Homework 6

CS 325

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1. <u>Shortest Paths using LP</u>: **(7 points)**

Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

max dt
subject to
    ds = 0
    dv – du ≤ w(u,v) for all (u,v)∈E

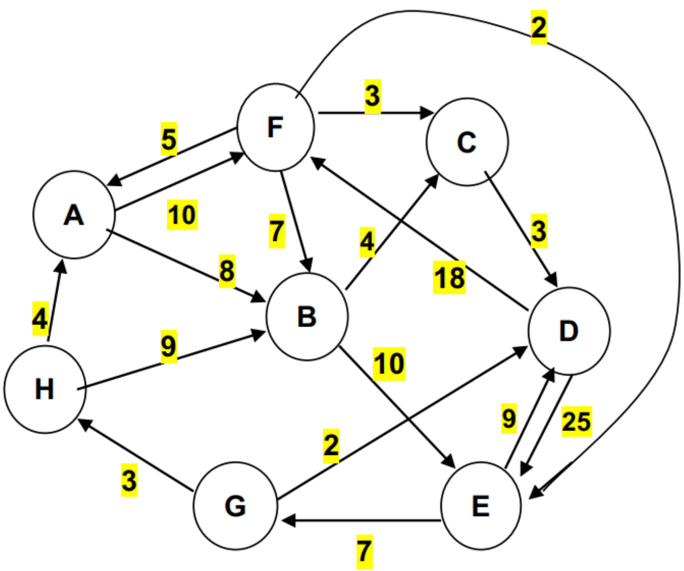
- We can compute the single-source by changing the objective function to

max $\sum_{v \in V} dv$

Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.

a) Find the distance of the shortest path from G to C in the graph below.

b) Find the distances of the shortest paths from G to all other vertices.



a) OBJECTIVE FUNCTION VALUE of dc (G to C) = 16.00000

Reports Window

```
 VARIABLE        VALUE          REDUCED COST
       DA         7.000000          0.000000
       DB        12.000000          0.000000
       DC        16.000000          0.000000
       DD         2.000000          0.000000
       DE        19.000000          0.000000
       DF        17.000000          0.000000
       DG         0.000000          0.000000
       DH         3.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          8.000000
       3)         0.000000          2.000000
       4)         3.000000          0.000000
       5)         0.000000          1.000000
       6)         3.000000          0.000000
       7)        17.000000          0.000000
       8)         8.000000          0.000000
       9)         3.000000          0.000000
      10)        26.000000          0.000000
      11)        26.000000          0.000000
      12)        15.000000          0.000000
      13)        12.000000          0.000000
      14)         4.000000          0.000000
      15)         0.000000          1.000000
      16)         0.000000          1.000000
      17)         0.000000          6.000000
      18)         0.000000          3.000000
      19)         0.000000          2.000000

NO. ITERATIONS=       0


LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)       16.00000

 VARIABLE        VALUE          REDUCED COST
       DC        16.000000          0.000000
       DG         0.000000          0.000000
       DF        17.000000          0.000000
       DA         7.000000          0.000000
       DB        12.000000          0.000000
       DE        19.000000          0.000000
       DD         2.000000          0.000000
       DH         3.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          0.000000
       4)         3.000000          0.000000
       5)         0.000000          1.000000
       6)         3.000000          0.000000
       7)        17.000000          0.000000
       8)         8.000000          0.000000
       9)         3.000000          0.000000
      10)        26.000000          0.000000
      11)        26.000000          0.000000
      12)        15.000000          0.000000
      13)        12.000000          0.000000
      14)         4.000000          0.000000
      15)         0.000000          0.000000
      16)         0.000000          0.000000
      17)         0.000000          1.000000
      18)         0.000000          0.000000
      19)         0.000000          1.000000

NO. ITERATIONS=       0
```

<untitled>

```
max dc
ST
        dg = 0
        df - da <= 10
        db - da <= 8
        dc - db <= 4
        de - db <= 10
        dd - dc <= 3
        de - dd <= 25
        df - dd <= 18
        dd - de <= 9
        dg - de <= 7
        da - df <= 5
        db - df <= 7
        dc - df <= 3
        de - df <= 2
        dd - dg <= 2
        dh - dg <= 3
        da - dh <= 4
        db - dh <= 9
```

max dc

ST

$dg = 0$

$df - da <= 10$

$db - da <= 8$

$dc - db <= 4$

$de - db <= 10$

$dd - dc <= 3$

$de - dd <= 25$

$df - dd <= 18$

$dd - de <= 9$

$dg - de <= 7$

$da - df <= 5$

$db - df <= 7$

$dc - df <= 3$

$de - df <= 2$

$dd - dg <= 2$

$dh - dg <= 3$

$da - dh <= 4$

$db - dh <= 9$

b)

Reports Window

```
 VARIABLE        VALUE          REDUCED COST
       DA         7.000000          0.000000
       DB        12.000000          0.000000
       DC        16.000000          0.000000
       DD        10.000000          0.000000
       DE        19.000000          0.000000
       DF        17.000000          0.000000
       DG         0.000000          0.000000
       DH         3.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          8.000000
       3)         0.000000          2.000000
       4)         3.000000          0.000000
       5)         0.000000          1.000000
       6)         3.000000          0.000000
       7)         9.000000          0.000000
       8)        16.000000          0.000000
       9)        11.000000          0.000000
      10)        18.000000          0.000000
      11)        26.000000          0.000000
      12)        15.000000          0.000000
      13)        12.000000          0.000000
      14)         4.000000          0.000000
      15)         0.000000          1.000000
      16)         0.000000          1.000000
      17)         0.000000          6.000000
      18)         0.000000          3.000000
      19)         0.000000          2.000000

NO. ITERATIONS=      10


LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      76.00000

 VARIABLE        VALUE          REDUCED COST
       DA         7.000000          0.000000
       DB        12.000000          0.000000
       DC        16.000000          0.000000
       DD         2.000000          0.000000
       DE        19.000000          0.000000
       DF        17.000000          0.000000
       DG         0.000000          0.000000
       DH         3.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          8.000000
       3)         0.000000          2.000000
       4)         3.000000          0.000000
       5)         0.000000          1.000000
       6)         3.000000          0.000000
       7)        17.000000          0.000000
       8)         8.000000          0.000000
       9)         3.000000          0.000000
      10)        26.000000          0.000000
      11)        26.000000          0.000000
      12)        15.000000          0.000000
      13)        12.000000          0.000000
      14)         4.000000          0.000000
      15)         0.000000          1.000000
      16)         0.000000          1.000000
      17)         0.000000          6.000000
      18)         0.000000          3.000000
      19)         0.000000          2.000000

NO. ITERATIONS=       0
```

```
<untitled>
max da + db + dc + dd + de + df + dg + dh
ST
        dg = 0
        df - da <= 10
        db - da <= 8
        dc - db <= 4
        de - db <= 10
        dd - dc <= 3
        de - dd <= 25
        df - dd <= 18
        dd - de <= 9
        dg - de <= 7
        da - df <= 5
        db - df <= 7
        dc - df <= 3
        de - df <= 2
        dd - dg <= 2
        dh - dg <= 3
        da - dh <= 4
        db - dh <= 9
```

Vertice		*Source Vertex = G*	
Start	**End**	**Variable**	**Shorest Path Distance**
G	**G**	DG	**0**
G	**A**	DA	**7**
G	**B**	DB	**12**
G	**C**	DC	**16**
G	**D**	DD	**10**
G	**E**	DE	**19**
G	**F**	DF	**17**
G	**H**	DH	**3**

max da + db + dc + dd + de + df + dg + dh

ST

dg = 0

df - da <= 10

db - da <= 8

dc - db <= 4

de - db <= 10

dd - dc <= 3

de - dd <= 25

df - dd <= 18

dd - de <= 9

dg - de <= 7

da - df <= 5

db - df <= 7

dc - df <= 3

de - df <= 2

dd - dg <= 2

dh - dg <= 3

da - dh <= 4

db - dh <= 9

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2. Product Mix: **(7 points)**
Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	Cost per yard	Yards available per month
Silk	$20	1,000
Polyester	$6	2,000
Cotton	$9	1,250

Product Information	Type of Tie			
	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	$6.70	$3.55	$4.31	$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	Type of Tie			
	Silk	Polyester	Blend 1 (50/50)	Blend 2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

Objective Function: Profit MAX 3.45s+2.32p+ 2.81b + 3.25c

Tie number constraints

s>=6000

s<=7000

p>=10000

p<=14000

b>=13000

b<=16000

c>=6000

c<=8500

Zero Constraints

s>=0

p>=0

b>=0

c>=0

Material in Yards available constraints are as follows:

0.125s<=1000

0.08p+0.05b+0.03c<=2000

0.05b+0.07c<=1250

LP OPTIMUM FOUND AT STEP 0

OBJECTIVE VALUE = 120196.000

	Tie Type	Number of Ties
	VARIABLE	**Value**
Silk s	S	7000
Poly p	P	13625
Blend1 b	B	13100
Blend 2 c	C	8500

Code:

MAX 3.45s+2.32p+ 2.81b + 3.25c

ST

s>=6000

s<=7000

p>=10000

p<=14000

b>=13000

b<=16000

```
c>=6000
c<=8500
s>=0
p>=0
b>=0
c>=0
!Material in Yards available constraints are as follows:
0.125s<=1000
0.08p+0.05b+0.03c<=2000
0.05b+0.07c<=1250
END
GIN s
GIN p
GIN b
GIN c
```

Reports Window

```
       OBJECTIVE FUNCTION VALUE

       1)      0.9457166E+08

 VARIABLE        VALUE          REDUCED COST
        S      7000.000000          0.000000
        P     13625.000000          0.000000
        B     13100.000000          0.000000
        C      8500.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)      1000.000000          0.000000
       3)         0.000000      13496.515625
       4)      3625.000000          0.000000
       5)       375.000000          0.000000
       6)       100.000000          0.000000
       7)      2900.000000          0.000000
       8)      2500.000000          0.000000
       9)         0.000000          0.476000
      10)      7000.000000          0.000000
      11)     13625.000000          0.000000
      12)     13100.000000          0.000000
      13)      8500.000000          0.000000
      14)       125.000000          0.000000
      15)         0.000000         29.000000
      16)         0.000000         27.200001

NO. ITERATIONS=       4


LP OPTIMUM FOUND AT STEP      0
OBJECTIVE VALUE =   120196.000


NEW INTEGER SOLUTION OF    120196.000     AT BRANCH      0 PIVOT
BOUND ON OPTIMUM:  120196.0
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       0

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)       120196.0

 VARIABLE        VALUE          REDUCED COST
        S      7000.000000          0.000000
        P     13625.000000          0.000000
        B     13100.000000          0.000000
        C      8500.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)      1000.000000          0.000000
       3)         0.000000          3.450000
       4)      3625.000488          0.000000
       5)       374.999603          0.000000
       6)        99.999756          0.000000
       7)      2900.000244          0.000000
       8)      2500.000000          0.000000
       9)         0.000000          0.476000
      10)      7000.000000          0.000000
      11)     13625.000000          0.000000
      12)     13100.000000          0.000000
      13)      8500.000000          0.000000
      14)       125.000000          0.000000
      15)         0.000000         29.000000
      16)         0.000000         27.200001

NO. ITERATIONS=       6
BRANCHES=    0 DETERM.=  1.000E    0
```

<untitled>

```
MAX 3.45s+2.32p+ 2.81b + 3.25c
ST
s>=6000
s<=7000
p>=10000
p<=14000
b>=13000
b<=16000
c>=6000
c<=8500
s>=0
p>=0
b>=0
c>=0
!Material in Yards available constraints are as follows:
0.125s<=1000
0.08p+0.05b+0.03c<=2000
0.05b+0.07c<=1250
END

GIN s
GIN p
GIN b
GIN c
```

3. Making Change (6 points)

Given coins of denominations (value) $1 = v_1 < v_2 < \ldots < v_n$, we wish to make change for an amount A using as few coins as possible. Assume that v_i's and A are integers. Since $v_1 = 1$ there will always be a solution. Solve the coin change using integer programming. For each of the following denomination sets and amounts, formulate the problem as an integer program with an objective function and constraints. Determine the optimal solution. What is the minimum number of coins used in each case and how many of each coin is used? Include a copy of your code.

a) V = [1, 5, 10, 25] and A = 202.
b) V = [1, 3, 7, 12, 27] and A = 293

a) Total Coins = 10 with
   2 coins of denomination 1
   8 coins of denomination 25

Code:

```
!Objective Function for a)

min a + b + c + d

ST

!Constraints

a>=0

b>=0

c>=0

d>=0

25d <= 202

10c <= 202

5b <= 202

a <= 202

a + 5b + 10c + 25d = 202

END

GIN a

GIN b

GIN c

GIN d
```

```
LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      10.00000

  VARIABLE        VALUE          REDUCED COST
         A         2.000000          1.000000
         B         0.000000          1.000000
         C         0.000000          1.000000
         D         8.000000          1.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         2.000000          0.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         8.000000          0.000000
        6)         2.000000          0.000000
        7)       202.000000          0.000000
        8)       202.000000          0.000000
        9)       200.000000          0.000000
       10)         0.000000          0.000000

NO. ITERATIONS=     412
BRANCHES=   208 DETERM.=   1.000E    0
```

```
<untitled>
!Objective Function for a)
min a + b + c + d
ST
!Constraints
a>=0
b>=0
c>=0
d>=0
25d <= 202
10c <= 202
5b <= 202
a <= 202
a + 5b + 10c + 25d = 202


END

GIN a
GIN b
GIN c
GIN d
```

b) Total Coins = 14 with

2 coins of denomination 7
3 coins of denomination 12
9 coins of denomination 27

Code:

```
!Objective Function for b)
min a + b + c + d + e
ST
!Constraints
a>=0
b>=0
c>=0
d>=0
e>=0
27e <= 293
12d <= 202
7c <= 202
3b <= 202
a <= 202
a + 3b + 7c + 12d + 27e = 293
END
GIN a
GIN b
GIN c
GIN d
GIN e
```

```
LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)       14.00000

 VARIABLE        VALUE          REDUCED COST
        A         0.000000          1.000000
        B         0.000000          1.000000
        C         2.000000          1.000000
        D         3.000000          1.000000
        E         9.000000          1.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          0.000000
       4)         2.000000          0.000000
       5)         3.000000          0.000000
       6)         9.000000          0.000000
       7)        50.000000          0.000000
       8)       166.000000          0.000000
       9)       188.000000          0.000000
      10)       202.000000          0.000000
      11)       202.000000          0.000000
      12)         0.000000          0.000000

NO. ITERATIONS=        83
BRANCHES=    26 DETERM.=    1.000E    0
```

```
<untitled>
!Objective Function for b)
min a + b + c + d + e
ST
!Constraints
a>=0
b>=0
c>=0
d>=0
e>=0
27e <= 293
12d <= 202
7c <= 202
3b <= 202
a <= 202
a + 3b + 7c + 12d + 27e = 293


END

GIN a
GIN b
GIN c
GIN d
GIN e
```

4. Consider the following linear program.

a) Write the following linear program in slack form. **(4 points)**

b) Please state what are the basic and non-basic variables in your slack form. **(1 points)**

Maximize $2x_1 - 6x_3$

Subject to

$x_1 + x_2 - x_3 \leq 7$

$3x_1 - x_2 \geq 8$

$-x_1 + 2x_2 + 2x_3 \geq 0$

$x_1 \geq 0$

$x_2 \geq 0$

$x_3 \geq 0$

a) Write the linear program in slack form.

- For slack form, we introduce a new variable s where $s = x_{n+i} = b_i - \sum_{j=1}^{n} a_{i,j} * x_j$, and $s \geq 0$. Let Z = the value of the objective function.
- For the left hand side (LHS),

- x_{n+i} are the **basic variables, n = number of unknowns**, and **i an integer where i ≥ 1**

- For the right-hand-side (RHS),
  - **RHS variables are called non-basic variables** and are the only variables that appear in the objective function.
  - Value b_i is the ith value in an m x 1 column vector in the equation $Ax \leq b$ where m = number of constraints and i = 1,2,...,m
  - x_j is the jth unknown where $x_j \geq 0$ and j = 1,2,..., n in the n x 1 column vector x.
  - $a_{i,j}$ is the (i + j)th inequality constraint where i = 1,2,...,m and j = 1,2,..., n in an m x n matrix A
- **Objective Function: Maximize $Z = 2x_1 – 6x_3$**
- Subject To (we substitute x_{n+i} for s)
  - $x_4 = 7 - x_1 – x_2 + x_3$
  - $x_5 = -8 + 3x_1 – x_2$
  - $x_6 = - x_1 + 2x_{2+} + 2x_3$
  - $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

b) The **basic variables (LHS) are x_4, x_5, x_6** and the **non-basic variables (RHS)** are x_1, x_2, x_3