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Dimension

(Almost) every physics quantity has a property called its *dimension*.
A *dimension* is the mathematical space in which a physics quantity exists.

Units:

Each dimension is represented by the units of that quantity. Often, we represent dimensions simply by writing the SI units of a quantity.

For example, *distance*, *displacement*, *length*, and *height* all have dimensions of *length* and they all have the SI unit of *meters*.

Time and time elapsed all have dimensions of *time* and have the SI unit of *seconds.*

Base Dimensions (base units)

(Almost) all physics dimensions can be written in terms of three base dimensions: length, mass, and time.

(Almost) all physics units can be written in terms of three basic units: meters, kilograms, and seconds.

(Almost) all physics units are created by *multiplying and dividing* the 3 base units.

The base units:

METERS
KILOGRAMS
SECONDS

Each of these quantities either has *base unit,* or has a unit actually written in terms of its base units. What are the units of each?

Time	Mass	Displacement
Velocity	Acceleration	

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Algebraically, simplify each of the following combinations of units:

1. $$\frac{\mathrm{m}}{\mathrm{kg}} \cdot \frac{\mathrm{s}}{\mathrm{m}^2}$$	**2.** $$\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \frac{\mathrm{m}^3}{\mathrm{s}^3} \cdot \frac{\mathrm{kg}^2}{\mathrm{m}^2}$$
3. $$\frac{\mathrm{m}^2}{\mathrm{kg}} \cdot \frac{\mathrm{kg}^3\mathrm{m}}{\mathrm{s}} \cdot \frac{\mathrm{kg}^2\mathrm{s}^3}{\mathrm{m}^3}$$	**4.** $$\frac{\mathrm{m}^4}{\mathrm{kg}} \cdot \frac{\mathrm{m}}{\mathrm{kg}^2} \cdot \frac{\mathrm{kg}}{(\mathrm{m \cdot s})^3}$$

Each of the following quantities are made by multiplying and dividing displacement, velocity, acceleration, What would be the units (dimensions) of the following quantities?

5. $$\frac{(\Delta x)a}{v^2}$$	**6.** $$\frac{(\Delta t)v^3}{(\Delta x)a^2}$$
7. [note, the *m* stands for *mass*, not meters] $$\frac{m^2(\Delta t)^5}{v^3}$$	**8.** $$\frac{a^3v^2}{m^2(\Delta x)^4}$$

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What is the SI unit for each of these quantities?

Force	Work	Kinetic Energy	Gravitational Potential Energy

You probably wrote either *Newtons* or *Joules* for each of the above units.
However, each of these units can also be written in terms of the base units, *meters*, *kilograms*, and *seconds*.
Use each of the following formulas to find the *true* unit of each of these quantities:

$$\text{Force} = (\text{Mass})(\text{Acceleration})$$

$$\text{Work} = (\text{Force})(\text{Distance})$$

$$\text{Kinetic Energy} = \frac{1}{2}(\text{mass})(\text{velocity})^2$$

(note: $\frac{1}{2}$ has no dimensions and doesn't need to be considered here)

$$\text{Gravitational Potential Energy} = (\text{Mass})(\text{Height})(\text{Free-Fall Acceleration})$$

9. What really is a *Newton*?

10. What really is a *Joule*?

11. Do all forms of energy (work, kinetic energy, and potential energy) have the same *dimensions*? Is that logical?

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Dimensions of the Kinematic Equations:

Whenever you add or subtract units in physics, they *must* have the same dimensions.

Below are the *four* kinematic equations. Your task is to prove that each equation is *dimensionally correct* by showing that each *term* of the equation has the same dimensions.

12.

Definition of Acceleration	$v_f = v_i + a \cdot \Delta t$

This equation has *three terms:*

	Write the term	What are the dimensions of this term?
Term #1		
Term #2		
Term #3		

13.

No-Time Equation	$v_f^2 = v_i^2 + 2a \cdot \Delta x$

The no-time equation

	Write the term	What are the dimensions of this term?
Term #1		
Term #2		
Term #3		

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14. The King of Kinematic Equations	$$\Delta x = v_i \cdot \Delta t + \frac{1}{2} a (\Delta t)^2$$

	Write the term	What are the dimensions of this term?
Term #1		
Term #2		
Term #3		

15. The Average Velocity Formula	$$\Delta x = \left(\frac{v_i + v_f}{2}\right) \Delta t$$

This one is slightly more complicated.
v_i and v_f can be added because they both have the same dimensions (m/s). When they are added together, the sum *still* has dimensions of m/s.

	Write the term	What are the dimensions of this term?
Term #1		
Term #2		

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16. Newton's Law of Universal Gravitation:

$$F_g = \frac{Gm_1m_2}{d^2}$$

This equation gives the *gravitational force* between any two objects.
F_g is a force and has units of Newtons.
m_1 and m_2 are masses. d is a distance and has units of meters.

Isaac Newton determined the proportionality relationships between each of the major quantities. However, a constant needed to be added that made the equation *dimensionally* correct.
The dimensions of the *right side* of the equation must be the same as the dimension of the *left side*.

Determine what the dimensions of G must be in order to make the equation work out dimensionally:

17. Coulomb's Law

$$F_e = \frac{kq_1q_2}{d^2}$$

Coulomb discovered the proportionality relationships, but once again a constant needed to be added that made the formula dimensionally correct. This constant is the variable k.
F_e is a force and has units of Newtons.
d is a distance and has units of meters.

q_1 and q_2 are both CHARGES. Their unit is coulombs (C). Coulombs a new *base unit*, just like, meters, kilograms, and seconds are.

Figure out what the constant k must be in order to make this equation dimensionally correct..
Your answer needs to be written in terms of meters, kilograms, seconds, and coulombs.

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Answers

1. $\frac{\text{s}}{\text{kg m}}$

2. $\frac{\text{m}^2\text{kg}^2}{\text{s}^5}$

3. kg^4s^2

4. $\frac{\text{m}^2}{\text{kg}^2\text{s}^3}$

5. $\frac{1}{\text{m}}$

6. s^2

7. $\frac{\text{kg}^2\text{s}^8}{\text{m}^3}$

8. $\frac{\text{m}}{\text{kg}^2\text{s}^8}$

9. A Newton is really $\frac{\text{kg m}}{\text{s}^2}$

10. A Joule is really $\frac{\text{kg m}^2}{\text{s}^2}$

11. All forms of energy have the same dimensions: $\frac{\text{kg m}^2}{\text{s}^2}$. This makes sense, as they are all the same thing and can be compared to each other.

12. All three terms have dimensions $\frac{\text{m}}{\text{s}}$.

13. All three terms have dimensions $\frac{\text{m}^2}{\text{s}^2}$.

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14. All three terms have dimensions m.

15. Both terms have dimensions m.

16. G has dimensions $\dfrac{\mathrm{m}^3}{\mathrm{kg\ s}^2}$

17. k has dimensions $\dfrac{\mathrm{kg\ m}^3}{\mathrm{C}^2 s^2}$

Packet on dimensionality

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The key issue

All units on the AP physics 1 exam can be written in terms of only four units:
 - meters, seconds, kilograms, and Coulmobs

The way you make this work:::

for each quantity ====
write a new unit

you can figure this out by looking at the core formulas that define units:::

- distance, position, and displacement, have units of *meters*.
- time has units of *seconds*
- mass has units of *kilograms*

The core formulas:

$$v = \frac{d}{t}$$

$$a = \frac{v_f - v_i}{t}$$

$$F = ma$$

$$p = mv$$

$$W = Fd$$

$$P = \frac{W}{t}$$

These equations define the units for speed (velocity), acceleration, force, momentum, work, and power.

Whenever two are added or subtracted

A crucial exercise::: go through each of these quantities to write each of these units in terms of meters, kilograms, and seconds (MKS).

How do analyze an equation *dimensionally*:
1 – The three core units are meters, kilograms, and seconds. Replace each quantity with the units for that quantity
2 – any two quantities being added or subtracted must have the *same units*. You can replace them with a single quantity when analyzing dimensions. if they don't' have the same units, then something is *wrong* with that formula!
3 – Always write units with a *clear* numerator and denominator. Whenever two quantities are multiplied, combine the numerators and denominators. Whenever two quantities are divided, flip the divisor and then combine the numerator and denominator.

[I think that a good *first exercise* is to dimensionally analyze the 4 kinematic equations!]

Good exercises:

- Complete a dimensional analysis of the 4 kinematic equations. Using dimensional analysis, show that each equation is dimensionally correct.
- Find the correct definitions of a Newton, a Joule, and a Watt in terms of the three core units.
- Look at the equations for all quantities defined in terms of *Joules*: work, kinetic energy, gravitational potential energy, and elastic potential energy. For each of these equations, show that they have the *same* dimensions in terms of the three core units.
- Find the units of the universal gravitational constant G. Expand our analysis to include Coulombs (the fourth core unit), and find the dimensions of the electrostatic constant.
- Analyze their definitions of current and voltage to find the dimensions of the Amp and the Volt. Use Ohm's Law to find the definition of the Ohm. Find the units of resistivity. Prove that the electric power formula (P= IV) is dimensionally correct.
- And then, just go through all the other formulas. [rotational motion, geometric formulas, formulas for period, formulas for forces]

Basically, kids should be able to do all of these things.
So, I need to make quizzes that ask students to *dimensionally analyze* every equation on the formula sheet.
[should I combine some of these issues?]

Basically, I need to make a long series of quizzes (maybe there will be as many as 10 – 20) on symbolic manipulation and dimensional analysis…and arrange them
These are all written down quizzes, so hopefully I will get the QR codes to work to massively increase my efficiency.

Related to these should be yet another set of quizzes which is about explaining the notation on the formula sheet *and* memorizing the useful equations that are not on the formula sheet.

these formulas need to be like the back of your hand

Dimensionally analyze the following equation:

$$\Delta x = v_i \Delta t + \frac{1}{2} a t^2$$

Write the units (in terms of meters, kilograms, and seconds) of each quantity. [one of them is dimensionless]

Δx	$=$	v_i	Δt	$+$	$\frac{1}{2}$	a	t^2

Find the dimensions of each of the three terms in the equation:

Δx	$=$	$v_i \Delta t$	$+$	$\frac{1}{2} a t^2$

$$v_f^2 = v_i^2 + 2a\Delta x$$

Write the units (in terms of meters, kilograms, and seconds) of each quantity. [one of them is dimensionless]

v_f^2	$=$	v_i^2	$+$	2	a	Δx

Find the dimensions of each of the three terms in the equation:

v_f^2	$=$	v_i^2	$+$	$2a\Delta x$

$v_f = v_i + a\Delta t$					
v_f	$=$	v_i	$+$	a	Δt
v_f	$=$	v_i	$+$	$a\Delta t$	

$$\Delta x = \left(\frac{v_f + v_i}{2}\right)\Delta t$$

Δx	$=$	$\left(\frac{v_f + v_i}{2}\right)$	Δt		