# Synthetic Control Estimation with Neural Networks

**David Arboleda-Cárcamo,[1] Daniel Lasso-Jaramillo,[2] Paula Torres-Higuera[3]**

[1] *Pre-doctoral Fellow, SciencesPo, Department of Economics.*
[2] *Consultant, World Bank, Development Economics Research Group (DECRG).*
[3] *Economist, Financial Superintendence of Colombia.*

*Synthetic control is a widely used causal inference methodology in recent research. Its main advantage lies in the ability to estimate treatment effects even when only a few units are assigned to the treatment group. In this approach, a direct counterfactual is constructed for these units using the convex combination of outcome variables from control units that best matches the time series of the treated units' outcome variable in the pre-treatment period. However, this strategy faces limitations in terms of predictive capacity, as it does not capture possible non-linearities in the relationship between predictor variables and the variable of interest. Furthermore, the methodology relies heavily on discretionary decisions when defining the predictor features. To address these limitations, we propose an alternative approach based on artificial neural networks for a non-parametric counterfactual estimation. Through simulations, we demonstrate that this new method significantly improves predictive capacity compared to the standard synthetic control approach. Furthermore, we apply neural networks to investigate the effect of German reunification on Germany's GDP and observe a remarkable improvement in pre-treatment fit. This result is suggestive of a more precise estimation of the reunification effect.*

***JEL Classification***: *C01, C14, C51.*
***Keywords***: *Econometric Theory, Synthetic Control, Machine Learning, Causal Inference.*

June 2023

## 1. Introduction

This paper proposes the estimation of synthetic control based on Artificial Neural Networks (ANNs). These networks have desirable features, including their universal approximation property and their ability to capture non-linear associations between variables. The dividing line between causal inference econometrics and time series analysis has been fading since the introduction of new methodologies that seek to combine both areas. Possibly, the most prominent of these is the synthetic control methodology, which emerged with the seminal work of Abadie y Gardeazabal (2003), in which the authors aimed to predict the GDP dynamics in the Basque Country in the absence of ETA terrorist attacks in order to estimate the effect of these attacks on economic growth. However, the use of synthetic control has encountered some drawbacks, as eloquently exposed by Abadie (2021). Nevertheless, synthetic control has remained the dominant methodology, even in scenarios where its problems are evident.

This paper sets out a procedure that uses ANNs to predict counterfactuals in contexts where synthetic control would typically be employed.

Furthermore, this procedure not only does not require additional information but also allows for estimation even in cases where the necessary data to estimate an adequate synthetic control is not available.

To validate the usefulness of the proposed procedure, two tests were conducted. Firstly, Monte Carlo experiments were carried out, which showed that the proposed neural network procedure represents a considerable improvement over synthetic control, to the extent that, on average, the RMSPE obtained by the methodology in this paper is about $60\,\%$ of that obtained by synthetic control. Secondly, one of the canonical works in the synthetic control literature was replicated, and it was demonstrated that the neural network methodology not only matches the "gold standard" but also does so without resorting to arbitrary decisions regarding the model construction, as is often noted in studies using synthetic control. Consequently, it is concluded that the proposed causal inference methodology in this paper represents a significant improvement over existing methodologies.

# 2. Literature Review

Intuitively, the synthetic control methodology seeks to construct, through a weighted combination of untreated units, a counterfactual for a treated unit to identify the treatment effect on said unit.

The synthetic control method was originally proposed by Abadie y Gardeazabal (2003) with the aim of estimating the effects of aggregate interventions. However, its use has been extended to other problems and applications such as the effect of tobacco taxes in California and cigarette sales (Abadie, Diamond y Hainmueller, 2010), and the effects of labor laws on migration (Bohn, Lofstrom y Raphael, 2014), among others.

## 2.1. Synthetic Control

Suppose there are $J+1$ units, where $j=1$ is the treated unit and $j=2,3,...,J+1$ are the donor or untreated units. We have a balanced panel with information for the same $T$ periods for all units, where unit 1 is exposed to the treatment during periods $T_0,...,T$, and the intervention has no effect during the periods before treatment $1,...,T_0$. Likewise, for each unit $j$ and period $t$, the outcome of interest $Y_{j,t}$ is observed, and we have $K$ explanatory variables.

Abadie, Diamond y Hainmueller (2015) define synthetic control as a weighted average of the donor units. This is represented by a weight vector of size $(J\times 1)$ with positions $W=(w_2,...,w_{J+1})$, where choosing a particular value for $W$ is equivalent to choosing a synthetic control. To do this, $W$ is selected in such a way that the characteristics of the donor units closely resemble those of the treated unit. Therefore, for every $m\in\{1,...,k\}$, with $k\leq T_0$, if we define $X_{1,m}$ as the value of the $m$-th characteristic of the treated unit before the intervention - which we want to adjust as closely as possible - and $X_{0,m}$ as a $(1\times J)$ vector collecting the $m$-th characteristic values of the $J$ donor units, the optimal weight ($W^*$) that minimizes the difference in pre-intervention characteristics between the synthetic control and the treated unit is chosen as:

$$\min_{V,W}\sum_{m=1}^{k} v_m (X_{1,m}-X_{0,m}W)^2$$

where $v_m$ is the relative importance given to the $m$-th characteristic when measuring the distance between $X_1$ and $X_0$. The choice of $v_m$ is usually made through cross-validation using pre-treatment period data.

Once the minimization problem has been solved, an estimator of the potential outcome of the treated unit in the absence of treatment, $Y_{1,t}(0)$, is constructed as follows:

$$\hat{Y}_{1,t}=\sum_{j=2}^{J+1} w_j * Y_{j,t}$$

The estimated effect in period $t$ is then given by the difference:

$$\tau_t = Y_{1,t} - \hat{Y}_{1,t}$$

However, there are some evident problems in using $\hat{Y}_{1,t}$ as an estimator of $Y_{1,t}(0)$. In particular, Abadie (2021) argues that although the synthetic control estimation is unbiased in the case where the data-generating process follows a VAR model, under other data-generating processes, the bias can only occasionally be bounded, and often such bounds turn out to be excessive. An example of this are linear factor models, defined as:

$$Y_{j,t}(0) = \delta_t + \theta_t Z_j + \lambda_t \mu_j + \varepsilon_{j,t}$$

where $\delta_t$ is a time trend, $Z_j$ and $\mu_j$ are vectors of observed and unobserved predictors of $Y_{j,t}(0)$, respectively, and $\varepsilon_{jt}$ are individual temporary shocks. Such models often closely approximate the dynamics that researchers usually expect from the data, so it becomes important to propose new methodologies that can at least partially alleviate the bias problems of synthetic control.

## 2.2. Artificial Neural Networks

Artificial neural networks aim to emulate the behavior of the human brain, characterized by learning through experience and extracting general knowledge from a dataset (López y Fernández, 2008). In its structure, an ANN consists of an input layer that initializes the data, one or more hidden layers, and an output layer. Each layer can have one or more neurons, connected in parallel, according to the optimal network architecture (Kohonen, 1988). Regarding the advantages of neural networks over other statistical models, it's worth noting that they can implicitly detect complex nonlinear relationships between the dependent variable and the covariates. Moreover, they have the ability to detect possible interactions among the explanatory variables, allowing for better fitting and the ability to uncover patterns in the data. In fact, they can approximate any function (universal approximation) (Tu, 1996).

## 2.3. Contribution

The methodology of ANN has been mainly implemented in the fields of artificial intelligence, machine learning, and deep learning with the aim of emulating the behavior of the human brain to recognize and solve patterns in various areas such as engineering and medicine (El Naqa y Murphy, 2015). Given the advantages of neural networks, the main contribution that this study makes to the literature is to apply neural network algorithms to the field of econometrics to improve estimates of causal effects. Specifically, to enhance the construction of synthetic counterfactuals by adopting a machine learning approach to the synthetic control methodology. Thus, it is expected that predictions of both simulated and real variables will be more accurate than those made by a synthetic control.

So far, there has been limited econometric knowledge built from machine learning. In particular, little has been studied regarding the ability of neural networks to estimate causal effects (Steinkraus, 2019). In fact, machine learning primarily focuses on predictive modeling, while the possibilities of causal analysis and subsequent decision-making have received less attention (Cui y Athey, 2022; Crown, 2019).

Among the few works that have been done to use neural networks for causal inference is the study by Steinkraus (2019), who both forecasted using a neural network and employed a synthetic control to estimate the effect of the construction of the Oresund Bridge on the local economy. The results of the study compared the effects under the two methodologies and concluded that the neural network outperforms the synthetic control by having greater predictive power on the training sample.

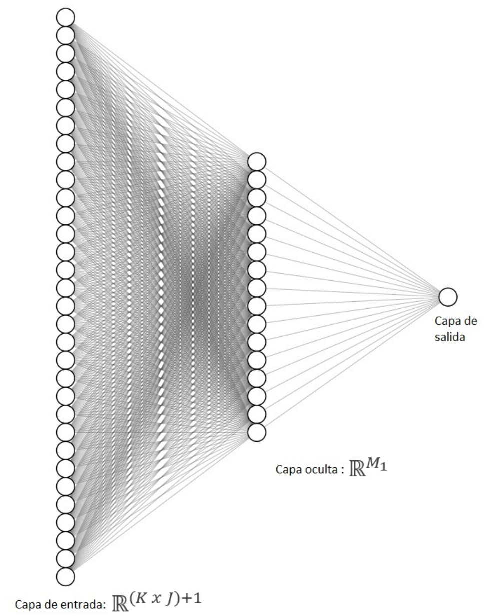
In this study, in addition to replicating the results of Abadie, Diamond y Hainmueller (2015) using a neural network, we include 1,000 simulations to provide greater validity to the comparison between the two methodologies. Therefore, this study contributes to bridging the gap between machine learning and causal inference, which has been minimal so far and has the potential to improve the prediction of causal estimates.

# 3. Methodology

This section presents the general structure of the neural network that will be used to predict the target counterfactual. During the description of this structure, general parameters will be used as the precise selection of these is part of the estimation process, as will be shown later. To begin, suppose there is information available for $K$ variables of the $J$ donor pool units. Then, each variable of each donor constitutes a neuron in the input layer, so that the total number of inputs in this layer is $D = J \times K + 1$, including the bias.

Subsequently, in a generic way, the $D$ inputs are combined through optimal weights and enter an activation function, possibly differentiable, to construct each of the $M_1$ neurons in the hidden layer. Each of these $M_1$ results can again be combined and transformed through activation functions to produce the $M_2$ neurons in the second hidden layer. The procedure is repeated until each of the $h$ hidden layers has been formed. Finally, the neurons in the $h$-th layer are weighted and pass through a single function to produce the prediction of $Y_t^1(0)$. The graphical representation of this procedure is shown in Figure 1.

**Figura 1:** *Generalized network diagram*




Two fundamental questions arise for the construction of the prediction model: (1) which activation functions should be used in each neuron? and (2) how many hidden layers should there be, and of what size? Although ultimately these unknowns depend on the experience of the programmer using the proposed procedure, two suggestions are presented to address these questions. First, the use of ReLU (rectified linear activation function) is suggested for two reasons. On the one hand, the use of this type of activation functions ensures that all inputs (such as the characteristics of the donor pool members in the construction of the first hidden layer, for example) are weighted with well-behaved weights (i.e., between 0 and 1), which eliminates potential extrapolation problems that may cause overfitting during model training. On the other hand, the decision to use ReLU functions is made based on intuitive aspects. In particular, synthetic control methods allow for some units to not contribute to the creation of the counterfactual, meaning that the weight of these units is “0". The ReLU function allows some neurons to remain inactive during the data propagation process, which intuitively opens the possibility for some donor units to not participate in the counterfactual creation process.

Second, the choice of the number and size of the hidden layers can become a problem of optimal model selection. Therefore, since these two qualities constitute hyperparameters, it is proposed to employ a cross-validation procedure using pretreatment period data, in which a grid of possible combinations of layer numbers and sizes is constructed, so that the final result is a model chosen based on a defined optimality criterion. The use of cross-validation has the property of providing approximately unbiased and consistent estimators of the evaluation metric. Asymptotically, the best model is selected based on the chosen goodness-of-fit measure.

# 4. Monte Carlo Experiments

## 4.1. General Procedure

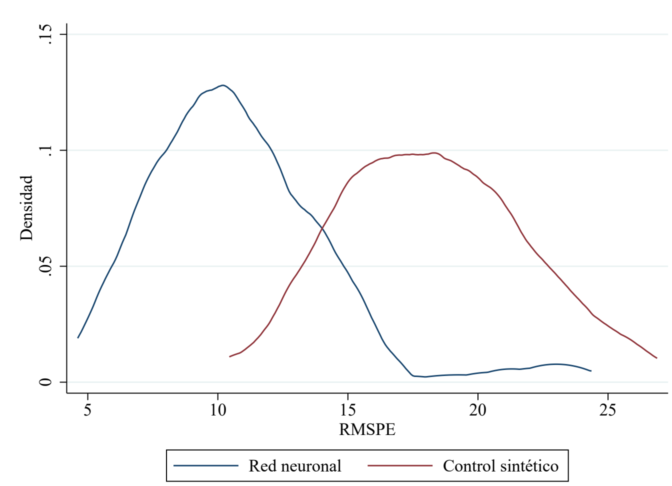
To empirically verify the goodness of fit of the methodology and compare its performance with the current gold standard (synthetic control), data simulations that follow a certain structure that plausibly approximates the observed variables in a panel can be used. To do this, the following procedure was followed:

1. A cross-sectional size $N$ and a temporal dimension $T$ are defined.
2. The number of independent variables $K$ and their data-generating process are determined. Given that the temporal dimension is of considerable importance in panel data, it is recommended that this process exhibit some pattern of temporal dependence. For this purpose, it is suggested to use processes of the ARIMA type, although other modeling approaches such as ARCH, GARCH, or VAR are equally appropriate.
3. The dependent variable of interest is constructed for each unit-period pair as a linear combination of the regressors, where the coefficients are not necessarily constant across unit-time but the data-generating process is stationary. Additionally, an idiosyncratic error term is included.
4. The panel is divided into two parts, with the pre-treatment period being everything below $T/2$ and the post-treatment period being the remaining period.
5. The proposed artificial neural network model is fitted using the pre-treatment period, and the synthetic control is estimated.
6. The estimated models are used to predict $Y_{1,t}(0)$, with which the root mean squared prediction error (RMSPE) is calculated for each model.
7. The goodness-of-fit metrics of both models are compared to evaluate whether the use of artificial neural networks represents a significant improvement over the synthetic control methodology in terms of the prediction capacity of potential outcomes.

## 4.2. Results

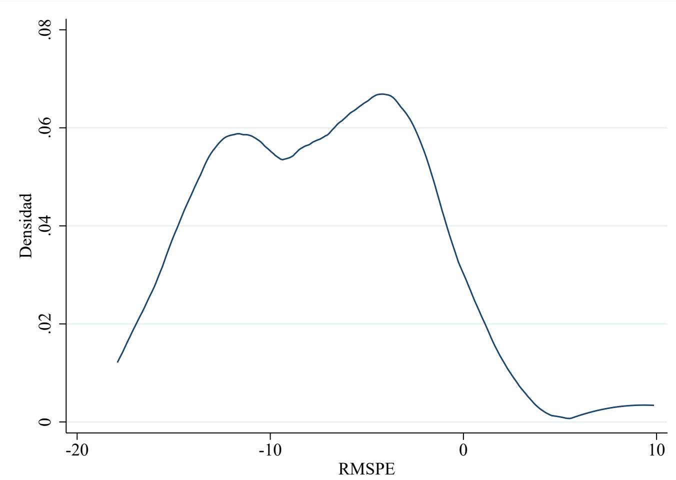
A total of $1,000$ simulations were conducted following the data-generating process structure described in the previous subsection. Specifically, $6$ predictor variables and one dependent variable were used. These variables were simulated for $16$ units, with the first unit being arbitrarily chosen as the object of the predictive procedure. The simulation was carried out for $53$ periods, of which the first $28$ were defined as the pre-treatment period and used for model calibration.

**Figura 2:** *Distribution of Post-treatment RMSPE, by Model*




As shown in Figure 2, the distribution of the RMSPE for the neural network model is considerably more centered towards the left compared to that of the synthetic control model. This implies that, in general, the neural network exhibits better goodness of fit for the post-treatment periods in the simulations. Furthermore, when making more precise comparisons by taking the difference between the RMSPE of the neural network and the synthetic control, we find that the previous intuition is reinforced, as demonstrated by Figure 3. This implies that in the simulations, where the theoretical value of the series is known with certainty, the ANN methodology learns and adjusts the value of the outcome variable to reality more effectively.

**Figura 3:** *Distribution of Post-treatment RMSPE Difference*




A more profound analysis reveals that the improvement due to the use of the proposed ANN model is indeed substantial. In approximately $95\,\%$ of the $1,000$ simulations, the proposed ANN model obtained a lower RMSPE than the estimator proposed by Abadie y Gardeazabal, 2003 for the post-treatment period. Moreover, on average, the RMSPE of the ANN model was about $60\,\%$ of that of the synthetic control, indicating an improvement of nearly $40$ percentage points in terms of goodness of fit. Consequently, the procedures described here strongly suggest that incorporating neural networks into causal inference methodologies that rely on predicting the evolution of variables over time can be highly relevant for obtaining more accurate estimators of the treatment effects of interest.

# 5. Case Study

As mentioned earlier, synthetic control is a methodology implemented in econometric research to construct counterfactuals in scenarios with few treated units. To compare the prediction of the synthetic control with that of the neural network proposed in this study, in addition to conducting simulations, the results of Abadie, Diamond y Hainmueller, 2015 were replicated. The authors studied the effect of the reunification of Germany, which occurred on October 3, 1990, on the economic performance of West Germany measured by per capita GDP in constant 2002 USD.

The authors constructed, using the synthetic control methodology, the counterfactual of the real per capita GDP of West Germany. In other words, they predicted the series that the per capita GDP of this region would have followed in the absence of German reunification.

To construct the counterfactual, the authors used annual panel data from 1960 to 2003 at the country level, with a sample of 16 non-treated countries from the OECD. The pre-reunification period characteristics used for the analysis include per capita GDP, inflation rate, industrial share in value-added, investment rate, schooling, and a measure of trade openness.

By implementing the synthetic control methodology, the authors constructed a synthetic version of West Germany with weights selected through cross-validation in such a way that the constructed counterfactual minimizes the RMSPE and replicates the per capita GDP of this region as closely as possible in the pre-reunification period. Therefore, the years from 1971 to 1980 are used as the testing period, and the years from 1981 to 1990 as the validation period.

Finally, the effect of German reunification on the per capita GDP of West Germany is estimated as the difference between the levels of per capita GDP between West Germany and the synthetic control for that region in the years following reunification. The authors find that during the first two years after reunification, there was no significant effect on the per capita GDP of West Germany. However, after 1992, the growth of per capita GDP in West Germany slowed down compared to the series presented by the synthetic control. Therefore, the authors conclude that German reunification had a negative and persistent effect on the growth path of per capita income in West Germany. In particular, during the period from 1990 to 2003, the per capita GDP of this region decreased by an average of 1,600 USD per year.

## 5.1. Artificial Neural Network Results

To compare the synthetic control methodology with artificial neural networks (ANNs) in constructing a counterfactual, the same exercise as Abadie, Diamond y Hainmueller (2015) was implemented, but using ANNs to compare the RMSPE of each model and understand which methodology provides a better fit to the data before treatment.

**Figura 4:** *Comparison of Synthetic West Germany between both estimations*

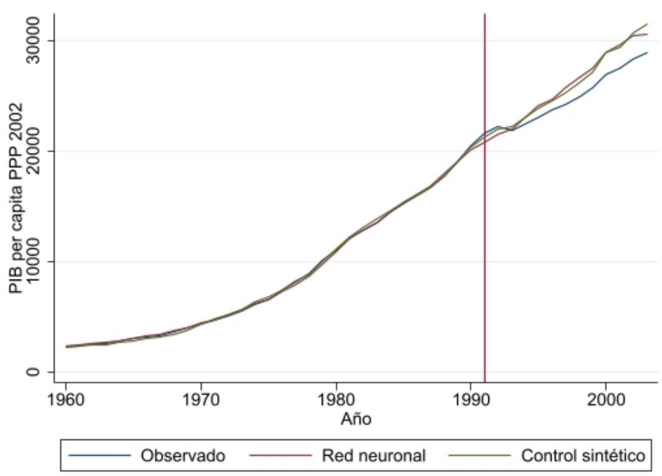
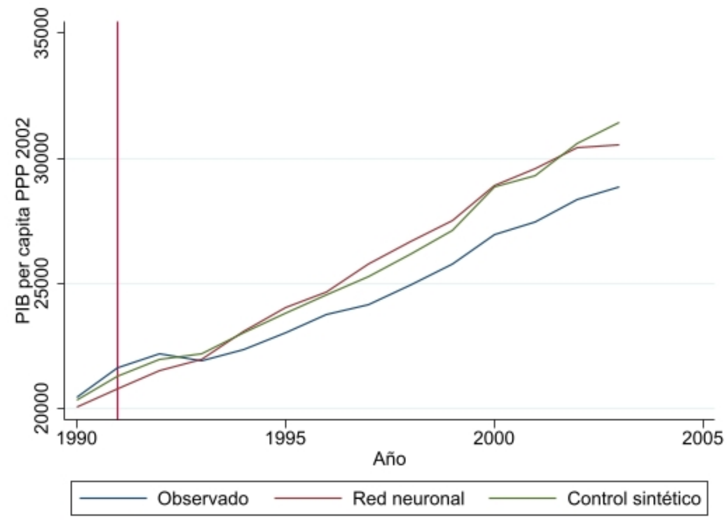



Figure 4 shows the comparison of the synthetic version of West Germany constructed using both methodologies. It was found that the synthetic control methodology of Abadie, Diamond y Hainmueller (2015) had a pre-treatment RMSPE of 399.96, while that of the proposed neural network was 370.72. If the learning of the series by the methodology suggests that the series interpolation will be correct after treatment, then the above implies that the ANN methodology offers better results in producing the counterfactual by replicating the variable of interest more accurately in the pre-treatment period.

**Figura 5:** *Comparison of both estimations in the post-treatment period*




On the other hand, Figure 5 provides a detailed look at the post-treatment periods following the reunification of Germany, where certain differences between both methodologies can be observed. While Abadie, Diamond y Hainmueller (2015) estimate an average reduction of 1,600 USD per capita per year, the ANN methodology estimates a decrease of 1,089 USD per capita per year. Based on the above, it can be established that the authors' results may overestimate the loss experienced by West Germany's reunification on per capita GDP. Furthermore, a clear difference is found in the counterfactual elaborated by ANN compared to the one elaborated through synthetic control during the years 1991-1992. This is because the synthetic control methodology is influenced by serial autocorrelation, which produces an inertial growth in the predicted series in the early periods, whereas this does not occur under the neural network methodology. Thus, based on the estimation presented in this study, it can be inferred that there was indeed a short-term positive effect of German reunification on GDP, which could be attributed to an increase in the labor force and migration to West Germany (Uhlig, 2006).

# 6. Conclusion

In this study, the possibility of using machine learning tools, specifically deep learning, in methodologies related to causal inference was explored. Specifically, the aim was to understand how neural networks can be used to construct synthetic controls that help approximate counterfactuals in contexts where there are few treated units and the series evolve nonlinearly over time.

Based on the results obtained through Monte Carlo experiments, it can be concluded that neural networks have a better fit in both pre-treatment and post-treatment periods compared to the estimation suggested by Abadie y Gardeazabal, 2003; Abadie, Diamond y Hainmueller, 2015. Due to the simulations, the behavior of the post-treatment series is observable, allowing the evaluation of the goodness of fit of both methodologies to these real cases. It was found that neural networks can be a better alternative to the estimator proposed by the aforementioned authors in 95 out of 100 simulations of autoregressive processes. This suggests that the methodology may be more suitable for estimating causal effects in scenarios where there is limited information on explanatory variables and when the data-generating process is nonlinear.

Similarly, the replication of the study by Abadie, Diamond y Hainmueller, 2015 provides a framework for the comparison and application of both methodologies. It was found that the neural network methodology offers a better fit for the pre-treatment period compared to the estimation proposed by Abadie y Gardeazabal, 2003; Abadie, Diamond y Hainmueller, 2015 and even suggests the possibility of overestimating the negative impact of Germany's reunification.

In this regard, the results of this study contribute to knowledge creation in the use of machine learning methodologies to estimate causal effects. This is particularly relevant considering that neural networks appear to be more accurate and less demanding in terms of required information and data-generating processes than synthetic control.

However, this is a preliminary work exploring the idea of using machine learning in the synthetic control methodology, and further research is needed to rigorously analyze the properties of using neural networks in estimation. This includes investigating the trade-off between bias and variance, the possibility of overfitting in neural networks, which may invalidate their generalization ability, the asymptotic properties of the estimation, and the mathematical development of the idea. Additionally, exploring statistical inference techniques is necessary, as this study does not present placebo studies that could be adapted to the neural network methodology. On this matter, the research paths are clear, and there have already been some advances in these fronts, which are expected to be further shared and better documented in a future version.

# Referencias


Abadie, Alberto (2021). “Using synthetic controls: Feasibility, data requirements, and methodological aspects”. En: *Journal of Economic Literature* 59.2, págs. 391-425.

Abadie, Alberto, Alexis Diamond y Jens Hainmueller (2010). “Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California’s Tobacco Control Program”. En: *Journal of the American Statistical Association* 105.490, págs. 493-505. DOI: `10.1198/jasa.2009.ap08746`. eprint: `https://doi.org/10.1198/jasa.2009.ap08746`. URL: `https://doi.org/10.1198/jasa.2009.ap08746`.

– (2015). “Comparative politics and the synthetic control method”. En: *American Journal of Political Science* 59.2, págs. 495-510.

Abadie, Alberto y Javier Gardeazabal (2003). “The economic costs of conflict: A case study of the Basque Country”. En: *American economic review* 93.1, págs. 113-132.

Bohn, Sarah, Magnus Lofstrom y Steven Raphael (2014). “DID THE 2007 LEGAL ARIZONA WORKERS ACT REDUCE THE STATE’S UNAUTHORIZED IMMIGRANT POPULATION?” En: *The Review of Economics and Statistics* 96.2, págs. 258-269. ISSN: 00346535, 15309142. URL: `http://www.jstor.org/stable/43554929` (visitado 04-06-2022).

Crown, William H (2019). “Real-world evidence, causal inference, and machine learning”. En: *Value in Health* 22.5, págs. 587-592.

Cui, Peng y Susan Athey (2022). “Stable learning establishes some common ground between causal inference and machine learning”. En: *Nature Machine Intelligence* 4.2, págs. 110-115.

El Naqa, Issam y Martin J Murphy (2015). “What is machine learning?” En: *machine learning in radiation oncology*. Springer, págs. 3-11.

Kohonen, Teuvo (1988). “An introduction to neural computing”. En: *Neural Networks* 1.1, págs. 3-16. ISSN: 0893-6080. DOI: `https://doi.org/10.1016/0893-6080(88)90020-2`. URL: `https://www.sciencedirect.com/science/article/pii/0893608088900202`.

López, R.F. y J.M.F. Fernández (2008). *Las Redes Neuronales Artificiales*. Metodología y Análisis de Datos en Ciencias Sociales. Netbiblo. ISBN: 9788497452465. URL: `https://books.google.com.co/books?id=X0uLwi1Ap4QC`.

Steinkraus, Arne (2019). “Estimating Treatment Effects With Artificial Neural Nets—A Comparison to Synthetic Control Method”. En: *Econ. Bull* 39.4, págs. 2778-2791.

Tu, Jack. V. (1996). “Advantages and disadvantages of using artificial neural networks versus logistic regression for predicting medical outcomes”. En: *Journal of Clinical Epidemiology* 49.11, págs. 1225-1231. ISSN: 0895-4356. DOI: `https://doi.org/10.1016/S0895-4356(96)00002-9`. URL: `https://www.sciencedirect.com/science/article/pii/S0895435696000029`.

Uhlig, Harald (2006). “Regional Labor Markets, Network Externalities and Migration: The Case of German Reunification”. En: *American Economic Review* 96.2, págs. 383-387. DOI: `10.1257/000282806777212260`. URL: `https://www.aeaweb.org/articles?id=10.1257/000282806777212260`.