AP Calculus BC Review Problems

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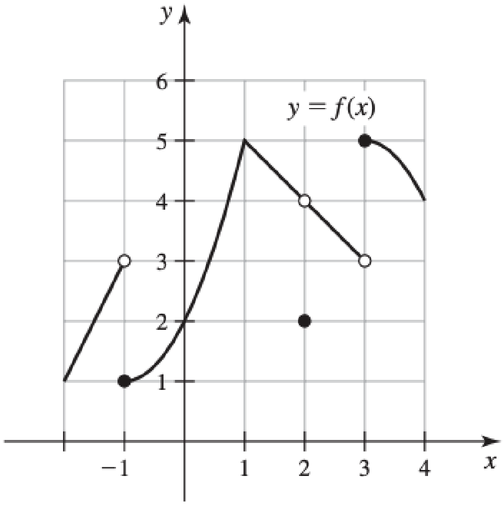
1. Use the graph of f in the figure to find the following values, if possible.



(a) $f(-1)$ **(b)** $\lim\limits_{x\to -1^-} f(x)$ **(c)** $\lim\limits_{x\to -1^+} f(x)$ **(d)** $\lim\limits_{x\to -1} f(x)$ **(e)** $f(1)$ **(f)** $\lim\limits_{x\to 1} f(x)$ **(g)** $\lim\limits_{x\to 2} f(x)$ **(h)** $\lim\limits_{x\to 3^-} f(x)$ **(i)** $\lim\limits_{x\to 3^+} f(x)$ **(i)** $\lim\limits_{x\to 3} f(x)$

2. Evaluate the following limits analytically (without using L'Hôpital's Rule!) or state that they do not exist.

   a) $\displaystyle\lim_{h\to 0} \frac{\sqrt{5x+h}-\sqrt{5x}}{h}$ where x is constant

   b) $\displaystyle\lim_{z\to\infty} \left(e^{-2z}+\frac{2}{z}\right)$

   c) $\displaystyle\lim_{x\to 4} \frac{x^3-7x^2+12x}{4-x}$

   d) $\displaystyle\lim_{x\to 3^-} \frac{x-4}{x^2-3x}$

   e) $\displaystyle\lim_{x\to 3} \frac{\sqrt{3x+16}-5}{x-3}$

   f) $\displaystyle\lim_{x\to 3} \frac{1}{x-3}\left(\frac{1}{\sqrt{x+1}}-\frac{1}{2}\right)$

   g) $\displaystyle\lim_{x\to 81} \frac{\sqrt[4]{x}-3}{x-81}$

h) $\displaystyle\lim_{x\to\pi/2} \frac{\frac{1}{\sqrt{\sin x}} - 1}{x + \pi/2}$

3. Find the intervals on which the following functions are continuous.

   a) $f(x) = \sqrt{x^2 - 5}$

   b) $g(x) = e^{\sqrt{x-2}}$

   c) $h(x) = \dfrac{2x}{x^3 - 25x}$

   d) $g(x) = \cos e^x$

4. Let

$$g(x) = \begin{cases} 5x - 2 & x < 1 \\ a & x = 1 \\ ax^2 + bx & x \geq 1 \end{cases}.$$

   Determine values of the constants a and b for which g is continuous at $x = 1$.

5. The amount of an antibiotic drug (in mg) in the blood t hours after an intravenous line is opened is given by

$$m(t) = 100(e^{-0.1t} - e^{-0.3t}).$$

   a) Use the Intermediate Value Theorem to show the amount of drug is 30 mg at some time in the interval $[0, 5]$ and again at some time in the interval $[5, 15]$.

   b) Is the amount of drug in the blood ever 50 mg?

6. Find an equation of the line tangent to the curve $y = f(x)$ at P for the following:

   a) $f(x) = \dfrac{x+3}{2x+1}$; $P = (0, 3)$

   b) $f(x) = \dfrac{1}{2\sqrt{3x+1}}$; $P = (0, 1/2)$

7. Use the definition of the derivative to compute $f'(x)$ given

   a) $f(x) = 2x^2 - 3x + 1$

   b) $f(x) = \sqrt{2x - 3}$

8. Evaluate and simplify the following derivatives.

   a) $\dfrac{d}{dx}(2x\sqrt{x^2 - 2x + 2})$

   b) $\dfrac{d}{dx}(5x + \sin^3 x + \sin(x^3))$

   c) $\dfrac{d}{dv}\left(\dfrac{v}{3v^2 + 2v + 1}\right)^{1/3}$

   d) $\dfrac{d}{dx}(2x \sin x\sqrt{3x - 1})$

   e) $\dfrac{d}{dx}(2^{x^2 - x})$

   f) $\dfrac{d}{dx}\sin^{-1}\left(\dfrac{1}{x}\right)$

g) $\dfrac{d}{dx}(\tan^{-1}(e^{-x}))$

9. Calculate $y'(x)$ for the following relations.
   a) $y = \dfrac{e^y}{1+\sin x}$
   b) $\sin x \cos(y-1) = \frac{1}{2}$

10. Find an equation of the line tangent to the following curves at the given point.
    a) $y + \sqrt{xy} = 6; (1,4)$
    b) $x^2y + y^3 = 75; (4,3)$

11. For what value(s) of x is the line tangent to the curve $y = x\sqrt{6-x}$ horizontal? Vertical?

12. Consider the following functions. In each case, without finding the inverse, evaluate the derivative of the inverse at the given point.
    a) $f(x) = 1/(x+1)$ at $f(0)$
    b) $f(x) = x^4 - 2x^2 - x$ at $f(0)$

13. Suppose $p(t) = -1.7t^3 + 72t^2 + 7200t + 80000$ is the population of a city t years after 1950.
    a) Determine the average rate of growth of the city from 1950 to 2000.
    b) What was the rate of growth of the city in 1990?

14. The distance between the head of a piston and the end of a cylindrical chamber is given by $x(t) = \frac{8t}{t+1}$ cm, for $t \geq 0$ (measured in seconds). The radius of the cylinder is 4 cm.
    a) Find the volume of the chamber, for $t \geq 0$.
    b) Find the rate of change of the volume $V'(t)$, for $t \geq 0$.
    c) Graph the derivative of the volume function. On what intervals is the volume increasing? Decreasing?

15. Two boats leave a dock at the same time. One boat travels south at 30 mi/hr and the other travels east at 40 mi/hr. After half an hour, how fast is the distance between the boats increasing?

16. A rope is attached to the bottom of a hot-air balloon that is floating above a flat field. If the angle of the rope to the ground remains 65° and the rope is pulled in at 5 ft/s, how quickly is the elevation of the balloon changing?

17. True or False? Explain why.
    a) For any function f, $\frac{d}{dx}|f(x)| = |f'(x)|$.
    b) If $f'(c) = 0$, then f has a local maximum or minimum at c.
    c) If $f''(c) = 0$, then f has an inflection point at c.
    d) If $\lim\limits_{x\to\infty} f(x) = \infty$ and $\lim\limits_{x\to\infty} g(x) = \infty$, then $\lim\limits_{x\to\infty}(f(x) - g(x)) = 0$.

18. Make a complete graph of the following functions on their domains or on the given interval.

a) $f(x) = \frac{1}{2}x^4 - 3x^2 + 4x + 1$

b) $f(x) = \frac{\cos(\pi x)}{1+x^2}$ on $[-2, 2]$

c) $f(x) = x^{2/3} + (x+2)^{1/3}$

19. A rectangular page in a textbook (with width x and length y) has an area of 98 square inches, top and bottom margins set at 1 in, and left and right margins set at $\frac{1}{2}$ in. The printable area of the page is the rectangle that lies within the margins. What are the dimensions of the page that maximize the printable area?

20. What point on the graph of $f(x) = \frac{5}{2} - x^2$ is closest to the origin?

21. For the following, find the linear approximation to f at the given point a. Then, use your answer to estimate the given function value.

    a) $f(x) = x^{2/3}; a = 27; f(29)$

    b) $f(x) = \sin^{-1} x; a = 1/2; f(0.48)$

22. Use linear approximation to estimate the following quantities while producing a small error.

    a) $1/4.2^2$

    b) $\tan^{-1} 1.05$

23. A bamboo shoot was 500 cm tall at $10:00$ a.m. and 515 cm at $3:00$ p.m.

    a) Compute the average growth rate of the bamboo shoot in cm/hr over the period of time from $10:00$ a.m. to $3:00$ p.m.

    b) What can you conclude about the instantaneous growth rate of bamboo between $10:00$ a.m. and $3:00$ p.m.?

24. Evaluate the following limits, using L'Hôpital's Rule when needed.

    a) $\displaystyle\lim_{x\to\infty} \frac{5x^2 + 2x - 5}{\sqrt{x^4 - 1}}$

    b) $\displaystyle\lim_{x\to\infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 - x})$

    c) $\displaystyle\lim_{y\to 0^+} \frac{\ln^{10} y}{\sqrt{y}}$

    d) $\displaystyle\lim_{x\to 0} \csc x \sin^{-1} x$

    e) $\displaystyle\lim_{x\to\infty} \frac{\ln^3 x}{\sqrt{x}}$

    f) $\displaystyle\lim_{x\to\pi/2^-} (\sin x)^{\tan x}$

    g) $\displaystyle\lim_{x\to 0^+} (\ln x)^x$

    h) $\displaystyle\lim_{x\to\infty} \left(\frac{2}{\pi}\tan^{-1} x\right)^x$

    i) $\displaystyle\lim_{x\to\infty} (\sqrt{x} + 1)^{1/x}$

    j) $\displaystyle\lim_{x\to 0^+} \frac{x}{\sqrt{1 - e^{-x^2}}}$

k) $\displaystyle\lim_{x\to 0^+} \frac{x^2}{1-e^{-x^2}}$

25. Consider the integral $\int_1^4 (3x-2)\,dx$. Evaluate the right and midpoint Riemann sums for the integral with $n=3$.

26. Evaluate the following limit by identifying the integral it represents:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(\frac{4k}{n}\right)^8 + 1\right]\left(\frac{4}{n}\right).$$

27. Let $f(x) = \int_0^x (t-1)^{15}(t-2)^9\,dt$.

    a) Find the intervals on which f is decreasing and the intervals on which f is increasing.

    b) Find the intervals on which f is concave down and the intervals on which f is concave up.

    c) Find values of x where f has a local minima, local maxima, or inflection point.

28. A particle moves along a line with a velocity given by $v(t) = 5\sin(\pi t)$ starting with an initial position $s(0)=0$. Find the displacement of and the distance traveled by the particle between $t=0$ and $t=2$.

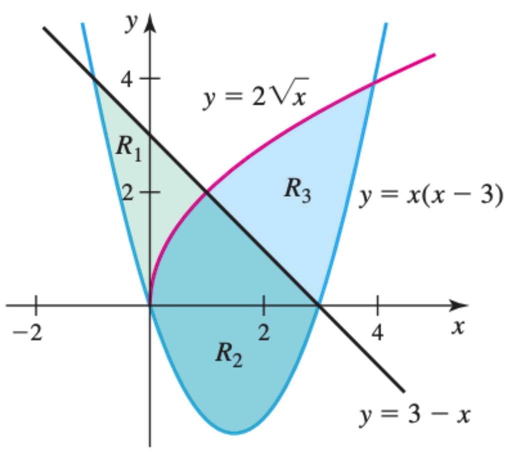
29. At $t=0$, a car begins decelerating from a velocity of 80 ft/s at a constant rate of 5 ft/s^2. Find its position function assuming $s(0)=0$.

30. Water flows out of a large tank at a rate (in m^3/hr) given by $V'(t) = 10/(t+1)$. If the tank initially holds 750 m^3 of water, when will the tank be empty?

31. Find the areas of the regions described.

    a) The region bounded by $y=x^2$, $y=2x^2-4x$, and $y=0$

    b) The regions R_1, R_2, and R_3 (separately) shown in the figure, which are formed by the graphs of $y=2\sqrt{x}$, $y=3-x$, and $y=x(x-3)$.



32. The region R is bounded by the curves $x=y^2+2$, $y=x-4$ and $y=0$.

    a) Write a single integral that gives the area of R.

b) Write a single integral that gives the volume of the solid generated when R is revolved about the x-axis.

c) Repeat (b) but revolved around y-axis.

33. The region bounded by the curves $y = 2e^{-x}$, $y = e^{x}$, and the y-axis is revolved around the x-axis. What is the volume of the solid that is generated?

34. Find the length of the following curves.

a) $y = \frac{x^3}{6} + \frac{1}{2x}$ on $[1, 2]$

b) $y = x^{1/2} - \frac{x^{3/2}}{3}$ on $[1, 3]$

35. Evaluate the following integrals.

a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

b) $\displaystyle\int_0^\infty xe^{-x}\,dx$

c) $\displaystyle\int x\sin(x^2)\cos^8(x^2)\,dx$

d) $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,dx$

e) $\displaystyle\int_0^1 \frac{x^2}{9-x^6}\,dx$

f) $\displaystyle\int \frac{\sqrt{t-1}}{2t}\,dt$

g) $\displaystyle\int \frac{x}{1+\sin x}\,dx$

h) $\displaystyle\int x\tan^{-1} x\,dx$

i) $\displaystyle\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx$

j) $\displaystyle\int \frac{2x^2+7x+4}{x^3+2x^2+2x}\,dx$

k) $\displaystyle\int \ln\sqrt{1+x^2}\,dx$

l) $\displaystyle\int \sin^{-1}(\sqrt{x})\,dx$