Danielle Mushillo
Michael Langer
COMP 251 : Algorithms and Data Structures
March 30, 2014

Assignment 4 : Q2

Algorithm:

$t(n)$ Find Closest Pair (x, y) {
$t(n\log n)$ $y' \leftarrow$ sort y
$t(c_1 n)$ Compute x_R, x_L, y_R, y_L // $y_R = y_R'$, $y_L = y_L'$
$t(n/2)$ Find Closest Pair (x_L, y_L)
$t(n/2)$ Find Closest Pair (x_R, y_R)
$t(c_2 n)$ Find Closest Crossing Pair (y')
c_3 Return min
}

$$t(n) = 2t(\tfrac{n}{2}) + n\log n + c_1 n + c_2 n + c_3$$

$k=1$ $$= 2\left(2t(\tfrac{n}{4}) + (\tfrac{n}{2})\log(\tfrac{n}{2}) + c_1(\tfrac{n}{2}) + c_2(\tfrac{n}{2}) + c_3\right) + n\log n + c_1 n + c_2 n + c_3$$

$$= 4t(\tfrac{n}{4}) + \underbrace{2(\tfrac{n}{2})\log(\tfrac{n}{2}) + 2nc_1 + 2nc_2 + 2c_3}_{\tau} + n\log n$$

$k=2$ $$= 4\left(2t(\tfrac{n}{8}) + \tfrac{n}{4}\log(\tfrac{n}{4}) + (c_1+c_2)\tfrac{n}{4} + c_3\right) + \tau + n\log n$$

$$= 8t(\tfrac{n}{8}) + n\log(\tfrac{n}{4}) + (c_1+c_2)n + 4c_3 + \tau + n\log n$$

$k=3$ $$= 8t(\tfrac{n}{8}) + \left(n(\log n + \log\tfrac{n}{2} + \log\tfrac{n}{4})\right) + 3nc_1 + 3nc_2 + (1+2+4)c_3$$

$k=k$ $$= 2^k t(\tfrac{n}{2^k}) + n\cdot\sum_{i=0}^{k-1}\log(\tfrac{n}{2^i}) + knc_1 + knc_2 + \sum_{i=0}^{k-1} 2^i c_3$$

Stops when $\frac{n}{2^k} = 1 \Rightarrow n = 2^k$
$k = \log n$

$$= n\,t(1) + n\sum_{i=0}^{\log n - 1}\log(\tfrac{n}{2^i}) + n\log n\, c_1 + n\log n\, c_2 + c_3\sum_{i=0}^{k-1} 2^i$$

$$= \frac{1-2^k}{1-2} = 2^k - 1$$

$$\log n/2^i = \log n - \log 2^i$$
$$= \log n - i$$

So: $$\sum_{i=0}^{\log n-1}\log(\tfrac{n}{2^i}) = \sum_{i=0}^{\log n-1} \log n - i = (\log n)\log n - \frac{\log n(\log n - 1)}{2} = (\log n)^2 - \frac{(\log n)^2}{2} + \frac{\log n}{2}$$

$$= nt(1) + n\left((\log n)^2 + \frac{(\log n)^2}{2} + \frac{\log n}{2}\right) + (c_1 + c_2)n\log n + c_3\underbrace{(2^k - 1)}_{\substack{2^{\log n} - 1 \\ = n - 1}}$$

$$\boxed{t(n) = nt(1) + \frac{n\log^2 n}{2} + \frac{n\log n}{2} + (c_1 + c_2)n\log n + c_3(n-1)}$$