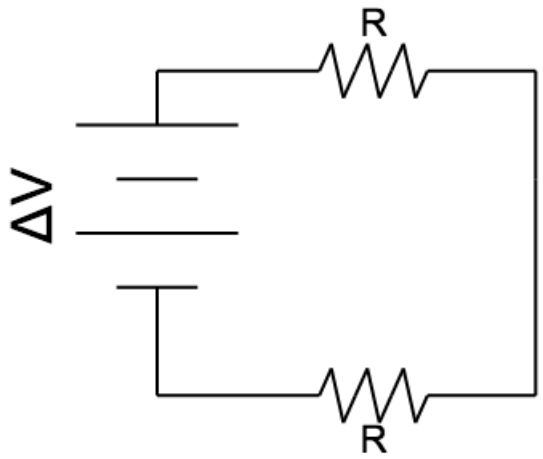
Part I: Solving Full Circuits Algebraically

When solving a full circuit algebraically, you can follow the same set of rules that would be followed for solving a circuit with numerical values!

I.1 You have a **series** circuit with a battery and two **identical** resistors. The potential difference across the battery is ΔV and the resistance of each resistor is R



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV and R by filling in the following table.

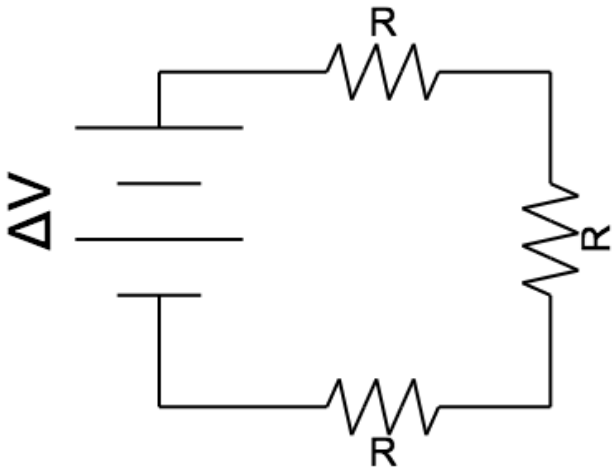
	Resistor A	Resistor B	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

Part I: Solving Full Circuits Algebraically

I.2 You have a **series** circuit with a battery and *three* **identical** resistors. The potential difference across the battery is ΔV and the resistance of each resistor is R**.**



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV and R by filling in the following table.

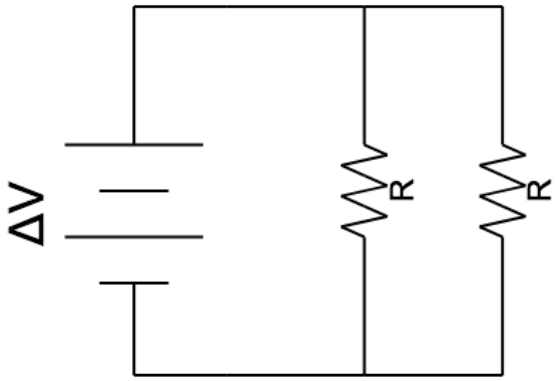
	Resistor A	Resistor B	Resistor C	Total (Battery)
Potential Difference				
Current				
Resistance				
Power				

b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B + P_C = P_{tot} = \Delta V \cdot I_{tot}$$

Part I: Solving Full Circuits Algebraically

I.3 You have a **parallel** circuit with a battery and two **identical** resistors. The potential difference across the battery is ΔV and the resistance of each resistor is R. Determine all relevant quantities for this circuit algebraically by filling in the following table.



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV and R by filling in the following table.

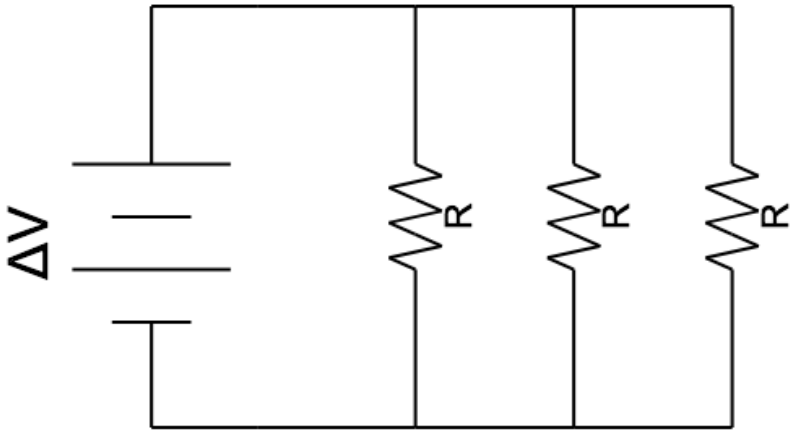
	Resistor A	Resistor B	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

Part I: Solving Full Circuits Algebraically

I.4 You have a **parallel** circuit with a battery and *three* **identical** resistors. The potential difference across the battery is ΔV and the resistance of each resistor is R. Determine all relevant quantities for this circuit algebraically by filling in the following table.



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV and R by filling in the following table.

	Resistor A	Resistor B	Resistor C	Total (Battery)
Potential Difference				
Current				
Resistance				
Power				

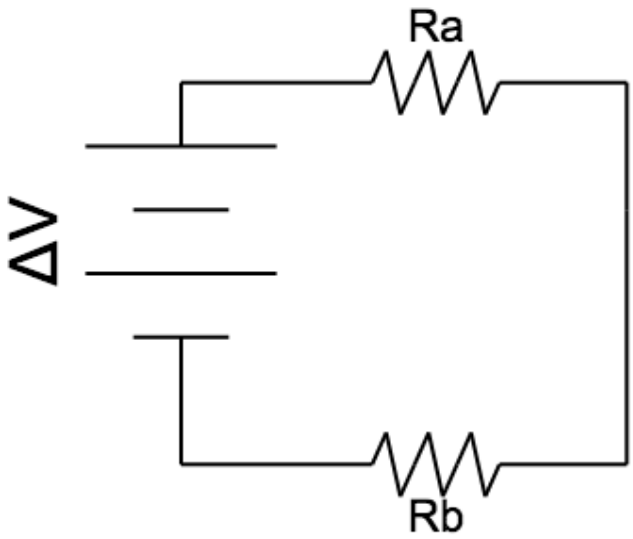
b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B + P_C = P_{tot} = \Delta V \cdot I_{tot}$$

Part I: Solving Full Circuits Algebraically

I.5

You have a **series** circuit with a battery and two resistors. **The two resistors are not identical**, they now have resistances of R_a and R_b, where $R_a \neq R_b$. The potential difference across the battery is ΔV.



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV, R_a, and R_b by filling in the following table.

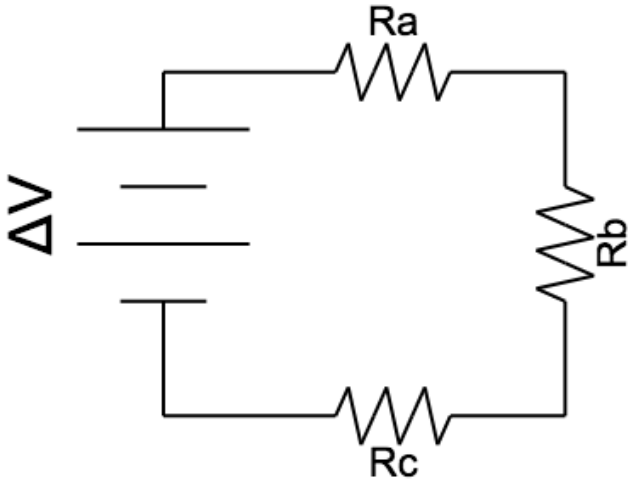
	Resistor A	Resistor B	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

Part I: Solving Full Circuits Algebraically

I.6
You have a **series** circuit with a battery and three resistors. **The three resistors are not identical**, they now have resistances of R_a, R_b and R_c, where $R_a \neq R_b \neq R_c$. The potential difference across the battery is ΔV.



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV, R_a, R_b, and R_c by filling in the following table.

	Resistor A	Resistor B	Resistor C	Total (Battery)
Potential Difference				
Current				
Resistance				
Power				

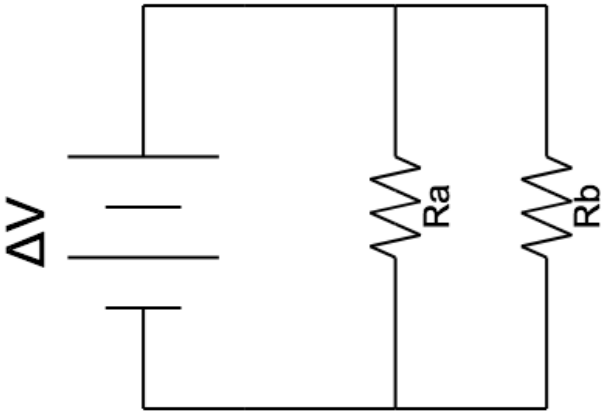
b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

Part I: Solving Full Circuits Algebraically

I.7

You have a **parallel** circuit with a battery and two resistors. **The two resistors are not identical**, they now have resistances of R_a and R_b, where $R_a \neq R_b$. The potential difference across the battery is ΔV.



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV, R_a, and R_b by filling in the following table.

	Resistor A	Resistor B	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B + P_C = P_{tot} = \Delta V \cdot I_{tot}$$

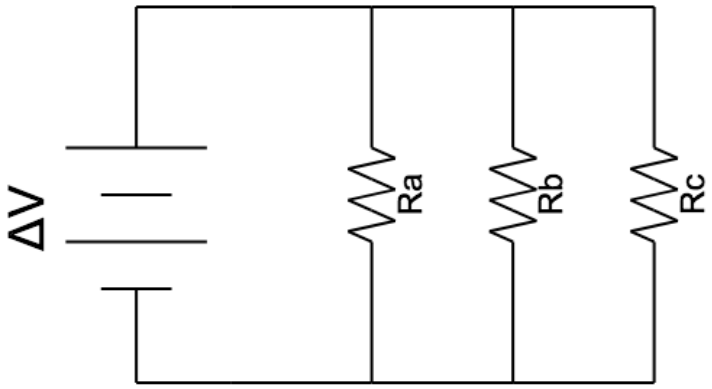
c) Problem **I.7** is actually the *derivation* of the formula for equivalent resistance of two resistors in parallel, which is always included in physics textbooks and used to solve combined circuits with two resistors in parallel:

$$\frac{1}{R_{tot}} = \frac{1}{R_A} + \frac{1}{R_B}$$

Show that your solution for *total resistance* matches this form.

Part I: Solving Full Circuits Algebraically

I.8
You have a **parallel** circuit with a battery and three resistors. **The three resistors are not identical**, they now have resistances of R_a, R_b and R_c, where $R_a \neq R_b \neq R_c$. The potential difference across the battery is ΔV.



a) Determine all relevant quantities for this circuit algebraically in terms of ΔV, R_a, R_b, and R_c by filling in the following table.

	Resistor A	Resistor B	Resistor C	Total (Battery)
Potential Difference				
Current				
Resistance				
Power				

b) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B + P_C = P_{tot} = \Delta V \cdot I_{tot}$$

c) Problem **I.8** is actually the *derivation* of the formula for equivalent resistance of two resistors in parallel, which is always included in physics textbooks and used to solve combined circuits with two resistors in parallel:

$$\frac{1}{R_{tot}} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}$$

Show that your solution for *total resistance* matches this form.

Part I: Solving Full Circuits Algebraically

I.9
You have a series circuit with three identical resistors.
The potential difference of the battery is ΔV, and the total current of the circuit is *I*.

Solve for each relevant element of the circuit in terms of ΔV and I (*not* in terms of the resistance!)

I.10
You have a series circuit with *n* identical resistors.
The potential difference of the battery is ΔV, the total current of the circuit is *I*, and each resistor has resistance *R*.

Derive an equation for I in terms of ΔV, n, and R.

I.11
You have a parallel circuit with three identical resistors.
The potential difference of the battery is ΔV, and the total current of the circuit is I_{tot}.

Solve for each relevant element of the circuit in terms of ΔV and I (*not* in terms of the resistance!)

I.12
You have a parallel circuit with *n* identical resistors.
The potential difference of the battery is ΔV, the total current of the circuit is I_{tot}, and each resistor has resistance *R*.

Derive an equation for I_{tot} in terms of ΔV, n, and R.

I.13
You have a parallel circuit with *n* identical resistors, each of resistance R.
Derive an equation for the total resistance of the circuit in terms of R and *n*.

Part I: Solving Full Circuits Algebraically

I.1 **ANSWER**

a)

	Resistor A	Resistor B	Total (Battery)
Potential Difference	$\frac{\Delta V}{2}$	$\frac{\Delta V}{2}$	ΔV
Current	$\frac{\Delta V}{2R}$	$\frac{\Delta V}{2R}$	$\frac{\Delta V}{2R}$
Resistance	R	R	$2R$
Power	$\frac{(\Delta V)^2}{4R}$	$\frac{(\Delta V)^2}{4R}$	$\frac{(\Delta V)^2}{2R}$

b)

Confirm that the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

$$\frac{(\Delta V)^2}{4R} + \frac{(\Delta V)^2}{4R} = \frac{(\Delta V)^2}{2R} = \frac{\Delta V}{1} \cdot \frac{\Delta V}{2R}$$

Part I: Solving Full Circuits Algebraically

I.2 ANSWER

a)

	Resistor A	Resistor B	Resistor C	Total (Battery)
Potential Difference	$\frac{\Delta V}{3}$	$\frac{\Delta V}{3}$	$\frac{\Delta V}{3}$	ΔV
Current	$\frac{\Delta V}{3R}$	$\frac{\Delta V}{3R}$	$\frac{\Delta V}{3R}$	$\frac{\Delta V}{3R}$
Resistance	R	R	R	$3R$
Power	$\frac{(\Delta V)^2}{9R}$	$\frac{(\Delta V)^2}{9R}$	$\frac{(\Delta V)^2}{9R}$	$\frac{(\Delta V)^2}{3R}$

b)

Confirm that the power of each resistor adds to the total power:

$$P_A + P_B + P_C = P_{tot} = \Delta V \cdot I_{tot}$$

$$\frac{(\Delta V)^2}{9R} + \frac{(\Delta V)^2}{9R} + \frac{(\Delta V)^2}{9R} = \frac{(\Delta V)^2}{3R} = \frac{\Delta V}{1} \cdot \frac{\Delta V}{3R}$$

Part I: Solving Full Circuits Algebraically

I.3 ANSWER

a)

	Resistor A	Resistor B	Total (Battery)
Potential Difference	ΔV	ΔV	ΔV
Current	$\frac{\Delta V}{R}$	$\frac{\Delta V}{R}$	$\frac{2(\Delta V)}{R}$
Resistance	R	R	$\frac{R}{2}$
Power	$\frac{(\Delta V)^2}{R}$	$\frac{(\Delta V)^2}{R}$	$\frac{2(\Delta V)^2}{R}$

b)

Confirm that the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

$$\frac{(\Delta V)^2}{R} + \frac{(\Delta V)^2}{R} = \frac{2(\Delta V)^2}{R} = \frac{\Delta V}{1} \cdot \frac{2(\Delta V)}{R}$$

Part I: Solving Full Circuits Algebraically

I.4 ANSWER

a)

	Resistor A	Resistor B	Resistor C	Total (Battery)
Potential Difference	ΔV	ΔV	ΔV	ΔV
Current	$\frac{\Delta V}{R}$	$\frac{\Delta V}{R}$	$\frac{\Delta V}{R}$	$\frac{3(\Delta V)}{R}$
Resistance	R	R	R	$\frac{R}{3}$
Power	$\frac{(\Delta V)^2}{R}$	$\frac{(\Delta V)^2}{R}$	$\frac{(\Delta V)^2}{R}$	$\frac{3(\Delta V)^2}{R}$

b)

Confirm that the power of each resistor adds to the total power:

$$P_A + P_B + P_C = P_{tot} = \Delta V \cdot I_{tot}$$

$$\frac{(\Delta V)^2}{R} + \frac{(\Delta V)^2}{R} + \frac{(\Delta V)^2}{R} = \frac{3(\Delta V)^2}{R} = \frac{\Delta V}{1} \cdot \frac{3(\Delta V)}{R}$$