI has area 2, the length CI can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is the value of $a+b+c$?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

8. (2007 AIME II #3) Square $ABCD$ has side length 13, and points E and F are exterior to the square such that $BE = DF = 5$ and $AE = CF = 12$. Find EF^2.



9. (2013 AIME II #8) A hexagon that is inscribed in a circle has side lengths 22, 22, 20, 22, 22, and 20 in that order. The radius of the circle can be written as $p + \sqrt{q}$, where p and q are positive integers. Find $p + q$.

ANSWERS ARE ON THE NEXT PAGE

§2.2 Answers

1. $5/2$
2. 3
3. 6
4. B
5. 14
6. 384
7. D
8. 578
9. 272

§3 Problem Set 3

§3.1 Problems

1. In $\triangle ABC$, point D is on BC such that $BD : CD = 5 : 2$ and point E is on AB such that $AE : BE = 4 : 3$. AD and EC intersect at point F. Find $AF : FD$ and $CF : FE$.

2. In $\triangle ABC$, point E is on AB such that $AE : BE = 4 : 3$ and point D is on BC such that $BD : CD = 5 : 2$. Point G is on AC such that $AG : CG = 3 : 7$. Cevian BG and transversal ED intersect at point F. Compute $EF : FD$ and $BF : FG$.

3. In $\triangle ABC$, let E be on AB such that $AE : EB = 1 : 3$, D on BC such that $BD : DC = 2 : 5$, and F on ED such that $EF : FD = 3 : 4$. Finally, let BF intersect AC at G. Find $AG : GC$ and $BF : FG$.

4. Point E is selected on side AB of triangle ABC in such a way that $AE : EB = 1 : 3$ and point D is selected on side BC such that $CD : DB = 1 : 2$. The point of intersection of AD and CE is F. Find $\frac{EF}{FC} + \frac{AF}{FD}$.

5. In $\triangle ABC$, C' is on the side AB such that $AC' : C'B = 1 : 2$, and B' is on side AC such that $AB' : B'C = 3 : 4$. If BB' and CC' intersect at P and ray AP and BC intersect at A', find $AP : PA'$.

6. In $\triangle ABC$, D is on AB and E is on BC. Let CD and AE intersect at K, and let ray BK and AC intersect at F. If $AK : KE = 3 : 2$ and $BK : KF = 4 : 1$, find $CK : KD$.

7. In $\triangle ABC$, angle bisectors AD and BE intersect at P. If the sides of the triangle are $BC = 3$, $AC = 5$, and $AB = 7$, find $BP : PE$.

8. In $\triangle ABC$, M is the midpoint of side BC, $AB = 12$, and $AC = 16$. Points E and F are taken on AC and AB, respectively, and lines EF and AM intersect at G. If $AE = 2AF$, find $EG : GF$.

9. In $\triangle ABC$, points D and E are on AB and AC, respectively. The angle bisector of $\angle A$ intersects DE at F and BC at T. If $AD = 1$, $DB = 3$, $AE = 2$, and $EC = 4$, compute the ratio $AF : AT$.

10. In $\triangle ABC$, $\angle CBA = 72^\circ$, E is the midpoint of side AC, D is a point on side BC such that $2BD = DC$, and AD and BE intersect at F. Find the ratio of the area of $\triangle BDF$ to the area of quadrilateral $FDCE$.

11. In $\triangle ABC$, cevians AD, BE, and CF intersect at point P. The areas of $\triangle PAF$, $\triangle PFB$, $\triangle PBD$, and $\triangle PCE$ are $40, 30, 35$, and 84, respectively. Find the area of $\triangle ABC$.

12. In $\triangle ABC$, D is on AB such that $AD : DB = 3 : 2$, and E is on BC such that $BE : EC = 3 : 2$. If ray DE and ray AC intersect at F, then find $DE : EF$.

13. In a triangle, segments are drawn from one vertex to the two trisection points of the opposite side. A median drawn from a second vertex is divided by these segments in the continued ratio of $x : y : z$, counted from the vertex. If $x \geq y \geq z$, then find $x : y : z$.

14. (1992 AIME #14) In triangle ABC, A', B', and C' are on the sides BC, AC, and AB, respectively. Given that AA', BB', and CC' are concurrent at the point O, and that $\frac{AO}{OA'} + \frac{BO}{OB'} + \frac{CO}{OC'} = 92$, find $\frac{AO}{OA'} \cdot \frac{BO}{OB'} \cdot \frac{CO}{OC'}$.

15. (1988 AIME #12) Let P be an interior point of triangle ABC and extend lines from the vertices through P to the opposite sides. Let a, b, c, and d denote the lengths of the segments indicated in the figure. Find the product abc if $a + b + c = 43$ and $d = 3$.

16. In $\triangle ABC$, let D and E be the trisection points of BC with D between B and E. Let F be the midpoint of AC, and let G be the midpoint of AB. Let H be the intersection of EG and DF. Find the ratio $EH : HG$.

Bonus (Difficult!) If the sides AB, BC, and CA of $\triangle ABC$ are divided at D, E, and F in the respective ratios of $1 : l$, $1 : m$, and $1 : n$, then prove that the cevians CD, AE, and BF form a triangle whose area is

$$\frac{(lmn - 1)^2}{(lm + l + 1)(mn + m + 1)(nl + n + 1)}$$

times the area of $\triangle ABC$. This is known as **Routh's Theorem**.

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§3.2 Answers

1. $AF : FD = 14 : 3$ and $CF : FE = 7 : 10$
2. $EF : FD = 9 : 35$ and $BF : FG = 75 : 79$
3. $AG : GC = 2 : 7$ and $BF : FG = 27 : 22$
4. $3 : 2$
5. $5 : 4$
6. $3 : 2$
7. $2 : 1$
8. $3 : 2$
9. $5 : 18$
10. $1 : 5$
11. 315
12. $1 : 2$
13. $5 : 3 : 2$
14. 94
15. 441
16. $2 : 3$