Dimensional Analysis:
In this section, we are going to improve our understanding of electric circuits by determining the units of electric circuits.
We are going to ask ourselves
What do Volts, Amps, Ohms, and Watts actually mean mathematically?
When considering what they mean, do our formulas make sense?

To this end, we are going to create *mathematically rigorous* definitions of voltage, current, resistance, and power.

Part 1: Base Units vs. Derived Units

The SI Unit System:
All of the *units* in physics are part of a system called the International System of Units, or the *Systeme International*, or the **SI unit system**.
According to this system, all of the units in physics, for all of our quantities, are derived from six units: seconds, meters, kilograms, amperes, kelvin, moles, and candelas.

Base Units vs. Derived Units:

In the SI unit system, there are two types of units: base units and derived units.
The base units are the six listed above: seconds, meters, kilograms, amperes, kelvin, moles, and candelas.
The derived units are any units derived from the base units.
For example, meters and seconds are *base units* but the unit meters per second is a derived unit, derived by dividing 1 meter by 1 second.

Basic Units for Electric Circuit:
We are going to set up a new unit system for electric circuits with three base units Joules, Coulombs, and Seconds.

Note that of these three, only *seconds* are an SI base unit.
In the SI unit system, a Coulomb is defined as 1 Ampere times 1 Second,
and a Joule is defined as 1 kilogram times 1 meter squared divided by 1 second squared.

However, it is easiest to understand electric circuits if we take Joules, Coulombs, and Seconds as our base units.

Joules:
A Joule is a unit for *energy*.

Coulomb:
A Coulomb is a unit for *charge*. Remember that in an electric circuit charge comes from electrons. If you have 1 Coulomb of charge, you have approximately 6 billion billon electrons.

Derived Units for Electric Circuits:

There are four derived units for our electric circuit unit system:
Volts, Amperes, Ohms, and Watts.
These four units are derived from our three base units.

A Volt is defined as 1 Joule divided by 1 Coulomb.
An Ampere is defined as 1 Coulomb dived by 1 second.
A Watt is defined as 1 Joule per 1 second.
An Ohm is defined as 1 Joule times 1 Second per 1 Coulomb squared.

Part 2: Mathematically Rigorous Definitions of Circuit Quantities:

Power:
The unit for power is Watts.
A Watt is equal to one Joule per second.
This means that some electric device, like a resistor or a light bulb, with a power of one Watt, emits 1 Joule of energy every second.
A battery with a power of 1 Watt inputs one Joule into the circuit every second.

Potential Difference:
The unit of potential difference is volts. A volt is equal to a Joule per Coulomb.
Potential difference is never defined for a single point but is always defined for *two points* within a circuit.
The potential difference across two points represents the amount of energy that a single unit of charge emits when it crosses between those points.

For example:
Look at the circuit above:
The potential difference between point A and point B is equal to 4 volts.
This means that when one Coulomb of charge crosses from point A to point B, it emits 4 Joules of electric
Or, to put it differently, when 6 billion billion electrons cross from point A to point B, they collectively emit 4 Joules of energy.

Current:
The current for a circuit is always defined for a single point in a circuit, and is defined with one direction as positive.
The unit for current is Amperes, and a Ampere is equal to 1 Coulomb per second.

This means that, if the current at point A is equal to 3 Amperes, then every second, 3 Coulombs of electric charge cross that point.
Or, every second, 9 * 6 = 54 billion billion electrons cross that point.

Current is always defined with an arrow, indicating the direction of positive current. Because current represents the motion of positive charge, if the current of a circuit is positive 4 Amps, it means that 54 billion billion electrons cross that point in the direction opposite the way that the arrow points.

[if you have learned about electron drift, note that this is the net movement of electrons. Electrons move both ways, but 54 billion cross one way.]

Resistance:
Resistance is defined by Ohms Law, and the unit of resistance is Ohms.

Resistance is defined as follows:
For some circuit elements, the current through the element and the potential difference across that element are directly proportional. These elements are called *Ohmic* elements, and the constant of proportionality between voltage and current for an Ohmic element is called the **resistance** of that element.

Thus, resistance is actually defined by the formula Ohm's Law:

$$\Delta V = IR$$

Part 3: Using Dimensional Analysis to Confirm Formulas

1. Demonstrate that the power formula (P = IV) is dimensionally correct.

2. The formula Ohm's Law (V = IR) is actually the *definition* of resistance.
Plug in the units for voltage and current in order to determine the unit for resistance in terms of Joules, Coulombs, and seconds.

3. Derive a formula for power in terms of current and resistance. Prove this formula is dimensionally correct.

4. Derive a formula for power in terms of potential difference and resistance. Prove this formula is dimensionally correct.

5. Derive a formula for current in terms of power and resistance. Prove this formula is dimensionally correct.

6. Imagine a circuit with a battery with a potential difference of ΔV and three resistors of resistance R_a, R_b, and R_c connected in series.
Derive a formula for the current of the circuit in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.

7. Derive a formula for the potential difference across resistor B in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.

8. Derive a formula for the power emitted by resistor B in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.

9. Derive a formula for the power inputted by the battery in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.

10. Imagine a circuit with a battery with a potential difference of ΔV and three resistors of resistance R_a, R_b, and R_c connected in parallel.
Derive a formula for the current of the circuit in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.

11. Derive a formula for the potential difference across resistor B in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.

12. Derive a formula for the power emitted by resistor B in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.

13. Derive a formula for the power inputted by the battery in terms of ΔV, R_a, R_b, and R_c. Prove your formula is dimensionally correct.