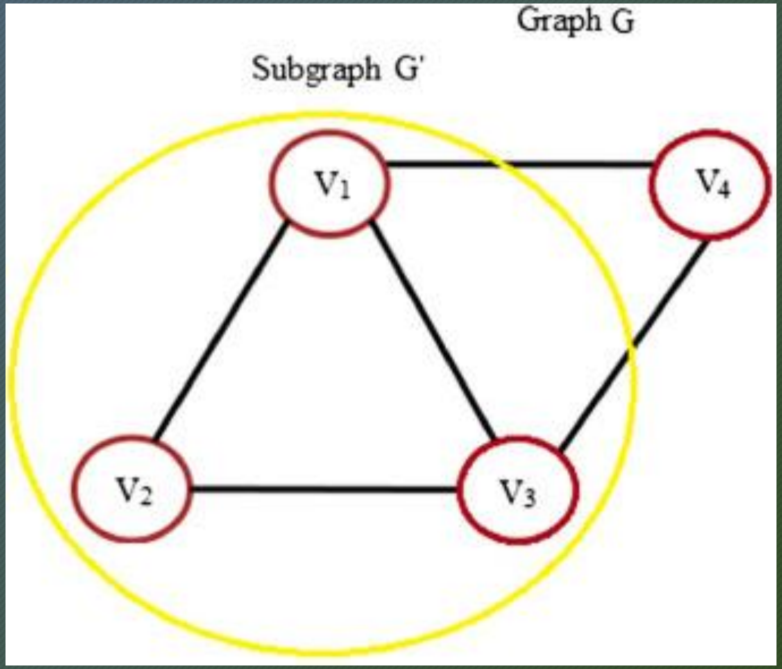
Subgraphs

- A subgraph G’ of graph G i.e., $(G' \subset G)$ is a graph, each of whose vertices $(V' \subset V)$ and edges $(E' \subset E)$

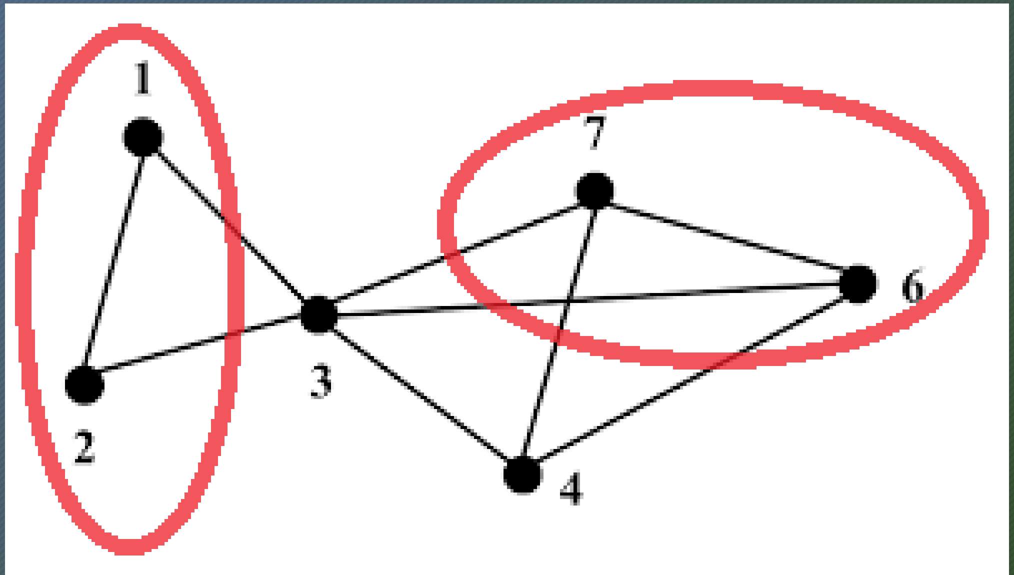
- **Transitivity rule:**

if $G'' \subset G' \subset G$, then $G'' \subset G$



Edge-disjoint Subgraphs

- Subgraphs G’ and G” of graph G are edge disjoint if G’ and G” do not have any edge in common

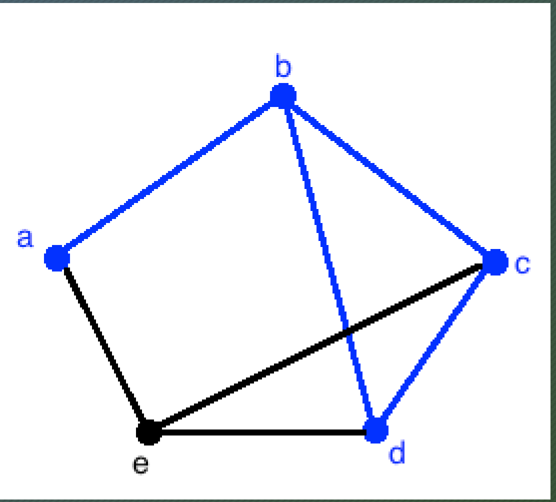


Walk (repeating edges or vertices is fine)

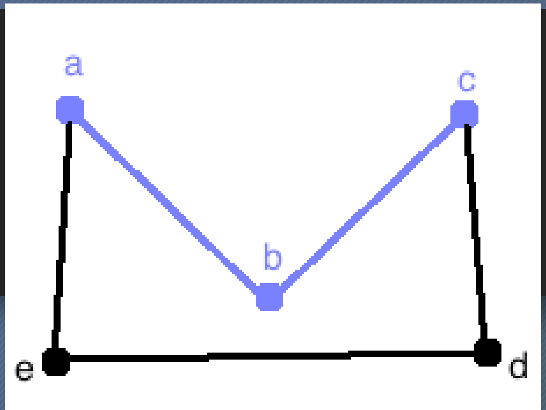
- A walk in a graph G = (V, E) is a finite sequence of vertices and edges that begins from any vertex V_0 and ends at any vertex V_k
  - E.g., abcdbcd is a walk

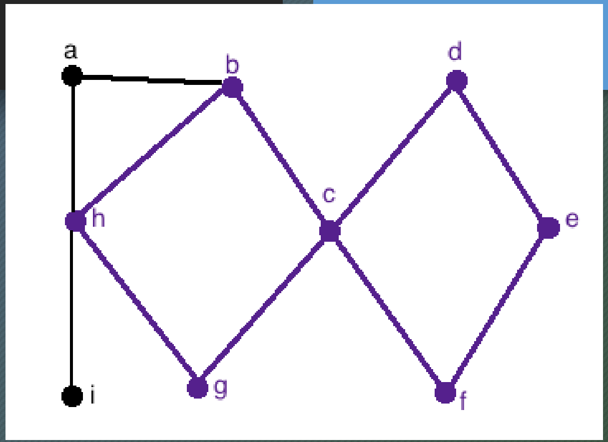
- **Open Walk:** A walk that has different starting and ending vertices i.e., $V_0 \neq V_k$
  - **E.g., abc is an open walk**

- **Closed Walk:** A walk that has the same starting vertex as its ending vertex i.e., $V_0 = V_k$
  - E.g., abcdba is a closed walk

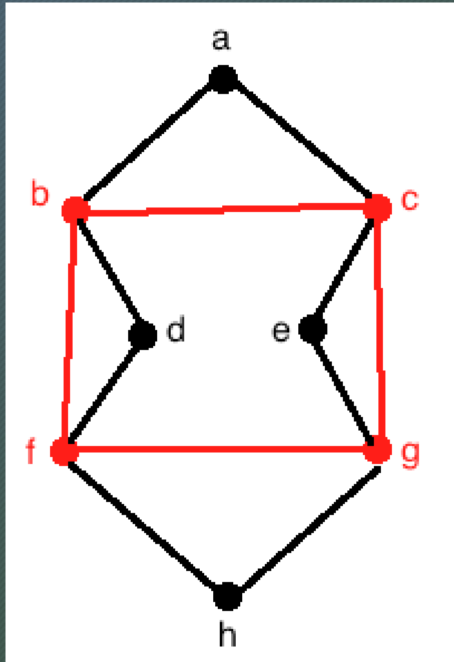


Trails, Circuits, Cycles



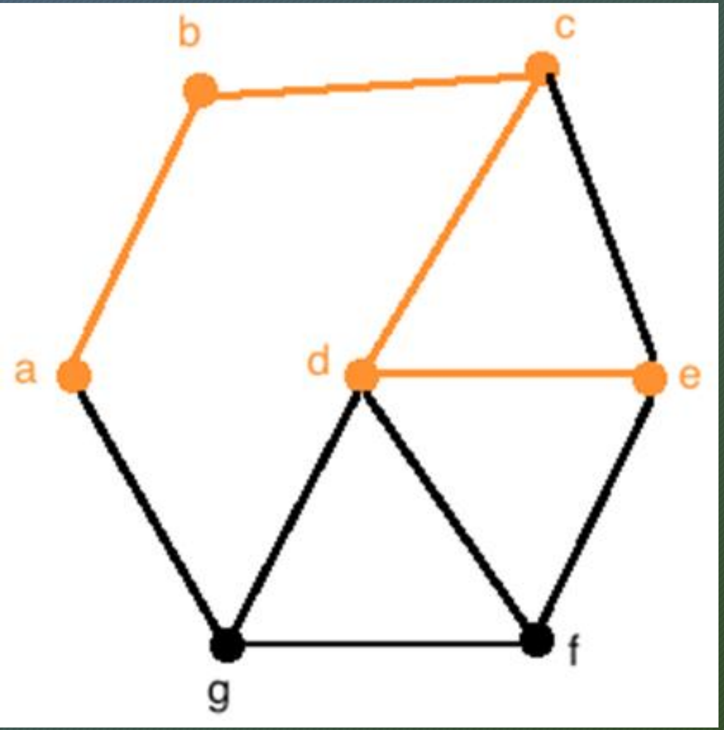


- Trails are walks with no edge repeated
  - E.g., abc is a trail
- Circuit is a closed trail having same starting and ending vertex, while in between vertices may repeat as well
  - E.g., hbcdefcgh
- Cycles are circuits with only one repeated vertex i.e., the starting vertex as its ending vertex
  - E.g., bcgfb



Paths

- A Path is a trial with no repeated vertex (so it has to be an open trail)
  - E.g., abcde is a path



Connected / Disconnected Graph

- A graph G is connected if there is at least one path between each pair of vertices in G. Otherwise its disconnected
- Each connected part in a disconnected graph is called a component or a community

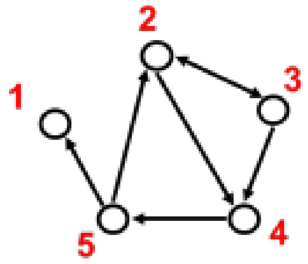
- A simple graph with N vertices and K components can have at most $\frac{(N-K)(N-K+1)}{2}$ edges

Adjacency, Incidences and Degree

- **Adjacency:**
  - Two vertices V_1 and V_2 are adjacent if there is an edge joining them
  - A vertex to vertex property
- **Incidence:**
  - If V_1 and V_2 are joined by an edge e, then both V_1 and V_2 are incident on edge e
  - **An edge to edge or edge to vertex property**
- **Degree:**
  - **Degree of a vertex V in graph G is the number of edges incident with V and is written as deg(V) or d(V)**
  - **A vertex property**

Adjacency Matrix, Edge List & Adjacency list

- Adjacency matrix



$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Edge List

2, 3
2, 4
3, 2
3, 4
4, 5
5, 2
5, 1

Adjacency List

1:
2: 3 4
3: 2 4
4: 5
5: 1 2