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While interned during the Second World War, John Kerrich tossed a coin 10,000 times and observed 5,067 tosses resulting in heads.

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'The probability that my best friend calls today is 30%'.

Such a 'subjective probability' is not based on experiments, and different people may assign different subjective probabilities to the same event.

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Rule for equally likely outcomes: If there are n possible outcomes and they are equally

likely, then 
$$P(A) = \frac{\text{number of outcomes in } A}{n}$$

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Multiplication rule: If A and B are independent, then P(A and B) = P(A) P(B)

Roll a die three times. What is  $P(at least one \square)$ ?

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We could write 'at least one i' as follows:

on the first roll or on the second roll or on the third roll.

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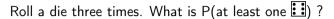
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=41.1%

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P(spam) = 20%. What is the probability that 'money' appears in an e-mail?

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 $= 0.08 \times 0.2 + 0.01 \times 0.8$ 

= 2.4%

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## Bayesian analysis

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- ▶ Before examining the e-mail, there is a *prior probability* of 20% that it is spam.
- ▶ After examining the e-mail for certain keywords such as 'money', the filter updates this prior probability using Bayes' rule to arrive at the *posterior probability* that the e-mail is spam.

1% of the population has a certain disease. If an infected person is tested, then there is a 95% chance that the test is positive.

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Know P(D)= 1%

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 =  $\frac{P(+|D) \ P(D)}{P(+|D) \ P(D) + P(+|no D) \ P(no D)}$  =  $\frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.02 \times 0.99}$  = 32.4%

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So the answer will be partly random: We don't know whether a 'yes' answer is due to the student cheating or to getting tails on the second toss. This should put the student at ease to answer truthfully.

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proportion of cheaters using all the answers collectively:

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$$P(yes) = \frac{27}{27 + 30} = 47\%$$

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So we estimate P(yes) = 
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 and get P(yes | Q1)=  $\frac{0.47-0.5\times0.5}{0.5} = 44\%$ 

#### Mini quiz

- ▶ A fair coin is tossed 5 times. Find the probability of getting at most 4 tails.
- ▶ 3% of all applicants to the Stanford Medical School are admitted. 70% of all applicants have a GPA of 3.6 or above. Of those who are admitted, 95% have a GPA of 3.6 or above.

What are the chances of being admitted if the GPA is 3.6 or above?