Kinematics and Calculus:

Objectives

- To show and demonstrate how kinematic graphs and kinematic equations reflect a rule called the *fundamental theorem of calculus*.

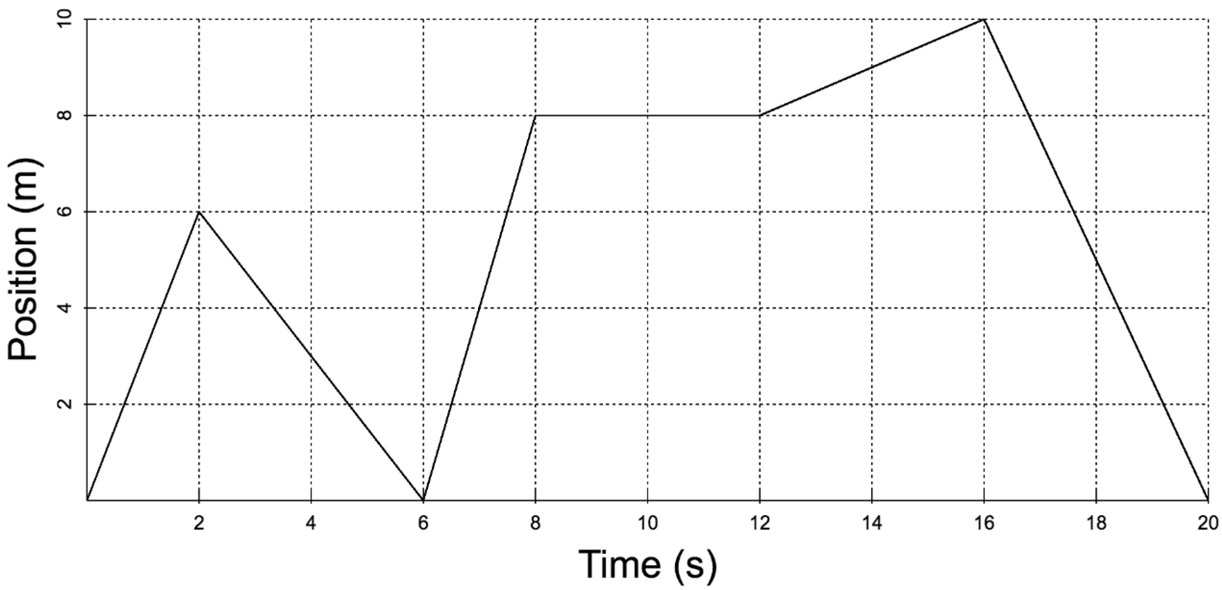
In math class, you would rigorously define derivatives and integrals in order to describe this

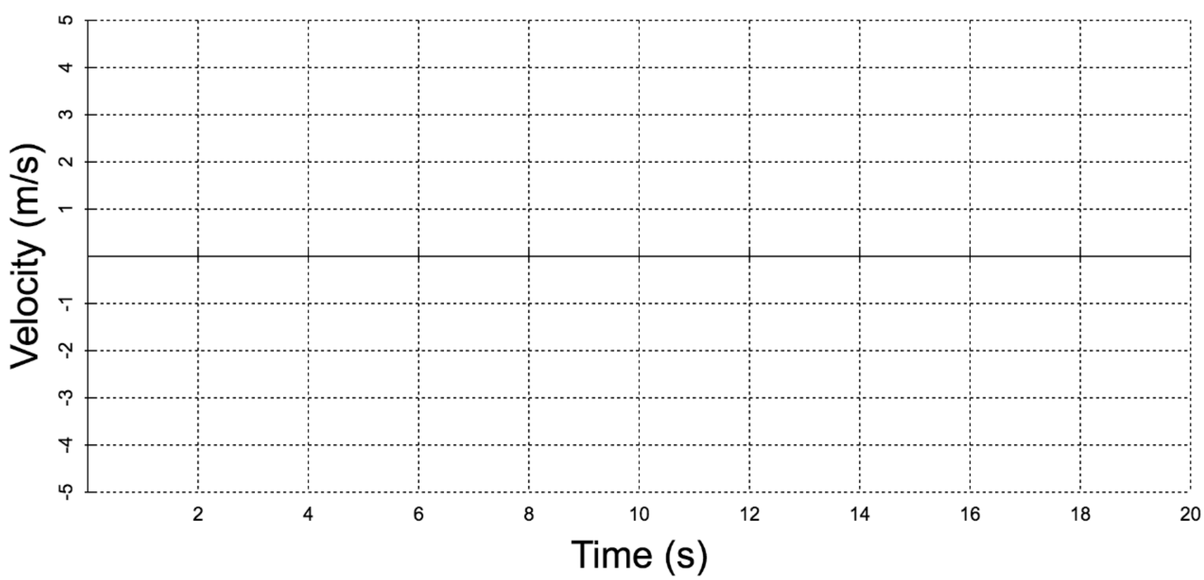
Some sections of this packet require some basic knowledge of calculus. The whole thing does not.

Part 1: Examining the Derivative
Copy this sentence three times:
"The slope of a position-time graph is the *velocity* at that time."

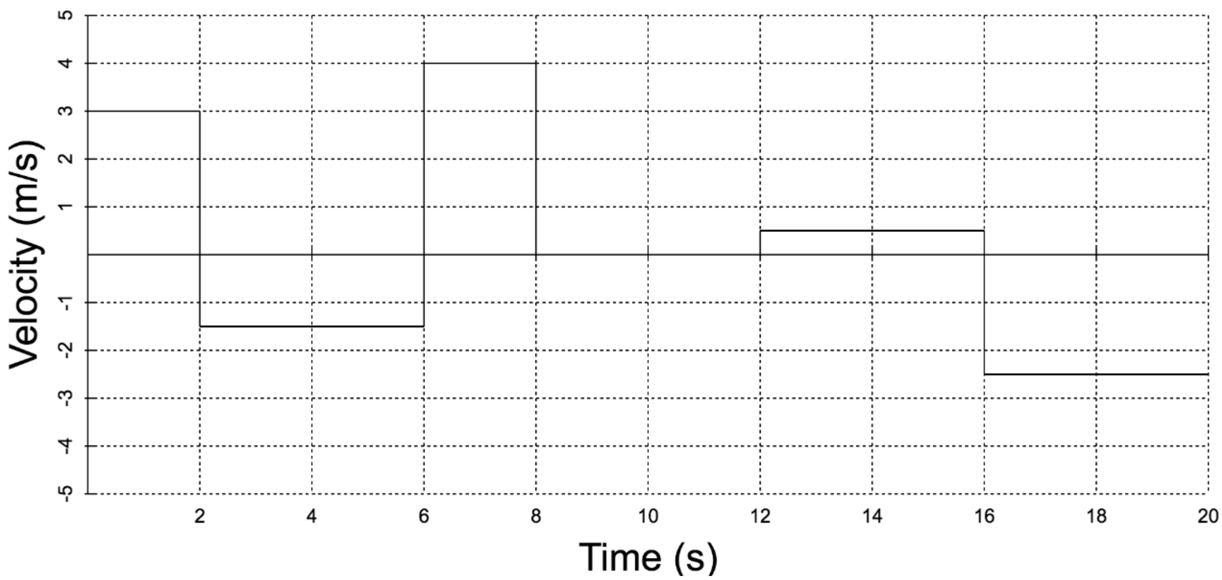
In mathematics, this relationship is stated:
"The velocity function is the *derivative* of the position function."

Look at the position-time graph. Create a corresponding velocity-time graph.





The answer:

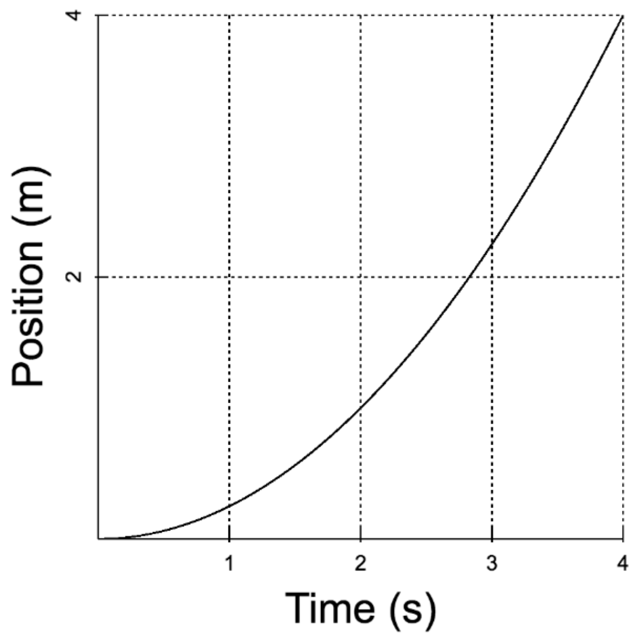


Conceptual questions:
Explain, in a few sentences, how you solved this problem:

What principle did you use to solve this problem?
[Hint: If you followed directions, you already copied it three times!]

Write a story to show, in real life, what this person is *doing*?
[hint: watch the video called ‘walk the graph’ on Flipping Physics]

Here is another position-time graph, in the shape of a *parabola*:



In real life, what would this person be *doing*?

Oh no! I want to make a graph of velocity-time, like we did before, but I can't do it so easily because the slope is changing!
Whatever will I do?

Tangent Line:

A *tangent line* is a line that just barely grazes a function. Geometrically, it touches the function at exactly one point.

For each point on a line, there is only one *tangent line*.

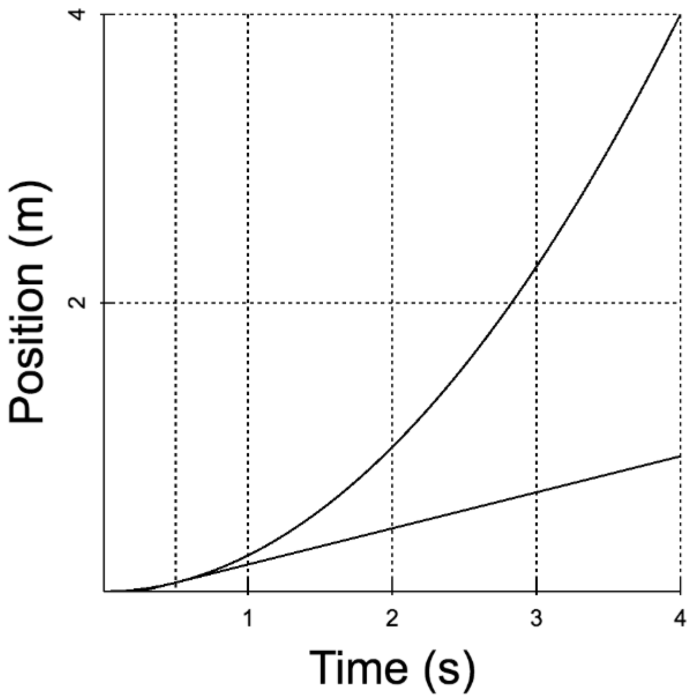
Instantaneous Velocity

The *instantaneous velocity* is the velocity at one particular moment in time.

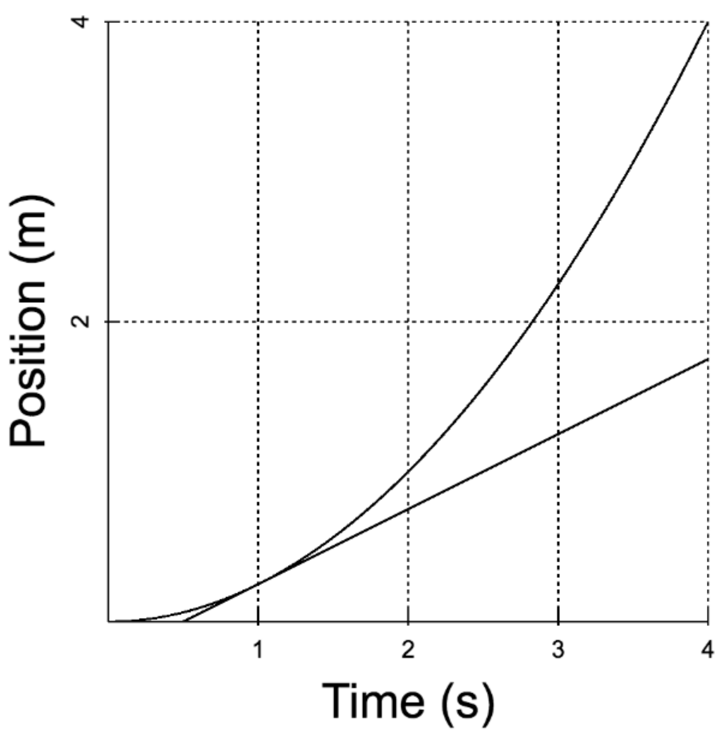
To find the *instantaneous velocity* from a position-time graph, take the *slope* of a *tangent line* to that position-time graph at one time.

For each of the *tangent lines* below, you can use a *ruler* to find the slope of the tangent line. Just measure and take RISE OVER RUN:

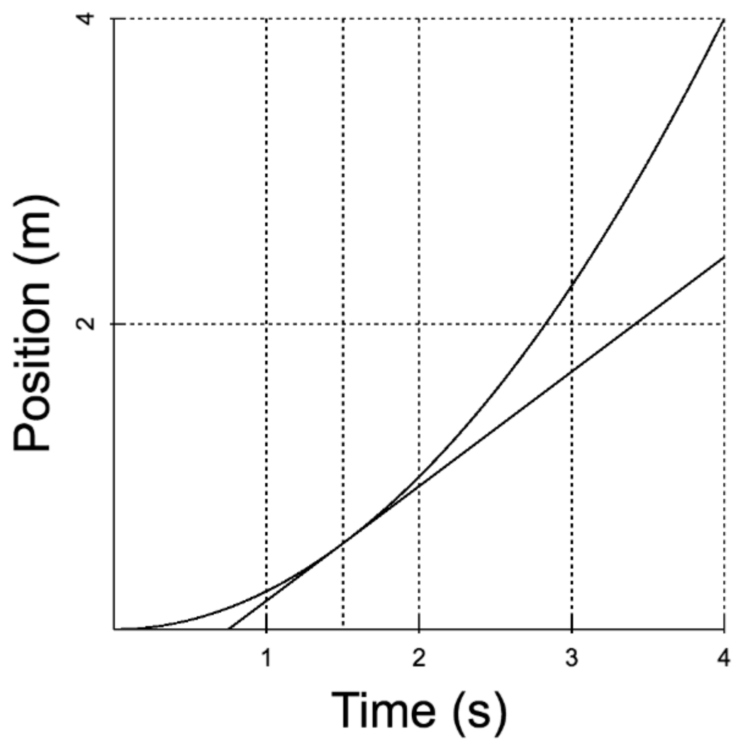
Tangent line is *drawn at* 0.5 seconds:



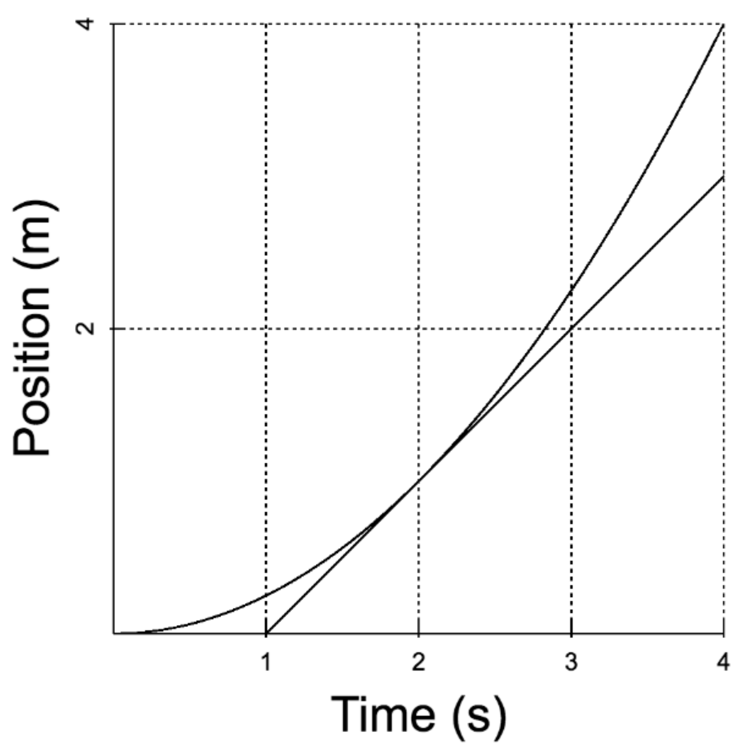
Tangent Line is drawn at 1.0 seconds:



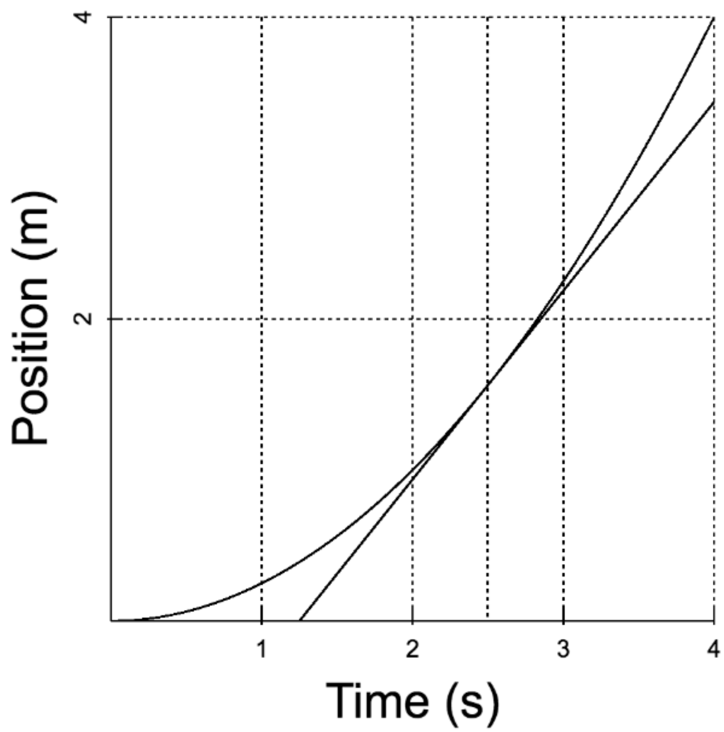
Tangent Line is drawn at 1.5 seconds:



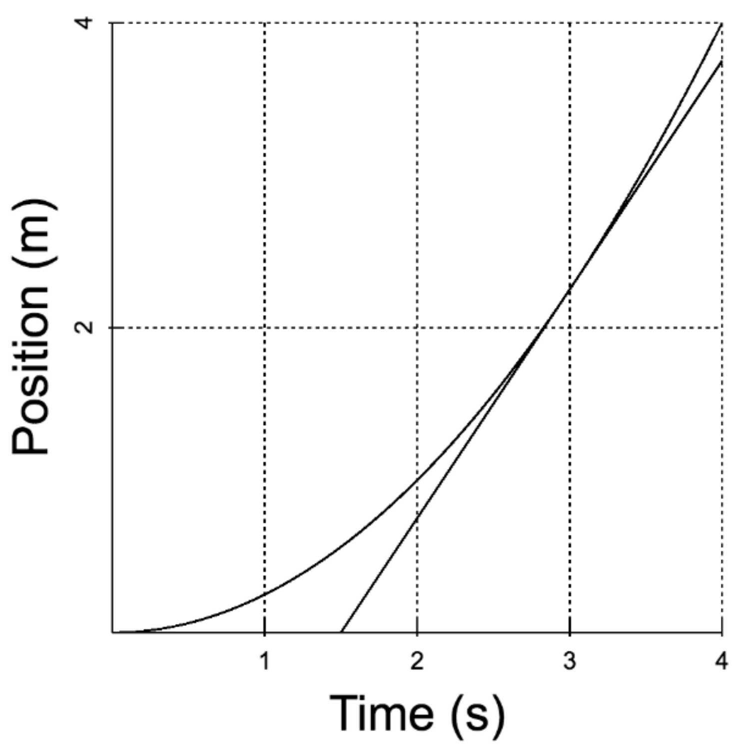
Tangent line is drawn at 2.0 seconds:



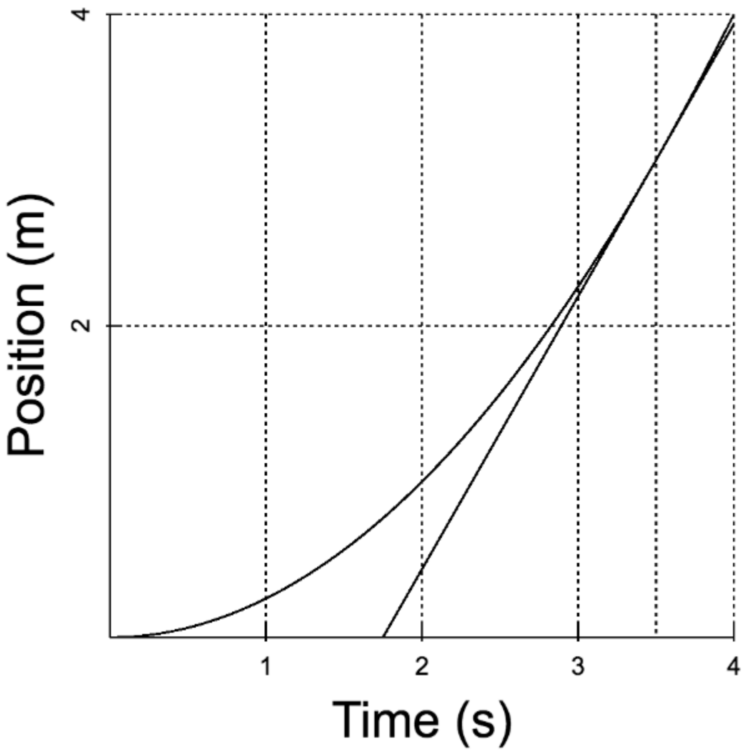
Tangent Line is drawn at 2.5 seconds:



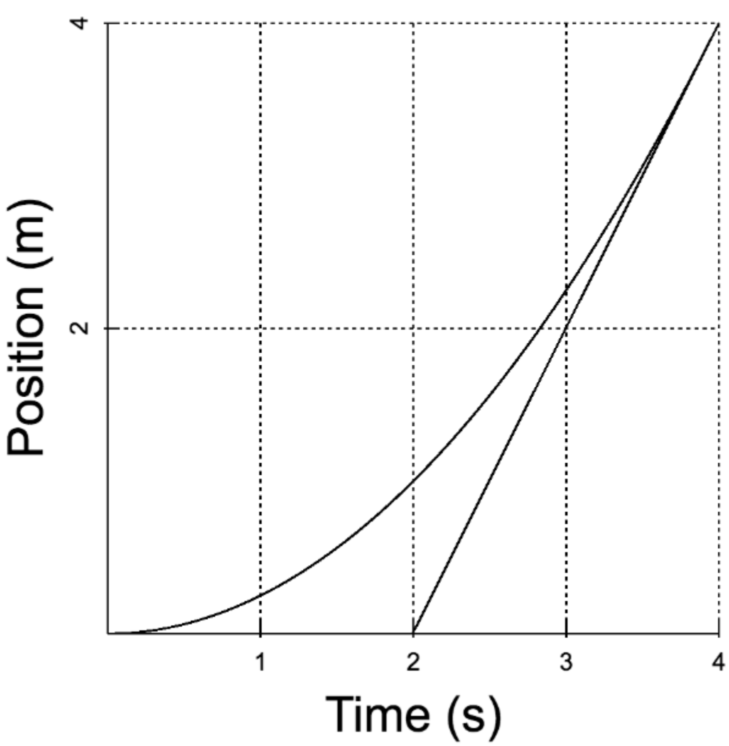
Tangent Line is drawn at 3.0 seconds:



Tangent line is drawn at 3.5 seconds:



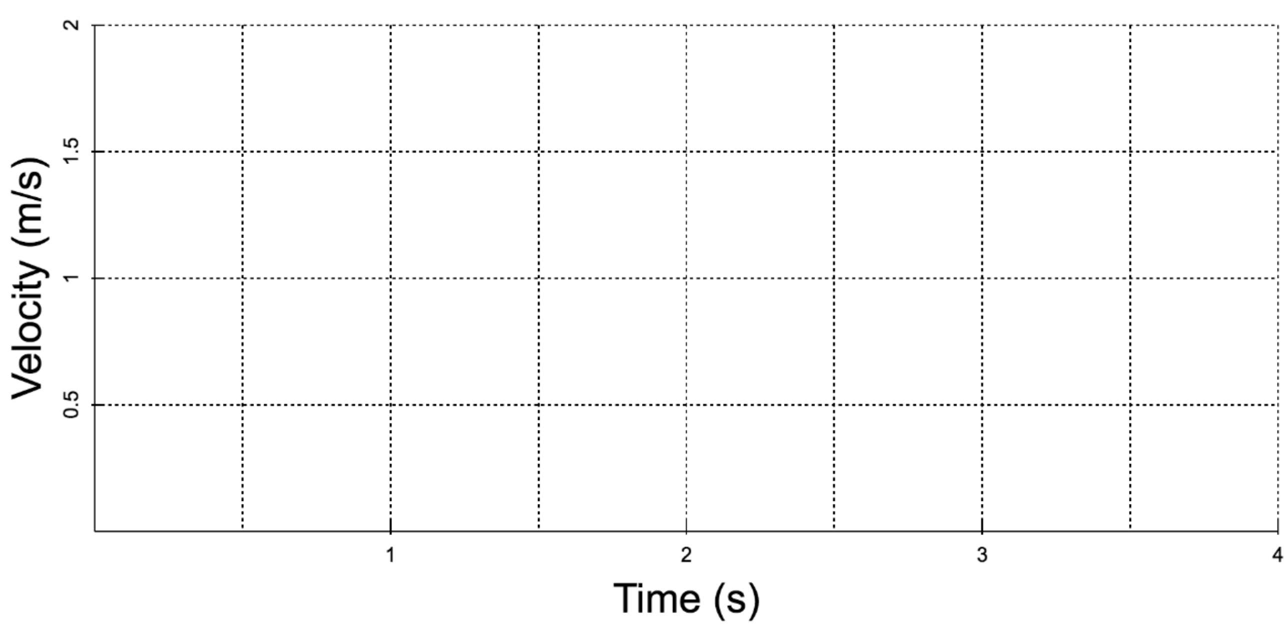
Tangent line is drawn 4.0 seconds:



The *slope of a tangent line* gives the *instantaneous velocity* at that time. Use all of the information to fill out a table below:

Time	Velocity
0	
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	

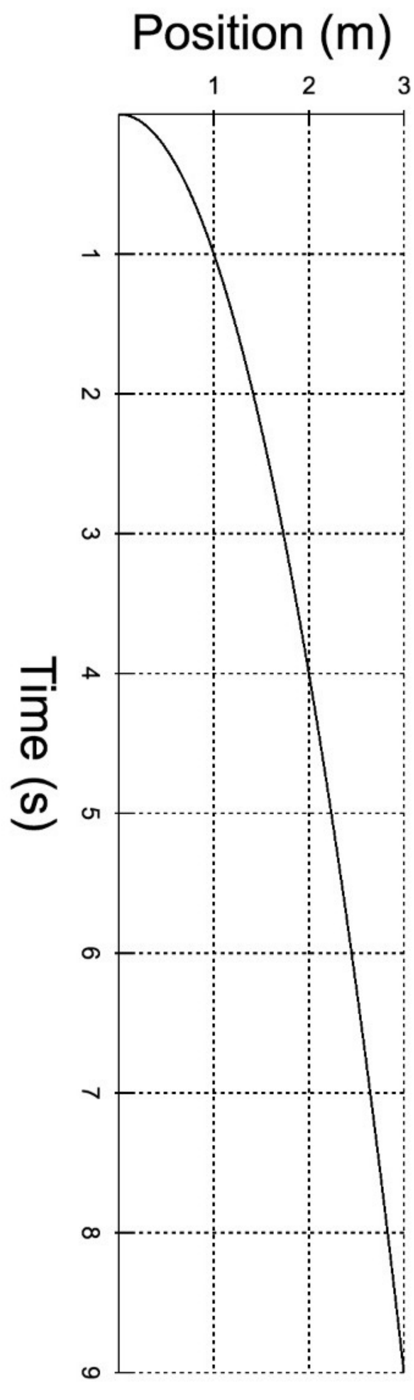
Now, use the information in the table to create a *velocity-time graph*:



The position-time graph represented a person moving with a *constant acceleration*.
Does the velocity time graph represent the same thing?
(If yes, you did it right!!!!)

Notice how, on the side of the graph, the x-scale and y-scale are the same.
Would you be allowed to use this ruler method if it weren't?
a) Yes, it would be exactly the same.
b) yes, but you would need to do more difficult math
c) no, you couldn't use it at all.

Here is another position-time graph, try drawing some tangent lines and figuring out what a velocity-time graph might look like.



Bonus: What function is this? (hint: look at x =1, x =4, and x = 9)

Part 2: Proving, nongraphically, that the kinematic equations follow the fundamental theory of calculus.
The kinematic equations describe motion under the condition that the acceleration is constant.

Redefine the king of kinematic equations as a function for position at a time [x(t)] in which initial position, initial velocity, and acceleration are constants. Label initial position by x_0 and initial velocity by v_0.

Redefine the definition of acceleration as a function for velocity at a time[v(t)] in which initial velocity and acceleration are constant.

If acceleration is constant, acceleration can be described by the simple equation
a(t) = a.

You now have three functions for x(t), v(t), and a(t), all under the condition that acceleration is constant.
Our goal is to demonstrate that these three functions satisfy the fundamental theorem of calculus. This means we need to show:

$$x'(t) = v(t)$$

$$v'(t) = a(t)$$

$$x(t) - x(t=0) = \int_0^t v(t')dt'$$

$$v(t) - v(t=0) = \int_0^t a(t')dt'$$

Note that, this by extension also means

$$x''(t) = a(t)$$

$$x(t) = \int_0^t \int_0^{t'} a(t'')dt''dt'$$

Prove the following are true:
If you take the derivative of your x(t) equation, you should receive the v(t) equation.

If you take the integral of the v(t) equation, you should receive the x(t) equation minus x(t=0). [Take a definite integral from 0 to t. Note that x(t = 0) is initial position.]

If you take the derivative of the v(t) equation, you should receive the a(t) equation.

If you take the integral of the a(t) equation, you should receive the v(t) equation minus v(t=0). Take a definite integral from (0) to (t). Note that v(t = 0) is initial velocity.

If you can do all that….you understand the basic idea of calculus and kinematics! Great job!