Multi-Step Algebra Problems Name ________________________

In each problem, solve for x.

1. $4 + 11 = 3x - 6$	**6.** $36 = 16 + 2 \cdot 12(x - 12)$
2. $55 = 12x^2 - 53$	**7.** $100 = 64 + 2 \cdot 10(x - 30)$
3. $19x = 24x - 15$	**8.** $81 = 25 + 2 \cdot 4(18 - x)$
4. $6 = \frac{24}{x}$	**9.** $64 = 16 + 2 \cdot 7(20 - x)$
5. $19 = \frac{31}{x}$	**10.** $100 = \frac{2 \cdot 4 \cdot 6}{x}$

11. $24 = 6\sqrt{\frac{x}{3}}$	**16.** $120 = \frac{3 \cdot 9 \cdot 12}{x^2}$
12. $40 = \frac{1}{4}\sqrt{\frac{x + 10}{3}}$	**17.** $300 = \frac{12 \cdot 3x}{9^2}$
13. $50 = \frac{2}{3}\sqrt{\frac{x + 13}{0.4}}$	**18.** $400 = \frac{3 \cdot 2 \cdot (10 - 2)}{x^2}$
14. $500 = \frac{1}{2} \cdot 4 \cdot x^2 + 4 \cdot 10 \cdot 12$	**19.** $\frac{1}{x} = \frac{1}{2} + \frac{1}{4}$
15. $200 = \frac{1}{2} \cdot x \cdot 5^2 + x \cdot 10 \cdot 14$	**20.** $\frac{1}{10} = \frac{1}{x} + \frac{1}{20}$

Answers:

1. 7
2. 3
3. 3
4. 4
5. 1.63
6. 12.83
7. 31.8
8. 11
9. 16.57
10. 0.48
11. 48
12. 76,790
13. 2,237
14. 3.16
15. 1.31
16. 1.64
17. 675
18. 0.34
19. 1.33
20. 20

Basic Symbolic Manipulation Name ___________________________

There will be a quiz on this content. You will be expected not only to write the answer, but demonstrate the steps to get to the answer!

Symbols, like X, Y, and Z, can be manipulated just like numbers can.

You can add, subtract, multiply, or divide both sides by a symbol.

To "solve an equation" for a symbol means that you will

For example, let's say I have the following equation:

$$Z = XY$$

I want to solve for Y. This means that I want to manipulate the terms until I have $Y =$ something.

In this specific case, I need to divide both sides by X, and I have

$$\frac{Z}{X} = Y$$

which is the answer!

One Step Equations:

1. Solve for B.

$A = BC$

2. Solve for E.

$D = E + F$

3. Solve for G.

$H = G - I$

4. Solve for N:

$M = \frac{N}{O}$

5. Solve for Q:

$P = 5Q$

Two Step Equations

6. Solve for L

$J = K - L$

It is very important to be able to solve equations where your variable is in the denominator. There are two ways to do it:

Solve for Z:

$X = \frac{Y}{Z}$

Method 1: first, multiply both sides by Z. Then, it's just like one of the equations above:

$$X = \frac{Y}{Z} \quad \Rightarrow \quad ZX = Y \quad \Rightarrow \quad Z = \frac{Y}{X}$$

Method 2: Put the left side of the equation over 1. Then, cross multiply to solve the problem:

$$X = \frac{Y}{Z} \quad \Rightarrow \quad \frac{X}{1} = \frac{Y}{Z} \quad \Rightarrow \quad XZ = 1Y \quad \Rightarrow \quad XZ = Y \quad \Rightarrow \quad Z = \frac{Y}{X}$$

7. Solve for R:

$\frac{S}{R} = T$

8. Solve for m:

$E = mgh$

9. Solve for h:

$E = mgh$

10. Solve for m:

$E = \frac{1}{2}mv^2$

Basic Symbolic Manipulation Name ___________________________

11. Solve for v:

$E = \frac{1}{2}mv^2$

12. Solve for F_1

$F_1 - F_2 = ma$

13. Solve for a

$F_1 - F_2 = ma$

14. Solve for v_i

$v_f = v_i + at$

15. Solve for a

$v_f = v_i + at$

16. Solve for t

$v_f = v_i + at$

17. Solve for v

$a = \frac{v^2}{r}$

Basic Symbolic Manipulation Name ___________________________

18. Solve for r

$$a = \frac{v^2}{r}$$

19. Solve for m_1

$$F = \frac{Gm_1m_2}{r^2}$$

20. Solve for m_2

$$F = \frac{Gm_1m_2}{r^2}$$

21. Solve for r:

$$F = \frac{Gm_1m_2}{r^2}$$

Basic Symbolic Manipulation Name ____________________

Answers:

1. $B = \frac{A}{C}$	**8.** $m = \frac{E}{gh}$	**15.** $a = \frac{v_f - v_i}{t}$
2. $E = D - F$	**9.** $h = \frac{E}{mg}$	**16.** $t = \frac{v_f - v_i}{a}$
3. $G = H + I$	**10.** $m = \frac{2E}{v^2}$	**17.** $v = \sqrt{ar}$
4. $N = MO$	**11.** $v = \sqrt{\frac{2E}{m}}$	**18.** $r = \frac{v^2}{a}$
5. $Q = \frac{P}{5}$	**12.** $F_1 = F_2 + ma$	**19.** $m_1 = \frac{Fr^2}{Gm_2}$
6. $L = K - J$	**13.** $a = \frac{F_1 - F_2}{m}$	**20.** $m_2 = \frac{Fr^2}{Gm_1}$
7. $R = \frac{T}{S}$	**14.** $v_i = v_f - at$	**21.** $r = \sqrt{\frac{Gm_1 m_2}{F}}$

Name: ______________________________ Date: ______________________

Slope from a Graph

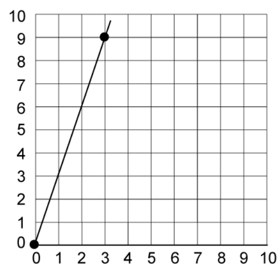
READ

To determine the slope of a line choose two points on the line. Then count how many steps up or down you must move to be on the same horizontal line as your second point. Multiply this number by the scale factor.

Put the result along with the positive or negative sign in the numerator of your slope ratio if the scale is one. Then count how many steps you must move right or left to land on your second point. Multiply the number of steps by the scale factor. Place the results in the denominator of your slope ratio.

EXAMPLES

A

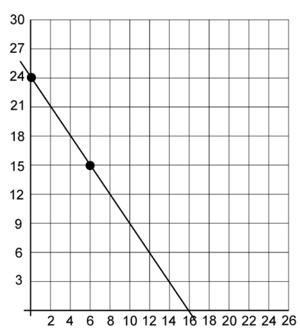


The chosen points for the example are (0, 0) and (3, 9). (There are many choices for this graph, but only one slope. If you have the point (0, 0), you should choose it as one of your points.)

It takes 9 vertical steps to move from (0, 0) to (0, 9). Put a 9 in the numerator of your slope ratio (or put 9 – 0). Then count the number of steps to move from (0, 9) to (3, 9). This is your denominator of your slope ratio. Again, you can do this by subtraction (3 - 0).

$$m = \frac{9}{3} = \frac{3}{1}$$

B



The two points that have been chosen for this example are (0, 24) and (6, 15). Be careful of the scales on each of the axes.

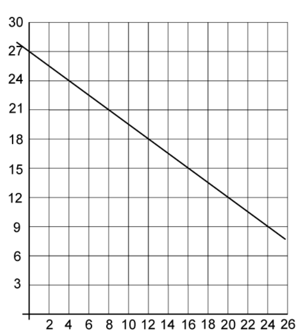
It takes 3 vertical steps to go from (0, 24) to (0, 15). But each of these steps has a scale of 3. So you put a -9 into the numerator of the slope ratio. It is *negative* because you are moving down from one point to the other. Then count the steps over to (6, 15). There are 3 steps but each counts for 2 so you put a 6 into the denominator of the slope ratio.

$$m = \frac{-9}{6} = \frac{-3}{2}$$

PRACTICE

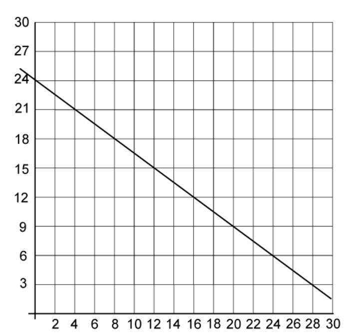
Find the slope of the line in each of the following graphs:

Graph #1:



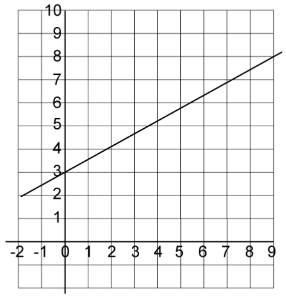
m = ______

Graph #2:



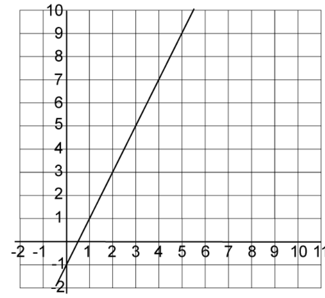
m = ______

Graph #3:



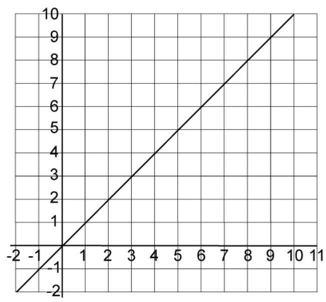
m = ______

Graph #4:



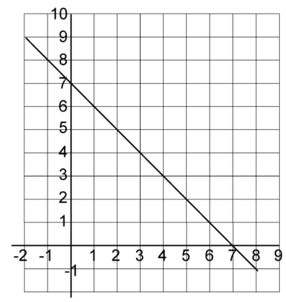
m = ______

Graph #5:



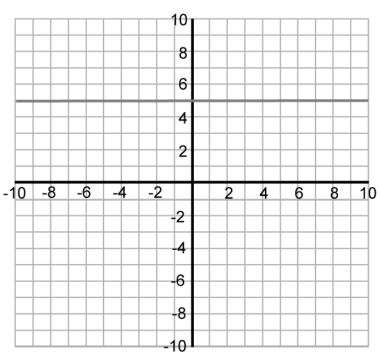
m = ______

Graph #6:



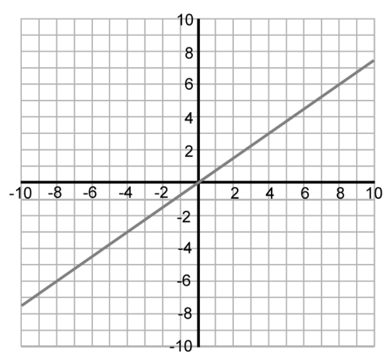
m = ______

Graph #7:



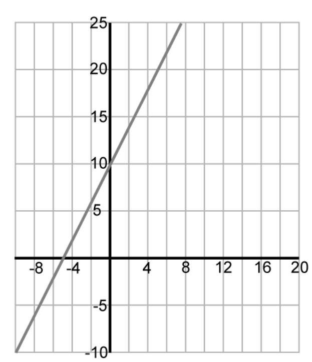
m = ______

Graph #8:



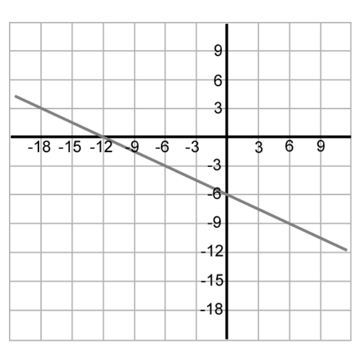
m = ______

Graph #9:



m = ______

Graph #10:



m = ______

Decimals and Place Value

1. Answers are:
   a. 0.14
   b. 4,600.025
   c. 1,000.001
   d. 985.63
   e. 8,035.4
2. Answers are:
   a. one hundred five and sixty-four thousandths
   b. twenty-three and forty nine ten-thousandths
   c. thirty-six and seven tenths
   d. forty-five and three thousandths
   e. seventy-four and nine hundred ninety-eight thousandths
3. Answers are:
   a. nineteen and thirty-two hundredths
   b. 43.49

Slope

1. m = $^{2}/_{4}$ = $^{1}/_{2}$
   As *x* changes 4 units to the left, *y* changes 2 units down. OR
   As *x* changes 4 units to the right, *y* changes 2 units up.
2. m = $^{7}/_{-3}$
   As *x* changes 3 units to the left, *y* changes 7 units up. OR
   As *x* changes 3 units to the right, *y* changes 7 units down.
3. m = $^{4}/_{7}$
   As *x* changes 7 units to the left, *y* changes 4 units down. OR
   As *x* changes 7 units to the right, *y* changes 4 units up.
4. m = $^{7}/_{-7}$ = m = -1
   As *x* changes 1 units to the left, *y* changes 1 units up. OR
   As *x* changes 1 units to the right, *y* changes 1 units down.
5. m = $^{1}/_{3}$
   As *x* changes 3 units to the left, *y* changes 1 units down. OR
   As *x* changes 3 units to the right, *y* changes 1 units up.

Slope from a Graph

Numbers correlate to graph numbers:

1. $m = \frac{-3 \times 3}{6 \times 2} = \frac{-3}{4}$
2. $m = \frac{-2 \times 3}{4 \times 2} = \frac{-3}{4}$
3. $m = \frac{5}{9}$
4. $m = \frac{4}{2} = 2$
5. $m = \frac{3}{3} = 1$
6. $m = \frac{-2}{2} = -1$
7. $m = 0$
8. $m = \frac{3}{4}$
9. $m = 2$
10. $m = \frac{-1}{2}$

Order of Operations, Part 1

1. 10
2. 50
3. 8.5
4. 7
5. 44
6. 5
7. 22
8. 32
9. 24
10. 12
11. 7
12. 24
13. 14
14. 19
15. 10
16. Answers are:
    a. \$25.00 + 2 × \$8.95
    b. \$42.90
17. Answers are:
    a. \$5.99 + 20 × \$0.95
    b. \$24.99
18. Answers are:
    a. 2 × 150 + 2 × 250
    b. 800 cells of bacteria
19. Answers are:
    a. 60 × \$7.50 + 70 × \$5.00 + 50 × \$7.50 + 90 × \$5.00
    b. \$1,625.00

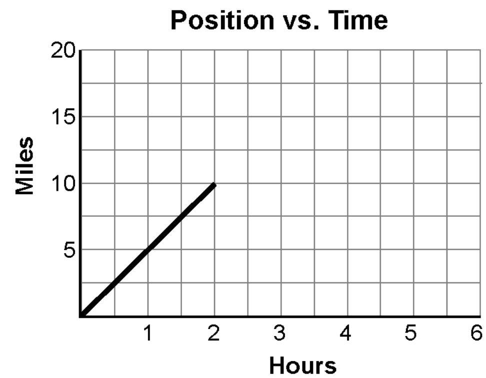
Name: ______________________________ Date: ______________

Analyzing Graphs of Motion With Numbers

READ

Speed can be calculated from position-time graphs and distance can be calculated from speed-time graphs. Both calculations rely on the familiar speed equation: $v = d/t$.

This graph shows position and time for a sailboat starting from its home port as it sailed to a distant island. By studying the line, you can see that the sailboat traveled 10 miles in 2 hours.



EXAMPLES

- **Calculating speed from a position-time graph**

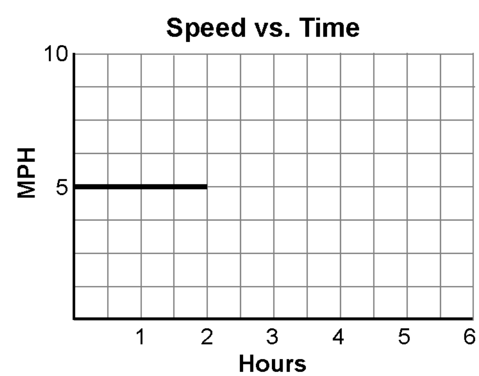
The speed equation allows us to calculate that the vessel speed during this time was 5 miles per hour.

$$v = d/t$$

$$v = 10 \text{ miles} / 2 \text{ hours}$$

$$v = 5 \text{ miles/hour, read as 5 miles per hour}$$

This result can now be transferred to a speed-time graph. Remember that this speed was measured during the first two hours.

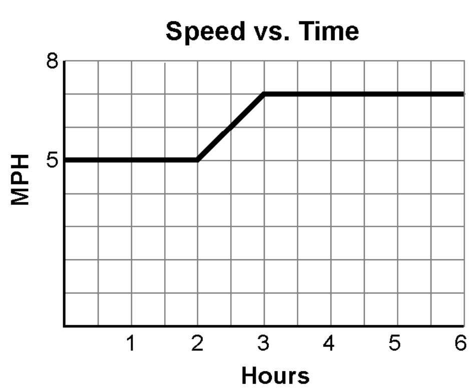


The line showing vessel speed is horizontal because the speed was constant during the two-hour period.

- **Calculating distance from a speed-time graph**

Here is the speed-time graph of the same sailboat later in the voyage. Between the second and third hours, the wind freshened and the sailboat increased its speed to 7 miles per hour. The speed remained 7 miles per hour to the end of the voyage.

How far did the sailboat go during this time? We will first calculate the distance traveled between the third and sixth hours.



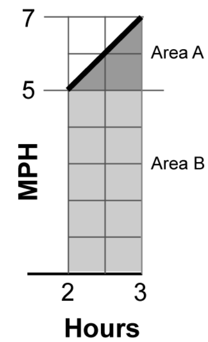
On a speed-time graph, distance is equal to the area between the baseline and the plotted line. You know that the area of a rectangle is found with the equation: $A = L \times W$. Similarly, multiplying the speed from the y-axis by the time on the x-axis produces distance. Notice how the labels cancel to produce miles:

$$\text{speed} \times \text{time} = \text{distance}$$

$$7 \text{ miles/hour} \times (6 \text{ hours} - 3 \text{ hours}) = \text{distance}$$
$$7 \text{ miles/}\cancel{\text{hour}} \times 3 \; \cancel{\text{hours}} = \text{distance} = 21 \text{ miles}$$

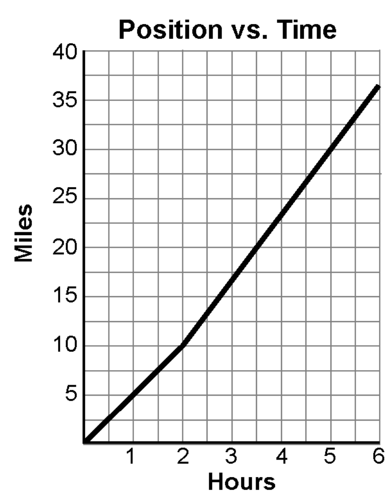
Now that we have seen how distance is calculated, we can consider the distance covered between hours 2 and 3.

The easiest way to visualize this problem is to think in geometric terms. Find the area of the rectangle labeled "1st problem," then find the area of the triangle above, and add the two areas.



Area of triangle A Geometry formula	The area of a triangle is one-half the area of a rectangle. $\text{speed} \times \frac{\text{time}}{2} = \text{distance}$ $(7 \text{ miles/hour} - 5 \text{ miles/hour}) \times \frac{(3 \text{ hours} - 2 \text{ hours})}{2} = \text{distance} = 1 \text{ mile}$
Area of rectangle B Geometry formula	$\text{speed} \times \text{time} = \text{distance}$ $5 \text{ miles/hour} \times (3 \text{ hours} - 2 \text{ hours}) = \text{distance} = 5 \text{ miles}$
Add the two areas	$\text{Area A} + \text{Area B} = \text{distance}$ $1 \text{ miles} + 5 \text{ mile} = \text{distance} = 6 \text{ miles}$

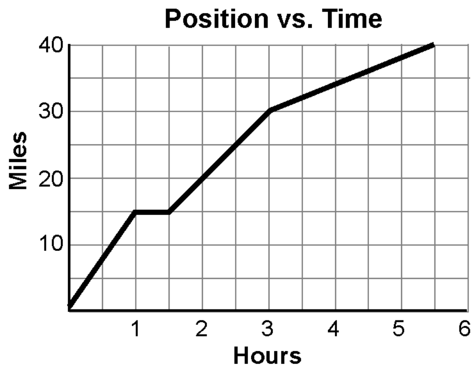
We can now take the distances found for both sections of the speed graph to complete our position-time graph:

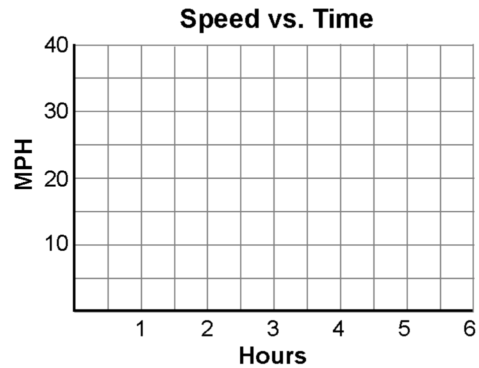


PRACTICE

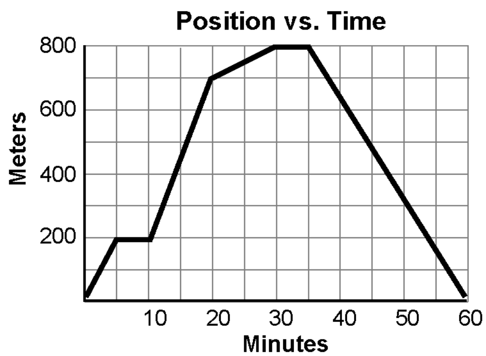
1. For each position-time graph, calculate and plot speed on the speed-time graph to the right.

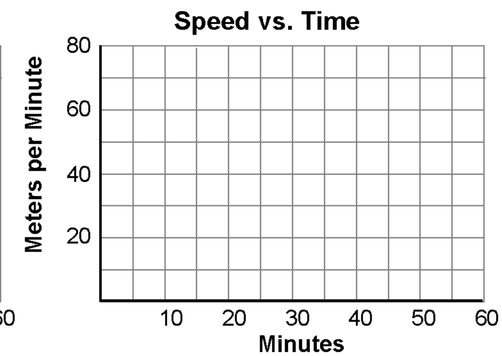
   a. The bicycle trip through hilly country



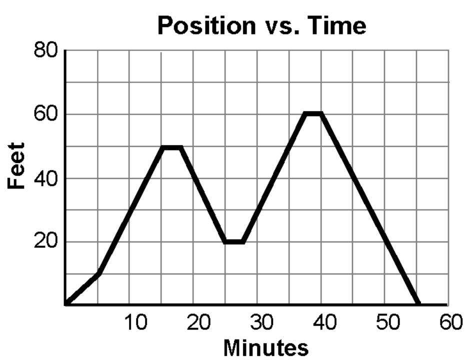


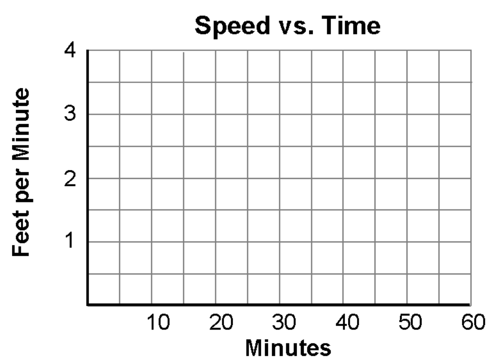
   b. A walk in the park





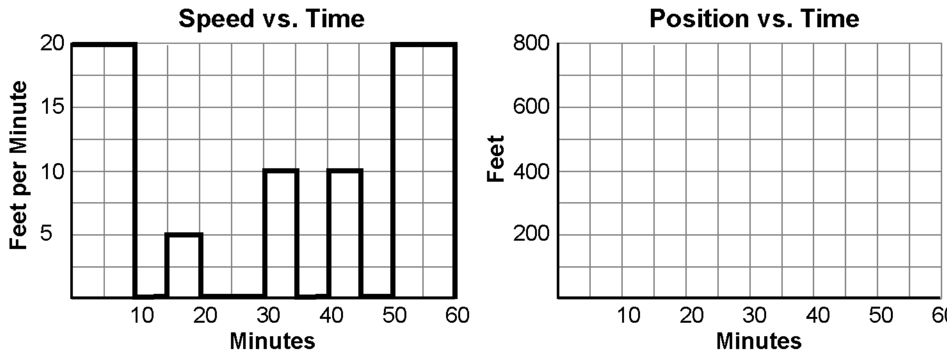
   c. Strolling up and down the supermarket aisles



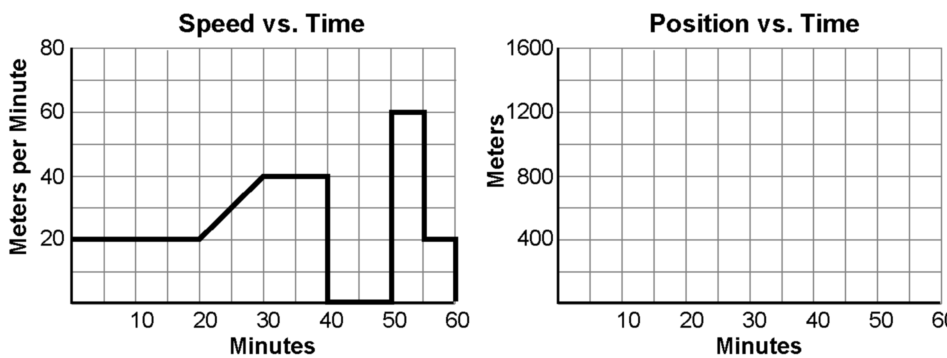


2. For each speed-time graph, calculate and plot the distance on the position-time graph to the right. For this practice, assume that movement is always away from the starting position.

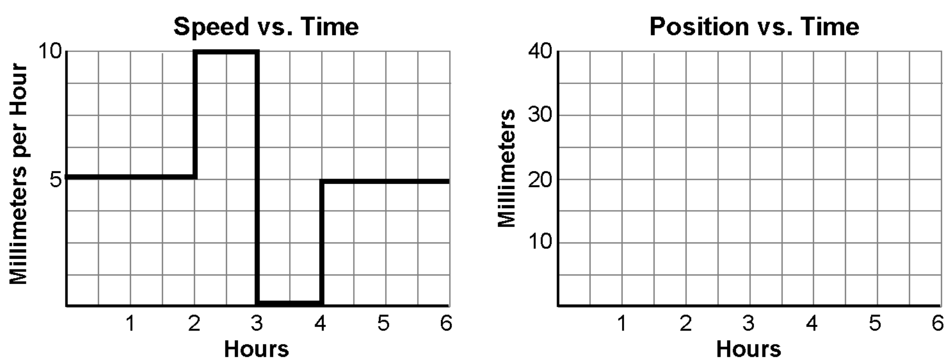
   a. The honey bee among the flowers



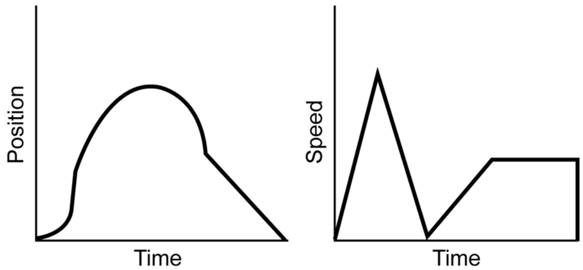
   b. Rover runs the street



   c. The amoeba



3. The Skyrocket. Graph the altitude of the rocket:

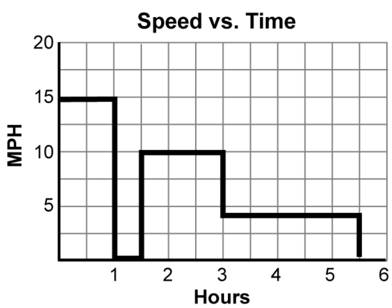


4. Each student story will include elements that are controlled by the graphs and creative elements that facilitate the story. Only the graph-controlled elements are described here.

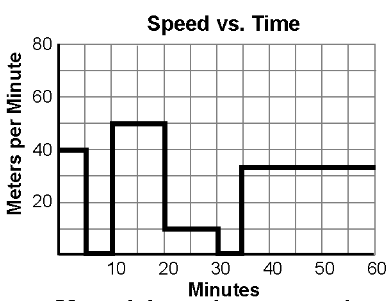
- The line begins and ends on the baseline, therefore Tim must start from and return to his house.
- The line rises toward the first peak as a downward curved line that becomes horizontal. This indicates that Tim's pace toward Caroline's house slowed to a stop.
- Then the line rises steeply to the first peak. This indicates that after his stop, Tim continues toward Caroline's house faster than before.
- The first peak is sharp, indicating that Tim did not spend much time at Caroline's house on first arrival.
- The line then falls briefly, turns to the horizontal, and then rises to a second peak. This indicates that Tim left, paused, and then returned quickly to Caroline's house.
- The line then remains at the second peak for a long time, then drops steeply to the baseline. This indicates that after spending a long time at Caroline's house, Tim probably ran home.

2.4 Analyzing Graphs of Motion With Numbers

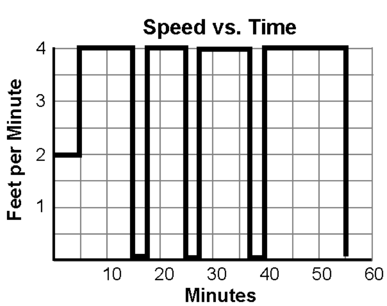
1. Answers are:
   a. The bicycle trip through hilly country.



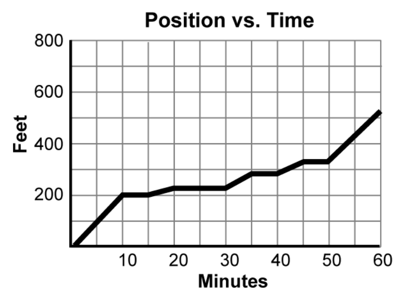
   b. A walk in the park.



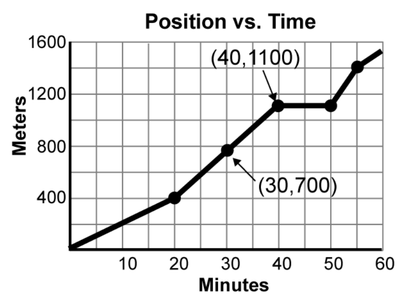
   c. Up and down the supermarket aisles.



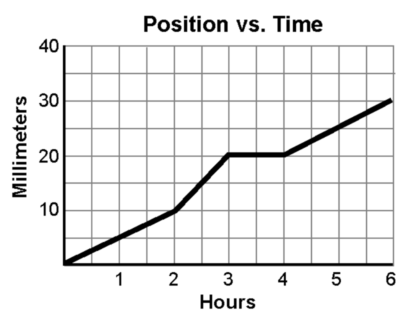
2. Answers are:
   a. The honey bee among the flowers.



   b. Rover runs the street.



   c. The amoeba.



2.4 Acceleration and Speed-Time Graphs

1. Acceleration = 5 miles/hour/hour or 5 miles/hour^2
2. Acceleration = -2 meters/minute/minute or -2 meters/minute^2
3. Acceleration = 0 feet/minute/minute or 0 feet/minute^2 or no acceleration
4. Answers are:
   Segment 1: Acceleration = 2 feet/second/second, or 2 feet/second^2
   Segment 2: Acceleration = 0.67 feet/second/second, or 0.67 feet/second^2
5. Distance = 1,400 meters
6. Distance = 700 meters
7. Distance = 75 kilometers