

<b>Circle</b> Geometrically, a circle is all points that are the same distance from one point.
<b>Radius</b>  The radius is the distance from the center of the circle to the outside. We also saw that a radius is a line segment from the center of a circle to the outside of the circle.
<b>Diameter</b>  The diameter of a circle is the distance from one end to another, going through the center. The diameter of a circle is twice the radius.  $D = 2r$
<b>Circumference</b>  The circumference of a circle is the distance all the way around the circle. (if an ant walked the entire way around the circle.
<b>Pi</b>  Pi ( $\pi$ ) is a Greek letter used to understand circles.  The circumference of a circle divided by the <i>diameter</i> is <i>always</i> equal to pi!  This is represented by the formula  $\pi = \frac{C}{D}$ or $C = 2\pi r$ <div style="text-align: right;"> <b>Make sure you know the significance and definition of pi!</b>  <b>It's not just a cool number, it means something!</b> </div>
<b>Pi to 20 digits:</b>  3.1415926358979323846  (Memorize at least <b>15</b> It will not take you long.)
<b>Area of a Circle</b>  The area of a circle is given by this formula:  $A = \pi r^2$
<b>Tangent</b>  A line <i>tangent to</i> a circle is a line that touches the circle in exactly one point (just barely grazes the outside of the circle). A tangent line and a radius that intersect are <i>always perpendicular</i> !

1. If a circle has a circumference of 500 meters, what is its diameter?
2. If a circle has a circumference of 20 meters, what is its diameter?
3. If a circle has a circumference of 10 meters, what is its radius?
4. If a circle has a circumference of 50 meters, what is its radius?
5. If a circle has a diameter of 60 meters, what is its circumference?
6. If a circle has a diameter of 90 meters, what is its circumference?
7. If a circle has a radius of 5 meters, what is its circumference?
8. If a circle has a radius of 9 meters, what is its circumference?
9. If a circle has a radius of 6 meters, what is its area?
10. If a circle has a radius of 17 meters, what is its area?

11. If a circle has a diameter of 23 meters, what is its area?
  
  
  
  
  
  
  
  
  
  
12. If a circle has a diameter of 39 meters, what is its area?
  
  
  
  
  
  
  
  
  
  
13. If a circle has a circumference of 40 meters, what is its area?
  
  
  
  
  
  
  
  
  
  
14. If a circle has a circumference of 3 meters, what is its area?
  
  
  
  
  
  
  
  
  
  
15. Write pi to 10 digits!!!

**ANSWERS**

**1.** 159 meters

**2.** 6.4 meters

**3.** 1.6 meters

**4.** 8.0 meters [or 7.958 meters]

**5.** 188.5 meters

**6.** 283 meters

**7.** 31.4 meters

**8.** 56.5 meters

**9.** 113 m<sup>2</sup>

**10.** 908 m<sup>2</sup>

**11.** 415 m<sup>2</sup>

**12.** 1195 m<sup>2</sup>

**13.** 127.3

**14.** 0.716

**15.** 3.141592653

**Triangle**

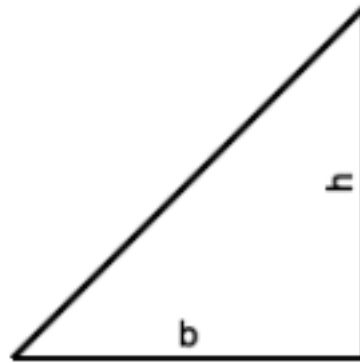
A triangle is any polygon with 3 sides.

**Right Triangle**

A right triangle has one angle equal to 90 degrees.

The area of a right triangle is equal to the one half the base times the height, in which the height is equal to two legs.

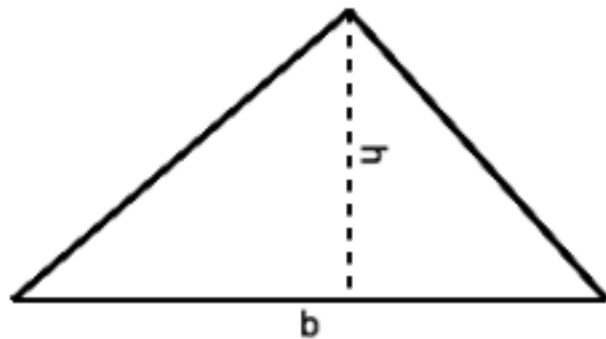
$$A = \frac{1}{2}bh$$

**Area of Any Triangle**

The area of any triangle is also one half the base times the height.

To find the area of a non-right triangle, draw a line from one vertex of the triangle that is perpendicular to the opposite side. This line is called an *altitude*. The “height” of the triangle is the length of the altitude, and the “base” is the length of the side it intersects.

$$A = \frac{1}{2}bh$$



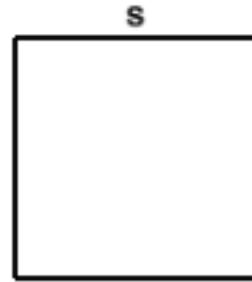
**Quadrilateral**

A quadrilateral is any polygon with 4 sides.

**Square**

A square is a quadrilateral in which all 4 sides and all 4 angles are equal (90 degrees).  
The area of a square the length of any side squared.

$$A = s^2$$

**Rectangle**

A rectangle is a quadrilateral in which all 4 angles are equal to 90 degrees, but all four sides are not necessarily equal. The length of each side is equal to the length of the opposite side.

The area of a rectangle is the base times the height.

$$A = bh$$



**Trapezoid**

A trapezoid is any quadrilateral with two parallel sides.

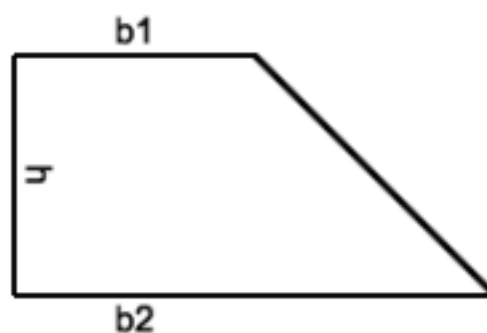
**Right Trapezoid**

A trapezoid is any quadrilateral with two parallel sides.

In a right trapezoid, one of the angles connecting these two parallel sides intersects both sides at a *right angle*.

The area of a right trapezoid is equal to the *average* of the two parallel sides, times the length of the side connecting them at right angles.

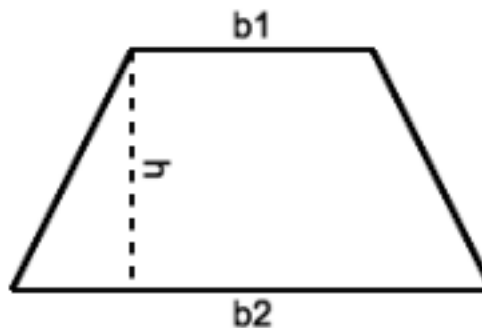
$$A = \left( \frac{b_1 + b_2}{2} \right) h$$

**Area of any Trapezoid**

As with triangles, the formula for the area for any trapezoid is a variation on the formula for the area of a right trapezoid.

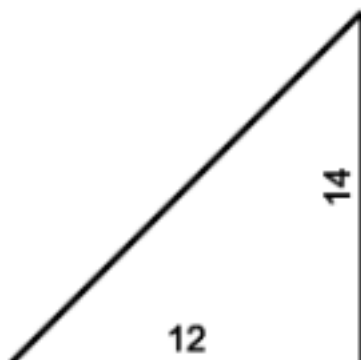
Draw a line that begins at one vertex, and intersects one parallel lines at a right angle. The area of the trapezoid is the average of the lengths of the two parallel sides, times the length of this connecting side.

$$A = \left( \frac{b_1 + b_2}{2} \right) h$$

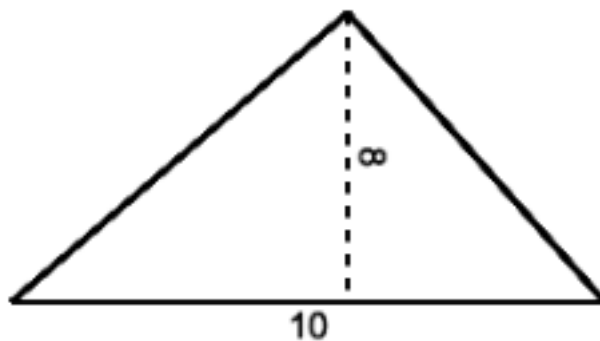


Find the area of each of the following shapes. Drawings are not to scale!

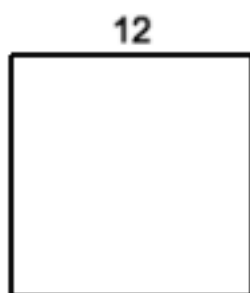
1.



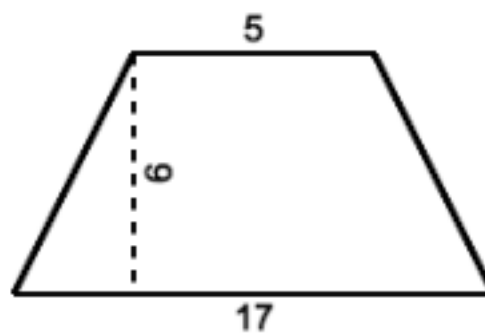
2.



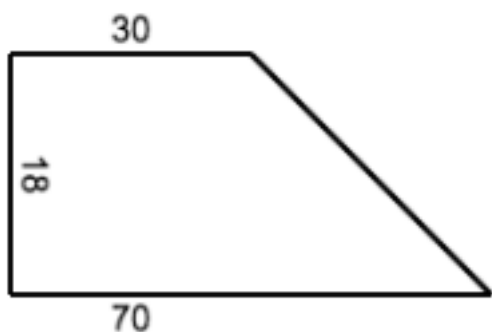
3.



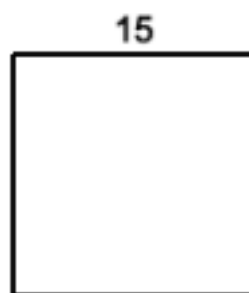
4.



5.

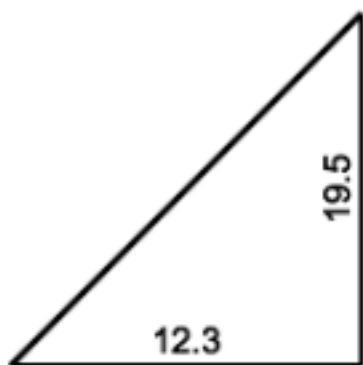


6.

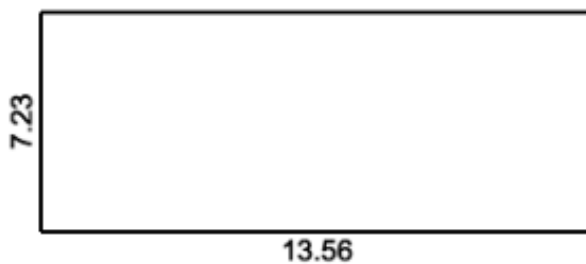




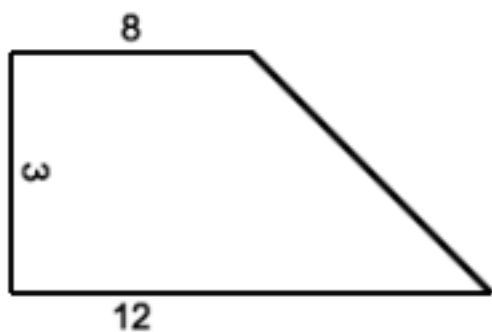
7.



8.



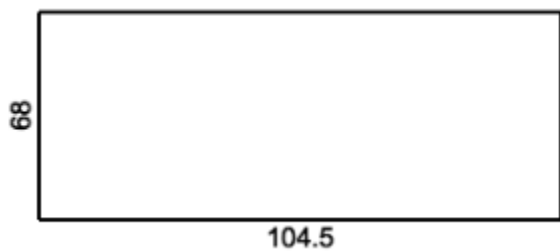
9.



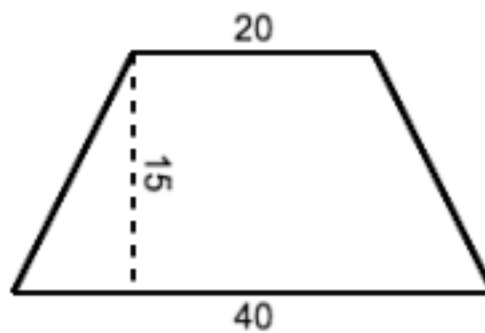
10.



11.



12.



**Answers:**

1. 84
2. 40
3. 144
4. 99
5. 900
6. 225
7. 239.85
8. 98.0
9. 30
10. 177.1
11. 7106 (or 7100, if rounded to 2 significant figures)
12. 450

## Trigonometry

### The GOAL

- Find...
- *every* side length and
- *every* angle of a right triangle.

### PURPOSE

- This is necessary to solve physics problems in *two dimensions*.
- Our first problem is to pilot robots into the correct locations!

### Part A: Pythagorean theorem problems

- A RIGHT TRIANGLE has at least one angle that is 90 degrees.
- The longest side of a right triangle is called the HYPOTENUSE. It is *always* opposite the 90 degree angle.

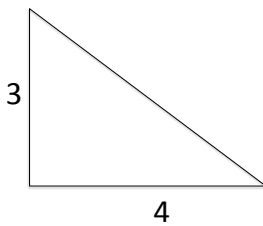
The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

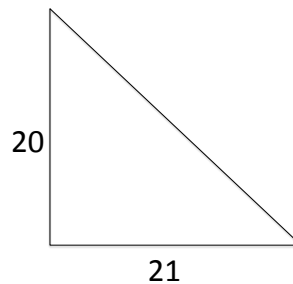
c is the length of the hypotenuse

a and b are the lengths of the two shorter sides.

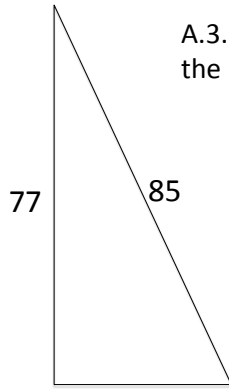
A.1. Find the length of the missing side.



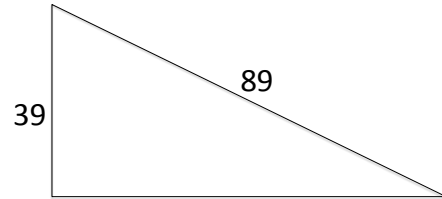
A.2. Find the length of the missing side.



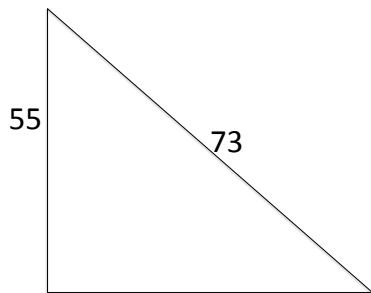
A.3. Find the length of the missing side.



A.4. Find the length of the missing side.



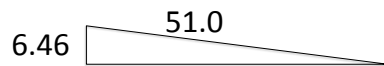
A.5. Find the length of the missing side.



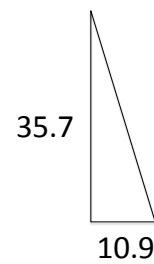
A.6. Find the length of the missing side.



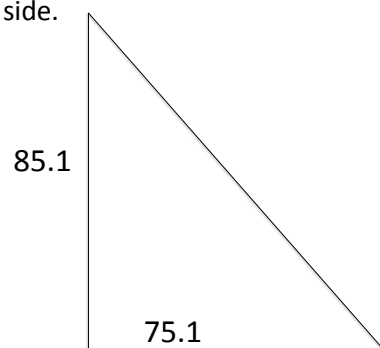
A.7. Find the length of the missing side.



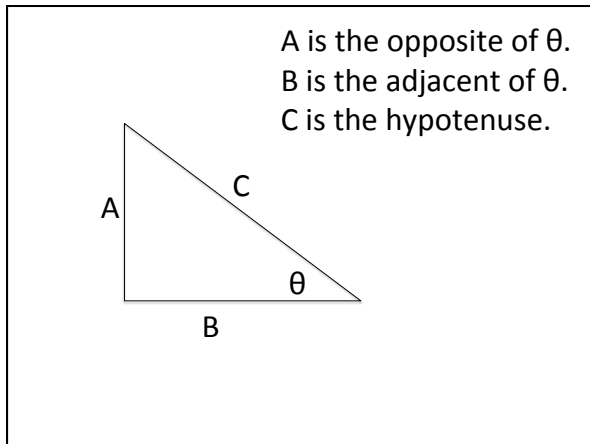
A.8. Find the length of the missing side.



A.9. Find the length of the missing side.



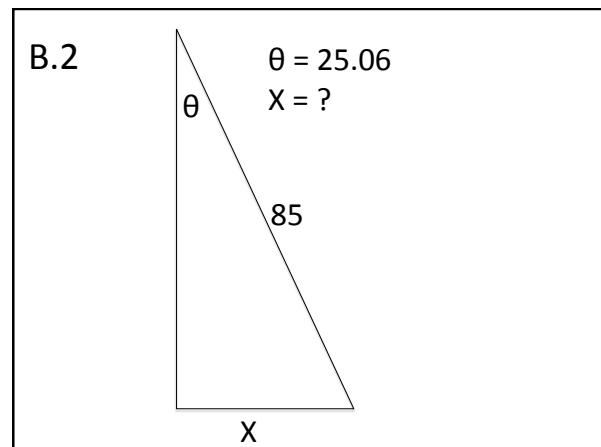
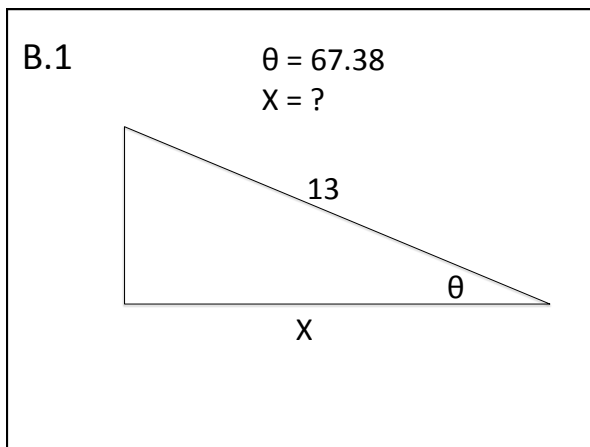
Part B: Finding side lengths from angles



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



B.3

$\theta = 81.20$

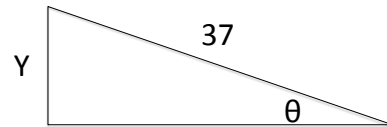
$Y = ?$



B.4

$\theta = 18.92$

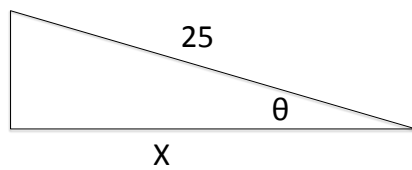
$Y = ?$



B.5

$\theta = 16.26$

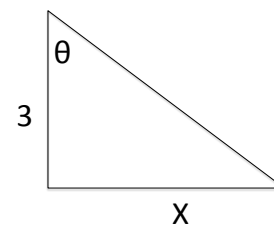
$X = ?$



B.6

$\theta = 53.13$

$X = ?$

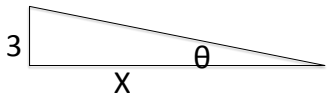




B.7

$\theta = 11.31$

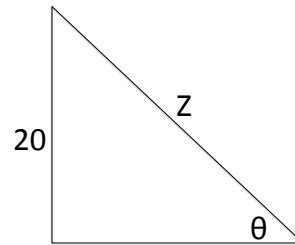
$x = ?$



B.8

$\theta = 43.60$

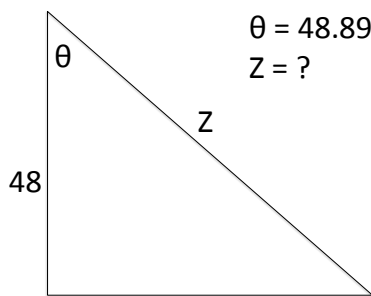
$z = ?$



B.9

$\theta = 48.89$

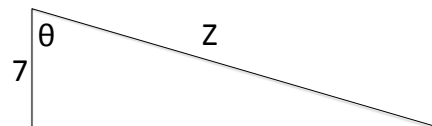
$z = ?$



B.10

$\theta = 73.74$

$z = ?$



### Answers

- A.1 5
- A.2 29
- A.3 36
- A.4 80
- A.5 48
- A.6 31.7
- A.7 50.6
- A.8 37.3
- A.9 113

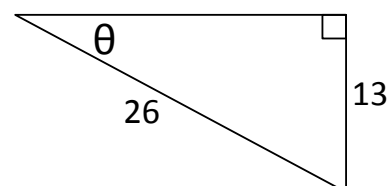
### ANSWERS

- B.1 5.0
- B.2 36
- B.3 13
- B.4 12
- B.5 24
- B.6 4.0
- B.7 15
- B.8 29
- B.9 73
- B.10 25

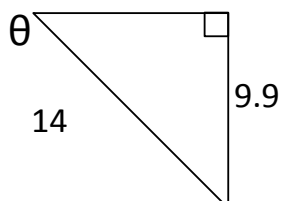
### Part C: Inverse Trigonometric Functions

Finding the measure of an angle from the sides

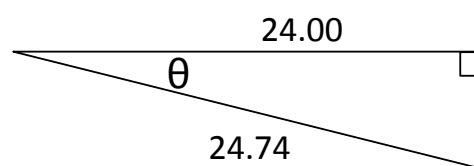
C.1



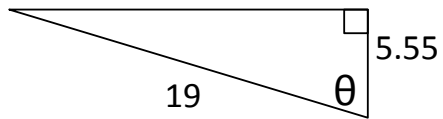
C.2



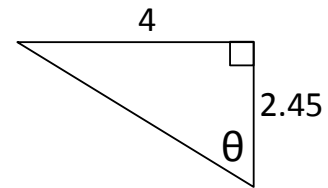
C.3



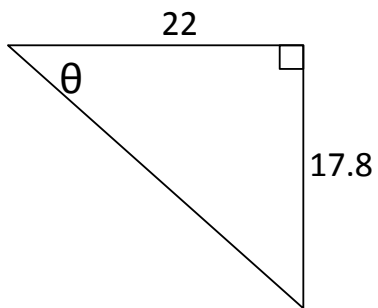
C.4



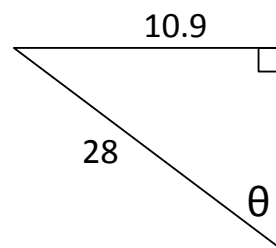
C.5



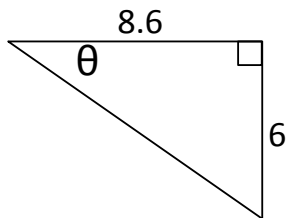
C.6



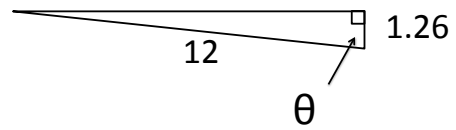
C.7



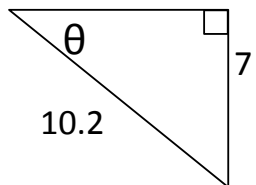
C.8



C.9



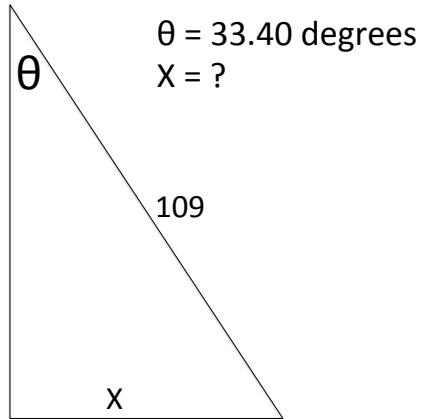
C.10



## Finding Side Length From Angles 2

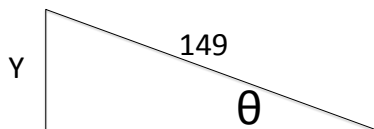
A trigonometric functions exercise

D.1



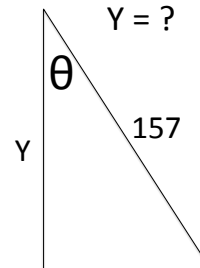
D.2

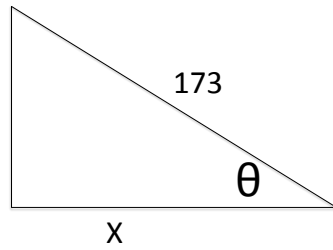
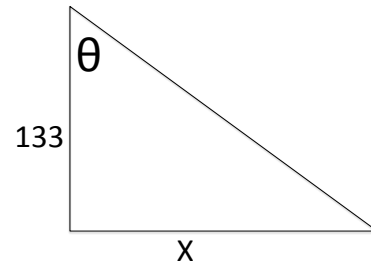
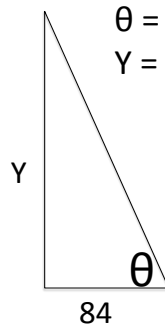
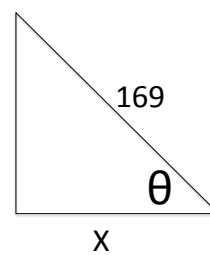
$\theta = 20.02$  degrees  
 $Y = ?$



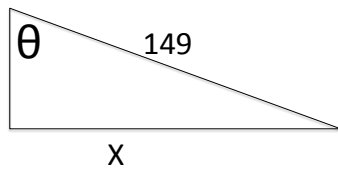
D.3

$\theta = 32.78$  degrees  
 $Y = ?$



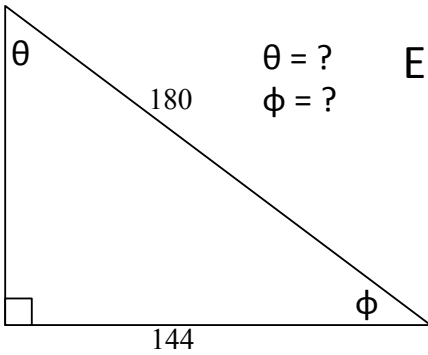
**D.4** $\theta = 17.49$  degrees  
 $X = ?$ **D.5** $\theta = 49.55$  degrees  
 $X = ?$ **D.6** $\theta = 65.50$  degrees  
 $Y = ?$ **D.7** $\theta = 52.88$  degrees  
 $X = ?$ 

D.8

 $\theta = 69.98 \text{ degrees}$   
 $X = ?$ 


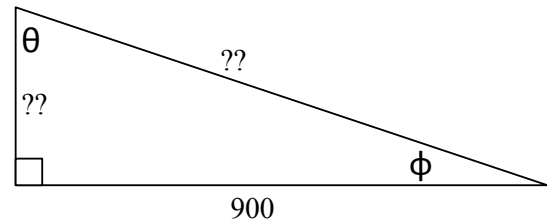
PART E: Solving for triangle  
completely

??


 $\theta = ?$   
 $\phi = ?$ 

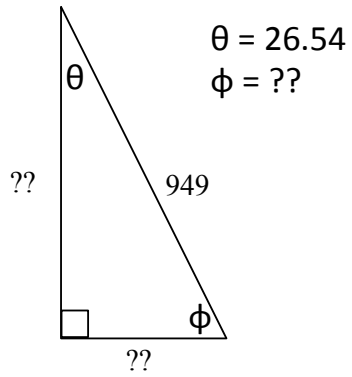
E.1

E.2

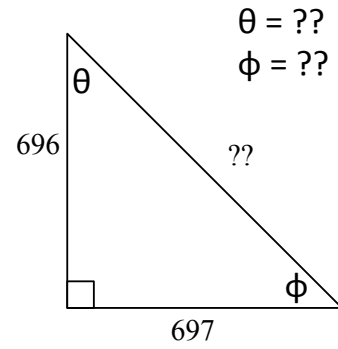
 $\theta = ??$   
 $\phi = 18.5$ 




E.3



E.4



## Answers

<u>C.1</u> 30	<u>C.6</u> 39
<u>C.2</u> 45	<u>C.7</u> 23
<u>C.3</u> 14	<u>C.8</u> 35
<u>C.4</u> 73	<u>C.9</u> 84
<u>C.5</u> 59	<u>C.10</u> 43

## Answers

• <u>D.1</u> 60	• <u>D.5</u> 156
• <u>D.2</u> 51	• <u>D.6</u> 184
• <u>D.3</u> 132	• <u>D.7</u> 102
• <u>D.4</u> 165	• <u>D.8</u> 140

