The normal curve

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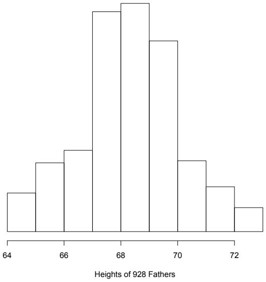

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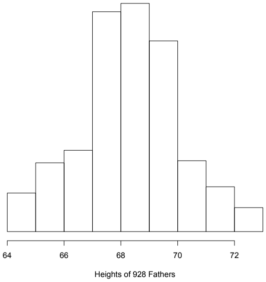

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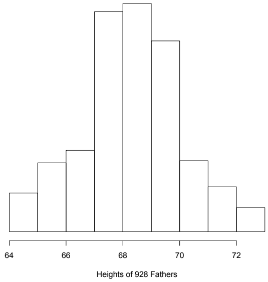

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But remember that some data have histograms that look quite different, e.g. incomes, house prices.

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Galton's measurements of heights of fathers have $\bar{x} = 68.3$ in and $s = 1.8$ in. Therefore about 95% of all heights are between 68.3 in -2×1.8 in $= 64.7$ in and 68.3 in $+2 \times 1.8$ in $= 71.9$ in.

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Recall that in a histogram, percentages are given by areas:

Standardizing data

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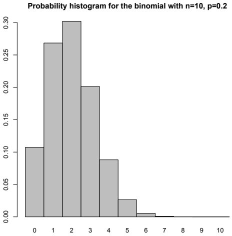

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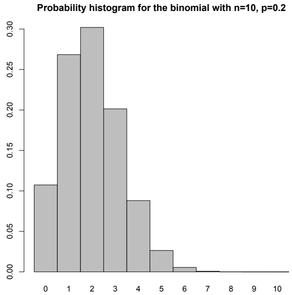

A histogram of data gives percentages for observed data. In contrast, a probability histogram is a theoretical construct: it visualizes probabilities rather than data that have been empirically observed.

Parameter and statistic

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- The formula for the standard error **does not depend on the size of the population**, only on the size of the sample.

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- The percentage of likely voters who approve is the percentage of 1s among the labels, which is the average of the labels.

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All of the above formulas are for sampling with replacement. They are still approximately true when sampling without replacement if the sample size is much smaller than the size of the population.

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The sampling distribution of S_n provides more detailed information about the chance properties of S_n than the summary numbers given by the expected value and the standard error.

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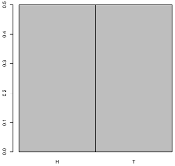

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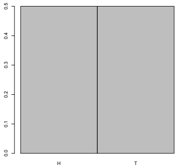
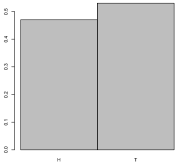


2. The histogram of the 100 observed tosses. This is an empirical histogram of real data:



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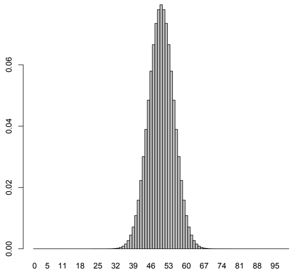

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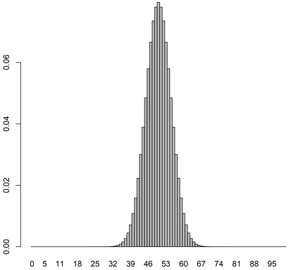

When doing statistical inference it is important to carefully distinguish these three histograms.

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More advanced versions of the law of large numbers state that the empirical histogram of the data (the histogram in 2. in the previous section) will be close to the probability histogram in 1. if the sample size is large.

The central limit theorem

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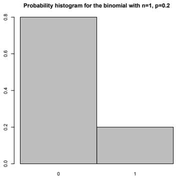

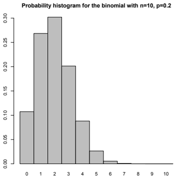

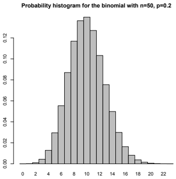

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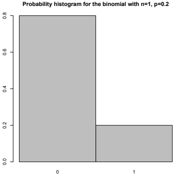

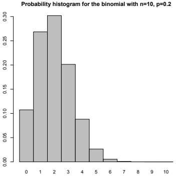

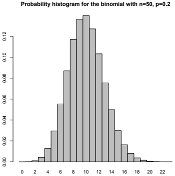

As n gets large, the probability histogram looks more and more similar to the normal curve. This is an example of the **central limit theorem**:

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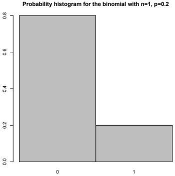
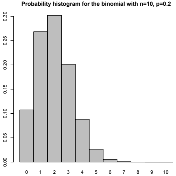
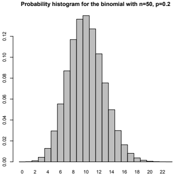
Recall the online game where you win with probability 0.2. We looked at the random variable $X =$'number of wins' in n gambles and found that X has the binomial distribution with that n and $p = 0.2$.





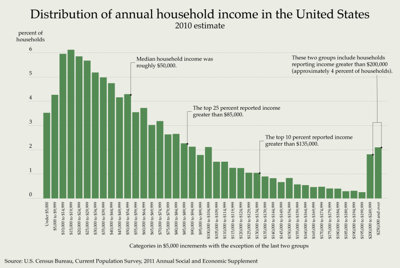


As n gets large, the probability histogram looks more and more similar to the normal curve. This is an example of the **central limit theorem**:

When sampling with replacement and n is large, then the sampling distribution of the sample average (or sum or percentage) approximately follows the normal curve. To standardize, subtract off the expected value of the statistic, then divide by its SE.

The central limit theorem

The key point of the theorem is that we know that the sampling distribution of the statistic is normal *no matter what the population histogram is*:



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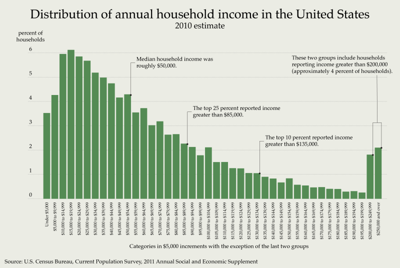

$$\mu = \$\,67,000$$
$$\sigma = \$\,38,000$$

If we sample n incomes at random, then the sample average $\bar{x}_n$ follows the normal curve centered at $\mathsf{E}(\bar{x}_n) = \mu = \$\,67,000$ and with its spread given by $\mathsf{SE}(\bar{x}_n) = \frac{\sigma}{\sqrt{n}} = \frac{\$\,38{,}000}{\sqrt{n}}$.

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For example, if we sample 100 incomes, then by the empirical rule there is about a 16% chance that $\bar{x}_n$ is larger than \$ 70,800:

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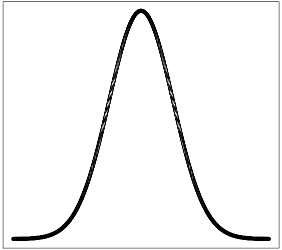

When does the central limit theorem apply?

For the normal approximation to work, the key requirements are:

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- We sample with replacement, or we simulate independent random variables from the same distribution.
- The statistic of interest is a sum (averages and percentages are sums in disguise).
- The sample size is large enough: the more skewed the population histogram is, the larger the required sample size n.
  (if there is no strong skewness then $n \geq 15$ is sufficient)

Mini quiz

1. There are two canidates running for governor in CA and they are said to have roughly equal support from the voters. To get a better idea who is ahead, a company polls 400 of the 20 million registered voters in California. Likewise, there are two candidates running for mayor in Palo Alto who are said to have roughly equal support, and the company polls 400 out of the 20,000 registered voters in Palo Alto. Will the first poll be more/equal/less accurate than the second?
2. The average taxable income reported on tax returns for the year 2016 is \$ 45,000, and the standard deviation of the taxable incomes is \$ 23,000. For each of the following two statements, state whether it is true or false:
   a. The percentage of taxable incomes that fall below \$ 30,000 can be computed from the above information using normal approximation.
   b. The chances that the sum of 100 randomly selected taxable incomes exceeds \$ 4 million can be computed from the above information using normal approximation.