Vertical Circles

Principles

- The centripetal force always points towards the center of the circle.
- The magnitude of the centripetal force is always given by this formula:

$$F_c = \frac{mv^2}{r}$$

- In a vertical circle, the centripetal force is not always the same force.

*note: the numbers used here are to make the math more straight

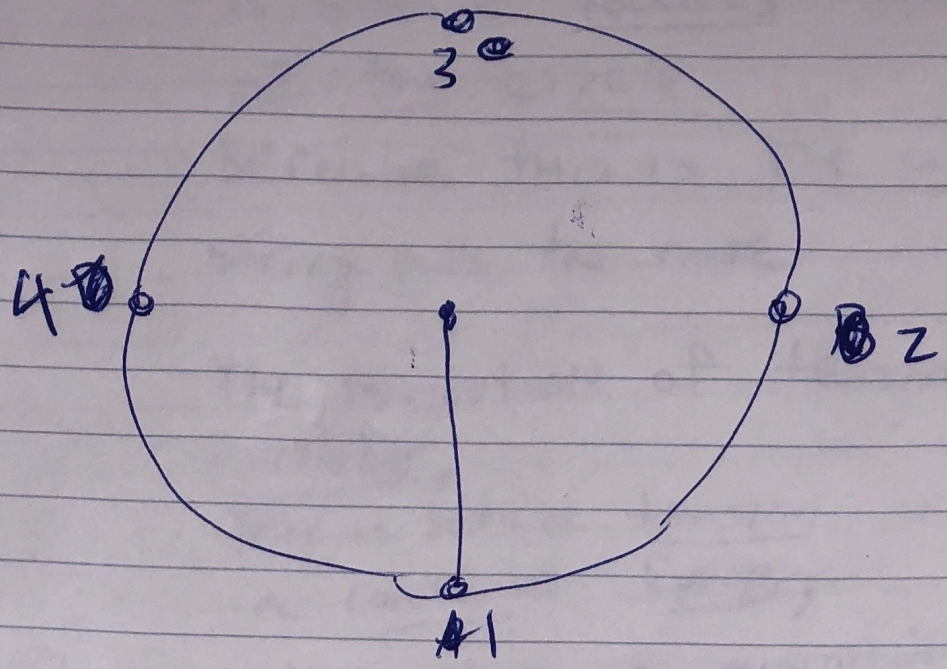
Walk Through Problem

- Somebody is swinging a rock in a vertical circle on a rope.
- Assume the rope has a length of 0.5m and the rock is moving at a constant speed of 4* m/s.
- The rock has a mass of 5* kg.

(a) Find the magnitude of centripetal force acting on the rock.

*note: the numbers used here are to make the math more straight forward and are NOT realistic values.

This diagram shows the motion of the rock ⇒



- 2 forces act on the rock:
  tension and gravity
  [there is also an applied force, for simplicity we will ignore it]

Gravity - The direction of gravity is always down.

- the magnitude of gravity is always given by
$F_g = m \cdot g$

Tension: - The direction of tension is always towards the center of the circle, because this is the way the string pulls the rope

- The magnitude of tension is variable, This is because tension is a constraint force, which adjusts its magnitude to the situation.

(b) Look at points 1, 2, 3, and 4 on the previous page.

At each point, draw a qualitative free-body diagram for the rock. On this diagram, include only tension + gravity + ignore any other forces.

① The 'centripetal force'

Consists of all of the forces pointing towards the center, subtracted by all of the forces pointing out from the center.

forces that are tangential are not included in the centripetal force.

Based on this description write a the centripetal force in terms of gravity and tension at each of points ①, ②, ③, and ④

(c) Using your equations from part (a)
Plug in the magnitudes
of the force of gravity
and the magnitude of the centripetal
force found in part (a).

Use these equations to determine
the magnitude of tension at
points (1), (2), (3), and (4).

(e) If you are able to safely, tie your keychain to a string and swing it in a vertical circle.

Where does the string feel the tightest (high tension)?

Where does the string feel the loosest (low tension)?

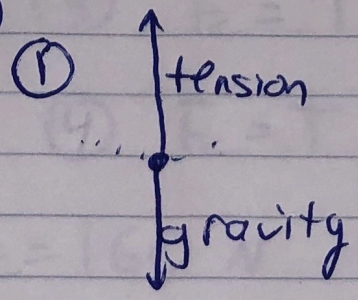
Does this agree with the mathematical results from part (d)?

Answers

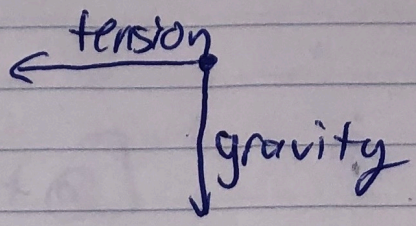
(a) $F_c = \frac{mv^2}{r} = \frac{5 \times 4^2}{0.5} = \frac{5 \times 16}{0.5} = 5 \times 32 = 160\,N$

(b)

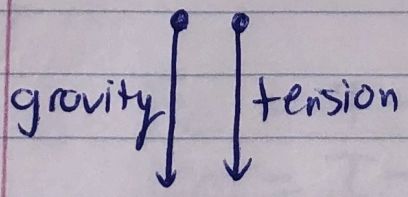
(1)



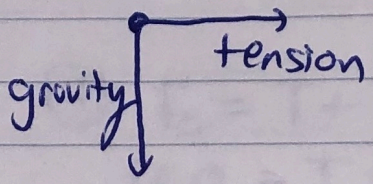
(2)



(3)



(4)



(c) (1) centripetal force = tension − gravity

(2) centripetal force = tension

(3) centripetal force = tension + gravity

(4) centripetal force = tension

(c) - continued

① $F_c = T - F_g$

② $F_c = T$

③ $F_c = T + F_g$

④ $F_c = T$

(d) $F_c = 160\ N$ [from part (a)]

$F_g = 49\ N$ $F_g = m \cdot g = 5 \times 9.8 = 49N$

① $F_c = T - F_g$

$160 = T - 49$

$T = 209\ N$

③ $F_c = T + F_g$

$160 = T + 49$

$T = 111\ N$

② $F_c = T$

$T = 160\ N$

④ $F_c = T$

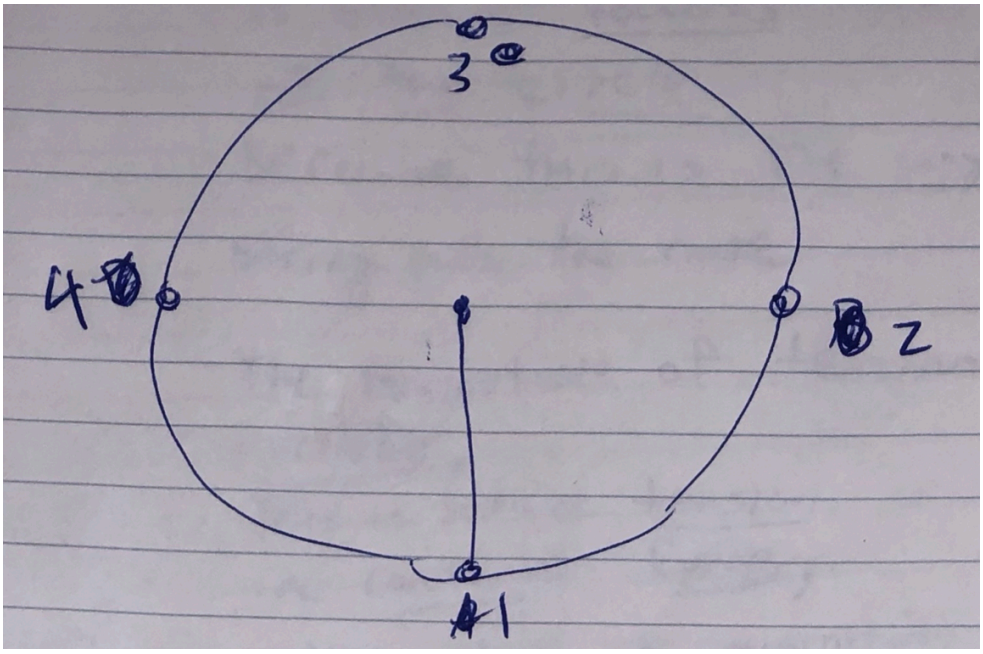
$T = 160\ N$

(e) The string should feel tightest at the bottom and looser at the top. In our mathematical analysis we found that tension was greatest at the bottom and ~~least~~ least at the top, complying with these results.

Vertical Circles Problems

1. A person is swinging a 2 kg rock in a circle of radius 0.3 m at a speed of 5 m/s.

a) Using the diagram to draw a qualitative free-body diagram of the block at points 1, 2, 3, and 4. Include only gravity and tension on the rock, and ignore any other applied forces.



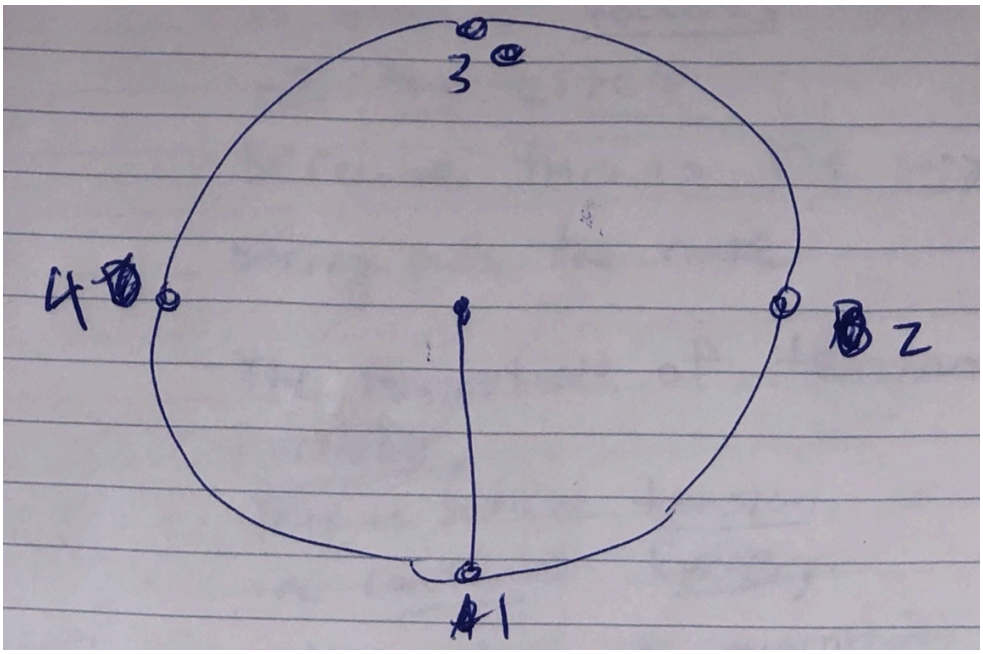
b) The centripetal force is defined as the force pointing *towards the center of the circle* subtracted by the force pointing away from the center of the circle.
For each of points 1, 2, 3, and 4, write the centripetal force F_c in terms of the gravitational force F_g and the tension T.

c) Solve for the magnitude of the centripetal force F_c and the gravitational force F_g using known formulas. Please copy each formula in its canonical form before plugging any numbers in.

d) Determine the magnitude of the tension at each of the four cardinal points of the circle, point 1, point 2, point 3, and point 4.

2. A person is swinging a 4 kg rock in a circle of radius 0.6 m at a speed of 3 m/s.

a) Using the diagram to draw a qualitative free-body diagram of the block at points 1, 2, 3, and 4. Include only gravity and tension on the rock, and ignore any other applied forces.



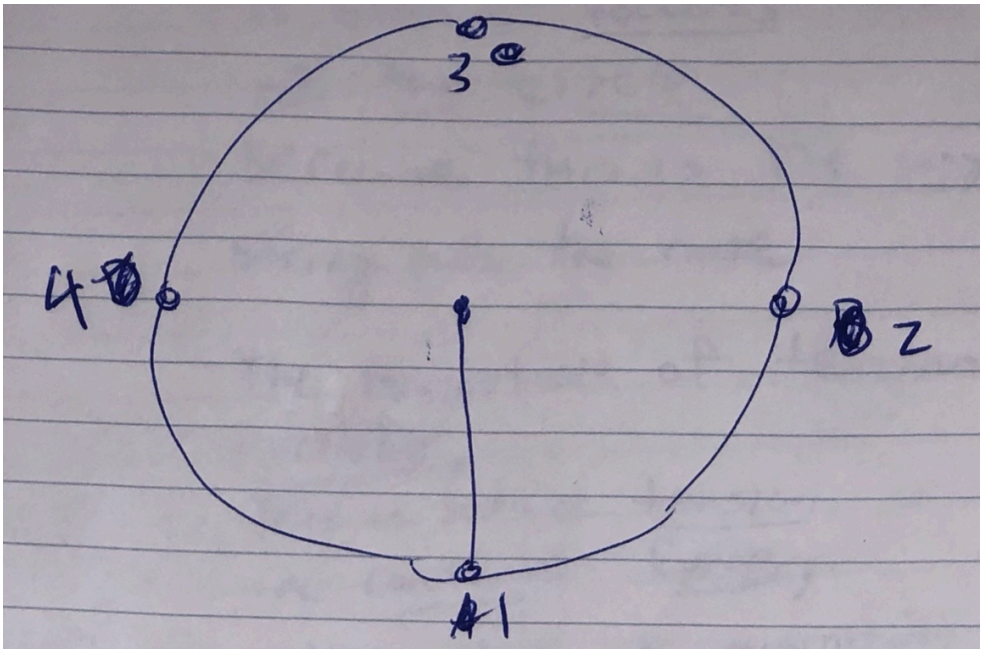
b) The centripetal force is defined as the force pointing *towards the center of the circle* subtracted by the force pointing away from the center of the circle.
For each of points 1, 2, 3, and 4, write the centripetal force F_c in terms of the gravitational force F_g and the tension T.

c) Solve for the magnitude of the centripetal force F_c and the gravitational force F_g using known formulas. Please copy each formula in its canonical form before plugging any numbers in.

d) Determine the magnitude of the tension at each of the four cardinal points of the circle, point 1, point 2, point 3, and point 4.

3. A person is swinging a 1.5 kg rock in a circle of radius 0.2 m. The rock takes a time of 0.4 s to get around the circle.

a) Using the diagram to draw a qualitative free-body diagram of the block at points 1, 2, 3, and 4. Include only gravity and tension on the rock, and ignore any other applied forces.



b) The centripetal force is defined as the force pointing *towards the center of the circle* subtracted by the force pointing away from the center of the circle.
For each of points 1, 2, 3, and 4, write the centripetal force F_c in terms of the gravitational force F_g and the tension T.

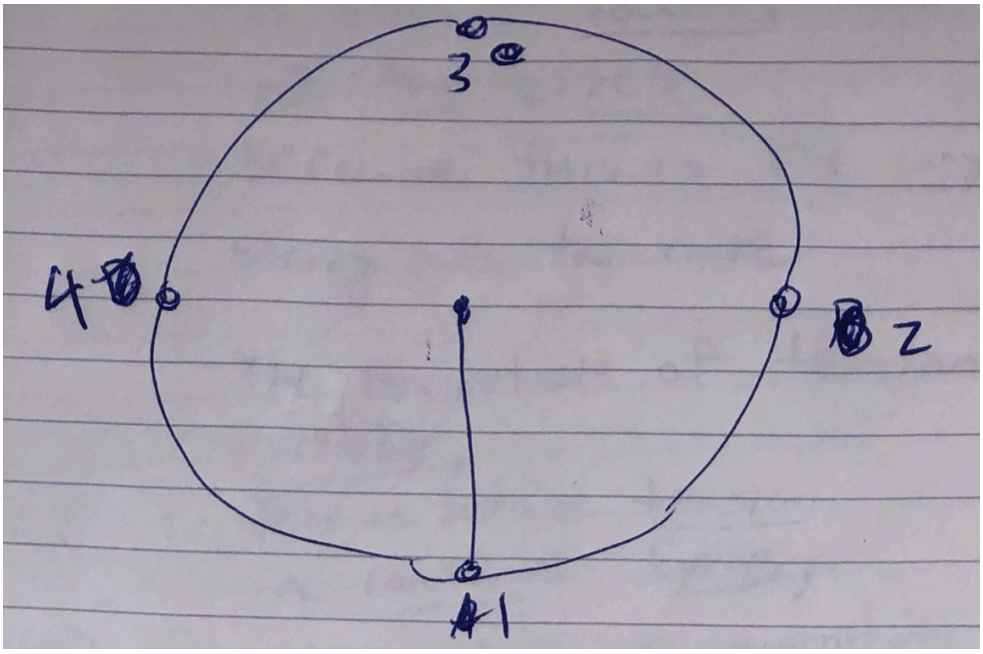
c) Find the speed of the rock as it moves around the circle.

d) Solve for the magnitude of the centripetal force F_c and the gravitational force F_g using known formulas. Please copy each formula in its canonical form before plugging any numbers in.

e) Determine the magnitude of the tension at each of the four cardinal points of the circle, point 1, point 2, point 3, and point 4.

4. A person is swinging a 1.8 kg rock in a circle of radius 0.1 m. The rock takes a time of 0.9 s to get around the circle.

a) Using the diagram to draw a qualitative free-body diagram of the block at points 1, 2, 3, and 4. Include only gravity and tension on the rock, and ignore any other applied forces.



b) The centripetal force is defined as the force pointing *towards the center of the circle* subtracted by the force pointing away from the center of the circle.
For each of points 1, 2, 3, and 4, write the centripetal force F_c in terms of the gravitational force F_g and the tension T.

c) Find the speed of the rock as it moves around the circle.

d) Solve for the magnitude of the centripetal force F_c and the gravitational force F_g using known formulas. Please copy each formula in its canonical form before plugging any numbers in.

e) Determine the magnitude of the tension at each of the four cardinal points of the circle, point 1, point 2, point 3, and point 4.