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Stellar Structure Project

Daniel Krista-Kelsey[1]

[1] *University of Massachusetts, Amherst, MA, 01003 USA*

1. INTRODUCTION

The physical effect in question that I will change is the metal fractions in a model star. Changing this fraction will also change the hydrogen fraction in the model star, and consequently, the amount of primary fuel that will be present. While hydrogen and helium are both sources of fuel, the latter being a more inefficient source, the amount of metal in a star will affect how much fuel there is overall, so I expect that having a higher fraction of metal will result in the model star producing a lower luminosity.

Another change I suspect will occur within the model star due to changing the fraction of metal is a change in the surface temperature and central temperature. Since luminosity is directly proportional to temperature, I expect that the same situation described prior for luminosity will have the same outcome; meaning that with higher metal fraction, the surface and core temperature of the star will be lower, and the opposite with lower metal fraction.

When the metal fraction increases, I expect the radius to increase like a red giant since there still needs to be enough pressure to keep the temperature high to cause fusion to occur, and therefore pressure has to increase too. Therefore, in the case of the fraction increasing, I expect the radius and core pressure to increase, and likewise, when the fraction is decreased, the radius and core pressure will decrease.

Understanding that opacity is exponentially and inversely proportional to the temperature of the star, I expect that if a star has a high core temperature, then the opacity of the star will be low as compared to a star with a cooler core temperature. As for emissivity, we learned that the luminosity of a star is proportional to the emissivity, so I expect that more luminous stars will have a higher emissivity, therefore, stars with lower metal fractions will have higher emissivities than larger metal fractions.

2. RESULTS

2.1. *Mass-Luminosity Plot*

Figure 1 shows the table of data produced by the stellar structure modeler code. Figure 1 (a) shows the control, where only the mass is changed and nothing else to see the results of the rest of the characteristics of the star. Figure 1 (b) shows only the change in the metal fraction of the star while keeping the mass of the star at $1M_\odot$. Figure 2 (c) shows the same input parameters as (b), except by plugging them into the model star without autofit, or in other words, a model that doesn't automatically try and find parameters to fit the radiative transfer equation at half the radius of the star.

Figure 2 shows the plots of the Mass-Luminosity relationship, given by the data of the control mass and luminosity values. (a) and (b) looks at the data using a linear-linear and log-log plot, respectively, while fitting a 4th-degree polynomial to the data to extract a relationship. (c) is also a log-log plot, but fitted with a linear relationship to extract the slope of the seemingly exponentially rising curve produced in the linear-linear plot. I calculated the slope of the linear fit of the log-log plot to be $slope = 4.176$. This means that the mass-luminosity relationship of this star with this current model star of polytrope n = 1 becomes:

$$L \approx M^{4.176}\,[\mathrm{L}_\odot]$$

where M is in $\mathrm{M}_\odot$. (d) is very similar to (c), but with the metal fraction Z changed to Z = 0.15, whereas the control sample had Z = 0.02. This change, in turn, has an effect on the scale of the mass-luminosity relationship: the new slope of the log-log linear fit slope is $slope = 4.612$, transforming the relationship to be:

$$L \approx M^{4.612}\,[\mathrm{L}_\odot]$$

The takeaway point here is that increasing the metal fraction Z in this model star results in the luminosity of the star having a higher dependence on the mass of the star. In other words, raising Z increases the luminosity of the star at a constant mass. This key point is demonstrated in Figure 1 (c) in the 'Luminosity' column.

This previous point is only true for, what seems to be, a condition of the star's mass. Comparing the $1M_\odot$ data point in Figure 1 (a) and (c), (c) raises the Z but diminishes the luminosity, and Figure 1 (b) shows that at $1M_\odot$, the luminosity decreases. Yet, the exponential relationship spanning all masses and luminosities at a constant Z increases with Z.

	Mass	Helium	Metal	Luminosity	T_eff	Radius	energy generation	opacity	mean mol weight	core pressure	core temp	core density
0	0.5	..24	0.02	-1.434	3.614	10.420	0.068	1.197	0.597	16.732	6.957	1.631
1	1.0	0.24	0.02	0.202	3.999	10.470	1.374	0.485	0.597	17.135	7.208	1.783
2	2.0	0.24	0.02	1.558	5.258	10.629	3.068	-0.015	0.597	17.101	7.350	1.607
3	5.0	0.24	0.02	3.063	4.485	10.928	4.209	-0.341	0.597	16.702	7.449	1.109
4	10.0	0.24	0.02	4.047	4.614	11.161	4.865	-0.423	0.597	16.370	7.517	0.710

(a)

	Mass	Helium	Metal	Luminosity	T_eff	Radius	energy generation	opacity	mean mol weight	core pressure	core temp	core density
0	1	0.24	0.04	0.018	3.940	10.496	1.198	0.693	0.606	17.031	7.188	1.705
1	1	0.24	0.06	-0.084	3.906	10.511	1.108	0.822	0.615	16.972	7.180	1.661
2	1	0.24	0.08	-0.146	3.886	10.520	1.065	0.909	0.625	16.934	7.178	1.632
3	1	0.24	0.10	-0.182	3.874	10.527	1.054	0.972	0.635	16.909	7.178	1.614
4	1	0.24	0.15	-0.210	3.863	10.535	1.130	1.067	0.661	16.875	7.187	1.588

(b)

	Mass	Helium	Metal	Luminosity	T_eff	Radius	energy generation	opacity	mean mol weight	core pressure	core temp	core density
0	0.5	0.24	0.15	-1.868	3.476	10.481	-0.359	1.807	0.661	16.489	6.940	1.450
1	1.0	0.24	0.15	-0.210	3.863	10.535	1.130	1.067	0.661	16.875	7.187	1.588
2	2.0	0.24	0.15	1.226	4.131	10.718	2.799	0.486	0.661	16.745	7.305	1.341
3	5.0	0.24	0.15	3.000	4.429	11.009	4.167	-0.102	0.661	16.379	7.413	0.867
4	10.0	0.24	0.15	4.150	4.603	11.235	4.983	-0.349	0.661	16.075	7.487	0.488

(c)

	Mass	Helium	Metal	Luminosity	T_eff	Radius	energy generation	opacity	mean mol weight	core pressure	core temp	core density
0	1	0.24	0.04	0.016	3.939	10.496	1.195	0.695	0.606	17.031	7.188	1.706
1	1	0.24	0.06	-0.085	3.907	10.511	1.107	0.823	0.615	16.972	7.180	1.661
2	1	0.24	0.08	-0.144	3.887	10.521	1.068	0.907	0.625	16.934	7.178	1.632
3	1	0.24	0.10	-0.183	3.874	10.527	1.053	0.973	0.635	16.909	7.178	1.614
4	1	0.24	0.15	-0.210	3.863	10.535	1.129	1.068	0.661	16.875	7.187	1.588

(d)

Figure 1. (a) Table of results for the control sample. (b) Table of results for the physical change in metal fraction. (c) Table of results for the physical change at one specific, distinct metal fraction with matching mass data points as the control sample. (d) Table of results for the physical change in metal fraction without autofit. All tables have these units: $M_\odot$, $L_\odot$, K, radius = cm, and (energy generation, opacity, mean weight, core pressure, core temp, and core density) are all in their appropriate CGS units. All columns in all tables are in logarithmic form.

2.2. *Overall Property Changes by Z*

Luminosity was heavily discussed in the previous section, but many of the other properties of the model star have changed too. Figure 3 shows (a) the change in core temperature and core pressure with evolving stellar mass without the change in Z, and (b) shows the change in radius and core density of the star with evolving stellar mass with an increased change in Z. Without the change, the core pressure peaks between $1M_\odot$ and $2M_\odot$ at z = 0.02, whereas the core temperature does not peak, though increasing the star's mass seems to slowly diminish the increase in temperature. Comparing this to the star with the higher Z, Z = 0.15, the core pressure peaks at a lower mass around $0.5M_\odot$, and the overall core pressure is lower with higher Z. The core temperature also seems to be relatively cooler than the star with lower Z, implying that increasing Z decreases star temperature and pressure.

The consensus of Figure 3 (b) is that with higher Z, there is, generally, a lower core density. There is no perfect model to represent the data collected, but the core density seems to peak around $1M_\odot$, regardless of

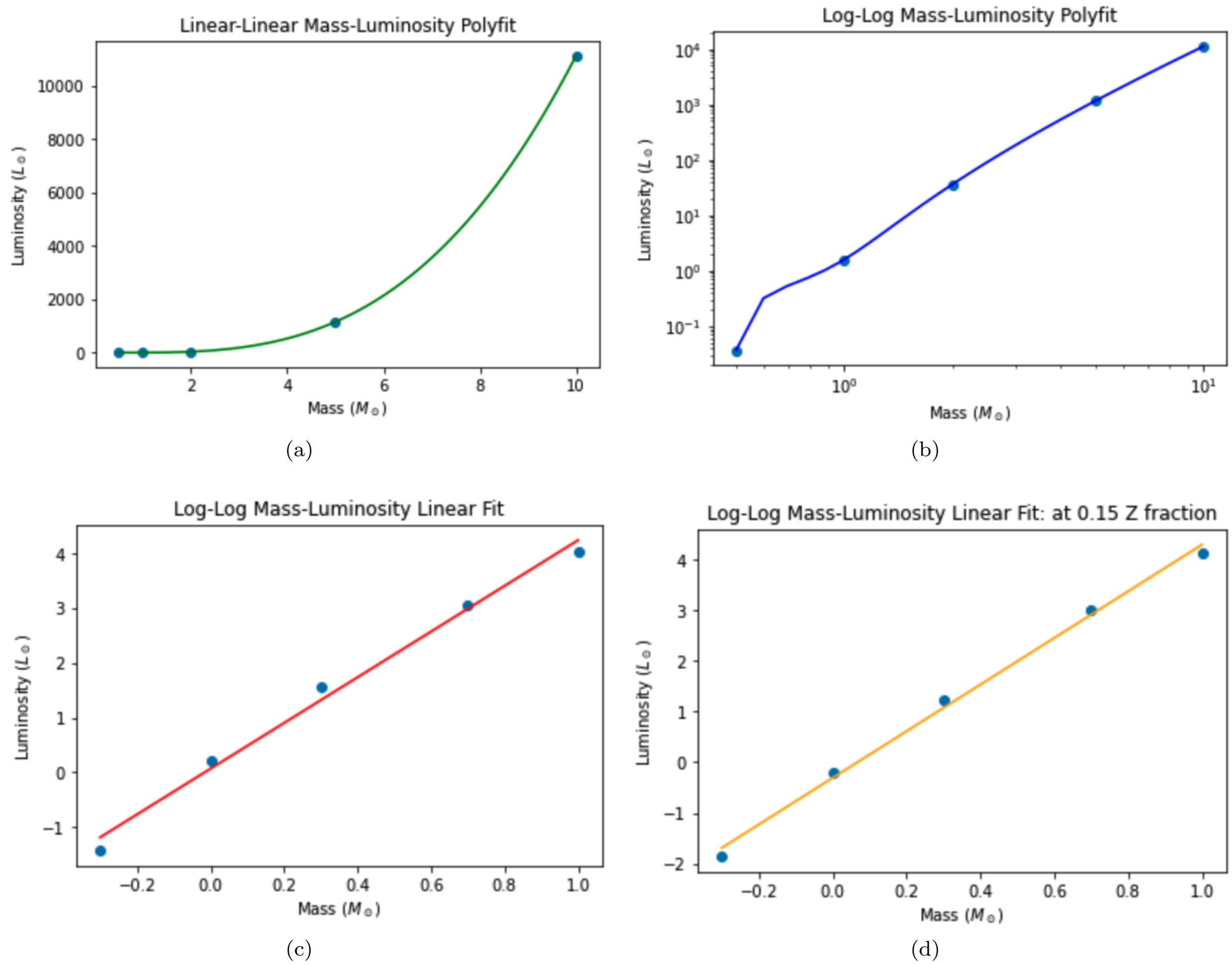


Figure 2. (a) A plot with linear axes showing mass v.s. luminosity using the control sample data set (refer to Figure 1 (a)). The polynomial fitted line suggests an exponential relationship. (b) The same data as (a), but plotted with logarithmic axes where the polynomial fit suggests a slight linear relationship. (c) The same data as (a) is plotted with logarithmic axes, but overlaid with a linear fit model. (d) A new set of data with the same masses as the data in (a) but with different Z, Z = 0.15, resulting in a slightly steeper log-log linear model, or a higher exponential when looking at the plot linearly (data can be found in Figure 1 (d)).

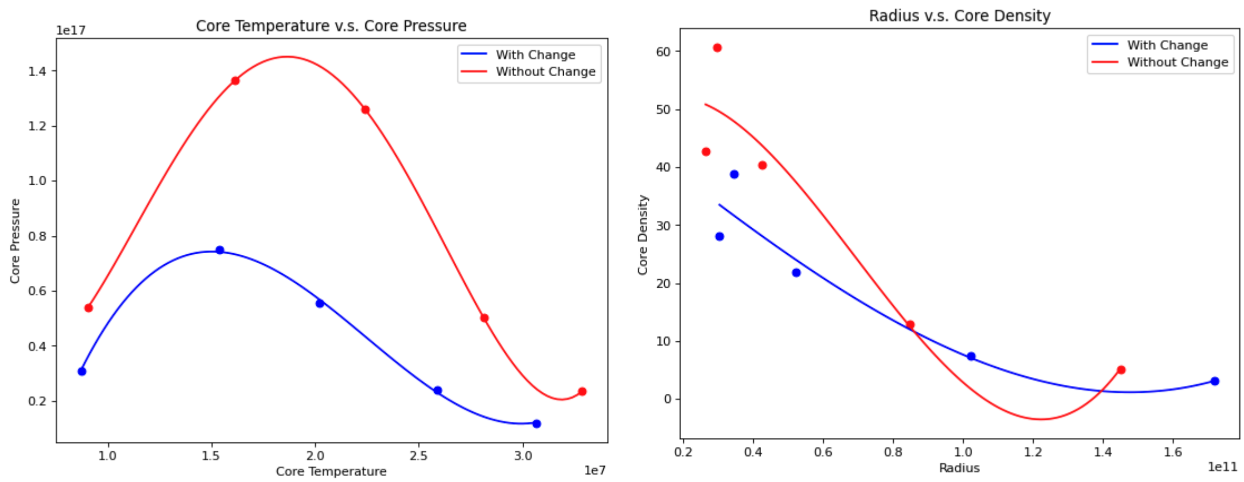
Z. The star with the higher Z also has, generally, a larger radius than the star with a lower Z.

2.3. *Changes Due to Mass and Z*

Combining the analysis from Figure 2 and Figure 3, the physical effect of metal fraction Z causes the other characteristics of the model star to highly vary with M. When Z changes, the other parameters have dramatic changes at every star mass, most notably the core pressure, luminosity, and radius of the star. This isn't to say that the other parameters aren't highly variable with mass and metal together as well.

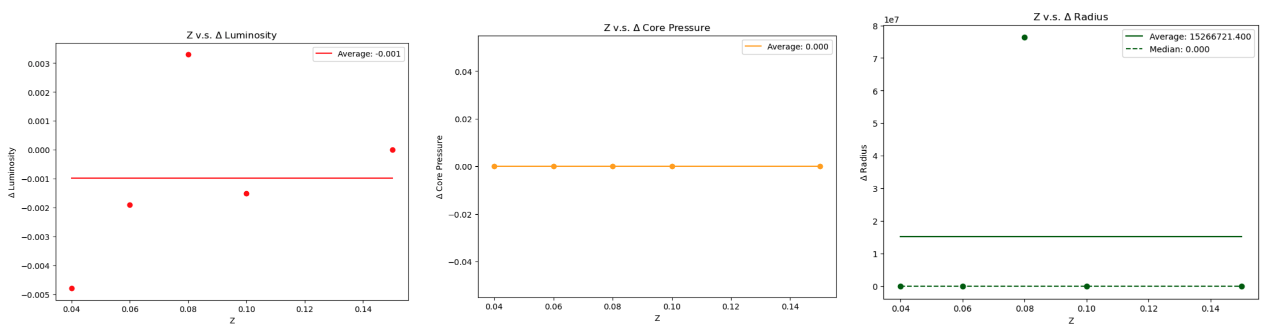
2.4. *Autofit v.s. No Autofit*

Understanding the difference between the autofit and the no autofit model is valuable. Inputting parameters into the no autofit model will result in the immediate changes to the star model without any other parameter of the star changing due to the rebalancing and readjustment to the balance of the star model. That is the key difference, where the autofit model automatically balances out the star due to a physical change- therefore other parameters not directly affected by a physical effect can still change due to other parameters changing. In this paper with the physical change being metal fraction, it is best to explore the most impacted parameters discussed before due to Z: luminosity, core pressure, and radius. This is done by utilizing Figure 1 (d), the change in Z in the no autofit model, and taking the difference from Figure 1 (b) so we can see the immediate differences changing Z has on our model star. Figure 4 shows the respective change in (a) luminosity, (b) core



(a) (b)

Figure 3. (a) Core Temperature is plotted against Core Pressure, with increasing star mass leading to an evolution in the +x axis direction. (b) Radius is plotted against Core Density, with increasing star mass leading to an evolution in the +x direction. Core pressure, core temperature, and core density are in their CGS units and radius is in cm.



(a) (b) (c)

Figure 4. (a) Shows the change in luminosity between the autofit and the no autofit models and its evolution over the metal fraction Z. Roughly no change is present; significantly small compared to the actual luminosity values. (b) Same as (a), but showing the change in core pressure between the two models over Z. The average error is exactly 0 for this, therefore both models represent core pressure the exact same. (c) Same as the other two, but now with the change in Radius. There is one outlier due to the radius at the 3rd data point being one one-thousandth off in log, but even the average error is nowhere relevant to the actual radius values.

pressure, (c) and radius with each step change of Z. As it turns out, there's no difference between the model stars and they produce practically equal results. This is significant because the metal content / fraction of the star's composition directly affects these three already very impacted parameters directly, and therefore have a direct relationship with these three parameters. To put it simply - the metal fraction directly affects luminosity, core pressure, and radius, and these changes aren't due to the rebalancing of the model star due to a change in another parameter.

3. INTERPRETATION AND DISCUSSION

3.1. *Distinguishing Stars With Different Z*

Though it may seem hard to tell between two stars apart, them being one with the likes on the control and one with the physical Z change, there is a way to tell. Figure 5 shows the luminosity and the surface temperature of the stars plotted against various masses. The red trendline is the line for the control star (Z = 0.02) and the blue trendline is the line for the star with the changed metal fraction (Z = 0.15). A star of any unknown mass has a unique temperature associated with its luminosity, so by finding it's match on the metal trendline that a star fits best, its metal fraction can be determined, and therefore, be distinguished.

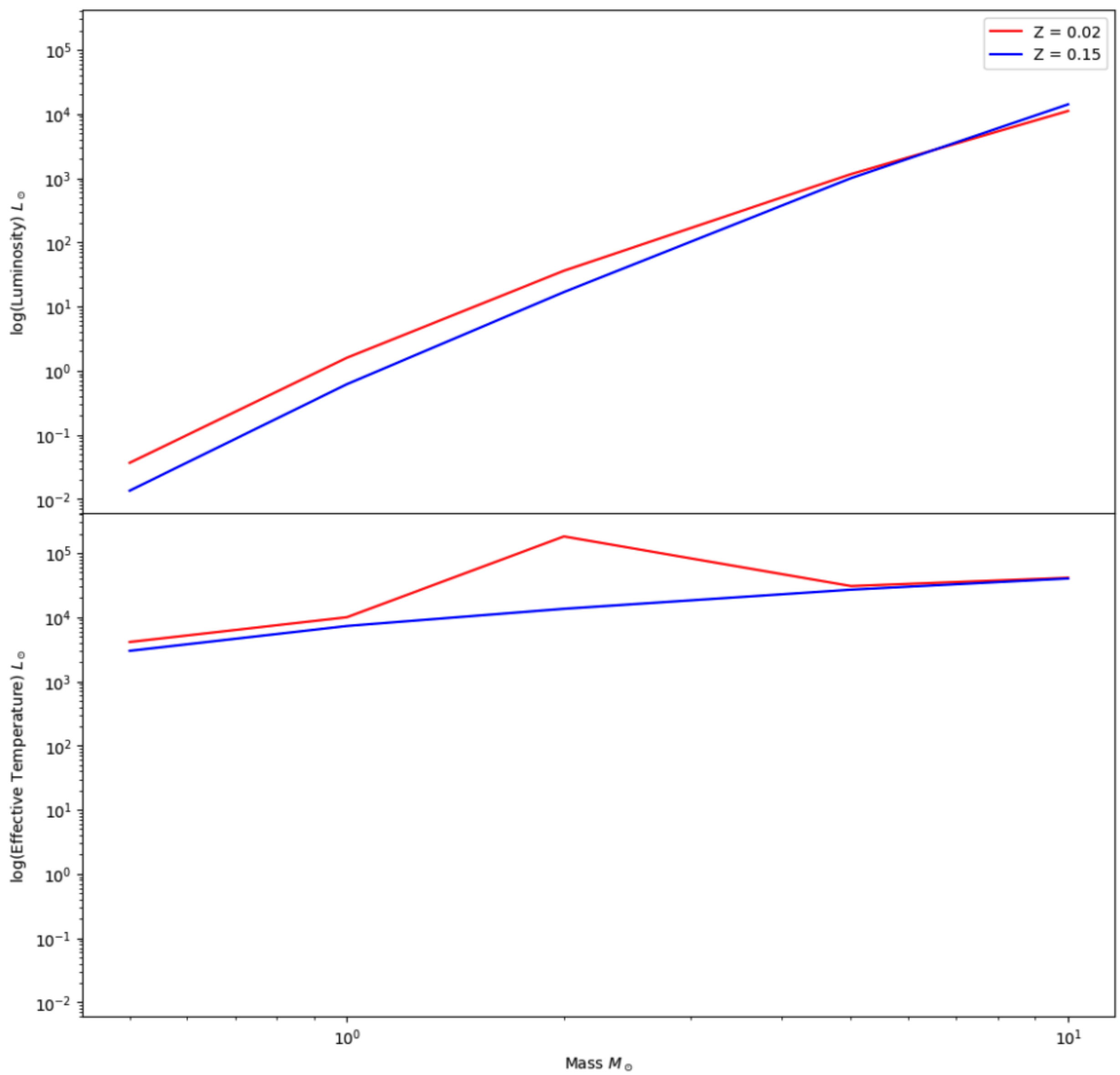


Figure 5. This plot shows the luminosity of the star and the outer temperature of the star plotted against the star. The different trendlines showcase the differences due to the physical effect made, the metal fraction. As visibly shown, the metal fraction of a star changes the luminosity and outer temperature of a star, so by matching up a measured luminosity and temperature, the metallicity can be extracted by seeing on which trendline the star falls on.

3.2. *Main-Sequence Discussion*

I expect the lifetime of a star on the main-sequence to be shorter if there is a higher metal fraction in a star. I say this because if there is a higher metal fraction in a star (and keeping into account that the helium fraction is not changing), there is going to be less relative hydrogen available as fuel in the star. With less hydrogen, there is going to be a shorter time for a star with high Z to burn its main fuel, then start evolving at the main-sequence turnoff once there is no more hydrogen. Once the star is on the turnoff, I do not believe that there will be a change in the luminosity. The change in luminosity between stars with different Z is already marginal, and this same trend will carry over into the turnoff - a marginal change but insignificant, since there is the same amount of helium to burn in both stars.

3.3. *Expected v.s. Actual Results*

In the introduction, I detailed what I expected which stellar quantities would change and in what manner when I changed the metal fraction. Almost every single point I brought up in the expectations were correct, and the changes happened as stated when the metal fraction increased/decreased. To restate - with an increase in the metal fraction, there will be an decrease in the luminosity, surface temperature, and core temperature, and an increase in radius and opacity. The two things I guessed wrong were the change in core pressure and emissivity/energy generation. When the metal fraction increased, the core pressure actually decreased and the energy generation actually increased.

3.4. *Relation to Stellar Structure Equations*

For a star to remain in balance, the following four equations balance each other out:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho$$

$$\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2}\rho$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \epsilon$$

$$\frac{1}{3}\frac{d(aT^4)}{dr} = -\frac{L(r)}{4\pi r^2}\frac{\bar{\kappa}}{c}\rho$$

For a star that has the physical change of changing the metal fraction, the first thing that is affected is the density of the star - with a higher metal fraction, the density of the core and the average density throughout can be safely assumed to change. When the density changes, $\frac{dM(r)}{dr}$, $\frac{dP(r)}{dr}$, $\frac{dL(r)}{dr}$, and $\frac{dT(r)}{dr}$ all change, including the opacity $\kappa = \kappa_0 \rho^a T^b$ and the emissivity $\epsilon = \epsilon_0 \rho^x T^y$. When $\frac{dM(r)}{dr}$ changes, $\frac{dP(r)}{dr}$ changes, and when $\frac{dL(r)}{dr}$ changes, $\frac{dT(r)}{dr}$ changes. When $\rho(r)$ changes, all of the equations change. Also, when either T or ρ changes, κ and ϵ changes. Once this continuous cycle of equations balancing each other starts to slowly converge, the star is then in balance. This is shown in the data obtained, seen in Figure (1), where when the physical change occurred, changing Z, every other parameter changed as well - this continuous cycle of changes snowballed throughout the entire star.

In the case of this specific report, again, assuming that density is the first parameter to change, we can explore how Z changes the entire star. At $1M_\odot$ with the change from Z = 0.02 to Z = 0.15, the core density changes from $log(\rho) = 1.783$ to $log(\rho) = 1.588$. Using my analysis of the stellar equations above, this means that, since mass is at a constant, radius increases in the $\frac{dM(r)}{dr}$ equation, which is shown to be true, as the radius increased to $log(r) = 10.535$ from $log(r) = 10.470$. In the $\frac{dP(r)}{dr}$ equation, ρ decreases and r increases, leading $\frac{dP(r)}{dr}$ to decrease shown by the core pressure reducing to $log(P_c) = 16.875$ from $log(P_c) = 17.135$. The same analysis goes for the rest of the stellar equations as well.

3.5. *Summary of Changes*

Simply put, the star changes in a predictable and intuitive way. When the metal fraction increases, the density lowers (I imagine higher-proton atoms have a larger radius, therefore there is less mass per volume), and this snowballs into changing every parameter, stated by the stellar structure changes. Again, generally, a higher metal fraction results in a decrease in the luminosity, surface temperature, and core temperature, and an increase in radius, opacity and energy generation increased.