G: Examining Newton's Laws Mathematically

In this quiz, you will examine Newton's Laws as mathematical statements.
This quiz is for students who already know the basics of Newton's Laws and how to apply them, to take a more rigorous look at the laws and their implications.

PART 1: NEWTON'S FIRST LAW

Newton's First Law - An object at rest will stay at rest unless acted upon by a *net*, *external* force. - An object in motion will stay in motion at a constant velocity unless acted upon by a *net*, *external* force.

1. If an object is not moving, what is the *net force* acting on this object?
2. If an object is moving at a constant velocity, what is the *net force* acting on this object?

A very important word: **net**

An object at rest does not begin moving only because a force acts on it, but because the *net* force acting on the object is greater than zero. In fact, every second of your life there has been at least one force acting on you (gravity), and typically many other forces as well (such as normal force and friction). But whenever you have been at rest, not moving, the sum total (net) of all forces acting on you has been zero.

3. Draw a free-body diagram of a person resting in a chair, doing nothing. There are two forces acting on this person. What must be the *net force* acting on this person? Why?

4. Draw any other example of a time in which several *forces* act on an object at rest, but the object does not begin moving because the *net force* is zero.

A second very important word: **external**

External Force A force that acts on a system from outside the system.
Internal Force A force that acts on a system from inside the system.

Examples of external and internal forces:
- Imagine a family in a car on a long road trip. The thrust of the car, the air resistance and friction acting on the car, and the gravity and normal force are all *external forces* acting on the car. If, inside the car, Jimmy pushes his little sister, that is an *internal force*. Each of the people inside the car is supported by their chair. Those are also internal forces.

Newton's First Law states that only *external forces* can cause a change in the velocity of s system.

5. Imagine you are at rest in space, trying to get to Mars, with no means of propulsion and nothing to turn off of. Can you begin moving towards Mars by twisting your body around? Explain you answer by referring to Newton's First Law and the concept of internal and external forces.

A very important phrase: **constant velocity**

Velocity Velocity is a *vector* that has *magnitude* and *direction*. The magnitude of velocity is how fast you are going. The direction of velocity shows the way you are traveling
Constant Velocity To have a constant velocity, you must have a *constant speed* and a *constant direction*. That means, you keep going the same speed and the same direction.

6. If an object is moving at a constant speed but it is changing direction, does it have a constant velocity?

7. A go-kart is moving in a circle with a constant speed of 8 m/s. Does it have a constant velocity?

Newton's First Law (restated) When the net external force acting on an object that is moving is zero, the object moves with a constant speed in a constant direction. For the object to change speed *or* to change direction, the net external force acting on the object must be greater than zero.

8. Can car turn if the net force on the car is zero?

9. If an astronaut is drifting through space with nothing to grab onto and no means of propulsion, can he move in a different direction? Why or why not?

PART 2: NEWTON'S SECOND LAW:

Newton's Second Law

$$\Sigma\vec{F} = m\vec{a}$$

Or

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

The way I have written the equation above, Net Force and Acceleration have an arrow above them because they are *vectors*. A *vector* has both magnitude and direction. Mass is a *scalar* because it has only magnitude and not direction.

10. If the mass of an object increases, is it harder or smaller to create acceleration according to Newton's Second Law?

11. In physics, we always write just *m* for mass, but there are actually two different definitions of mass: gravitational mass and inertial mass, that are equal.
Does the *m* in this formula refer to gravitational mass or inertial mass, and how do you know?

12.
The definition of acceleration is change in velocity divided by time:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

Combine this equation with Newton's Second Law to determine a formula for change in velocity in terms of net force, mass, and change in time.

Newton's Second Law (restated)

$$\overrightarrow{\Delta v} = \frac{\Sigma\vec{F}}{m} \cdot \Delta t$$

We are going to find the relationship between the direction of *net force* and the *change in the magnitude of velocity of an object* to prove the following table:

Direction of Net Force	**Change in Motion of An Object**
The same direction as velocity	The *magnitude* of velocity increases.
The opposite direction as velocity	The *magnitude* of velocity decreases.
Perpendicular to velocity	The *direction* of velocity changes.

Head-To-Tail Vector Addition

To do head-to-tail vector addition with velocity, I draw an arrow for initial velocity, then draw an arrow for *change in velocity* that begins exactly where initial velocity left off. The *final velocity* is then between the tail of initial velocity and the head of change in velocity.

(This is difficult to understand well, but you can Google it for more images, or look at the images in the answer section of this packet.)

13. Net force is always in the same direction as change in velocity.
If net force is in the same direction as velocity for some small time interval, demonstrate using *head-to-tail* vector addition that the magnitude of velocity increases.
[If you don't understand how to do this, make sure you look at the answer section!]

14. If the net force is in the opposite direction as velocity for a short time, demonstrate using *head-to-tail* vector addition that the magnitude of velocity decreases.

15. If the net force is in a direction *perpendicular to velocity* for a short time, demonstrate using *head to tail* vector addition that the direction of velocity changes.

16. Newton's First Law, mathematically, can actually be deduced from Newton's Second Law. Explain how.

17. True or false: An object moves in the direction of the force on that object. If true, defend. If False, come up with a *counterexample*.

PART 3: Newton's Third LAW!

Newton's Third Law:

If A exerts a force on B, then B exerts a force on A called the *reaction force* with equal magnitude and opposite direction.

Newton's Third Law is frequently written as follows:
"For Every Action there is an Equal and Opposite Reaction"

This statement however, is not mathematically rigorous enough to be used to solve difficult physics problems. We need to make the following clarifications to use this principle in physics class.
a) The term "action" actually means force.
b) The terms "equal" and "opposite" mean that the reaction force has equal magnitude and opposite direction. This makes sense because force is a vector.
c) We need information on the bodies exerting the forces. The first force is a force that A exerts on B, and the reaction force is a force that B exerts on A.

We will learn more about Newton's Third law later. It is really a topic onto itself. For now, we will focus on the mathematical details of the first and second laws and the basics of the third law.

18. What are the problems with the statement "For every action there is an equal and opposite reaction"? What do we stay instead?

Answers:

1. Zero

2. Zero

3. There are two forces of equal magnitude: normal force up and gravity down.

4. Any free-body diagram with net force = 0.

5. NO! The person can move around as much as they like, but any force they exert will be an *internal force*, a force exerted by one part of their body on another part. The net *external force* will still be zero, and thus they cannot begin moving according to Newton's First Law.

6. NO! If direction is changing, the object is changing velocity.
7. NO! The go-kart is moving at a constant speed but is changing direction as it goes around in a circle, so its velocity is changing.

8. NO! If the net force on the car is zero, according to Newton's First law, it must move at a constant velocity, meaning it must have a constant direction, no cannot turn.

9. NO! There are no external forces acting on the astronaut, so according to Newton's First Law he can move only at a constant velocity. This means he cannot change direction.

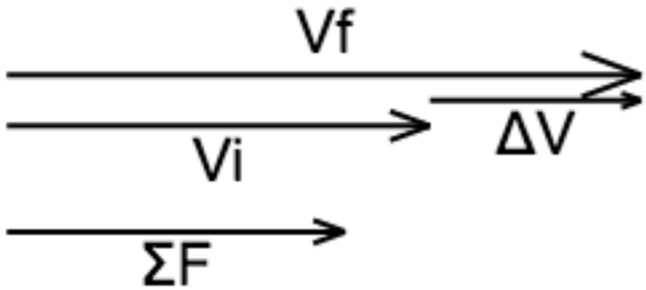
10. Higher mass means it is harder to carte acceleration.

11. The variable *m* in Newton's Second Law refers to inertial mass because it represents resistance to changes in velocity.

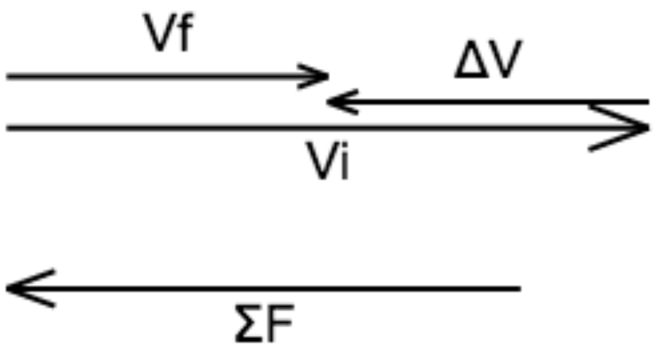
12.

$$\overrightarrow{\Delta v} = \frac{\Sigma \vec{F}}{m} \cdot \Delta t$$

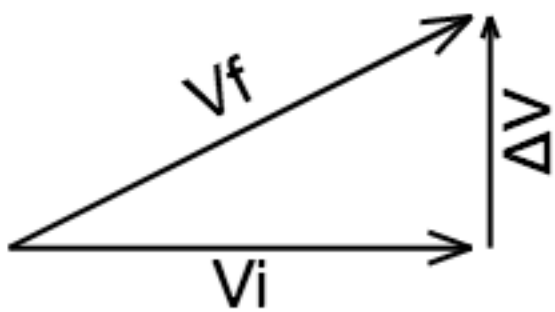
13. If net force is in the same direction as initial velocity, then the *change in velocity* is in the same direction as initial velocity, and the arrow for final velocity will be *longer* than the arrow for initial velocity:



14. If the net force is in the opposite direction as the initial velocity then by Newton's Second Law the *change in velocity* will be in the opposite direction as initial velocity, and final velocity arrow will be *shorter* than the initial velocity arrow.



15. If the net force is in the perpendicular direction as the initial velocity, then by Newton's Second Law the *change in velocity* will be in the same direction, and the final velocity arrow will be in a different direction.
[We will learn much more about this particular example when we reach *circular motion*.]



16. Mass can never be zero for matter, so if net force is zero, the only possible option is that acceleration is also zero. This means that an object either is at rest or moves at a constant velocity.

17. FALSE! For example, if the net force acting on an object is in the opposite direction of the object's velocity, the object continues to move forward, even though the force is acting on it the other way.

18. Many of the terms included in the statement are ill-defined.

Instead we say:
"If A exerts a force on B, then B exerts a force on A called the *reaction force* with equal magnitude and opposite direction."