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Roll-No : 20p-0750
Section : "C"
Assignment : "1"

Solution:.

a) A Says "C" is a knave.
B Says "A" is a knight.
C Says "I am the Spy."

There will be total nine proposit

p_1 = A is knight
p_2 = A is knave
p_3 = A is Spy

q_1 = B is knight
q_2 = B is knave
q_3 = B is Spy

r_1 = C is knight
r_2 = C is knave
r_3 = C is Spy

Now make some conditions.

i) γ_2
ii) P_1
iii) γ_3

→ 27 possible cases by making truth table we chose only those where two knights and one knave.

A	B	C
K	K	K
K	K	N
K	K	S
K	N	K
K	N	N
K	N	S
K	S	K
K	S	N
K	S	S
K	K	K
K	K	N
K	K	S
K	N	K
K	N	N
K	N	S
K	S	K
K	S	N
K	S	S

Case: i

A	B	C
Knight	Knight	Knave

1, $r_2 = F$

2, $p_1 = F$

3, $r_3 = T$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$r_1 = F$	$r_2 = T$	$r_3 = T$

by putting values of r_2, p_1, r_3 check the condition in eq, (1), (2), (3).

1, $r_2 = F$

$T = F$

Case doesn't exist ans:

$T \neq F$

Case: "ii"

A	B	C
knight	knave	knight

i) $r_2 = F$
ii) $p_1 = T$
iii) $r_3 = F$

$p_1 = T \quad p_2 = F \quad p_3 = F$
$q_1 = F \quad q_2 = T \quad q_3 = F$
$r_1 = T \quad r_2 = F \quad r_3 = F$

by putting values in equations.

i) $F = F$ ✓
ii) $T = T$ ✓
iii) $F = F$ ✓

Therefore A is knight, B is knave, and C is knight.

Case: "iii"

A	B	C
knave	knight	knight

i) $r_2 = T$
ii) $p_1 = F$
iii) $r_3 = F$

$p_1 = F$ $\quad p_2 = T$ $\quad p_3 = F$

$q_1 = T$ $\quad q_2 = F$ $\quad q_3 = F$

$r_1 = T$ $\quad r_2 = F$ $\quad r_3 = F$

from eq (i):

$$F = T$$

Case doesn't exist.

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b) A says "I am knight"
B says "I am knave"
C says "B is knight"

p_1 = A is knight
p_2 = A is knave
p_3 = A is Spy

q_1 = B is knight
q_2 = B is knave
q_3 = B is Spy

r_1 = C is knight
r_2 = C is knave
r_3 = C is Spy

making equations:

i) p_1
ii) q_2
iii) q_1

Truth * Table

A	B	C
K	K	K
K	K	N
K	K	S
K	K	K
K	N	K
K	N	N
K	N	S
K	N	K
K	S	N
K	S	S
K	S	K
N	K	N
N	K	S
N	K	K
N	N	N
N	N	S
N	N	K
N	S	N
N	S	S
N	S	K
S	K	N
S	K	S
S	K	K
S	N	N
S	N	S
S	N	K
S	S	N
S	S	S
S	S	K
S	S	N
S	S	S

Case: i

A	B	C
knight	knight	knave

i $p_1 = F$

ii $q_2 = F$

iii $q_1 = T$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$r_1 = F$	$r_2 = T$	$r_3 = F$

i $T = F$

Case doesn't exist ans:

$T \neq F$

Case : ii

A	B	C
knight	knave	knight

i $p_1 = F$

$q_2 = T$

$q_1 = F$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = F$	$q_2 = T$	$q_3 = F$
$r_1 = T$	$r_2 = F$	$r_3 = F$

i $T = F$

Case doesn't exist ans:

$T \neq F$

$p_1 = F$ $p_2 = T$ $p_3 = T$

$q_1 = T$ $q_2 = F$ $q_3 = F$

$r_1 = F$ $r_2 = T$ $r_3 = F$

i

ii

iii

Case: iii

A	B	C
Knave	Knight	Knight

i $p_1 = T$

$q_2 = F$

$q_1 = F$

$p_1 = F$ $p_2 = T$ $p_3 = F$

$q_1 = T$ $q_2 = F$ $q_3 = F$

$r_1 = T$ $r_2 = F$ $r_3 = F$

i $F = T$

Case doesn't exist as:

$F \neq T$

c) A says "I am knave"
B says "I am knave"
C says "I am knave"

P_1 : A is knave
P_2 : A is knight
P_3 : A is spy

q_1 : B is knave
q_2 : B is knight
q_3 : B is spy

r_1 : C is knave
r_2 : C is knight
r_3 : C is spy

Conditions

i) P_1
ii) q_1
iii) r_1

Case : i.

A	B	C
knight	knight	knave

i $p_1 = F$
ii $q_1 = F$
iii $r_1 = T$

$p_1 = F \quad p_2 = T \quad p_3 = F$
$q_1 = F \quad q_2 = T \quad q_3 = F$
$r_1 = T \quad r_2 = F \quad r_3 = F$

i $F = F$ ✓
ii $F = F$ ✓
iii $T = T$ ✓

Therefore A is knight, B is knight and C is knave

Case : ii

A	B	C
knight	knave	knight

i $p_1 = F$
ii $q_1 = T$
iii $r_1 = F$

$p_1 = F \quad p_2 = T \quad p_3 = F$
$q_1 = T \quad q_2 = F \quad q_3 = F$
$r_1 = F \quad r_2 = T \quad r_3 = F$

i) F = F ✓
ii) T = T ✓
iii) F = F ✓

Therefore: A is knight, B is knave and C is knight.

Case: iii

A	B	C
knave	knight	knight

i) $P_1 = T$
ii) $q_1 = F$
iii) $\gamma_1 = F$

$P_1 = T \quad P_2 = F \quad P_3 = F$
$q_1 = F \quad q_2 = T \quad q_3 = F$
$\gamma_1 = F \quad \gamma_2 = T \quad \gamma_3 = F$

i) T = T ✓
ii) F = F ✓
iii) F = F ✓

Therefore A is knave, B is knight and C is knight.

d) A says "I am knight"
B says "A is telling truth"
C says "I am spy"

p_1 = A is knight
p_2 = A is knave
p_3 = A is spy

q_1 = B is knight
q_2 = B is knave
q_3 = B is spy

r_1 = C is knight
r_2 = C is knave
r_3 = C is spy

equations:

i) p_1
ii) p_1
iii) r_3

Case: i

A	B	C
knight	knight	knave

i) $p_1 = F$
ii) $p_1 = F$
iii) $\gamma_3 = T$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$\gamma_1 = F$	$\gamma_2 = T$	$\gamma_3 = F$

i) ~~# # #~~ $T = F$

Case does not exist ~~obs~~
~~# # # T ≠ F~~

Case: ii

A	B	C
knight	knave	knight

i) $p_1 = F$
ii) $p_1 = T$
iii) $\gamma_3 = F$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = F$	$q_2 = T$	$q_3 = F$
$\gamma_1 = T$	$\gamma_2 = F$	$\gamma_3 = F$

i) $T = F$

Case does not exist Ans:
$T \neq F$

Case : iii

A	B	C
knave	knight	knight

i $p_1 = T$

ii $p_1 = F$

iii $\gamma_3 = F$

$p_1 = F$	$p_2 = T$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$\gamma_1 = T$	$\gamma_2 = F$	$\gamma_3 = F$

i $F = \overline{T}$

Case doesnit exist as

$F \neq T$