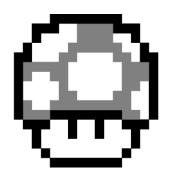
#### **CS1010S Programming Methodology**

# Lecture 6 Working with Sequences

18 Feb 2014

### 2D & 3D Rune Contest

#### 2D Runes - Notable Mention











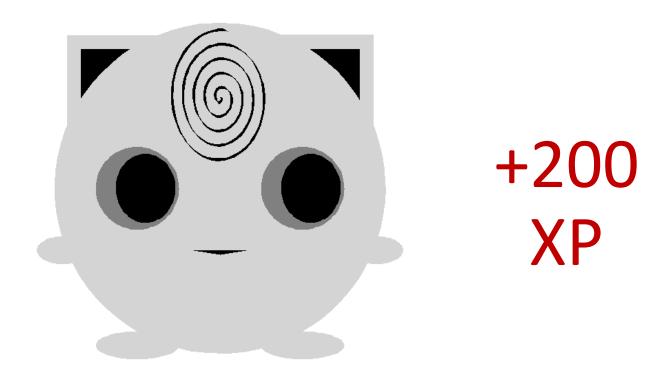




- Huang Weiqi Victor
- Guan Xibeijia
- Adrian Tan Hong Ji
- Zhang Peixu

+100 XP

#### 2D Runes - Third Place



Wang Si Qi

#### 2D Runes – Second Place



+300 XP

**Zhang Peixu** 

#### 2D Runes – First Place



Guan Xibeijia

## 3D Anaglyphs – Notable Mentions





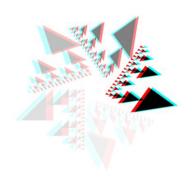


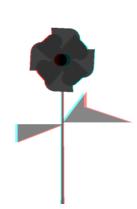




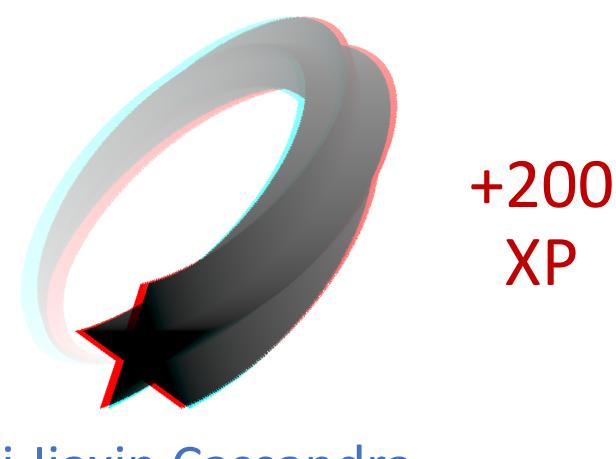
- Wang Si Qi
- Tan Wei Liang
- Benjamin Ong
- Yeo Xin Yi
- Muhammad Haikal

+100 XP



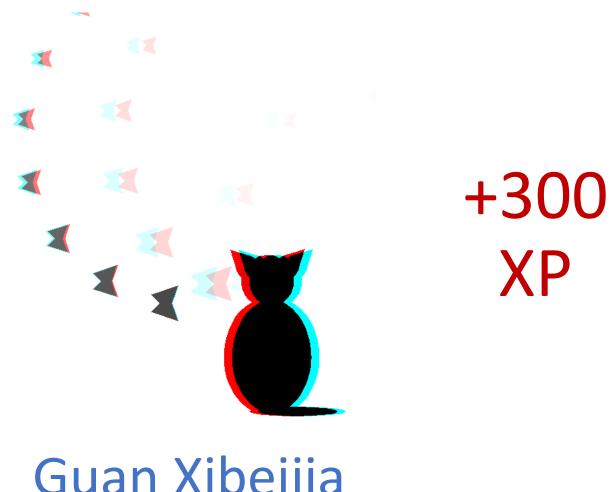


#### 3D Anaglyphs – Third Place



Li Jiaxin Cassandra

#### 3D Anaglyphs – Second Place



Guan Xibeijia

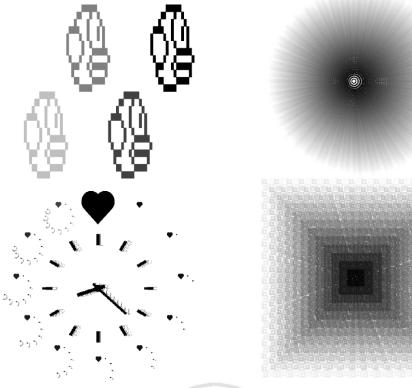
#### 3D Anaglyphs – First Place



+400 XP

Jacinda Siew Xinying

#### 3D Hollusion – Notable Mentions

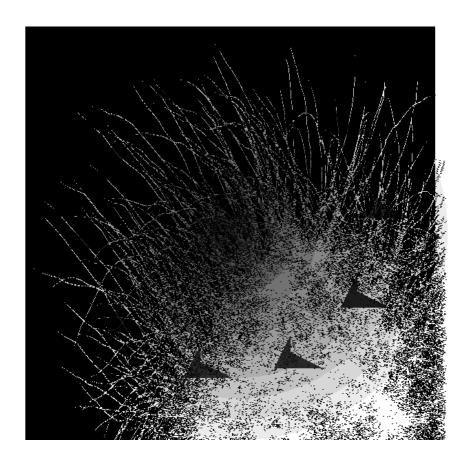


- Fang Gian Yao
- Samuel Tan
- Lee Jia Hui
- Liang Tian Ze
- Lester Sim

+100 XP



#### 3D Hollusion – Third Place



+200 XP

Franklin Leonardo

#### 3D Hollusion – Second Place



**Zhang Xiaoyue** 

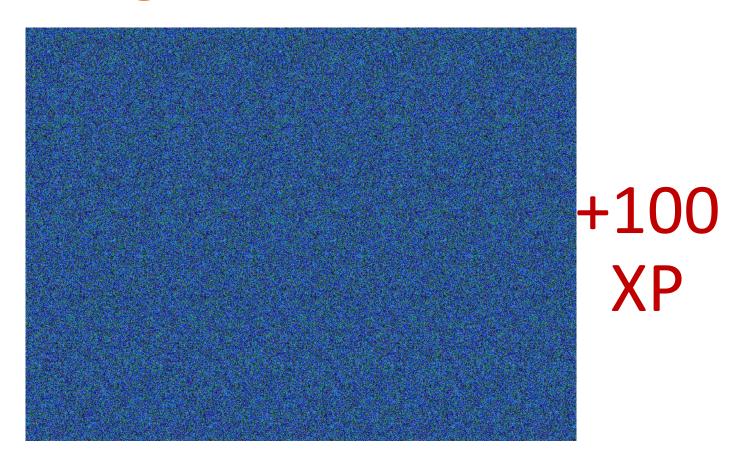
#### 3D Hollusion – First Place



+400 XP

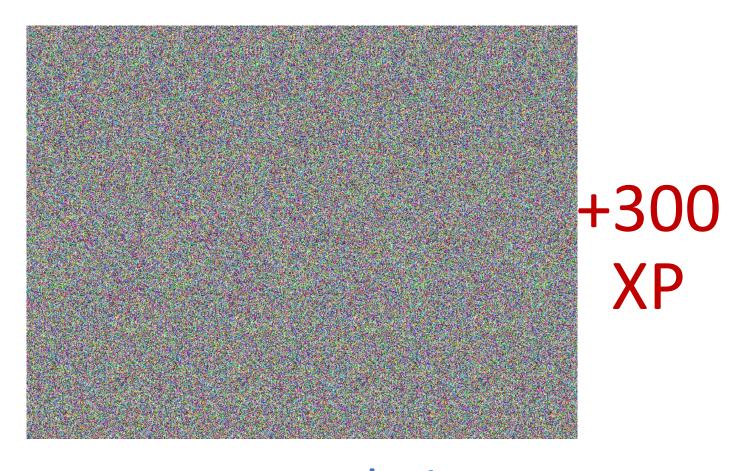
Yu Yingru

#### 3D Stereogram - Notable Mention



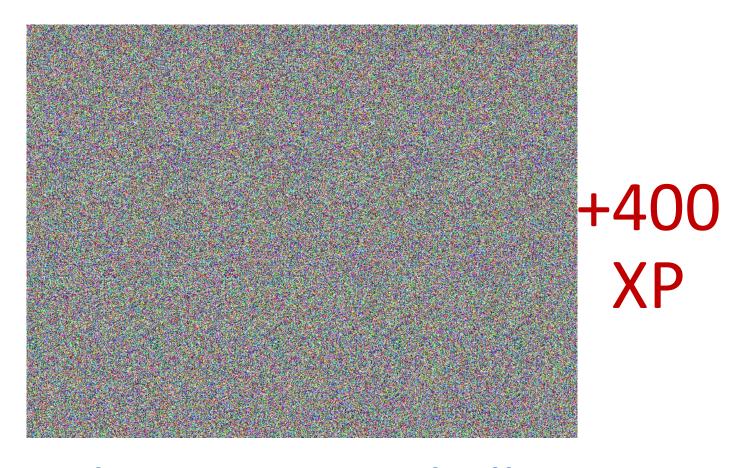
**Zhang Peixu** 

#### 3D Stereogram – Second Place



Darren Wee Thai Yuan

#### 3D Stereogram – First Place



Chan Jia Hui Isabella

#### Make-up Recitation

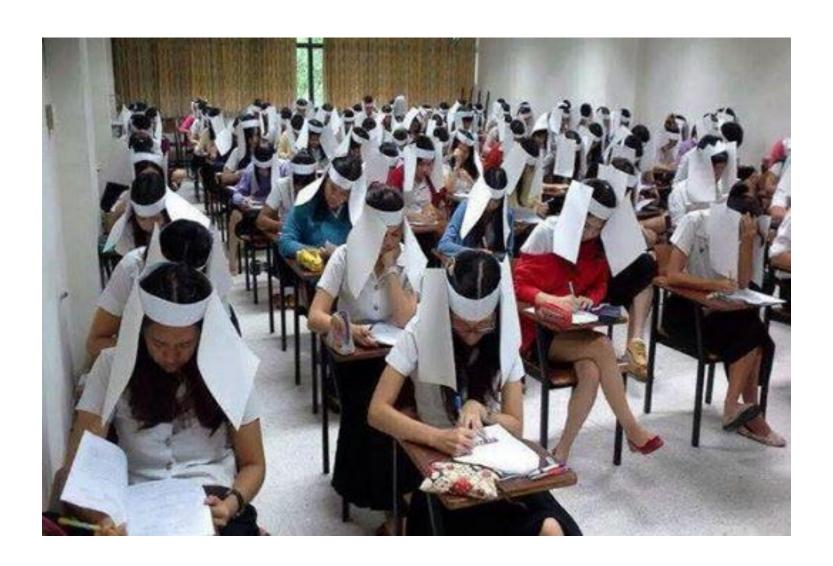
- Monday 23 Feb (COM2-04-02)
  - 11 12 noon
  - -1-2 pm
- Tuesday 24 Feb (COM1-02-01)
  - 10 11 am
  - 11 12 noon
  - -1-2 pm
- Wednesday 25 Feb (COM1-02-04)
  - 10 11 am

#### E-Learning Week



#### Midterm Exam

- Venue: MPSH 1
- Open-sheet exam (no laptops!)
  - 1 x A4 sheet (both sides)
- Scope: everything up to and including Lecture 5 (Data Abstraction)
- Past Year Exams have been uploaded to Coursemology



#### Midterm Exam

- Python Expressions
- Solving Problems with Recursion/Iteration
  - Order of Growth
- Higher Order Functions
- Data Abstraction
  - Define new Abstract Data Type +
     Operations

## Only 15%

Don't Stress

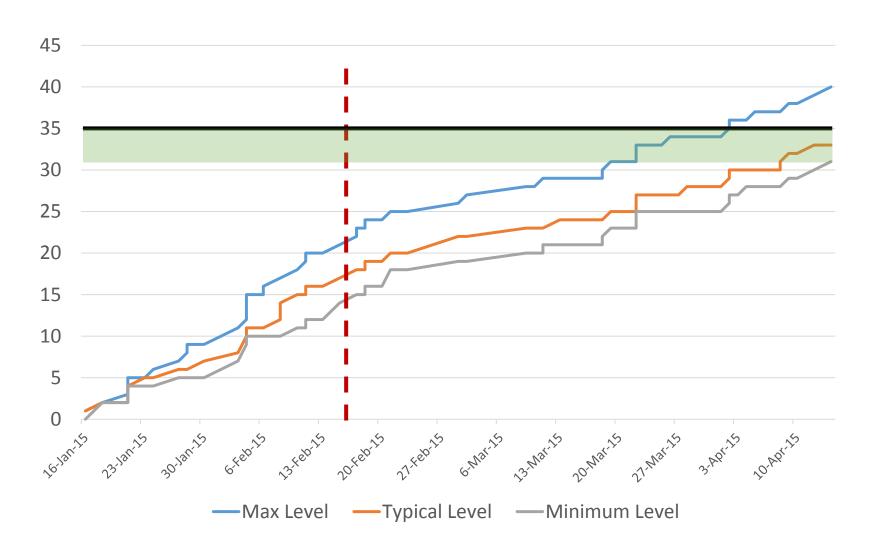
#### Help is Coming

- Remedial Sessions
  - 24 Feb (Tue), 6:30 8:30 pm
  - 27 Feb (Sat), 10:30 12 pm
- Exam Review
  - 26 Feb (Thurs)
- "Desperado" Session
  - 3 Mar (Tues)

#### No Tutorials & Recitations

- No Recitations on midterm week
- Tutors will still be at the lab on Mon/Tues during tutorial times for consultation

#### **EXP** curve



#### Today's Agenda

- Processing Sequences
  - Recursion & Iteration
- Tree as nested sequences
  - Hierarchical structures
- Signal-processing view of Computations
- Working with Files

#### Recap: Data Abstraction

- Abstracts away irrelevant details, exposes what is necessary
- Separates usage from implementation.
- Captures common programming patterns
- Serves as a building block for other compound data.

#### Key idea

- Decide on an internal representation of the Abstract Data Type (ADT) Tuple!
- Write functions that operate on that new ADT

Key insight: nobody needs to know your internal representation to use your ADT

# Guidelines for Creating Compound Data

- Constructors
  - To create compound data from primitive data
- Selector (Accessors)
  - To access individual components of compound data
- Predicates
  - To ask (true/false) questions about compound data
- Printers
  - To display compound data in human-readable form

#### Sequences

- Sequential data, represented by tuples
- Get the first element of the list:

```
seq[0]
```

Get the rest of the elements:

```
seq[1:]
```

If a list is a tuple containing a single integer 4:

```
seq = (4,)
seq[0] \rightarrow 4
seq[1:] \rightarrow ()
```

#### Reversing a Sequence

- Notice that (seq[0],) is a tuple and not an integer
- Can only concatinate tuples with tuples

#### Orders of Growth

```
def reverse(seq):
                              Iterative
      result = ()
     for item in seq:
          result = (item,) + result
      return result
tuple1 + tuple2 takes len(tuple1) + len(tuple2) steps!
 Orders of growth:
                         Time
                                 Space
                        O(n^2)
                                 O(n^{2})
   - Recursive version:
```

 $O(n^2)$ 

O(n)

Iterative version:

### Key Idea:

# Handle the First Element and then the Rest

Iterate down the sequence!

#### Scaling a sequence

Suppose we want to scale all the elements of a sequence by some factor  $scale_seq((1, 2, 3, 4), 3) \rightarrow (3, 6, 9, 12)$ def scale\_seq(seq, factor): **if** seq == (): Time?  $O(n^2)$ return () Space?  $O(n^2)$ else: return (seq[0] \* factor,) + scale seq(seq[1:], factor)

#### Scaling a sequence (iterative)

Suppose we want to scale all the elements of a sequence by some factor

```
scale_seq((1, 2, 3, 4), 3) \rightarrow (3, 6, 9, 12)
def scale_seq(seq, factor):
    result = ()
    for element in seq:
        result = result + (element * factor,)
    return result
                            Time? O(n^2)
                            Space? O(n)
```

# Squaring a sequence

Given a sequence, we want to return a sequence of the squares of all elements.

```
square_seq((1, 2, 3, 4)) \rightarrow (1, 4, 9, 16)

def square_seq(seq):
    if seq == ():
        return ()
        Space? O(n^2)
        Space? O(n)

        return (seq[0] ** 2, ) +
             square_seq(seq[1:])
```

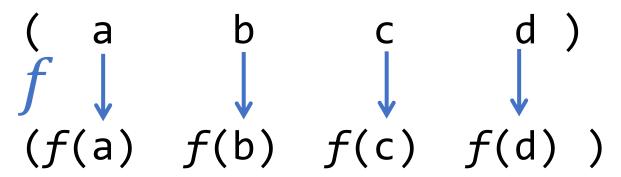
Homework: Do this iteratively

# Looking for patterns ....

```
def scale_seq(seq, factor):
    if seq == ():
        return ()
    else:
        return (seq[0] * factor,))+
                scale seq(seq[1:], factor)
def square_seq(seq):
                                     Higher-order
    if seq == ():
                                       function!!
        return ()
    else:
        return (seq[0]
                square_seq(seq[1:])
```

# Mapping

Often, we want to perform the same operation on every element of a list.



This is called *mapping*.

# Mapping

#### Scaling a list by a factor

```
def scale_seq(seq, factor):
    return map(lambda x: x * factor, seq))
```

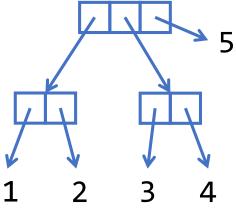
# Examples

```
map(abs, (-10, 2.5, -11.6, 17))
\rightarrow (10, 2.5, 11.6, 17)
map(square, (1, 2, 3, 4))
\rightarrow (1, 4, 9, 16)
map(cube, (1, 2, 3, 4))
\rightarrow (1, 8, 27, 64)
```

# **Trees**

Trees are sequences of sequences and single elements

- This is possible because of the closure property: we can include a sequence as an element of another sequence
- This allows us to build hierarchical structures, e.g. trees.



# Examples

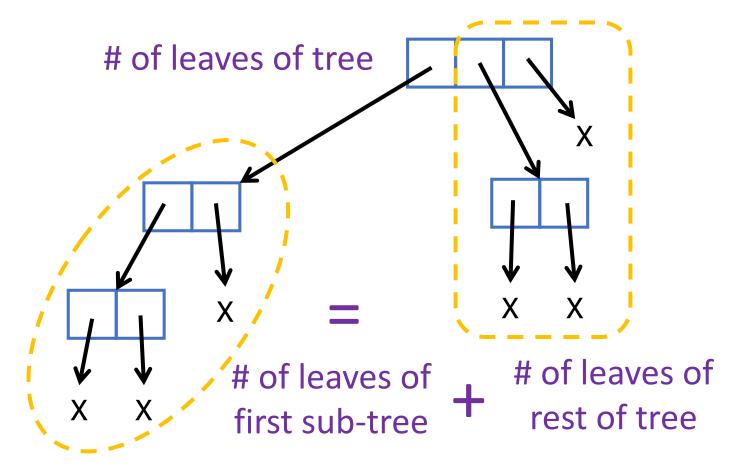
((1, 2), 3, 4)

```
x = ((1, 2), 3, 4)
len(x) \rightarrow 3
count_leaves(x) \rightarrow 4
(x, x)
\rightarrow (((1, 2), 3, 4), ((1, 2), 3, 4))
len((x, x))
\rightarrow 2
count leaves((x, x))
```

# How would we count the leaves? RECURSION!

# Recurrence Relation

#### Observation:



# Recursion

```
In other words,
count_leaves(tree) =
count_leaves(tree[0]) +
count_leaves(tree[1:])
```

# Base Case: If tree is empty Zero!

# **Another Base Case**

#### Observe:

Possible for the head or tail to be a leaf!

Leaf 
$$\Rightarrow$$
 +1

# Summary

# Strategy:

- If tree is empty, then 0
- Another base case:
  - tree is a leaf, then count as 1
- Count this, and add to:
  - tail also a tree, so recursively count this

# **Count Leaves**

```
def count leaves(tree):
    if tree == ():
        return 0
    elif(is_leaf(tree):)
        return 1
    else:
        return count leaves(tree[0])
        + count leaves(tree[1:])
```

# What are leaves

```
Remember type() in Lecture 1:
>>> t = (1, 2, 3)
>>> type(t)
<class 'tuple'>
>>> type(t) == tuple
True
def is leaf(item):
    return type(item) != tuple
```

# Mapping over trees

Suppose we want to scale each leaf by a factor, i.e.

```
mytree \rightarrow (1, (2, (3, 4), 5), (6, 7))

scale_tree(mytree, 10)
\rightarrow (10, (20, (30, 40), 50), (60, 70))
```

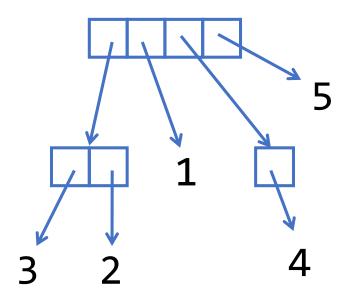
# Strategy

- Since tree is a sequence of sequences, we can map over each element in a tree.
- Each element is a subtree, which we recursively scale, and return sequence of results.
- Base case: if tree is a leaf, multiply by factor

# Mapping over trees

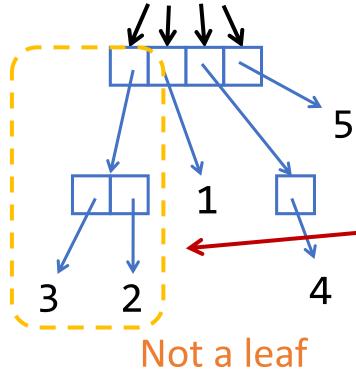
```
def scale tree(tree, factor):
    def scale_func(subtree):
        if is_leaf(subtree):
            return factor * subtree
        else:
            return scale_tree(subtree, factor)
    return map(scale_func, tree)
Compare with:
def count_leaves(tree):
    if tree == ():
        return 0
    elif is leaf(tree):
        return 1
    else:
        return count leaves(tree[0]) + count leaves(tree[1:])
```

```
tree = ((3, 2), 1, (4,), 5)
```

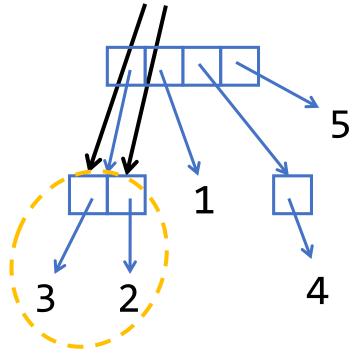


Suppose we do scale\_tree(tree, 2)

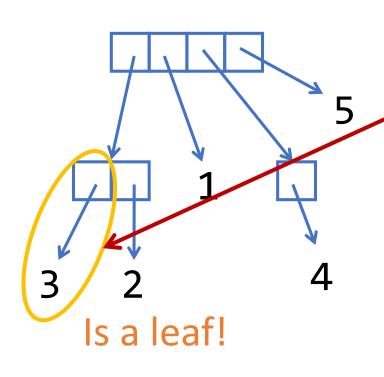
tree = 
$$((3, 2), 1, (4,), 5)$$



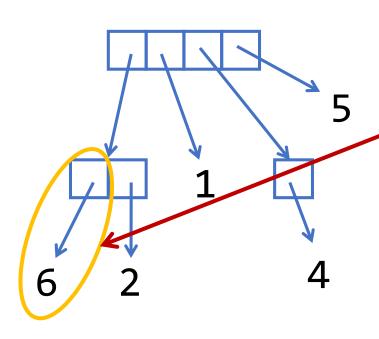
tree = 
$$((3, 2), 1, (4,), 5)$$



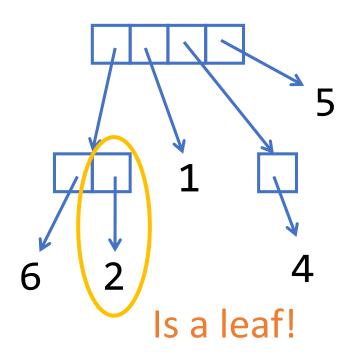
tree = 
$$((3, 2), 1, (4,), 5)$$



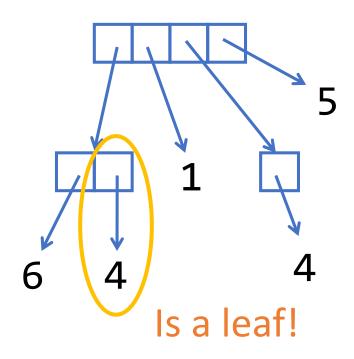
tree = 
$$((3, 2), 1, (4,), 5)$$



tree = 
$$((3, 2), 1, (4,), 5)$$

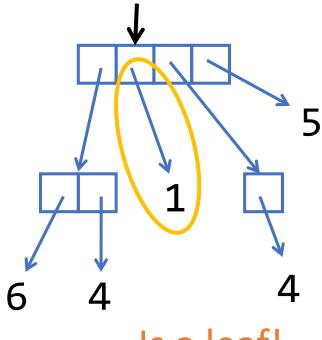


tree = 
$$((3, 2), 1, (4,), 5)$$



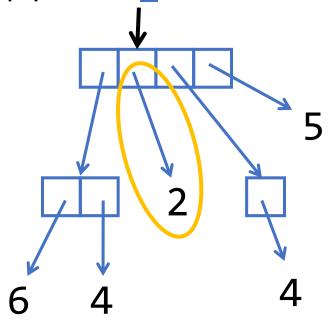
tree = 
$$((3, 2), 1, (4,), 5)$$

Apply scale\_func to each element

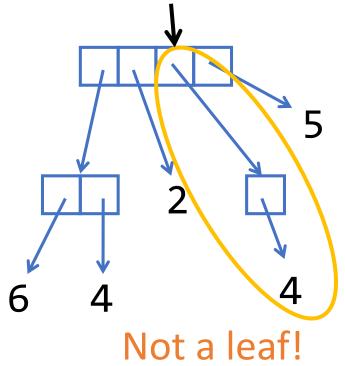


Is a leaf!

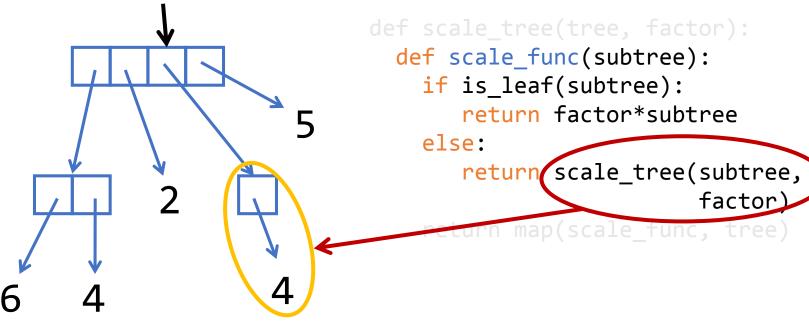
tree = 
$$((3, 2), 1, (4,), 5)$$



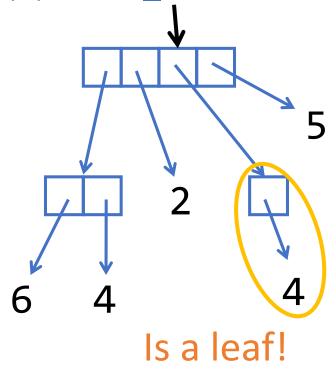
tree = 
$$((3, 2), 1, (4,), 5)$$



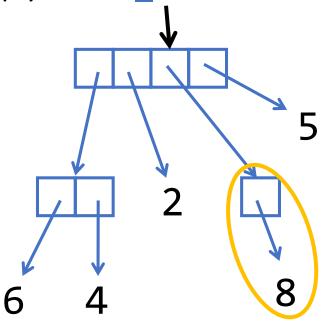
tree = 
$$((3, 2), 1, (4,), 5)$$



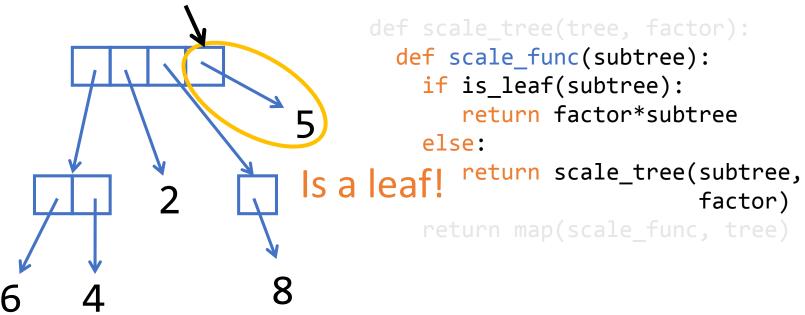
tree = 
$$((3, 2), 1, (4,), 5)$$



tree = 
$$((3, 2), 1, (4,), 5)$$

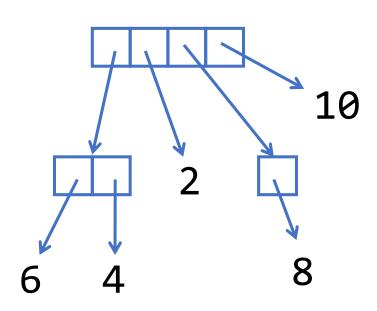


tree = 
$$((3, 2), 1, (4,), 5)$$



tree = 
$$((3, 2), 1, (4,), 5)$$

Done applying scale\_func to each element

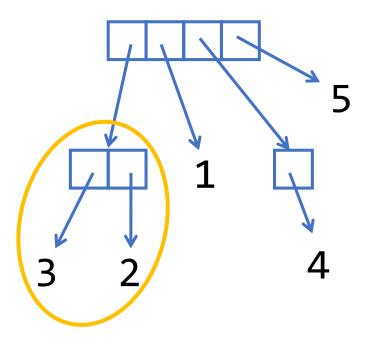


# Let's compare with count\_leaves

```
tree = ((3, 2), 1, (4,), 5)
                               def count_leaves(tree):
                                   if tree == ():
                                      return 0
                                   elif is_leaf(tree):
                                      return 1
                                   else:
                                      return count_leaves(tree[0])
                                      + count_leaves(tree[1:])
                   count_leaves
count leaves +
```

# Let's compare with count\_leaves

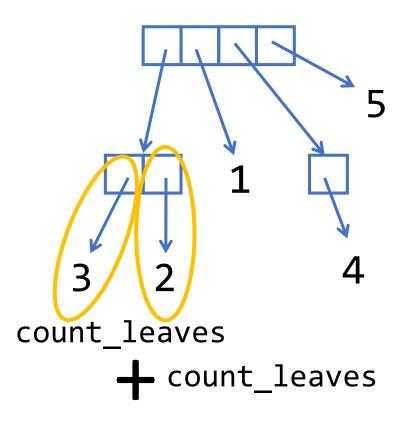
tree = 
$$((3, 2), 1, (4,), 5)$$



```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

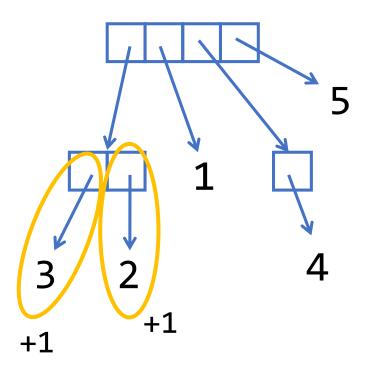
# Let's compare with count\_leaves

tree = 
$$((3, 2), 1, (4,), 5)$$



```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

tree = 
$$((3, 2), 1, (4,), 5)$$



```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

```
tree = ((3, 2), 1, (4,), 5)
               def count_leaves(tree):
                   if tree == ():
                       return 0
                   elif is leaf(tree):
                       return 1
                   else:
                       return count_leaves(tree[0])
                       + count_leaves(tree[1:])
    count leaves
```

```
tree = ((3, 2), 1, (4,), 5)
```

```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

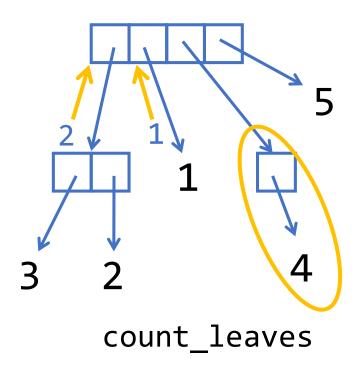
```
tree = ((3, 2), 1, (4,), 5)
```

```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

```
tree = ((3, 2), 1, (4,), 5)
```

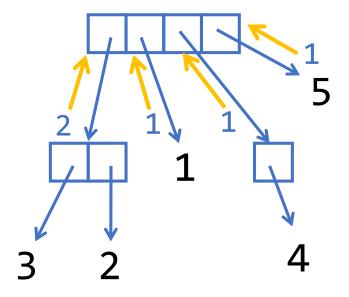
```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

tree = 
$$((3, 2), 1, (4,), 5)$$



```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

```
tree = ((3, 2), 1, (4,), 5)
```



```
def count_leaves(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        return 1
    else:
        return count_leaves(tree[0])
        + count_leaves(tree[1:])
```

# Key Idea: Traverse tree with recursion

Check for leaf!

## Sanity Check (QOTD)

How do you write a function copy\_tree that takes a tree and returns a copy of that tree?

# Sanity Check (QOTD)

```
def copy_tree(tree):
    return tree # is NOT acceptable!

>>> t = (1, 2, 3)
>>> t_copy = copy_tree(t)
t == t_copy → True
t is t copy → False
```

### Listening to Music

- Signal goes through various stages of "processing".
- Additional component can be inserted.
- Easy to change component.
- Components interface via signals.



# Modeling Computation as Signal Processing

- Producer (enumerator) creates signal.
- Filter removes some elements.
- Mapper modifies signal.
- Consumer (accumulator) consumes signal.

#### Benefits

- 1. Modularity: each component independent of others; components may be re-used.
- 2. Clarity: separates data from processes
- 3. Flexibility: new component can be added

# Example: Sum of squares of odd leaves

Given a tree, want to add the squares of (only) leaves of odd numbers:

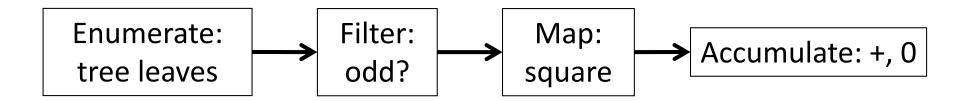
```
sum odd squares(((1, 2), (3, 4))) \rightarrow 10
```

# Example: Sum of squares of odd leaves

```
def sum odd squares(tree):
    if tree == ():
        return 0
    elif is_leaf(tree):
        if tree % 2 == 0:
            return 0
        else:
            return tree ** 2
    else:
        return sum odd squares(tree[0]) +
               sum odd squares(tree[1:])
```

# Alternative Approach

View it as signal processing computation!



### How to represent "signals"?

- Sequences

# Enumerating leaves

What does the following function do? def enumerate tree(tree): **if** tree == (): return () elif is leaf(tree): return (tree,) else: return enumerate\_tree(tree[0]) + enumerate\_tree(tree[1:]) enumerate\_tree((1, (2, (3, 4)), 5))  $\rightarrow$  (1, 2, 3, 4, 5) Also known as flattening the tree.

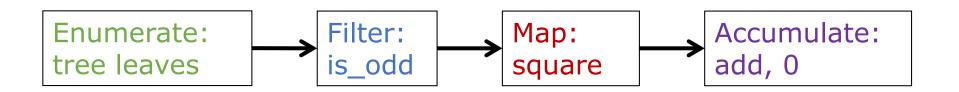
# Filtering a sequence

```
def filter(pred, seq):
                          Note: we are overwriting
    if seq == ():
                          the default Python filter function!
         return ()
    elif pred(seq[0]):
         return (seq[0],)
                 + filter(pred, seq[1:])
    else:
         return filter(pred, seq[1:])
is odd = lambda x:x%2 != 0
filter(is odd, (1, 2, 3, 4, 5)) \rightarrow (1, 3, 5)
```

# Accumulating a sequence

```
def accumulate(fn, initial, seq):
    if seq == ():
         return initial
    else:
        return fn(seq[0],
                    accumulate(fn, initial,
                                seq[1:]))
add = lambda x, y: x+y
accumulate(add, 0, (1, 2, 3, 4, 5))
accumulate(lambda x, y:(x, y), (),
            (1, 2, 3, 4, 5))
\rightarrow (1, (2, (3, (4, (5, ()))))) \rightarrow 15
```

# Putting it together



# Putting it together

```
(1, 2, (3, 4))
enumerate leaves |
            (1, 2, 3, 4)
       filter odd?
               (1, 3)
     map square 👃
accumulate add, 0
```

# Another Example: Tuple of even Fib

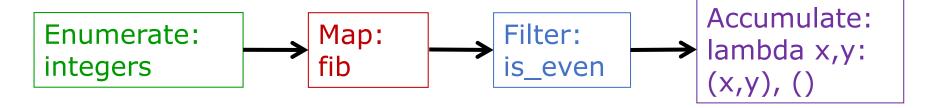
Want a list of even fib(k) for all k up to given integer n.

# "Usual" Way

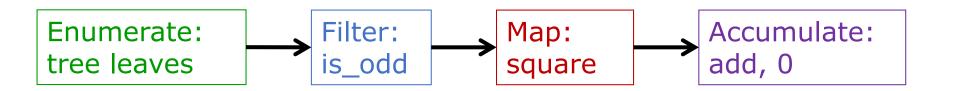
```
def even fibs(n):
    result = ()
    for k in range(1, n + 1):
        f = fib(k)
        if is even(f):
            result = result + (f, )
    return result
is\_even = lambda x: x % 2 == 0
>>> even_fibs(30)
(2, 8, 34, 144, 610, 2584, 10946, 46368, 196418,
832040)
```

#### Signal processing view

Even fibs



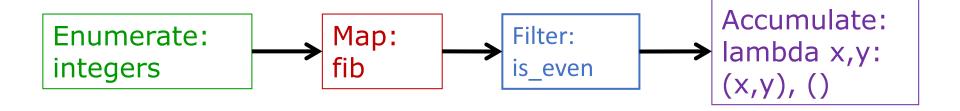
Compare: sum square odd leaves



#### **Enumerate integers**

```
def enumerate_interval(low, high):
    return tuple(range(low,high+1))
enumerate_interval(2, 7)
→ (2, 3, 4, 5, 6, 7)
```

#### **Even Fibs**



#### Signal Processing View

- Modular components:
  - Enumerate, Filter, Map, Accumulate
  - Each is independent of others.
  - Modularity is a powerful strategy for controlling complexity.

#### Signal Processing View

- Build a library of components.
- Sequences used to interface between components.

#### Re-using components

 Want a sequence of squares of fib(k)

#### Other Uses

- Suppose we have a sequence of personnel records.
- Want to find salary of highest-paid programmer.

# Default Python map and filter functions

Returns an iterable instead of tuple, but you can force it into a tuple.

```
>>> a = (1,2,3,4,5)
>>> b = filter(lambda x: x%2 == 0, a)
>>> b
<filter object at 0x02EC4710>
```

```
>>> for i in b:
         print(i)
>>> c = tuple(b)
>>> C
```

```
>>> b = filter(lambda x: x%2 == 0, a)
>>> b
<filter object at 0x02E42C10>
>>> c = tuple(b)
>>> C
(2, 4)
>>> for i in c:
        print(i)
```

# Working with Files

#### Reading a File:

```
input = open('inputfilename.txt', 'r')
some_line = input.readline()
```

We can check for end of file by checking whether some\_line == '' #empty string

#### Writing to a File:

```
output = open('outputfilename.txt', 'w')
output.write('HELLO WORLD')
```

# Example

```
def metrics(dictfile):
    dict = open(dictfile, 'r')
    currword = dict.readline()
    longest word = currword
    shortest word = currword
    while currword != '':
        if(len(currword) < len(shortest word)):</pre>
                                                   Find longest and
            shortest word = currword
                                                   shortest word
        if(len(currword) > len(longest_word)):
            longest word = currword
        currword = dict.readline()
    output = open("output.txt", "w")
    output.write("longest word: " + longest_word)
                                                        write to file
    output.write("shortest word: " + shortest_word
```

### Example

```
dictionary.txt >>
CS1010S
BEST
MODULE
WORLD
metrics("dictionary.txt")
output.txt >>
longest word: CS1010S
shortest word: BEST
```

## Summary

- Data often comes in the form of sequences
  - Easy to manipulate using recursion/iteration
  - Can be nested
- Closure property allows us to build hierarchical structures, e.g. trees, with tuples
  - Can use recursion to traverse such structures

### Summary

- "Signal-processing" view of computation.
  - Powerful way to organize computation.
  - Sequences as interfaces
  - Components: (i) Enumeration, (ii) Map,
     (iii) Filter, (iv) Accumulate

#### WebEx Test Session

