We are all connected!

"We are caught in an inescapable network of mutuality, tied in a single garment of destiny. Whatever affects one directly, affects all indirectly." Martin Luther King Jr. (January 15, 1929 – April 4, 1968)

Networks

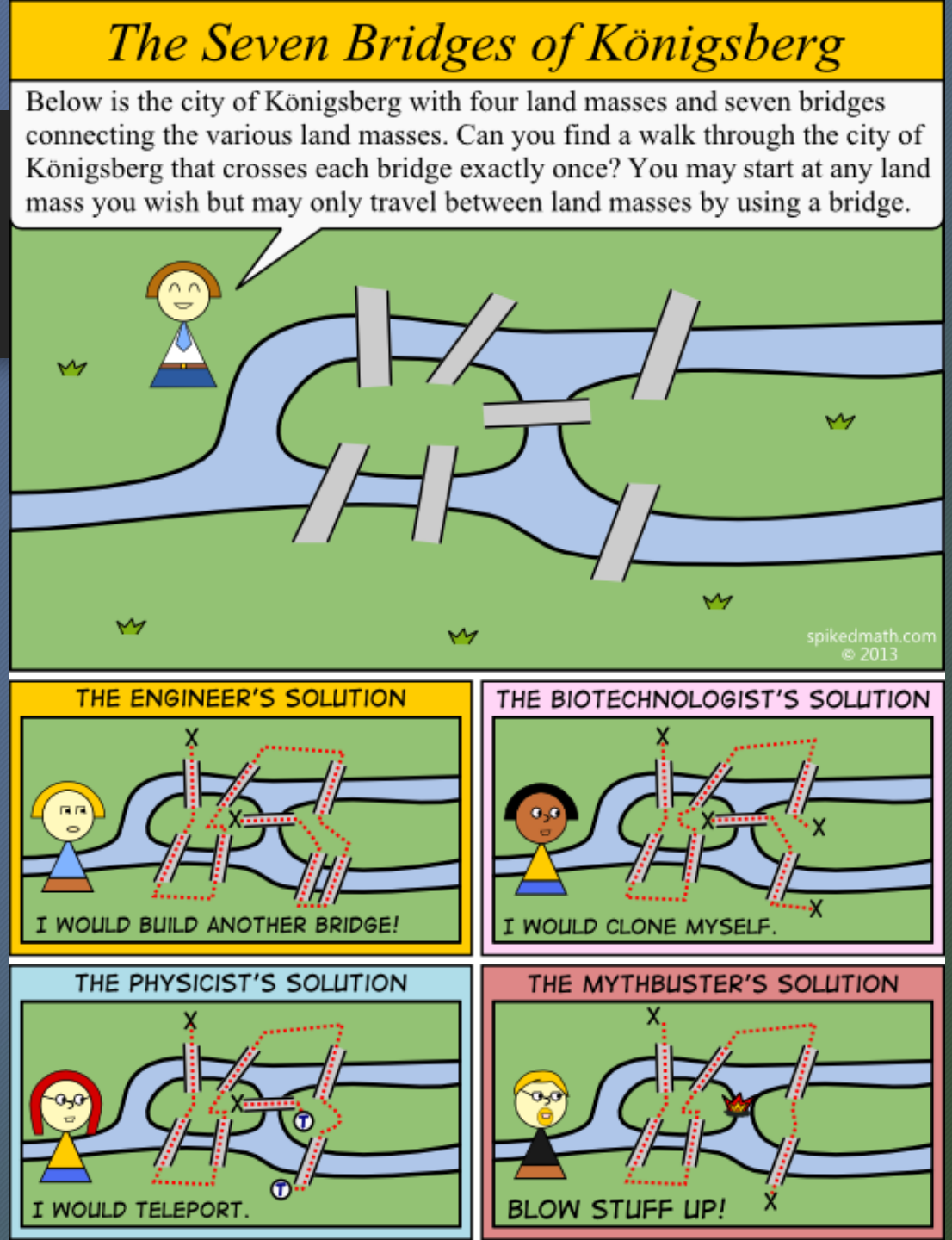
- Many real world systems can be viewed as a network i.e., collection of nodes inter-linked to one another

- Node: A unit, can be a person, organization, computer etc.
- Interaction: relationship, friendship, colleagues… (directed or not)

- World Wide Web (WWW)
  - 2.4 billion users, 50 billion web pages, 600 million web servers, 20 million DNS servers …

Konigsberg's Bridges

- Solved by Leonhard Euler solved it in 1736, using a graph
- Became the pioneering work in graph theory



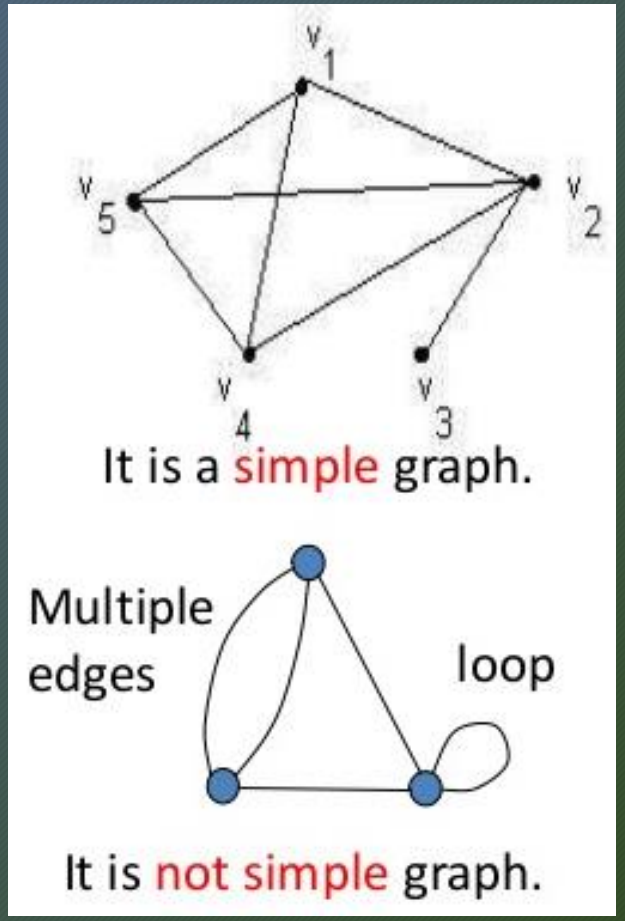
Simple (General) Graphs

- G = (V, E) is a graph consisting of V and E
- V = {v_1, v_2, v_3, …, v_N} is a set of objects called vertices
- E = {e_1, e_2, e_3, …, e_k} is a set of links called edges

- Such that an edge e_{ij} exists between two vertices (v_i, v_j)

But have no

- Self edges i.e., the a vertex sharing an edge with itself e_{aa}
- Some graphs may allow multiple edges between the same vertices!

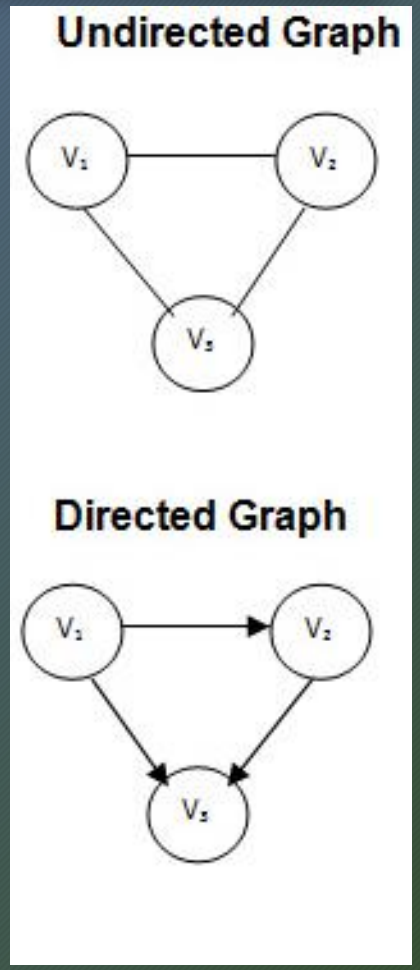


Directions in Graphs

- Graphs may be
  - Unidirectional (Directed graphs)

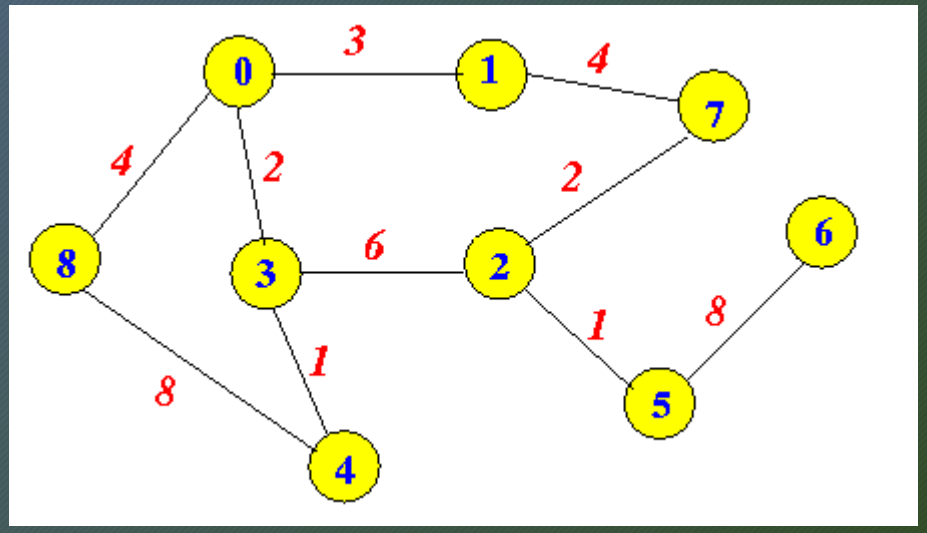
or

  - Bidirectional (Undirected graphs)



Weighted Graph

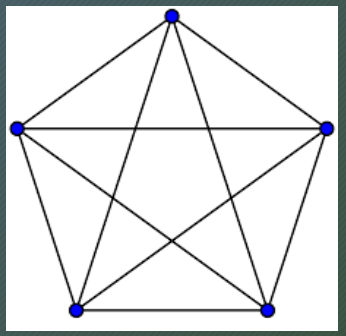
- A **weighted graph** is a **graph** in which each branch is given a numerical **weight**. A **weighted graph** is therefore a special type of labeled **graph** in which the labels are numbers (which are usually taken to be positive).



Complete Graph

- $G = (V, E)$ where each pair of distinct vertices are adjacent and total number of vertices > 1
- The total edges in the graph, K_N are

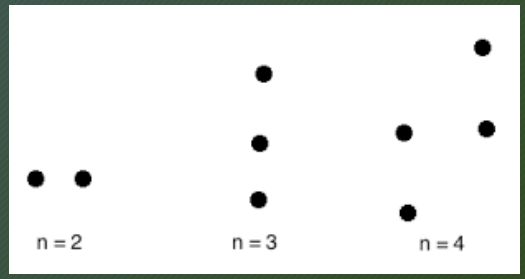
$$K_N = \frac{N(N-1)}{2}$$



Null Graph

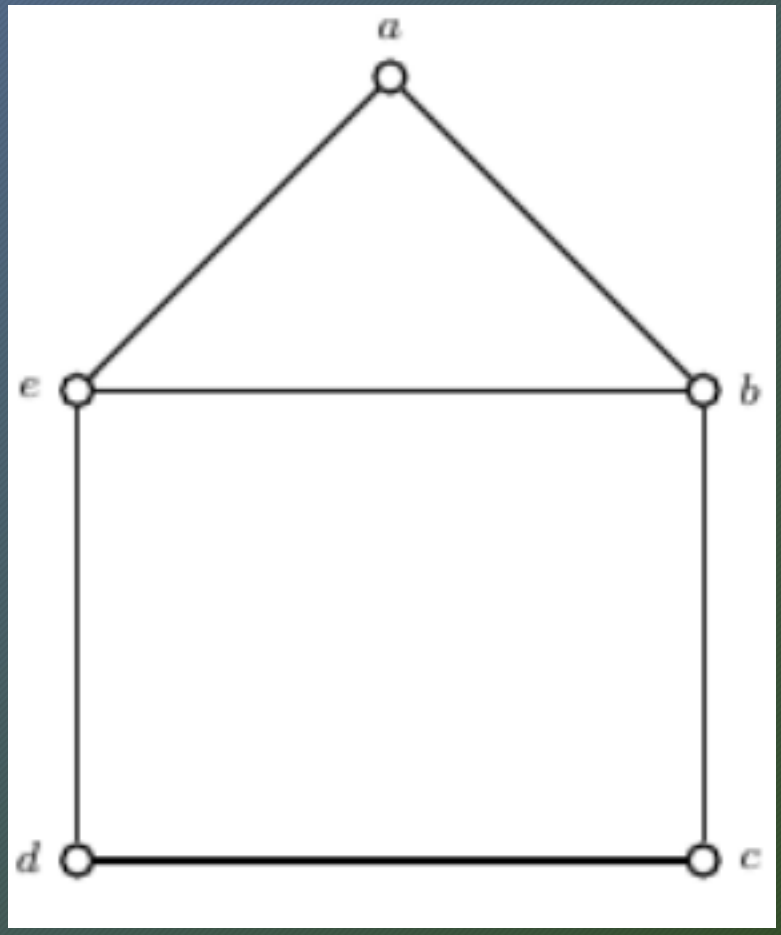
- $G = (V, E)$ where E is a null set

The vertices are there but not linked to one another



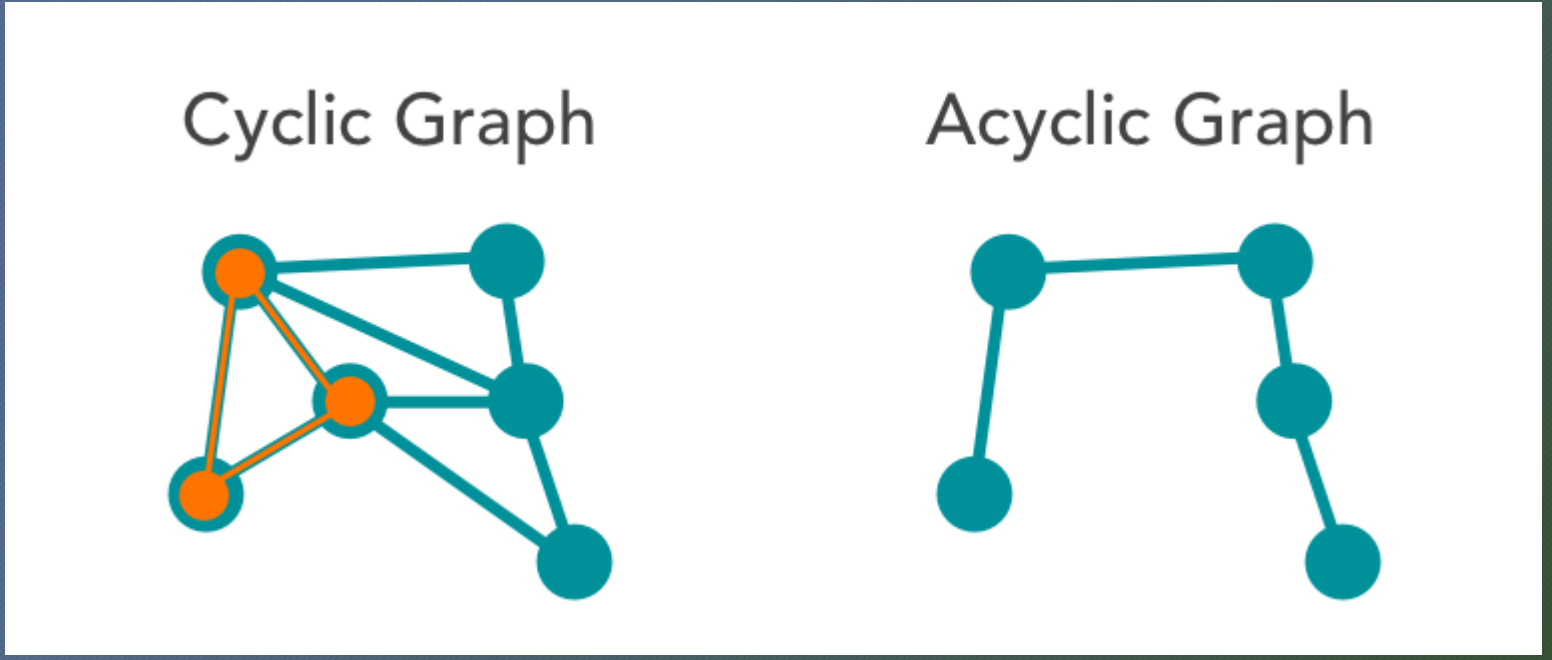
Example

- List all vertices and edges
- List all vertices that are adjacent to each other i.e., sharing a direct edge
- Which vertex has the most number of edges



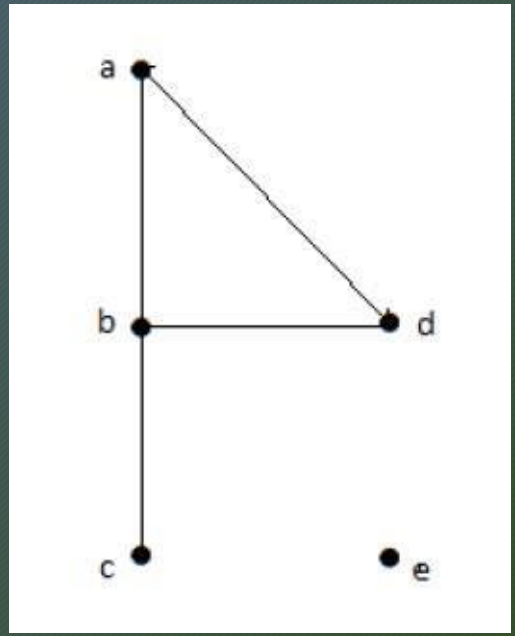
Cyclic or Acyclic Graphs

- A graph having cycles or loops is called cyclic graph
- Graphs without cycles or loops is called acyclic graph



Isolated, Pendant Vertex

- An isolated vertex v is the one having no incident edge, i.e., $d(v) = 0$.
- A vertex v having $d(v) = 1$ is called a pendant vertex.



Theorems

- The sum of degrees of all vertices in a graph G, is twice the number of edges in G,

$$\sum_{i=1}^{N} \deg(v_i) = 2e$$

- The number of vertices of odd degree in a graph G, is always even