Full Structural Equation Modeling

Theory Construction and Statistical Modeling



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Outline

Structural Equation Modeling Measurement Model Structural Model

Mediation with SEM

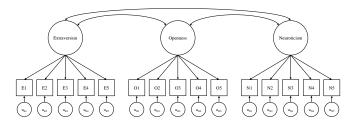


Full SEM

A full structural equation model (SEM) simply combines path analysis and CFA.

 SEM allows us to model complicated structural relations among latent variables.

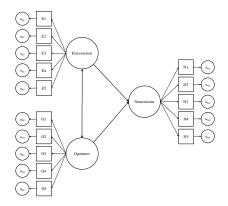
Let's consider a simple, three-factor CFA model.



$CFA \rightarrow SEM$

We first evaluate the validity of the measurement model via CFA.

 We then convert the CFA to an SEM by converting some covariances to latent regression paths.



```
## Load the lavaan package and some data:
library(lavaan)
data(bfi, package = "psych")
## Specify the CFA model:
cfaMod <- '
extra = E1 + E2 + E3 + E4 + E5
open = 01 + 02 + 03 + 04 + 05
neuro = N1 + N2 + N3 + N4 + N5
## Estimate the model:
cfaOut <- cfa(cfaMod, data = bfi, missing = "fiml", std.lv = TRUE)
## Check the fit:
fitMeasures(cfaOut,
           c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr")
                  pvalue cfi tli
  chisq
          df
                                             rmsea
                                                       srmr
2251.679 87.000
                   0.000 0.809 0.769
                                            0.094
                                                      0.081
```

```
partSummary(cfaOut, 7)
Latent Variables:
                    Estimate
                               Std.Err
                                        z-value P(>|z|)
  extra =~
    E1
                       0.973
                                 0.032
                                         30,607
                                                    0.000
    E2
                       1.163
                                 0.030
                                         38.171
                                                    0.000
    E3
                      -0.815
                                 0.027
                                        -30.358
                                                    0.000
    E4
                      -0.979
                                 0.028
                                        -35.254
                                                    0.000
    E5
                      -0.714
                                 0.027
                                        -26.638
                                                    0.000
  open =~
    01
                       0.630
                                 0.025
                                         24.886
                                                    0.000
    02
                      -0.605
                                 0.036
                                        -16.781
                                                    0.000
    03
                       0.897
                                 0.029
                                         30.765
                                                    0.000
    Π4
                       0.290
                                 0.028
                                         10.402
                                                    0.000
    05
                      -0.602
                                 0.031
                                        -19.734
                                                    0.000
  neuro =~
    N1
                       1.272
                                 0.027
                                         47.254
                                                    0.000
    N2
                       1.218
                                 0.026
                                         46,491
                                                    0.000
    NЗ
                       1.157
                                 0.029
                                         40.195
                                                    0.000
    N4
                       0.892
                                 0.030
                                         29.356
                                                    0.000
    N5
                       0.823
                                 0.031
                                          26.163
                                                    0.000
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```

```
partSummary(cfaOut, 8)
Covariances:
                           Std.Err z-value P(>|z|)
                  Estimate
 extra ~~
                   -0.444 0.024 -18.472
                                              0.000
   open
                    0.240
                             0.023 10.551
                                              0.000
   neuro
 open ~~
                    -0.117
                             0.025
                                     -4.667
                                              0.000
   neuro
```

```
partSummary(cfaOut, 9)
Intercepts:
                    Estimate
                               Std.Err
                                         z-value
                                                   P(>|z|)
   .E1
                        2.974
                                 0.031
                                          96.223
                                                     0.000
   .E2
                        3.143
                                 0.030
                                         103,424
                                                     0.000
   .E3
                       4.002
                                 0.026
                                         156.117
                                                     0.000
   .E4
                       4.421
                                 0.028
                                         160.350
                                                     0.000
   .E5
                       4.417
                                 0.025
                                         174,595
                                                     0.000
   .01
                       4.816
                                 0.021
                                         224.964
                                                     0.000
   .02
                        2.713
                                 0.030
                                          91.745
                                                     0.000
   .03
                       4.436
                                 0.023
                                         191.555
                                                     0.000
   .04
                       4.892
                                 0.023
                                         211.519
                                                     0.000
   .05
                        2.490
                                 0.025
                                          98.932
                                                     0.000
   .N1
                        2.932
                                 0.030
                                          98.589
                                                     0.000
   .N2
                        3.508
                                 0.029
                                         121.459
                                                     0.000
   .N3
                        3,217
                                 0.030
                                         106,147
                                                     0.000
   .N4
                                 0.030
                                                     0.000
                        3.185
                                         106.894
   .N5
                        2,969
                                 0.031
                                          96,663
                                                     0.000
                        0.000
    extra
                        0.000
    open
                        0.000
    neuro
 8 of 25
```

```
partSummary(cfaOut, 10)
Variances:
                     Estimate
                                Std.Err
                                         z-value
                                                   P(>|z|)
   .E1
                        1.713
                                  0.054
                                          31.442
                                                      0.000
   .E2
                        1.224
                                  0.049
                                          24.952
                                                      0.000
   .E3
                        1.163
                                 0.038
                                          30.388
                                                      0.000
   .E4
                        1.166
                                  0.041
                                          28.522
                                                      0.000
   .E5
                        1.272
                                  0.039
                                          32,789
                                                      0.000
   .01
                        0.878
                                  0.031
                                          28.320
                                                      0.000
   .02
                        2.083
                                  0.062
                                          33,705
                                                      0.000
   .03
                        0.686
                                  0.043
                                          16.130
                                                      0.000
   .04
                        1.407
                                  0.039
                                          36.236
                                                      0.000
   .05
                        1.401
                                  0.044
                                          31.837
                                                      0.000
   .N1
                        0.848
                                  0.037
                                          23.029
                                                      0.000
   .N2
                        0.842
                                  0.035
                                          24.184
                                                      0.000
   .N3
                        1.228
                                  0.043
                                          28,308
                                                      0.000
   .N4
                        1.666
                                  0.051
                                          32.808
                                                      0.000
   .N5
                        1.942
                                  0.056
                                           34,465
                                                      0.000
                        1.000
    extra
                        1.000
    open
                        1.000
    neuro
 9 of 25
```

```
## Add structural paths:
semMod <- '
extra = E1 + E2 + E3 + E4 + E5
open = 01 + 02 + 03 + 04 + 05
neuro = N1 + N2 + N3 + N4 + N5
neuro ~ extra + open
## Estimate the model:
semOut <- sem(semMod, data = bfi, missing = "fiml", std.lv = TRUE)</pre>
## Check the fit:
fitMeasures(semOut,
           c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr")
  chisq
             df pvalue cfi tli
                                            rmsea
                                                      srmr
2251.679 87.000 0.000
                          0.809 0.769
                                                     0.081
                                            0.094
```

```
partSummary(semOut, 7)
Latent Variables:
                    Estimate
                               Std.Err
                                        z-value P(>|z|)
  extra =~
    E1
                       0.973
                                 0.032
                                         30,607
                                                    0.000
    E2
                       1.163
                                 0.030
                                         38.172
                                                    0.000
    E3
                      -0.815
                                 0.027
                                        -30.358
                                                    0.000
    E4
                      -0.979
                                 0.028
                                        -35.254
                                                    0.000
    E5
                      -0.714
                                 0.027
                                        -26.638
                                                    0.000
  open =~
    01
                       0.630
                                 0.025
                                         24.886
                                                    0.000
    02
                      -0.605
                                 0.036
                                        -16.781
                                                    0.000
    03
                       0.897
                                 0.029
                                         30.765
                                                    0.000
    Π4
                       0.290
                                 0.028
                                        10.402
                                                    0.000
    05
                      -0.602
                                 0.031
                                        -19.734
                                                    0.000
  neuro =~
    N1
                       1.235
                                 0.027
                                         45.916
                                                    0.000
    N2
                       1.183
                                 0.026
                                         45,360
                                                    0.000
    NЗ
                       1.123
                                 0.028
                                         39.976
                                                    0.000
    N4
                       0.866
                                 0.029
                                         29.605
                                                    0.000
    N5
                       0.799
                                 0.031
                                         26.204
                                                    0.000
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```

```
partSummary(semOut, 8:9)
Regressions:
                          Std.Err z-value P(>|z|)
                 Estimate
 neuro ~
   extra
                   0.241 0.030
                                  8.169
                                            0.000
                  -0.014 0.031 -0.448
                                            0.654
   open
Covariances:
                 Estimate Std.Err z-value P(>|z|)
 extra ~~
                  -0.444 0.024 -18.472
                                            0.000
   open
```

```
partSummary(semOut, 10)
Intercepts:
                    Estimate
                               Std.Err
                                         z-value
                                                   P(>|z|)
   .E1
                        2.974
                                 0.031
                                          96.223
                                                     0.000
   .E2
                        3.143
                                 0.030
                                         103,424
                                                     0.000
   .E3
                       4.002
                                 0.026
                                         156.117
                                                     0.000
   .E4
                       4.421
                                 0.028
                                         160.350
                                                     0.000
   .E5
                       4.417
                                 0.025
                                         174,595
                                                     0.000
   .01
                       4.816
                                 0.021
                                         224.964
                                                     0.000
   .02
                        2.713
                                 0.030
                                          91.745
                                                     0.000
   .03
                       4.436
                                 0.023
                                         191.555
                                                     0.000
   .04
                       4.892
                                 0.023
                                         211.520
                                                     0.000
   .05
                        2.490
                                 0.025
                                          98.932
                                                     0.000
   .N1
                        2.932
                                 0.030
                                          98.589
                                                     0.000
   .N2
                        3.508
                                 0.029
                                         121,459
                                                     0.000
   .N3
                        3,217
                                 0.030
                                         106,146
                                                     0.000
   .N4
                                 0.030
                                                     0.000
                        3.185
                                         106.894
   .N5
                        2,969
                                 0.031
                                          96,663
                                                     0.000
                        0.000
    extra
                        0.000
    open
                        0.000
   .neuro
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```

```
partSummary(semOut, 11)
Variances:
                     Estimate
                                Std.Err
                                         z-value
                                                   P(>|z|)
   .E1
                        1.713
                                  0.054
                                          31.442
                                                      0.000
   .E2
                        1.224
                                  0.049
                                          24.952
                                                      0.000
   .E3
                        1.163
                                 0.038
                                          30.388
                                                      0.000
   .E4
                        1.166
                                  0.041
                                          28.522
                                                      0.000
   .E5
                        1.272
                                  0.039
                                          32,789
                                                      0.000
   .01
                        0.878
                                 0.031
                                          28.320
                                                      0.000
   .02
                        2.083
                                  0.062
                                          33,705
                                                      0.000
   .03
                        0.686
                                  0.043
                                          16.130
                                                      0.000
   .04
                        1.407
                                  0.039
                                          36.236
                                                      0.000
   .05
                        1.401
                                  0.044
                                          31.837
                                                      0.000
   .N1
                        0.848
                                  0.037
                                          23.029
                                                      0.000
                                          24.184
   .N2
                        0.842
                                  0.035
                                                      0.000
   .N3
                        1.228
                                  0.043
                                          28,308
                                                      0.000
   .N4
                        1.666
                                  0.051
                                          32.808
                                                      0.000
   .N5
                        1.942
                                  0.056
                                          34,465
                                                      0.000
                        1.000
    extra
                        1.000
    open
                        1.000
   .neuro
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```

Why SEM?

With SEM we get to model the types of complex relations we can specify via path models while leveraging all the strengths of latent variables.

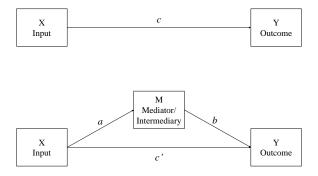
- Multiple-group SEM models moderation by group.
 - The latent variables give us the ability to evaluate measurement invariance across groups.
 - We'll see more of these ideas in the next lecture.
- Path analysis and SEM lend themselves especially well to mediation analysis and conditional process analysis.

MEDIATION WITH SEM



Boring Model

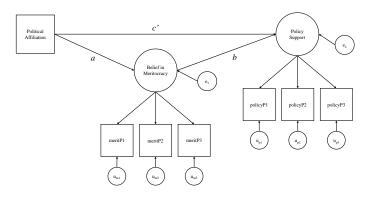
In Week 3, all of our models have looked something like:



But there is no reason that we need to restrict ourselves to observed variables.

Better Model

We can (and should) test for indirect effects using full SEMs such as:



Measurement error can be a big problem for mediation analysis, so latent variable modeling is highly recommended.

```
dat1 <- readRDS("../data/adamsKlpsData.rds") %>% select(-merit, -policy)
## Specify the CFA model:
mod5.1 <- '
merit = meritP1 + meritP2 + meritP3
policy = policyP1 + policyP2 + policyP3
## Fit the CFA and check model:
out5.1 <- cfa(mod5.1, data = dat1, std.lv = TRUE)
## Check model fit:
fitMeasures(out5.1.
           c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr")
chisq df pvalue cfi tli rmsea
                                           srmr
16.869 8.000 0.031 0.922 0.853 0.113 0.065
```

```
partSummary(out5.1, 7)
Latent Variables:
                             Std.Err z-value P(>|z|)
                   Estimate
  merit =~
    meritP1
                      0.690
                               0.134
                                        5.155
                                                  0.000
    meritP2
                      0.968
                              0.142
                                        6.830
                                                  0.000
    meritP3
                      0.748
                               0.137
                                        5.458
                                                  0.000
  policy =~
    policyP1
                      0.851
                               0.186
                                        4.570
                                                  0.000
    policyP2
                      0.996
                               0.167
                                        5.967
                                                  0.000
    policyP3
                      1.121
                                         6.339
                               0.177
                                                  0.000
```

```
partSummary(out5.1, 8:9)
Covariances:
                  Estimate
                           Std.Err z-value P(>|z|)
 merit ~~
   policy
                    -0.336
                             0.131
                                     -2.563
                                               0.010
Variances:
                  Estimate
                           Std.Err
                                    z-value
                                             P(>|z|)
   .meritP1
                     0.865
                             0.165
                                      5.248
                                               0.000
   .meritP2
                    0.445
                           0.201 2.211
                                               0.027
   .meritP3
                    0.833 0.172 4.857
                                               0.000
   .policyP1
                    1.836
                           0.324 5.671
                                               0.000
                            0.256
                                               0.000
   .policyP2
                    0.942
                                      3.683
   .policyP3
                    0.857
                            0.297
                                      2.882
                                               0.004
   merit
                     1,000
   policy
                     1,000
```

```
partSummary(out5.2, 7:8)
Latent Variables:
                            Std.Err z-value P(>|z|)
                  Estimate
 merit =~
   meritP1
                     0.545
                             0.125
                                      4.347
                                               0.000
   meritP2
                     0.858
                            0.378
                                      2.271
                                               0.023
   meritP3
                     0.609
                            0.118
                                      5.171
                                               0.000
 policy =~
   policyP1
                     0.799
                            0.188
                                      4.258
                                               0.000
   policyP2
                     0.924 1.281
                                      0.722
                                               0.471
   policyP3
                     1.001
                              1.036
                                      0.966
                                               0.334
Regressions:
                  Estimate
                            Std.Err
                                     z-value P(>|z|)
 policy ~
   merit
              (b)
                    -0.195
                             0.203
                                     -0.961
                                               0.336
   polAffil
                     0.169
                             0.137
                                      1.236
                                               0.217
 merit ~
   polAffil
              (a)
                    -0.411
                             0.100
                                     -4.122
                                               0.000
```

```
partSummary(out5.2, 9:10)
Variances:
                  Estimate
                           Std.Err z-value
                                             P(>|z|)
   .meritP1
                     0.922
                             0.181
                                      5.086
                                               0.000
   .meritP2
                    0.341
                           7.972
                                      0.043
                                               0.966
   .meritP3
                    0.869 0.182
                                      4.785
                                              0.000
   .policyP1
                     1.801 0.326 5.520
                                              0.000
   .policyP2
                    0.918
                            70.763
                                      0.013
                                               0.990
   .policyP3
                    0.922
                            41.876
                                      0.022
                                               0.982
   .merit
                    1,000
   .policy
                     1.000
Defined Parameters:
                           Std.Err
                                    z-value
                                             P(>|z|)
                  Estimate
   ab
                     0.080
                             0.089
                                      0.894
                                               0.371
```

```
parameterEstimates(out5.2, boot.ci.type = "bca.simple") %>%
select(c("label", "est", "ci.lower", "ci.upper")) %>%
filter(label != "")

label    est ci.lower ci.upper
1    b -0.195    -0.620    0.182
2    a -0.411    -0.631    -0.245
3    ab    0.080    -0.074    0.289
```