Cut Sets

A set of edges whose removal results in a disconnected graph, provided that no sub-set of the set also results in a disconnected graph.

A cut set reduces the rank of a graph by one.

Rank $R = N - k$

Nullity $\mu = E - N + k$

Application: Finding weak spots in communication networks & strengthening roads and communication networks

Theorem: Every cut-set in a connected graph G must contain at least 1 branch of every spanning tree of G

Theorem: Every circuit has an even number of edges common with any cut-set.

Fundamental Cut Set

A cut set that contains one and only one branch of a spanning tree of graph G.

Theorem: A chord that creates a fundamental circuit α occurs in every fundamental cut set associated with the branches of α.

Edge Connectivity: The number of edges in smallest cut-set

Vertex Connectivity: Minimum number of vertices whose removal disconnects a graph

Cut Vertex: The vertex whose removal results in a disconnected graph

Theorem: A vertex v3 is a cut vertex if and only if there exists two vertices v1 and v2 such that every path between v1 and v2 passes through v3.

Theorem: Edge connectivity of a graph G cannot be more than a vertex in G having minimum degree.

Theorem: Vertex connectivity of a graph G cannot be more than the edge connectivity.

Theorem: Maximum vertex connectivity in a graph G is the integer part of relation 2e/n

$$\text{Vertex connectivity} \le \text{Edge connectivity} \le 2e/n$$