Electromagnetic Wave Proportionality

Introduction to Proportionality for Electromagnetic Waves

This pod is entirely about understanding these concepts of electromagnetic waves:

- Wavelength and Frequency of an electromagnetic wave are inversely proportional
- Frequency and energy of an electromagnetic wave are directly proportional
- Wavelength and energy of an electromagnetic wave are inversely proportional

How this looks in practice:

If wavelength is multiplied by 2,
frequency and energy are divided by 2.

If wavelength is divided by 2,
frequency and energy are multiplied by 2

If frequency is multiplied by 2,
energy is also multiplied by 2,
but wavelength is divided by 2.

If frequency is divided by 2,
energy is also divided by 2,
but wavelength is multiplied by 2.

If energy is multiplied by 2,
frequency is also multiplied by 2,
but wavelength is divided by 2.

If energy is divided by 2,
frequency is also divided by 2,
but wavelength is multiplied by 2.

We can use proportionality to calculate wave quantities much more quickly and to generally increase our knowledge on how these quantities are related.

Questions:

1. If wavelength is divided by 3, what happens to frequency? What happens to energy?

2. If frequency is divided by 8, what happens to wavelength? What happens to energy?

3. If frequency is multiplied by 5, what happens to wavelength? What happens to energy?

4. If wavelength is multiplied by 7, what happens to frequency? What happens to energy?

5. If energy is multiplied by 9, what happens to wavelength? What happens to frequency?

6. If energy is divided by 4, what happens to wavelength? What happens to frequency?

More Practice

[note: I selected numbers to be simply and easy to work with, not numbers that realistically reflect electromagnetic waves. But they'll be plenty of realistic numbers later

7. A wave has a wavelength of 3 meters and a frequency of 20 hertz.
If the wavelength increases to 6 meters, what is the new frequency?

8. A wave has a wavelength of 20 meters and a frequency of 9 hertz.
If the wavelength increases to 60 meters, what is the new frequency?

9. A wave has a wavelength of 80 centimeters and a frequency of 10 Hertz.
If the wavelength decreases to 20 centimeters, what is the new frequency?

10. A wave has a wavelength of 10 centimeters and a frequency of 7 Hertz.
If the wavelength decreases to 2 centimeters, what is the new frequency?

11. A wave has a frequency of 3 Hertz and an energy of 50 Joules.
If the frequency increases to 6 Hertz, what is the new energy?

12. A wave has a frequency of 2 Hertz and an energy of 4 Joules.
If the frequency increases to 8 Hertz, what is the new energy?

13. A wave has a frequency of 9 Hertz and an energy of 12 Joules.
If the frequency decreases to 3 Hertz, what is the new energy?

14. A wave has a wavelength of 10 centimeters, a frequency of 4 Hertz, and an energy of 8 Joules.
If the wavelength increases to 40 centimeters, what is the new frequency and energy?

15. A wave has a wavelength of 5 centimeters, a frequency of 9 Hertz, and an energy of 27 Joules.
If the frequency decreases to 1 Hertz, what is the new wavelength and energy?

16. A wave has a wavelength of 3 centimeters, a frequency of 16 Hertz, and an energy of 6 Joules.
If the energy decreases to 3 Joules, what is the new wavelength and frequency?

Why is this true?

This is true because of the formulas relating wavelength, frequency, and energy.

$c = \lambda f$

We know that *c*, the speed of light, is a constant value. Two numbers that multiply to equal a constant value are always *inversely proportional*.

$E = hf$

We know that *h*, Planck's constant, is a constant (duh). If a value equals a constant times another value, those two variable values are always *directly proportional*.

Solving Problems
Two different ways to use proportionality.
1. The official formula
2. Your intuition

The Official Formula
Imagine you have two waves, Wave 1 and Wave 2. Their frequencies and wavelengths are related by this formula:

$$\frac{\lambda_1}{\lambda_2} = \frac{f_2}{f_1} = \frac{E_2}{E_1}$$

$\lambda_1 =$ wavelength of wave 1 $\quad f_1 =$ frequency of wave 1 $\quad E_1 =$ Energy of wave 1
$\lambda_2 =$ wavelength of wave 2 $\quad f_2 =$ frequency of wave 2 $\quad E_2 =$ Energy of wave 2

For this formula, you can use *any units*. They do not need to be the SI units! This can help you solve a problem much more quickly, without doing a bunch of conversions.
However, there is one rule. Whatever units you use for wave 1, you need to use the same units for wave 2.
For example, if you measure the wavelength of wave 1 in nanometers, but the wavelength of wave 2 in centimeters, you are breaking the rules! You need to convert them both into the same unit in order to use the formula.

Using Your Intuition

If you have a strong grasp on how proportionality works, you can calculate necessary quantities much more quickly by using your intuition instead of the 'official formulas.' There isn't really a specific guide to how to do this, as it is your intuition, or sense of how things work without thinking much.

Practice Using the Formula (or your intuition)

Please figure out the following problems by using the proportionality formula (or your intuition). Please DO NOT use the formulas from the previous section. The goal here is to learn a different way of approaching these problems.

For each problem, show your work, to show you understand how to use proportionality.

17. A wave has a wavelength of 300 nanometers and frequency of 1000 Terahertz.
Another wave has a wavelength of 400 nanometers. What is its frequency?

18. A wave has a wavelength of 250 nanometers and frequency of 1200 Terahertz.
Another wave has a wavelength of 500 nanometers. What is its frequency?

19. A wave has a wavelength of 580 nanometers and frequency of 517 Terahertz.
Another wave has a frequency of 530 Terahertz. What is its wavelength?

20. A wave has a wavelength of 120 nanometers and frequency of 2.5 Petahertz.
Another wave has a frequency of 2.8 Petahertz. What is its wavelength?

21. A wave has a wavelength of 940 nanometers, a frequency of 319 Terahertz, and an energy of 1.32 electron volts.
If the wavelength changes to 840 nanometers, what is the frequency and energy?

22. A wave has a wavelength of 200 nanometers, a frequency of 1500 Terahertz, and an energy of 6.20 electron volts.
If the wavelength changes to 220 nanometers, what is the frequency and energy?

Explaining Categories of Waves Using Proportionality

23. Please fill out the following table using only proportionality relationships:
Which is a much, much faster method than using the formulas from the previous pod:

Color	Wavelength (nm)	Frequency (THz)	Photon Energy (eV)
Red	670 nm	448 THz	1.85 eV
Orange	610 nm		
Yellow	575 nm		
Green	555 nm		
Blue	485 nm		
Violet	425 nm		

24.
Look at your answer. And make sure that it connects with what we already learned about wavelength and frequency.
As the wavelength gets shorter, does the frequency get higher?
As the frequency gets higher, does the energy also get higher?

25.
The following table gives examples of two violet and two ultraviolet waves

Color (two are visible, and two are invisible)	Wavelength	Frequency	Photon Energy (eV)
A violet wave	465 nm		
Another violet wave	430 nm		2.89 eV
Near Ultraviolet Number 1	420 nm		
Near Ultraviolet Number 2	400 nm	750 THz	

26. Microwaves

Color (all invisible colors)	**Wavelength**	**Frequency**	**Photon Energy**
Microwave L		1.5 GHz	6.20 micro electron volts
Microwave S		3 GHz	
Microwave C	5 cm	6 GHz	
Microwave X		10 GHz	

27. X-Rays
(four randomly determined X-Ray waves)

Color (all invisible colors)	Wavelength	Frequency	Energy
X-Ray #1		60 PHz	
X-Ray #2	2 nm		
X-Ray #3	0.8 nm		412.5 eV
X-Ray #4		600 PHz	660 eV

28. Infrared Waves:

Color (all invisible)	Wavelength	Frequency	Photon Energy
Infrared O	1310 nm		
Infrared E	1410 nm		
Infrared S		201 THz	0.830 eV
Infrared C			0.801 eV
Infrared L	1595 nm		0.778 eV
Infrared U			0.752 eV

Answers:

1. Frequency and energy are both multiplied by 3.
2. Wavelength is multiplied by 8; energy is divided by 8.
3. Wavelength is divided by 5; energy is multiplied by 5.
4. Frequency and energy are both divided by 5.
5. Wavelength is divided by 9; frequency is multiplied by 9.
6. Wavelength is multiplied by 4; frequency is divided by 4.
7. 10 Hertz
8. 3 Hertz
9. 40 Hertz
10. 35 Hertz
11. 100 Joules
12. 16 Joules
13. 4 Joules
14. Frequency = 1 Hertz; energy = 2 Joules
15. Wavelength = 45 centimeters; energy = 3 Joules
16. Wavelength = 6 centimeters; frequency = 8 Hertz
17. 750 Terahertz
18. 600 Terahertz
19. 595 nanometers
20. 107 nanometers

21.
Frequency = 285 Terahertz
Energy = 1.18 eV

22.
Frequency =1363 Terahertz
Energy = 5.63 eV

23.

Color	**Wavelength (nm)**	**Frequency (THz)**	**Photon Energy (eV)**
Red	670 nm	448 THz	1.85 eV
Orange	610 nm	492 THz	2.03 eV
Yellow	575 nm	522 THz	2.16 eV
Green	555 nm	541 THz	2.23 eV
Blue	485 nm	619 THz	2.56 eV
Violet	425 nm	706 THz	2.92 eV

24.
They do! And that means you did it right!

25.

Color (two are visible, and two are invisible)	**Wavelength**	**Frequency**	**Photon Energy (eV)**
A violet wave	465 nm	645 THz	2.67 eV
Another violet wave	430 nm	698 THz	2.89 eV
Near Ultraviolet Number 1	420 nm	714 THz	2.95 eV
Near Ultraviolet Number 2	400 nm	750 THz	3.102 eV

26.

Color (all invisible colors)	**Wavelength**	**Frequency**	**Photon Energy**
Microwave L	20 cm	1.5 GHz	6.20 micro electron volts
Microwave S	10 cm	3 GHz	12.4 μeV
Microwave C	5 cm	6 GHz	24.8 μeV
Microwave X	3 cm	10 GHz	41.3 μeV

27.

Color (all invisible colors)	Wavelength	Frequency	Energy
X-Ray #1	5 nm	60 PHz	66 eV
X-Ray #2	2 nm	150 PHz	165 eV
X-Ray #3	0.8 nm	375 PHz	412.5 eV
X-Ray #4	0.5 nm	600 PHz	660 eV

28.

Color (all invisible)	Wavelength	Frequency	Photon Energy
Infrared O	1310 nm	229 THz	0.947 eV
Infrared E	1410 nm	213 THz	0.880 eV
Infrared S	1495 nm	201 THz	0.830 eV
Infrared C	1548 nm	194 THz	0.801 eV
Infrared L	1595 nm	188 THz	0.778 eV
Infrared U	1650 nm	182 THz	0.752 eV