Home assignment number 2, 2020, in SF2863 Systems Engineering.

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Questions should be posed in the discussion group in canvas.

The home assignments are a mandatory part of the course. In total, there are two home assignments and you need to collect 4.0 out of 6.0 points in order to pass this part of the course. Home assignment 2 can give up to 4.0 points. The home assignments can, in addition, give bonus points on the exam. If a home assignment is handed in before the deadline, then the points awarded on the assignment will also count as bonus points on the exam (hence, home assignment 2 can give up to 4.0 bonuspoints on the exam). In order to get bonus points on the exam the report should be submitted to canvas before 23:59, Tuesday, December 15, 2020.
This home assignment should be done in groups of 2 persons.
See the information on canvas on how to form groups and submit your results. One report per group is to be handed in.
In the report your should describe in your own words how the problem was solved. In particular, carefully describe how you defined your states, modelled the problem and implemented the simulations. Do not explain the code, but rather the mathematical ideas. You should not need more than 6 pages for the report. The answers to the questions in the assignment should be given in the main report and numbered as in this text. In particular, it should not be necessary to look at your code to understand how you define and calculate things. Relevant print-outs and plots should be included in the report. You may use any computer program to facilitate the computations, but you have to include your computer code in the submission. We use automated plagiarism-control tools and will also make random checks that you have not copied parts of another groups report or computer code!
Questions should be posed in the discussion group in canvas.
State your name, and email adress on the front of the report.

The purpose of this home assignment is to stimulate the understanding of how the spare parts optimization theory can be applied to a small test-example.

Problem statement

We consider the case where aircrafts are equipped with 9 different “line replaceable units”, $LRU_1, \cdots, LRU_9$ representing wheels, brakes, hydraulics, electric actuators, fuel systems, landing gears, engines, radio, and radar.
Consider one basis to which there is a (random) arrival of airplanes with an LRU_j unit in need of repair. A malfunctioning LRU_j unit arrives with intensity λ_j, described by the vector

$$\lambda = [50, \quad 40, \quad 45, \quad 51, \quad 25, \quad 48, \quad 60, \quad 35, \quad 15]/1000$$

When a malfunctioning LRU_j unit arrives to a base it is replaced immediately with a functioning LRU_j unit from the local inventory, as long as it is not empty. If there is no functioning LRU_j unit in the local inventory to replace the malfunctioning LRU_j unit with, then there will be a local inventory queue, a backorder is issued and the aircraft is grounded and has to wait for a functioning LRU_j unit to arrive from the repair shop.
The malfunctioning LRU_j unit is directly sent to the local repair shop. The repair time for a malfunctioning LRU_j unit is assumed, on average, to be T_j hours, where

$$T_1 = 4,\ T_2 = 7,\ T_3 = 14,\ T_4 = 5,\ T_5 = 10,\ T_6 = 18,\ T_7 = 24,\ T_8 = 8,\ T_9 = 12.$$

The purchase cost per unit for LRU_j is c_j where

$$c_1 = 14,\ c_2 = 19,\ c_3 = 25,\ c_4 = 15,\ c_5 = 10,\ c_6 = 45,\ c_7 = 80,\ c_8 = 33,\ c_9 = 30.$$

Today there are no spare LRU units, and as a consequence it happens regularly that planes are grounded. This is now going to change, some money is reserved to purchase spare LRUs and you should implement the process.

Assignments

The assignment consist of solving the spare parts optimization problem using two different approaches. The first using Marginal Allocation and the second using Dynamic programming. Each part of the home assignment (Marginal Allocation/Dynamic programming) can yield up to 2.0 points. The theory of the Dynamic Programming comes a bit later in the course and you may have to wait a bit before you can start working on that part.

Marginal Allocation

1. Define functions f and g used for solving the given problem, and show that they satisfy all required assumptions to apply Marginal Allocation.

2. Use the marginal allocation algorithm to determine all the efficient points of the multiobjective optimization problem to minimize the EBO and total cost for spares when the total budget is bounded by $C_{\text{budget}} = 500$.

Start with no spares and determine how the expected number of grounded airplanes decrease when adding each new spare LRU unit.

3. Plot the efficient curve and determine in a table the efficient solutions, the corresponding objective function values and the marginal decrease in EBO for each new unit of cost spent on the LRU spare units.

Dynamic Programming

Now assume that we want to determine the best spare parts setup for a certain specified budget, *i.e.*, we want to solve the problem

$$\left[\begin{array}{ll} \min\limits_{s} & \text{EBO}(s) \\ s.t. & C(s) \leq C_{\text{budget}} \end{array}\right]$$

where $C_{\text{budget}} = 500$.
This is an integer programming problem, but with some nice properties. Use dynamic programming to solve the problem for the specified budget, by determining the solutions to all problems of the same type but with a smaller budget.

4. Define carefully the states, stages, decisions, state-update equations, value functions, and recursive relation of the value function.

Explain your notation and your solution algorithm with great care. We will not decode your implementation.

5. Solve the optimization problem, using dynamic programming based on the setting in 4, for every choice of $C_{\text{budget}} = 0, 1, \cdots, 500$.

Because there are so many solutions you do NOT have to describe all of them in a table in the report. It is enough to tabulate optimal allocations, total costs and EBO, for $C_{\text{budget}} = 0, 100, 150, 350, 500$ in the report.

6. Finally, compare the solutions from this problem to the efficient solutions determined in the first part of the assignment by plotting them in the same EBO *vs.* cost graph. Is the result reasonable?

Please note that the Dynamic programming approach has to be applied, solutions based on total enumeration of all possible combinations will not be rewarded any points.

Good luck!