K: Solve Circuits with Symbolic Algebra
Level 6
Prerequisite: Solve Complete Circuits with Nontraditional Information
Points To:

Objectives:

- Using Ohm's Law and the Power Formula, create formulas for voltage, current, resistance, and power in terms of combinations of the other variables.

- Given a series or parallel circuit with known resistance and a battery of known voltage, solve for the voltage, current, resistance, and power of each element and the total circuit.

- Conduct dimensional analyses of electric circuit problems to prove that algebraic formulas you derive are dimensionally correct.

Part B: Solving Circuit Equations Algebraically

Theoretical physicist are scientists and mathematicians who construct theories that explain how the universe functions. They rarely work with actual numbers, because to explain the universe you must use the most general methods. Thus, it is important to learn to how to solve physics problems algebraically, with *symbols* for numbers.

"In terms of"
This phrase is used very frequently when describing algebraic physics problems.

To solve for "A in terms of B and C" means you derive an equation
A = _______
in which the right side of the equation includes only B, C, numbers, operations (like plus, minus, square root, or log), and known constants (like the speed of light, the charge of an electron, the mass of a proton, etc.).

To solve for "J in terms of K, L ,and M" means you derive an equation
J = _______
in which the right side of the equation includes of K, L, M, numbers, operations, and known constants.

<u>Solving Ohm's Law Algebraically</u>

V = IR

B.1 Solve for current in terms of voltage and resistance.

B.2 Solve for resistance in terms of current and voltage.

<u>Solving Ohm's Law and the power formula algebraically.</u>

V = IR P = IV

Solve for each variable in terms of the known quantities. Two of the answers will be the equations themselves!

B.3 Variable: voltage Known quantities: current, power

B.4 Variable: voltage Known quantities: current, resistance

B.5 Variable: voltage Known quantities: resistance, power

B.6 Variable: power Known quantities: current, voltage

B.7 Variable: power Known quantities: current, resistance

B.8 Variable: power Known quantities: voltage, resistance

B.9 Variable: current Known quantities: voltage, resistance

B.10 Variable: current Known quantities: power, resistance

B.11 Variable: current Known quantities: voltage, power

B.12 Variable: Resistance Known quantities: voltage, power

B.13 Variable: Resistance Known quantities: voltage, current

B.14 Variable: Resistance Known quantities: current, power

Part C: Dimensional Analysis Introduction

A crucial part of any algebraic analysis is a *dimensional analysis*. In a dimensional analysis, you need to prove that the new formula you created has quantities of the correct dimension.
The Base units (when studying electrical circuits) are:
JOULES* –(for energy)
SECONDS – (for time)
COULOMBS – (for charge)

Unit	Abbreviation	Quantity
Joule	J	Energy
Second	s	Time
Coulomb	C	Charge

C.1. What is the abbreviation for Coulomb?

C.2. What is the abbreviation for Joule?

*Note that a Joule is not actually a base unit and is constructed of three other units (1 Joule = kg m^2 / s^2), However, for the purposes of electric circuit analysis, you do not need to break a Joule into separate units.

Simplify each of the following combinations of units:

C.3. $$\frac{C^2}{s^3}\cdot\frac{s}{C}$$	**C.4.** $$\frac{C}{s^5}\cdot\frac{s}{J}\cdot\frac{s^2}{J^5}$$
C.5. $$\frac{s^3}{J}\cdot\frac{J^2}{s^6}\cdot\frac{C^3}{J^4}$$	**C.6.** $$\frac{(Cs)^2}{J}\cdot\frac{C}{s}\cdot\frac{C^4}{(Js)^3}$$

C.7. $$\frac{(\mathrm{C/s})}{\mathrm{J}^2}\cdot\frac{\mathrm{s}^3}{(\mathrm{JC})^2}\cdot\frac{(\mathrm{J/C})}{\mathrm{J}}$$	**C.8.** $$\frac{(\mathrm{J/s})^3}{\mathrm{s}}\cdot\frac{\mathrm{C}^4}{(\mathrm{C/J})^2}\cdot\frac{(\mathrm{Cs})^4}{\mathrm{J}^3}$$

Base Units:

Quantity	Mathematical Definition	SI Unit	SI Unit in base units
Voltage	$V = \frac{\Delta E}{\Delta q} = \frac{\text{energy}}{\text{charge}}$	Volts	$\mathrm{V} = \frac{\text{Joule}}{\text{Coulomb}}$
Current	$I = \frac{\Delta q}{\Delta t} = \frac{\text{charge}}{\text{time}}$	Amps	$\mathrm{A} = \frac{\text{Coulomb}}{\text{second}}$
Resistance	$R = \frac{V}{I} = \frac{\text{voltage}}{\text{current}}$	Resistance	$\mathrm{R}=\frac{\text{Joule}\cdot\text{second}}{(\text{Coulomb})^2}$
Power	$P = \frac{\Delta E}{\Delta t} = \frac{\text{energy}}{\text{time}}$	Watts	$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}}$

Each of the following nonsense quantities are created by multiplying the four electrical quantities. For each quantity, determine what unit it would have.

Steps:

1. replace each quantity with the units of that quantity.
2. simplify the units as in problems C.1 – C.6.

C.9. $$\text{coolness} = V^2 I^3$$	**C.10.** $$\text{awesomeness} = \frac{V^3}{P^4}$$
C.11. $$\text{tublarity} = \frac{(VR)^2}{I^3}$$	**C.12.** $$\text{gnarliness} = \frac{P^2}{(VI)^3}$$
C.13. $$\text{grooviness} = \frac{(P/R)^3}{(VR)^2}$$	**C.14.** $$\text{swellness} = \frac{(R/I)^3}{(V/R)^4}$$

Part D: Using Dimensional Analysis to Verify Equations

Coming Soon!

Part E: Solving Full Circuits Algebraically

When solving a full circuit algebraically, you can follow the same set of rules that would be followed for a typical circuit.

E.1 You have a **series** circuit with a battery and two **identical** resistors. The potential difference across the battery is ΔV and the resistance of each resistor is R

a) Draw the circuit:

b) Determine all relevant quantities for this circuit algebraically by filling in the following table.

	Resistor A	Resistor B	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

c) Use dimensional analysis to confirm the quantities in boxes have appropriate units. You can confirm all 12 boxes if you like, but focus on the three boxes below:

Total Current	Potential Difference of Resistor A	Power of Resistor B

d) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

E.2 You have a **parallel** circuit with a battery and two **identical** resistors. The potential difference across the battery is ΔV and the resistance of each resistor is R. Determine all relevant quantities for this circuit algebraically by filling in the following table.

a) Draw the circuit:

b) Determine all relevant quantities for this circuit algebraically by filling in the following table.

	Resistor A	Resistor B	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

c) Use dimensional analysis to confirm the quantities in boxes have appropriate units. You can confirm all 12 boxes if you like, but focus on the three boxes below:

Current through resistor A	Power of Resistor B	Total Resistance

d) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

E.3

You have a **series** circuit with a battery and two resistors. **The two resistors are not identical**, they now have resistances of R_1 and R_2, where $R_1 \neq R_2$. The potential difference across the battery is ΔV.

a) Draw the circuit:

b) Determine all relevant quantities for this circuit algebraically by filling in the following table.

	Resistor 1	Resistor 2	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

c) Use dimensional analysis to confirm the quantities in boxes have appropriate units. You can confirm all 12 boxes if you like, but focus on the three boxes below:

Total Current	Potential Difference across Resistor 2	Power of Resistor 1

d) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

E.4

You have a **parallel** circuit with a battery and two resistors. **The two resistors are not identical**, they now have resistances of R_1 and R_2, where $R_1 \neq R_2$. The potential difference across the battery is ΔV.

a) Draw the circuit:

b) Determine all relevant quantities for this circuit algebraically by filling in the following table.

	Resistor 1	Resistor 2	Total (Battery)
Potential Difference			
Current			
Resistance			
Power			

c) Use dimensional analysis to confirm the quantities in boxes have appropriate units. You can confirm all 12 boxes if you like, but focus on the three boxes below:

Total Current	Potential Difference across Resistor 2	Power of Resistor 1

d) Confirm that your answer follows the *conservation of energy*. That is, the power of each resistor adds to the total power:

$$P_A + P_B = P_{tot} = \Delta V \cdot I_{tot}$$

e) Problem **E.4** is actually the *derivation* of the formula for equivalent resistance of two resistors in parallel, which is always included in physics textbooks and used to solve combined circuits with two resistors in parallel:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Show that your solution for *total resistance* matches this form. [Also, remember where this formula came from….because it's very important to know that formulas like this one come from someplace!]

Part F: Algebraically Solving Circuits with Different Information

Coming Soon!

Answers

B.1 $I = \frac{V}{R}$	B.4 $V = IR$	B.7. $P = I^2R$	B.10. $I = \sqrt{\frac{P}{R}}$	B.13. $R = \frac{V}{I}$
B.2 $R = \frac{V}{I}$	B.5 $V = \sqrt{PR}$	B.8. $P = \frac{V^2}{R}$	B.11. $I = \frac{P}{V}$	B.14. $R = \frac{P}{I^2}$
B.3 $V = \frac{P}{I}$	B.6. $P = IV$	B.9. $I = \frac{V}{R}$	B.12. $P = \frac{V^2}{R}$	

C.1 C	C.4 $\frac{\mathrm{C}}{\mathrm{s^2J^6}}$	C.7 $\frac{\mathrm{s^2}}{\mathrm{J^4C^3}}$	C.10 $\frac{\mathrm{s^4}}{\mathrm{C^4J}}$	C.13 $\frac{\mathrm{C^{12}}}{\mathrm{s^8J^4}}$
C.2 s	C.5 $\frac{\mathrm{C^3}}{\mathrm{J^3s^3}}$	C.8 $\mathrm{C^6J^2}$	C.11 $\frac{\mathrm{J^4s^5}}{\mathrm{C^6}}$	C.14 $\frac{\mathrm{C^{18}}}{\mathrm{J^{14}s^4}}$
C.3 $\frac{\mathrm{C}}{\mathrm{s^2}}$	C.6 $\frac{\mathrm{C^7}}{\mathrm{J^3s^2}}$	C.9 $\frac{\mathrm{J^2C}}{\mathrm{s^3}}$	C.12 $\frac{\mathrm{s^2}}{\mathrm{J^2}} = \frac{1}{\mathrm{W^2}}$	