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Confidence intervals give a more precise statement.

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95% is called the **confidence level**.

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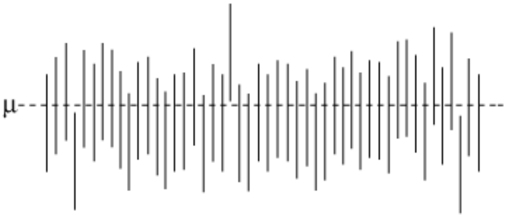

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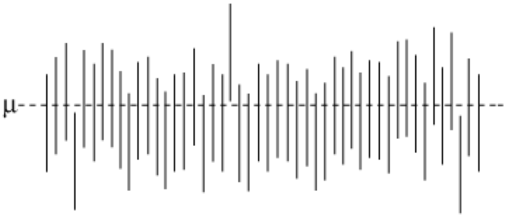

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Keep in mind that the interval varies from sample to sample, while the population percentage is a fixed number.

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SE is the standard error of the estimate. If the estimate is an average (e.g. a percentage), then we know that

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   So a 95% confidence interval for p is

$$58\% \;\pm\; 2\frac{0.49}{\sqrt{1000}}, \qquad \text{which is } [54.9\%, 61.1\%]$$

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   The measurement error follows a probability histogram that is unknown to us. We estimate the standard deviation σ of this probability histogram by the standard deviation s of the sample of 30 measurements.

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- There is an easy to remember formula for a 95% confidence interval for a percentage:

  $$\text{estimated percentage} \pm \frac{1}{\sqrt{n}}$$

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- It is journalistic convention to use a 95% confidence level unless stated otherwise.

Mini quiz

1. A random sample of 500 sales prices of recently purchased homes in a county is taken. From that sample a 90% confidence interval for the average sales price of all homes in the county is computed to be \$ 215,000 +/- \$ 35,000. True or false:
   a. About 90% of all home sales in the county have a sales price in the range \$ 215,000 +/- \$ 35,000.
   b. There is a 90% chance that the average sales price of all homes in the county is in the range \$ 215,000 +/- \$ 35,000.
2. Based on a sample of 500 salaries in a large city we want to find a confidence interval for the average salary in that city.
   a. Is it possible to do this using the formula 'average +/- z SE'? (Keep in mind that the histogram of salaries is not normal but quite skewed.)
   b. The margin of error for this confidence interval turns out to be \$ 5,400. How many salaries do we need to sample in order to shrink the margin of error to about \$ 2,000?

3. You are interested what the current starting salary for jobs in data science is. You solicit feedback on an online forum about data science and you get 230 replies with salary numbers. Can you use the formula ‘average +/- z SE’ to find a confidence interval for the average starting salary?