

Euler Graphs, Paths and Circuits

Certain graph problems deal with finding a path between two vertices such that each edge is traversed exactly once, or finding a path between two vertices while visiting each vertex exactly once. These paths are better known as Euler path and Hamiltonian path respectively.

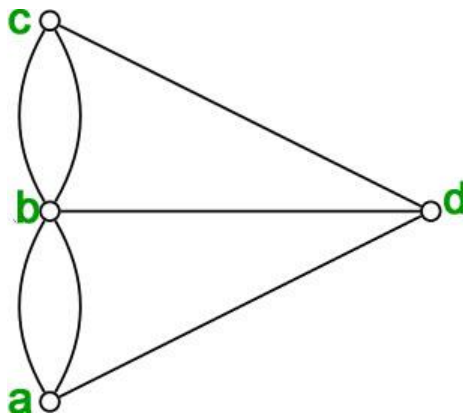
Definition of Euler Line and Graph: If some closed walk in a graph contains all the edges of the graph, then the walk is called an Euler Line, and the graph an Euler Graph, provided all vertices are of even degree.

Definition of Unicursal Line and Graph: If some open walk in a graph contains all the edges of the graph, then the walk is called a Unicursal Line, and the graph a Unicursal Graph provided it has exactly two vertices of odd degree.

Euler Paths and Circuits:

- An Euler path is a path that uses every edge of a graph exactly once.
- An Euler circuit is a circuit that uses every edge of a graph exactly once.
- An Euler path starts and ends at different vertices.
- An Euler circuit starts and ends at the same vertex.

The Königsberg bridge problem's graphical representation:



There are simple criteria for determining whether a multigraph has a Euler path or a Euler circuit. For any multigraph to have a Euler circuit, all the degrees of the vertices must be even.

Theorem: A given graph is an Euler graph if and only if all vertices of G are of even degree.

Theorem: A connected multigraph (and simple graph) with at least two vertices has a Euler circuit if and only if each of its vertices has an even degree.

Finding Euler Lines (if any) in a given graph – Done in Class.