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Why did the study find a statistically significant result?

The study looked at 800 different health effects:

There were 800 statistical tests involved.

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When analyzing large amounts of data it is easy to fall into this trap because there are so many potential relationships to explore, which leads to **data snooping** (=**data dredging**).

Reproducibility and Replicability

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- 'How science goes wrong' in The Economist (10/13/2013)
- 'Why most published research findings are false' by J. Ioannidis (2005)

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As a consequence the adjusted p-values may not be significant any more even if a noticeable effect is present.

Accounting for multiple testing

Alternatively, we can try to control the **False Discovery Proportion (FDP)**:

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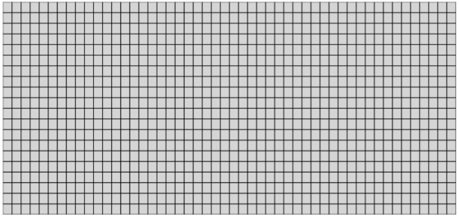

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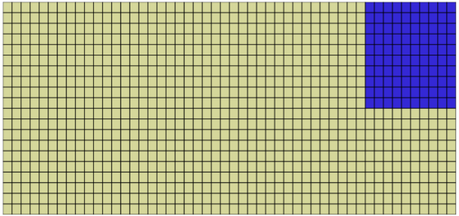

In 900 cases the null hypothesis is true ("Nothing is going on"), and in 100 cases an alternative hypot hesis is true ("There is an effect: something is going on").

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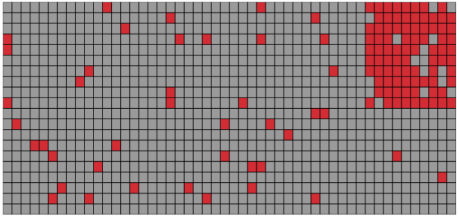

Doing 1,000 tests results in Discoveries and Non-discoveries.

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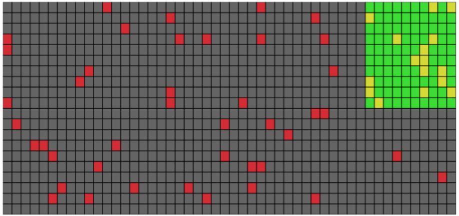

We made 80 true discoveries and 41 false discoveries. The false discovery proportion is 41/121=0.34.

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3. Declare discoveries for all tests i from 1 to k.

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Then test this hypothesis on the validation set.

This approach requires strict discipline: You are not allowed to look at the validation set during the exploratory step!

Mini quiz

1. A medical study examines whether there is a significant correlation between any of 12 lifestyle choices and high blood pressure. It doesn't find any significant correlation, but upon further examination the researchers find a highly significant (p-value <0.5%) correlation between two of the lifestyle choices. This correlation seems not to have been noticed before. Which of the following three statements is an appropriate summary of these findings:

i) The correlation between these two lifestyle choices is highly significant and should be reported as such.

ii) The seemingly significant correlation was found as a consequence of data snooping and therefore the p-value is not valid. The researchers shouldn't report anything.

iii) The seemingly significant correlation was found as a consequence of data snooping and therefore the p-value is not valid. However, this could potentially be a significant new finding. The researchers can report it as such, pointing out that they cannot attach a valid p-value to this finding. It can serve as a hypothesis for a future study with new data, which would then allow for statistically valid conclusions.

2. 1,000 tests were evaluated with the Bonferroni correction. 31 tests had corrected p-values smaller than 5%. Which of the following three statements are an appropriate conclusion:

i) There is a 95% probability that all of these 31 null hypotheses are false.

ii) This is sufficient evidence to reject all of these 31 null hypotheses, because there is only a 5% chance that any of these 31 p-values would be this small if the null hypothesis were true.

iii) If we reject these 31 null hypotheses then we can expect that about 5% of them are rejected in error.

3. 1,000 tests were evaluated with the FDR at the 5% level, which resulted in 31 discoveries. Which of the above statements (i)-(iii) are an appropriate conclusion?