Kinematic Equation Derivations

Objective:

From basic mathematical principles, derive each of the kinematic equations under the condition that *acceleration is constant*.

Name	**Equation**
Definition of Acceleration	$v_f = v_i + a \cdot \Delta t$
The King of Kinematic Equations	$\Delta x = v_i \cdot \Delta t + \frac{1}{2} a (\Delta t)^2$
The Average Velocity Formula	$\Delta x = \left(\frac{v_i + v_f}{2}\right) \Delta t$
No-Time Equation	$v_f^2 = v_i^2 + 2a \cdot \Delta x$

1. Derive the definition of acceleration.

Definition of Acceleration	$v_f = v_i + a \cdot \Delta t$

Acceleration in physics is defined as change in velocity divided by change in time:

$$a = \frac{\Delta v}{\Delta t}$$

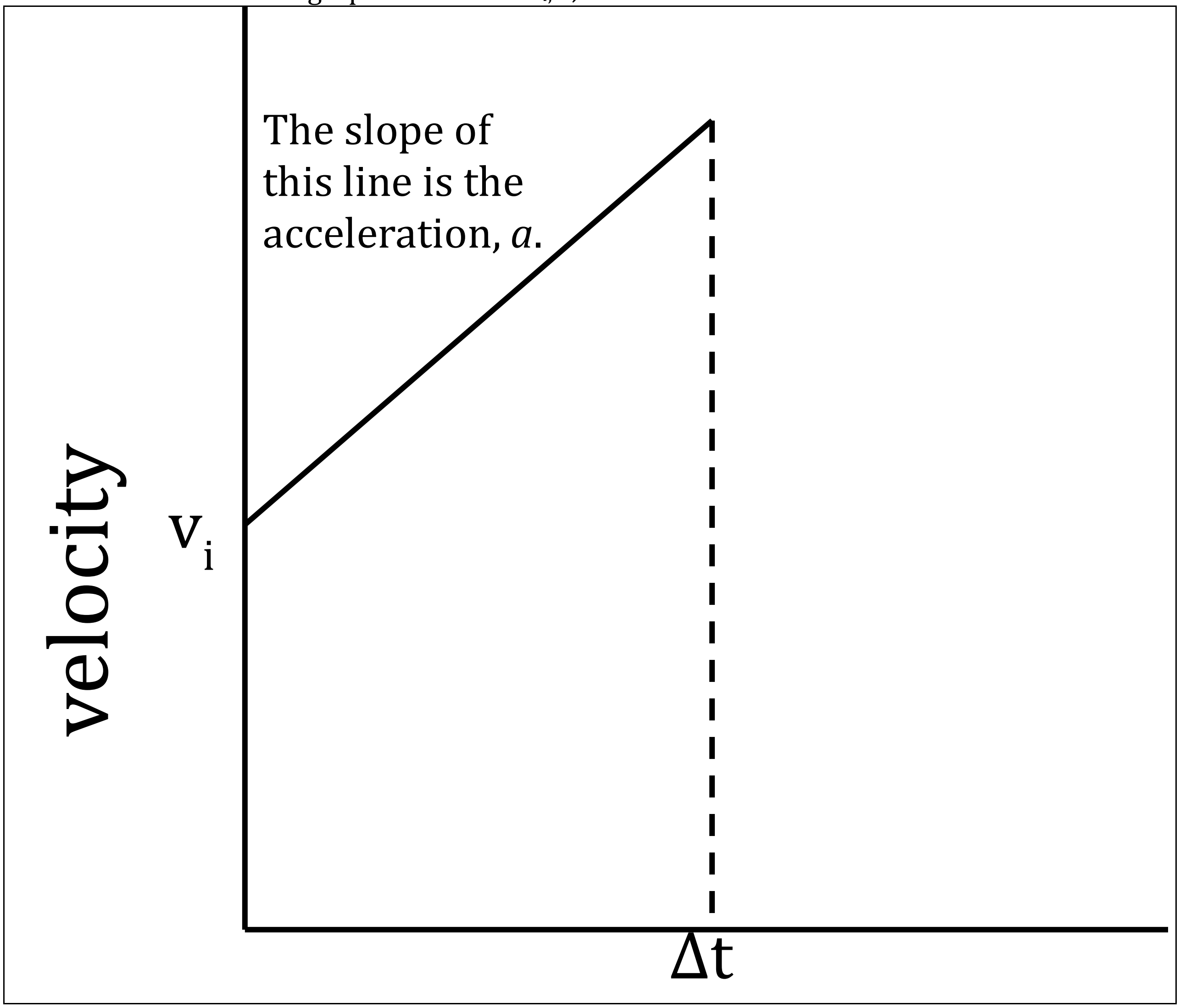
From that formula, derive the kinematic equation for the definition of acceleration.

2. The king of kinematic equations [not its real name!]:

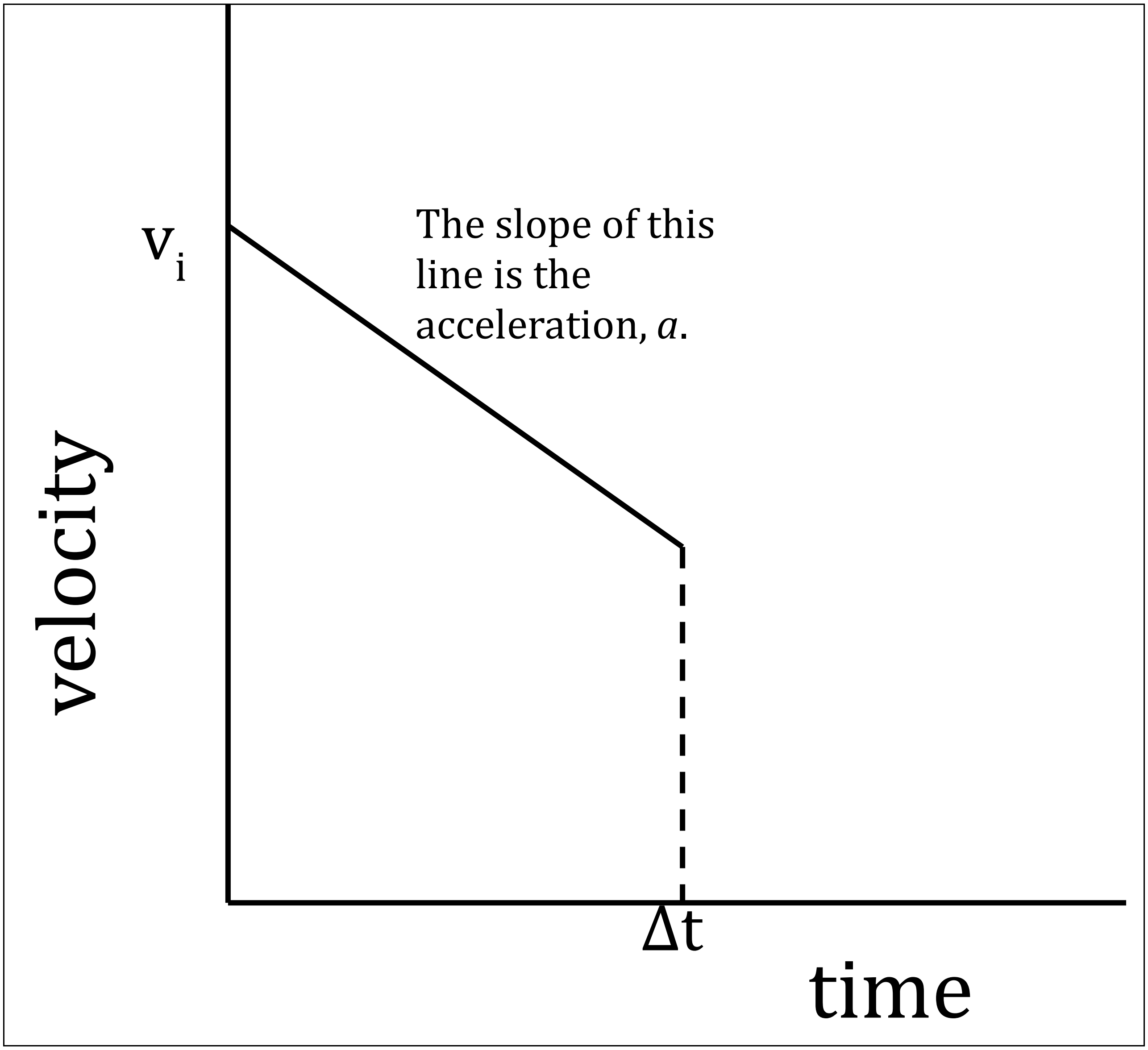
The King of Kinematic Equations	$\Delta x = v_i \cdot \Delta t + \frac{1}{2} a (\Delta t)^2$

The fundamental theorem of calculus tells us that *displacement* (change in position) is equal to the *area under the curve* of a *velocity-time graph*.

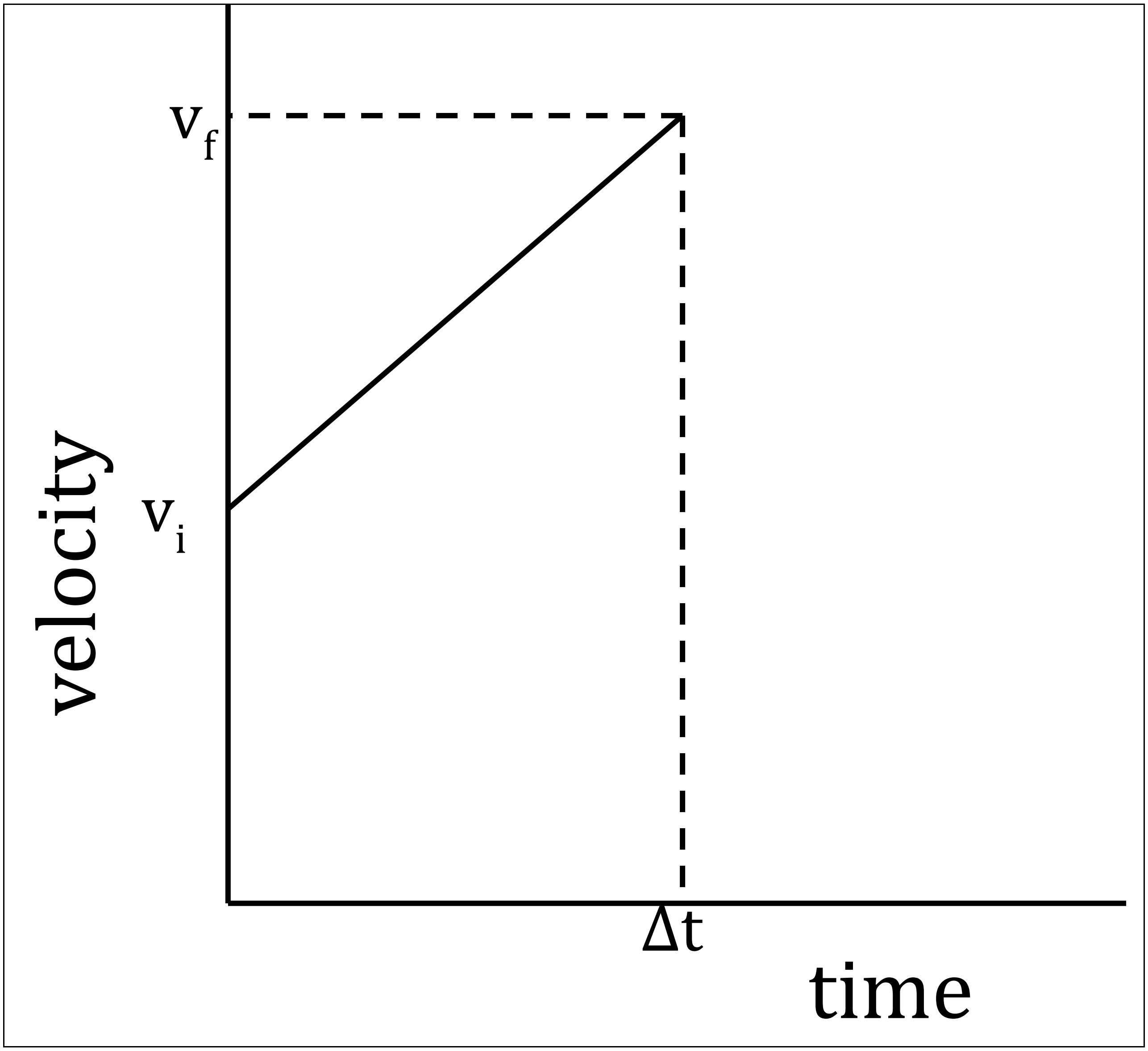
Consider the following *velocity-time graph*, which depicts an object that begins with a velocity of v_i and accelerates at a rate a for a time of Δt. Derive an equation for the area under the curve of this graph in terms of v_i, a, and Δt.



To be come complete, reconsider the same situation in which, in this case, the object is slowing down. You should derive the *same equation*!



3. The average velocity formula.
Consider the following position-time graph, which depicts an object that accelerates from initial velocity v_i to a final velocity v_f in a time Δt. Derive an equation for the *area under the curve* of this graph, which should equal the *displacement* of the object.



Other ways to consider the *average velocity equation*

- The average velocity equation is identical to the equation for the *area of a trapezoid*.

- The average velocity equation treats velocity, quite literally, as an average. Two velocities are added and divided by two to determine the average.

This is another equation for average velocity that applies whenever acceleration is constant:

$$\text{average velocity} = \frac{v_i + v_f}{2}$$

E.1 What is the average of 6 and 12?

E.2 What is the average of 3, 8, 10, and 11?

E.3 Explain how to calculate the average in mathematics.

E.4 Now, explain the average velocity formula written above. How does make sense?

4. The no-time equation

The no-time equation can be derived by combining the *definition of acceleration* into the *average velocity* formula in a way that eliminates *time* from the equation. Do this!

There is another, very similar method you can use to find the *no-time equation*. Figure it out, and do it!

5. Reducing to a simpler case.

The four kinematic equations are used when there is *constant acceleration*.
If there is a constant velocity, then there is only one equation that determines the motion of an object:

$$v = \frac{\Delta t}{\Delta x}$$

[This equation is titled the definition of velocity.]
Prove that, if acceleration is zero, all of the kinematic equations reduce to either this equation or *redundant equations*.
Note that, if acceleration is zero, then $v_i = v_f = v$.