Assignment 3
Advanced Algorithms & Data Structures PS

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Proof. Zu zeigen: Ist D_{ij} ungerade so ist $D^{'}_{kj} \leq D^{'}_{ij}$ für alle Nachbarn k von i.

$$D_{ij}(mod2) = 1 \tag{1}$$

$$\implies \exists l \in N : D_{ij} = 2l - 1 \tag{2}$$

und es gilt:

$$D_{ij} = 2D^{'}_{ij} - 1 \tag{3}$$

aus (2) und (3) folgt:

$$2l - 1 = 2D^{'}_{ij} - 1 \implies D^{'}_{ij} = l \tag{4}$$

Der Knoten k ist direkter Nachbar von i also gilt:

$$D_{ij} - 1 \leq D_{kj} \leq D_{ij} + 1 \tag{5}$$

Fall 1: $D_{kj} = D_{ij} - 1$

$$D_{kj} = D_{ij} - 1 = 2l - 2 \tag{6}$$

$$D_{kj}(mod2) = 0 \implies D_{kj} = 2D^{'}_{kj} \tag{7}$$

$$2l - 2 = 2D^{'}_{kj} \implies D^{'}_{kj} = l - 1 \tag{8}$$

$$D^{'}_{ij} = l \wedge D^{'}_{kj} = l - 1 \implies D^{'}_{kj} \leq D^{'}_{ij} \tag{9}$$

Fall 2: $D_{kj} = D_{ij}$

$$D_{kj} = D_{ij} = 2l - 1 \tag{10}$$

$$D_{kj}(mod2) = 1 \implies D_{kj} = 2D^{'}_{kj} - 1 \tag{11}$$

$$2l - 1 = 2D^{'}_{kj} - 1 \implies D^{'}_{kj} = l \tag{12}$$

$$D^{'}_{ij} = l \wedge D^{'}_{kj} = l \implies D^{'}_{kj} \leq D^{'}_{ij} \tag{13}$$

Fall 3: $D_{kj} = D_{ij} + 1$

$$D_{kj} = D_{ij} + 1 = 2l \tag{14}$$

$$D_{kj}(mod2) = 0 \implies D_{kj} = 2D_{kj}^{'} \tag{15}$$

$$2l = 2D_{kj}^{'} \implies D_{kj}^{'} = l \tag{16}$$

$$D_{ij}^{'} = l \wedge D_{kj}^{'} = l \implies D_{kj}^{'} \leq D_{ij}^{'} \tag{17}$$

$\square$

Proof. Zu zeigen: D_{ij} ist gerade genau dann, wenn $\sum_{k\in\Gamma(i)} D^{'}_{kj} \geq D^{'}_{ij} * deg(i)$.

$$D_{ij}(mod2) = 0 \tag{18}$$

$$\implies \exists l \in N : D_{ij} = 2l \tag{19}$$

und es gilt:

$$D_{ij} = 2D^{'}_{ij} \tag{20}$$

aus (2) und (3) folgt:

$$2l = 2D^{'}_{ij} \implies D^{'}_{ij} = l \tag{21}$$

Der Knoten k ist direkter Nachbar von i also gilt:

$$D_{ij} - 1 \leq D_{kj} \leq D_{ij} + 1 \tag{22}$$

Fall 1: $D_{kj} = D_{ij} - 1$

$$D_{kj} = D_{ij} - 1 = 2l - 1 \tag{23}$$

$$D_{kj}(mod2) = 1 \implies D_{kj} = 2D^{'}_{kj} - 1 \tag{24}$$

$$2l - 1 = 2D^{'}_{kj} - 1 \implies D^{'}_{kj} = l \tag{25}$$

Fall 2: $D_{kj} = D_{ij}$

$$D_{kj} = D_{ij} = 2l \tag{26}$$

$$D_{kj}(mod2) = 0 \implies D_{kj} = 2D^{'}_{kj} \tag{27}$$

$$2l = 2D^{'}_{kj} \implies D^{'}_{kj} = l \tag{28}$$

Fall 3: $D_{kj} = D_{ij} + 1$

$$D_{kj} = D_{ij} + 1 = 2l + 1 \tag{29}$$

$$D_{kj}(mod2) = 1 \implies D_{kj} = 2D^{'}_{kj} - 1 \tag{30}$$

$$2l + 1 = 2D^{'}_{kj} - 1 \implies D^{'}_{kj} = l + 1 \tag{31}$$

Es gilt also:

$$\forall k \in \Gamma(i) : D^{'}_{kj} \geq l \tag{32}$$

Sei $m = \big|\Gamma(i)\big| = deg(i)$, dann:

$$\sum_{k\in\Gamma(i)} D^{'}_{kj} \geq m * l = D^{'}_{ij} * deg(i) \tag{33}$$

□