CS1010S Programming Methodology

Lecture 7
Searching & Sorting

11 March 2015

Today's Agenda

- Python Lists
- Searching
  - Linear search
  - Binary search
- Sorting
  - Basic sorting algorithms
  - Properties of sorting

Why Lists?

- Recall: tuples are immutable.

```
int_tuple = (1, 2, 3)
int_tuple[0] = 5
```

TypeError: 'tuple' object does not support item assignment

- What about lists?

```
int_list = [1, 2, 3] # this is a list
int_list[0] = 5     # [5, 2, 3]
```

Mutable!!

a.k.a. Arrays

Lists are sequences
So are tuples and strings

Sequence Operations

Tuple	**List**
`()`	`[]`
`(1, 2, 3)`	`[1, 2, 3]`
`type((1, 2, 3))`	`type([1, 2, 3])`
`<class 'tuple'>`	`<class 'list'>`
`t = tuple(range(5))`	`l = list(range(5))`
→ `(0, 1, 2, 3, 4)`	→ `[0, 1, 2, 3, 4]`
`t[4]` → `4`	`l[4]` → `4`
`t[2:]` → `(2, 3, 4)`	`l[2:]` → `[2, 3, 4]`

Sequence Operations

Tuple

```
a = (1, 2, 3, 4)
b = (5, 6, 7, 8)
c = a + b
c → (1, 2, 3, 4, 5,
     6, 7, 8)
```

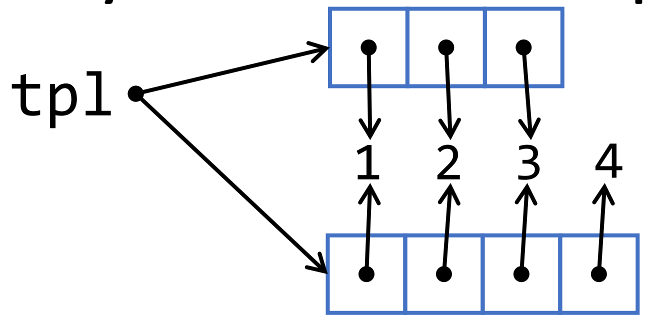
List

```
a = [1, 2, 3, 4]
b = [5, 6, 7, 8]
c = a + b
c → [1, 2, 3, 4, 5,
     6, 7, 8]
```

Appending element to list

With Tuples, we can only create new tuples

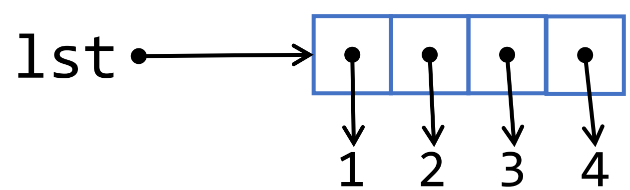
```
tpl = (1, 2, 3)
tpl = tpl + (4,)
```



With Lists, we can directly **append** to the list

```
>>> lst = [1, 2, 3]
>>> lst.append(4)
>>> lst
[1, 2, 3, 4]
```

element



No reassignment necessary

Extending list with list

We can also directly **extend** an existing list

```
lst = [1, 2, 3]        list
lst.extend([4, 5, 6])
lst → [1, 2, 3, 4, 5, 6]
```

This is **equivalent** to the following

```
lst = [1, 2, 3]
lst += [4, 5, 6]
lst → [1, 2, 3, 4, 5, 6]
```

Mutable versus Immutable

```
lst = [1, 2, 3]
lst2 = lst
lst == lst2          →True
lst is lst2          →True

lst += [4, 5, 6]
lst                  →[1, 2, 3, 4, 5, 6]
lst2                 →[1, 2, 3, 4, 5, 6]
lst == lst2          →True
lst is lst2          →True
```

Mutable versus Immutable

```
tup = (1, 2, 3)
tup2 = tup
tup == tup2                 → True
tup is tup2                 → True

tup += (4, 5, 6)
tup                         (1, 2, 3, 4, 5, 6)
tup2                        (1, 2, 3)
tup == tup2                 → False
tup is tup2                 → False
```

Deleting Elements

```
>>> a = [-1, 1, 66.25, 333, 333, 1234.5]
>>> del a[0]
>>> a
[1, 66.25, 333, 333, 1234.5]

>>> del a[2:4]
>>> a
[1, 66.25, 1234.5]

>>> del a[:]
>>> a
[]
```

What if we do `del a`?

Sequence Operations

Some other handy functions that will work on sequences

```
len([1, 2, 3, 4])    → 4
min([1, 2, 3, 4])    → 1
max([1, 2, 3, 4])    → 4
1 in [1, 2, 3]       → True
5 not in [1, 2, 3]   → True
[1, 2, 3] * 2        → [1, 2, 3, 1, 2, 3]
```

List Operations

```
lst = [1, 2, 3, 4]
```

```
lst.copy()
```

returns a shallow copy of a list

```
lst.insert(<pos>, <element>)
```

inserts `element` into position `pos`

List Operations

`lst.pop(<pos>)`

returns and remove element at position `pos`. If `pos` is omitted, removes the last element

`lst.remove(<element>)`

removes first occurrence of `element` from list, error if `element` is not in the list.

`lst.clear()`

clears the list

List Operations

```
s = [1, 2, 3, 4, 5]
t = s.copy()          # t = [1, 2, 3, 4, 5]
s.reverse()           # s = [5, 4, 3, 2, 1]
                      # t = [1, 2, 3, 4, 5]
s.insert(0, 1)        # s = [1, 5, 4, 3, 2, 1]
s.pop()               # 1
                      # s = [1, 5, 4, 3, 2]
s.pop(1)              # 5
                      # s = [1, 4, 3, 2]
s.insert(2, 2)        # s = [1, 4, 2, 3, 2]
s.remove(4)           # s = [1, 2, 3, 2]
s.clear()             # s = []
```

Iterating on Lists

We can iterate through lists using `for` operator

```
>>> s = [1, 2, 3, 4, 5]
>>> for element in s:
    print(element)
1
2
3
4
5
```

List Comprehension

We can iterate also generate new lists

```
>>> s = [1, 2, 3, 4, 5]
>>> t = [n ** 2 for n in s]
[1, 4, 9, 16, 25]
```

Does this look familiar?

It should. This is equivalent to:

```
t = list(map(lambda n: n**2, s))
```

Python Lists: Summary

- Lists are sequences
  - can be used with all the sequence operations
- Lists are mutable
  - has mutable operations which are not common to tuples and strings

Searching

- You have a list.
- How do you find something in the list?

- Basic idea: go through the list from start to finish one element at a time.

Linear Search

- Idea: go through the list from start to finish

5	2	3	4

- Example: Search for 3

5	2	3	4

3 not found, move on

5	2	3	4

3 not found, move on

5	2	3	4

Found 3.

Linear Search

Idea: go through the list from start to finish

```
# equivalent code
for i in [5, 2, 3, 4]:
    if i == 3:
        return True
```

Linear Search

Implemented as a function:

```
def linear_search(value, lst):
    for i in lst:
        if i == value:
            return True
    return False
```

What kind of performance can we expect?

Large vs small lists?

Sorted vs unsorted lists?

$$O(n)$$

Can we do better?

Of course la!

Searching

IDEA:

If the elements in the list were sorted in order, life would be much easier.

Why?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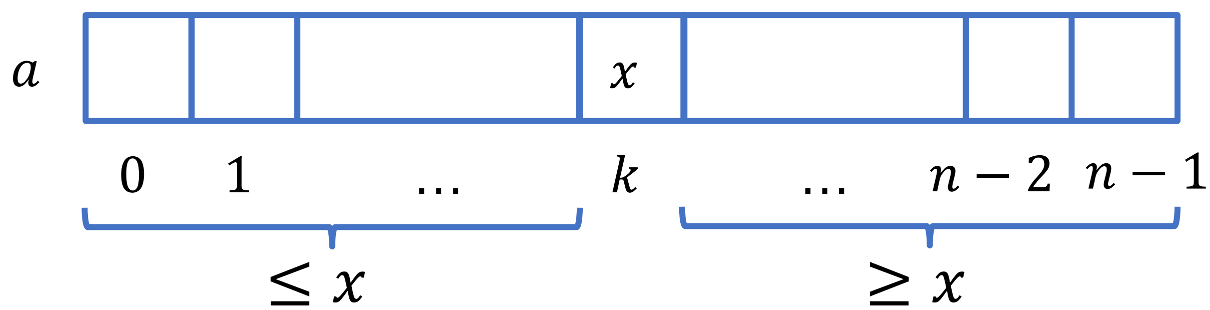
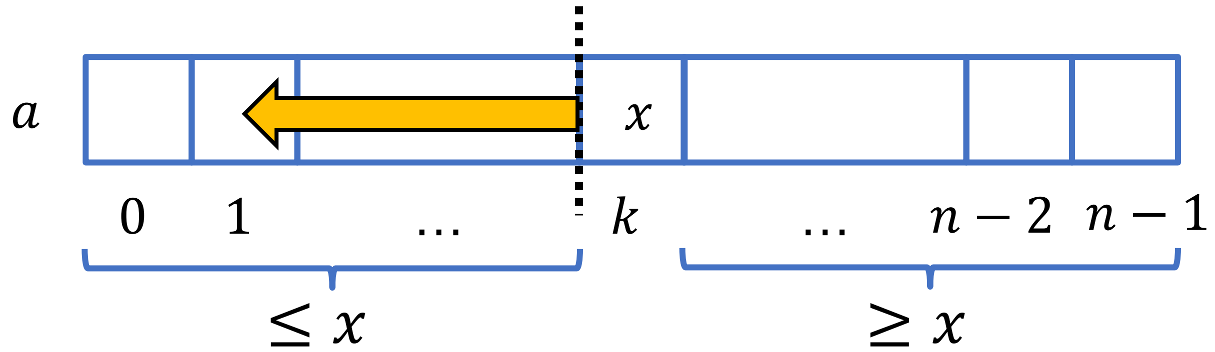
IDEA

If list is sorted, we can "divide-and-conquer"

Assuming a list is sorted in ascending order:





if the k^{th} element is larger than what we are looking for, then we only need to search in the indices $< k$

Binary Search

1. Find the middle element.
2. If it is what we are looking for (key), return True.
3. If our key is smaller than the middle element, repeat search on the left of the list.
4. Else, repeat search on the right of the list.

Binary Search

Looking for 25 (key)

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

Find the middle element: 34

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

Not the thing we're looking for: $34 \neq 25$

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

$25 < 34$, so we repeat our search on the left half:

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

Binary Search

Find the middle element: 12

5	9	12	18	25	34	85	100	123	345

25 > 12, so we repeat the search on the right half:

5	9	12	18	25	34	85	100	123	345

Find the middle element: 25

5	9	12	18	25	34	85	100	123	345

Great success: 25 is what we want

5	9	12	18	25	34	85	100	123	345

Binary Search

"Divide and Conquer"

In large sorted lists, performs much better than linear search <u>on average</u>.

Binary Search

Algorithm (assume sorted list):

1. Find the middle element.
2. If it is we are looking for (key), return True.
3. A) If our key is smaller than the middle element, repeat search on the left of the element.
   B) Else, repeat search on the right of the element.

Binary Search

```
def binary_search(key, seq):
    if seq == []:
        return False
    mid = len(seq) // 2
    if key == seq[mid]:
        return True
    elif key < seq[mid]:
        return binary_search(key, seq[:mid])
    else:
        return binary_search(key, seq[mid+1:])
```

Binary Search

```
def binary_search(key, seq): # seq is sorted
    def helper(low, high):
        if low > high:
            return False
        mid = (low + high) // 2 # get middle
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Binary Search

Now let's try searching for 11:

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

```
def binary_search(key, seq):
    def helper(low, high):
        if low > high:
            return False
        mid = (low + high) // 2
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 1. Find the middle element.

Binary Search

Now let’s try searching for 11:

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

```
def binary_search(key, seq):
    def helper(low, high):
        if low > high:
            return False
        mid = (low + high) // 2
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

key → 11

helper(0, 10-1)

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 0, 9
        if low > high:
            return False
        mid = (low + high) // 2 # mid=4
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 1. Find the middle element.

Binary Search

Now let's try searching for 11:

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

```
def binary_search(key, seq):
    def helper(low, high): # 0, 9
        if low > high:
            return False
        mid = (low + high) // 2 # mid=4
        if key == seq[mid]: # 11 == 25
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 2. If it is what we are looking for, return True

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 0, 9
        if low > high:
            return False
        mid = (low + high) // 2 # mid=4
        if key == seq[mid]: # 11 == 25
            return True
        elif key < seq[mid]: # 11 < 25
            return helper(low, mid-1) # helper(0, 4-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 3a. If key is smaller, look at left side

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 0, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=4
        if key == seq[mid]: # 11 == 25
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 3a. If key is smaller, look at left side

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 0, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=1
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 1. Find the middle element

Binary Search

Now let’s try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 0, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=1
        if key == seq[mid]: # 11 == 9
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 2. If it is what we are looking for, return True

Binary Search

Now let's try searching for 11:

5	**9**	**12**	**18**	**25**	**34**	**85**	**100**	**123**	**345**

```
def binary_search(key, seq):
    def helper(low, high): # 0, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=1
        if key == seq[mid]: # 11 == 9
            return True
        elif key < seq[mid]: # 11 < 9
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 3a. If key is smaller, look at left side

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 0, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=1
        if key == seq[mid]: # 11 == 9
            return True
        elif key < seq[mid]: # 11 < 9
            return helper(low, mid-1)
        else:
            return helper(mid+1, high) # helper(1+1, 3)
    return helper(0, len(seq)-1)
```

Step 3b. Else look at right side

Binary Search

Now let’s try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 2, 3
        if low > high:
            return False
        mid = (low + high) // 2
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 3b. Else look at right side

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 2, 3
        if low > high:
            return False
        mid = (low + high) // 2
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 3b. Else look at right side

Binary Search

Now let’s try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 2, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=2
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 1. Find the middle element

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 2, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=2
        if key == seq[mid]: # 11 == 12
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 2. If it is what we are looking for, return True

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 2, 3
        if low > high:
            return False
        mid = (low + high) // 2 # mid=2
        if key == seq[mid]: # 11 == 12
            return True
        elif key < seq[mid]: # 11 < 12
            return helper(low, mid-1) # helper(2, 2-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 3a. If key is smaller, look at left side

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 2, 1
        if low > high:
            return False
        mid = (low + high) // 2
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Step 3a. If key is smaller, look at left side

Binary Search

Now let's try searching for 11:

5	9	12	18	25	34	85	100	123	345

```
def binary_search(key, seq):
    def helper(low, high): # 2, 1
        if low > high: # 2 > 1
            return False
        mid = (low + high) // 2
        if key == seq[mid]:
            return True
        elif key < seq[mid]:
            return helper(low, mid-1)
        else:
            return helper(mid+1, high)
    return helper(0, len(seq)-1)
```

Key cannot be found. Return False

Binary Search

- Each step eliminates the problem size by half.
  - The problem size gets reduced to 1 very quickly
- This is a simple yet powerful strategy, of halving the solution space in each step
- What is the order of growth?

$$O(\log n)$$

Wishful Thinking

We assumed the list was sorted.

Now, let's deal with this assumption!

Sorting

- High-level idea:
  1. some objects
  2. function that can order two objects

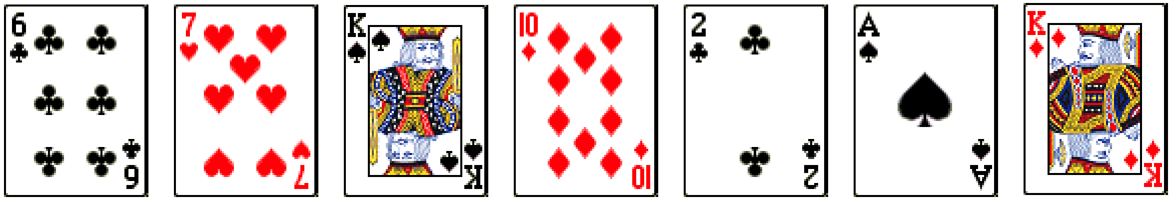
  $\Rightarrow$ order all the objects

How Many Ways to Sort?

Too many. ☺

Example

Let's sort some playing cards?

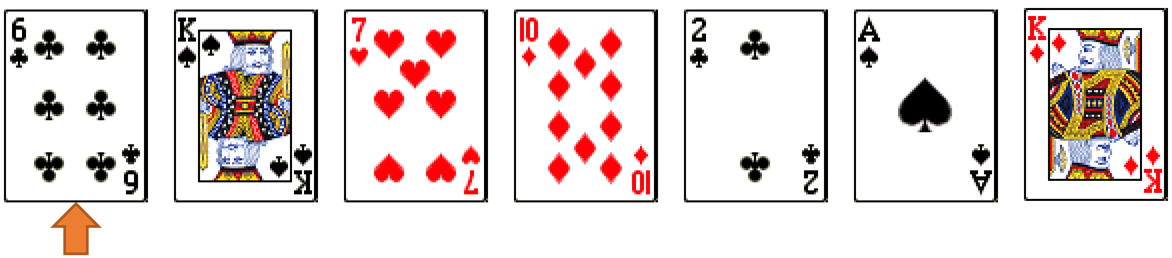


What do you do when you play cards?

Obvious Way

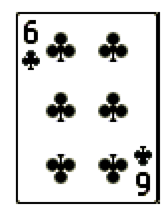
Find the smallest card not in hand (SCNIH), and put it at the end of your hand. Repeat.

Unsorted



Sorted

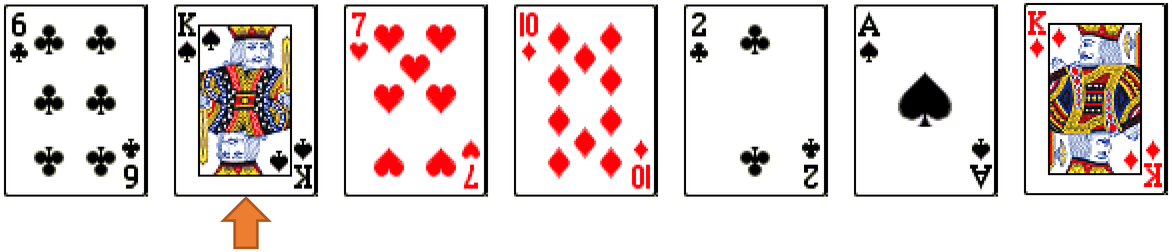
Smallest



Obvious Way

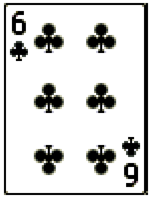
Find the smallest card not in hand (SCNIH), and put it at the end of your hand. Repeat.

Unsorted



Sorted

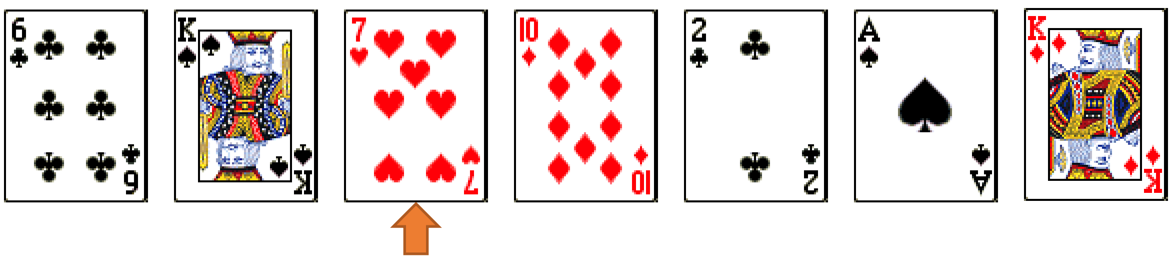
Smallest



Obvious Way

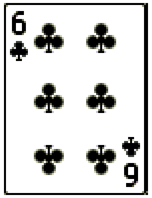
Find the smallest card not in hand (SCNIH), and put it at the end of your hand. Repeat.

Unsorted



Sorted

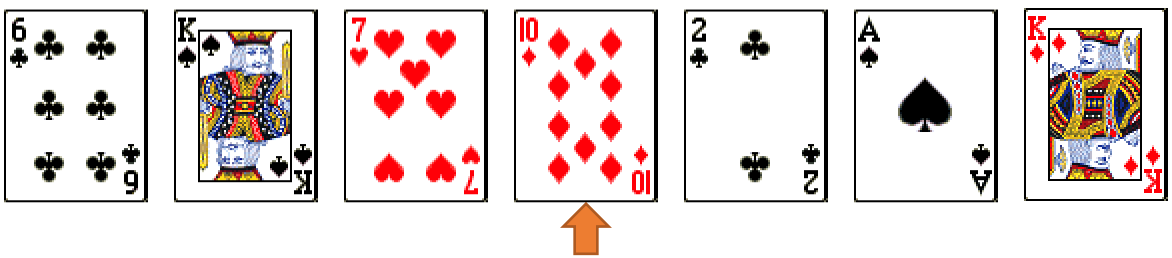
Smallest



Obvious Way

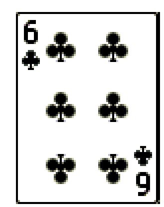
Find the smallest card not in hand (SCNIH), and put it at the end of your hand. Repeat.

Unsorted



Sorted

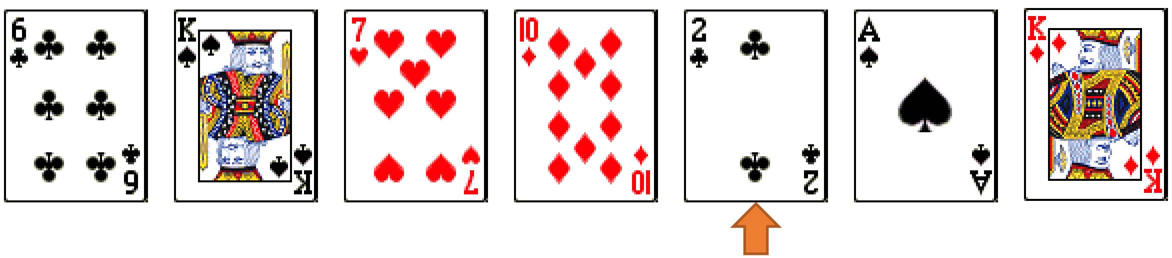
Smallest



Obvious Way

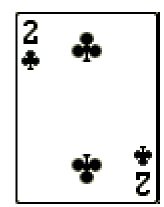
Find the smallest card not in hand (SCNIH), and put it at the end of your hand. Repeat.

Unsorted



Sorted

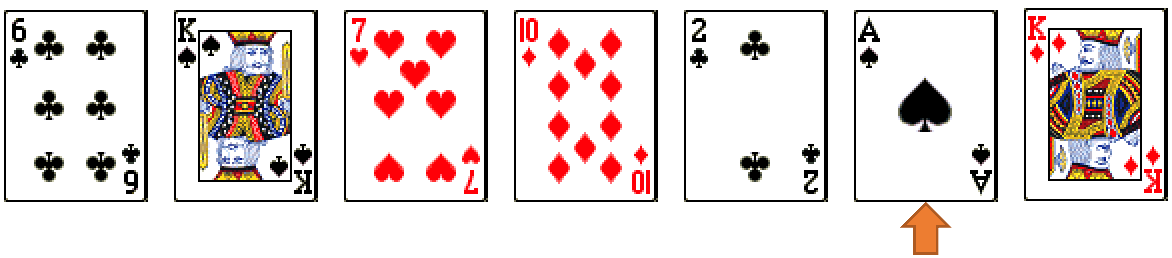
Smallest



Obvious Way

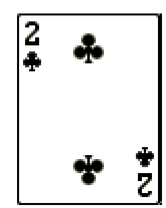
Find the smallest card not in hand (SCNIH), and put it at the end of your hand. Repeat.

Unsorted



Sorted

Smallest



Obvious Way

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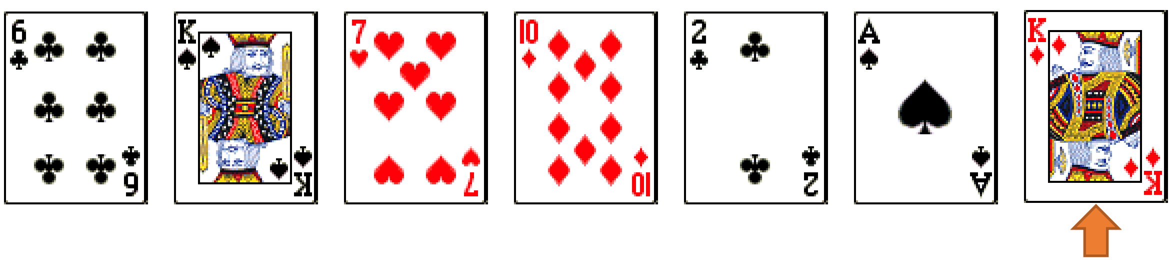
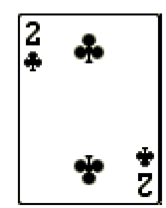
Unsorted



Sorted

Smallest



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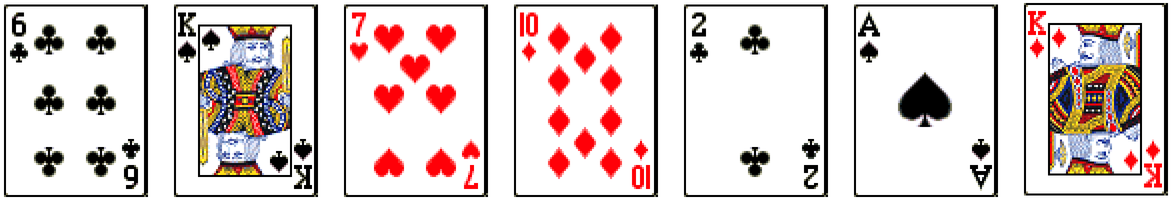
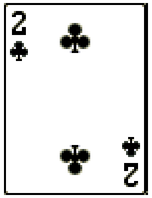


Sorted

Smallest



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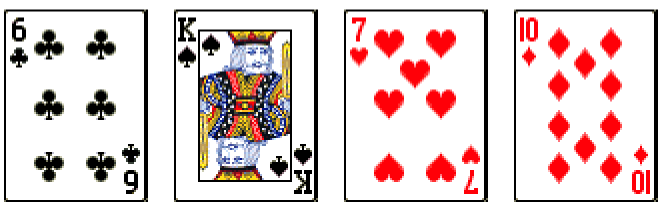
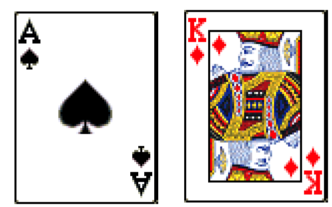
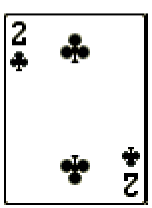




Sorted

Smallest



Obvious Way

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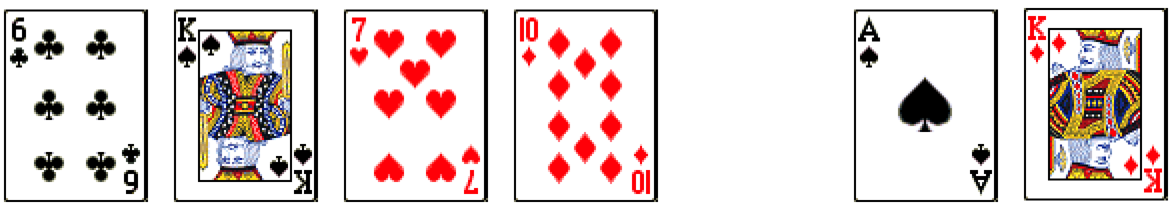
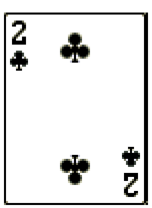
Repeat

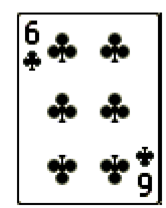
Unsorted



Sorted



Smallest



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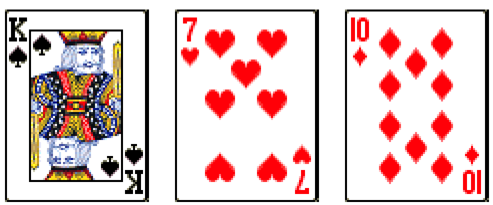
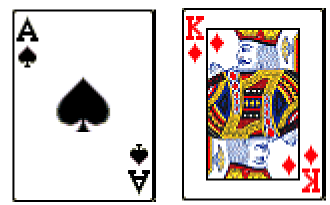
Repeat

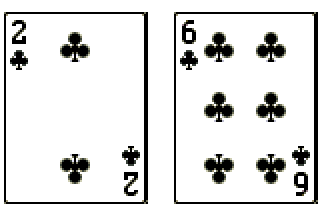
Unsorted





Sorted

Smallest



Obvious Way

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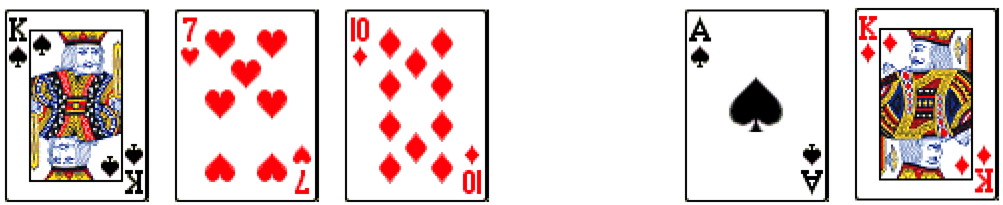
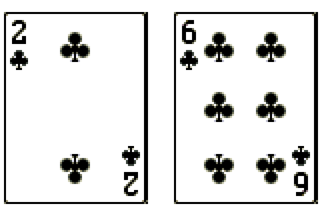
Repeat

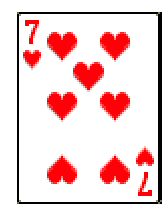
Unsorted



Sorted



Smallest



Obvious Way

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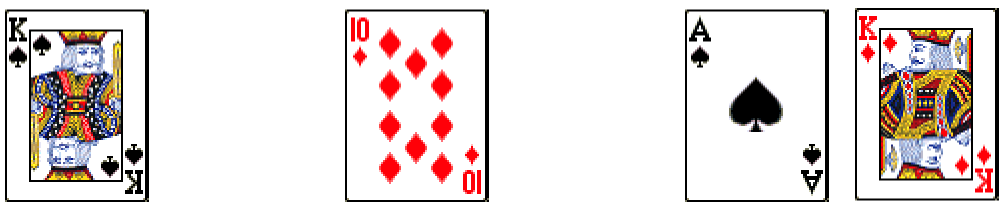
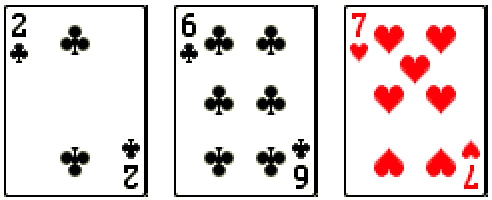
Repeat

Unsorted



Sorted

Smallest



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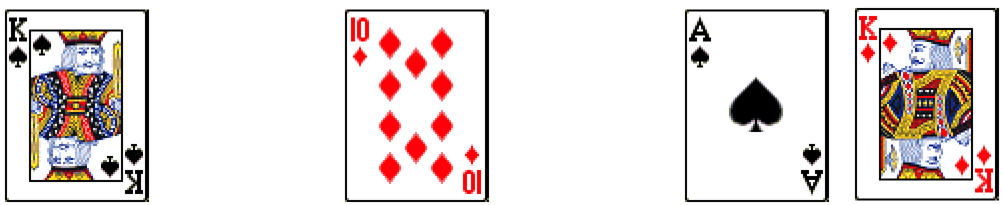
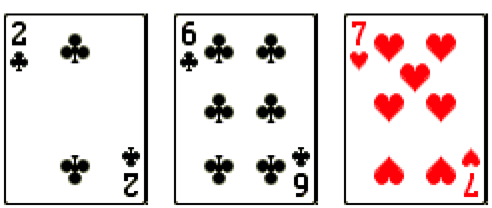
Repeat

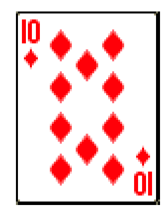
Unsorted



Sorted



Smallest



Obvious Way

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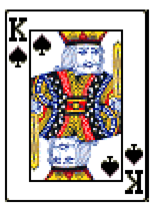
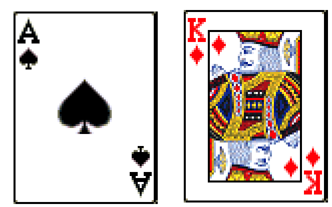
Repeat

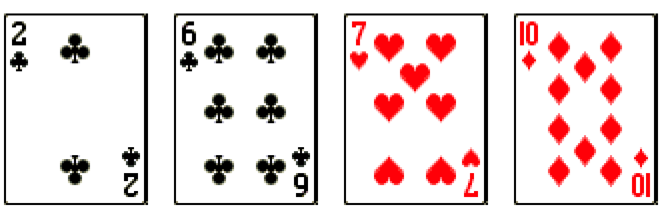
Unsorted





Sorted

Smallest



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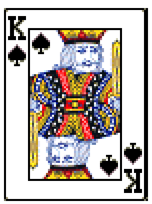
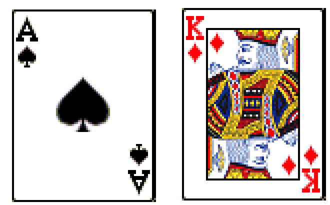
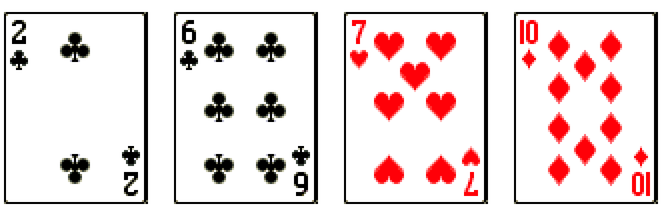
Repeat

Unsorted

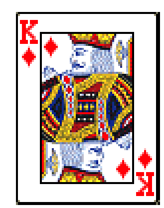




Sorted



Smallest



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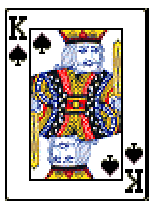
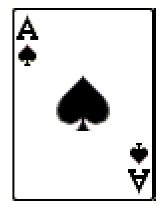
Unsorted





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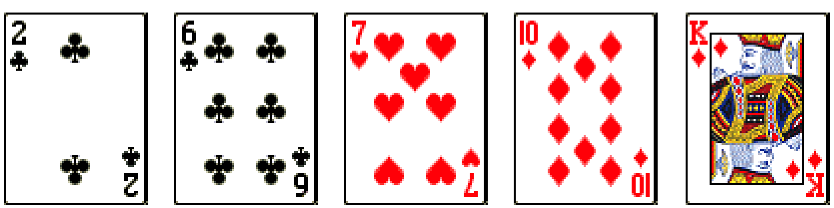
Smallest



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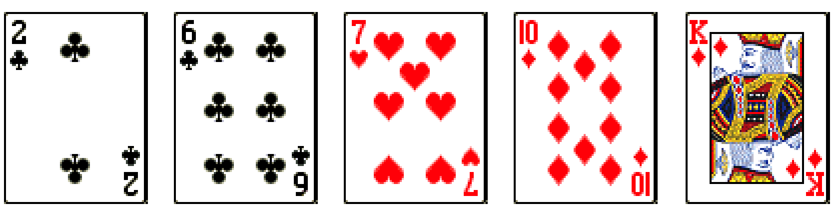
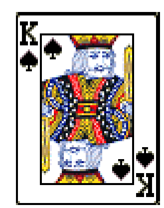
Repeat

Unsorted

Sorted



Smallest



Obvious Way

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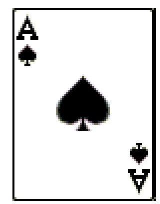
Repeat

Unsorted



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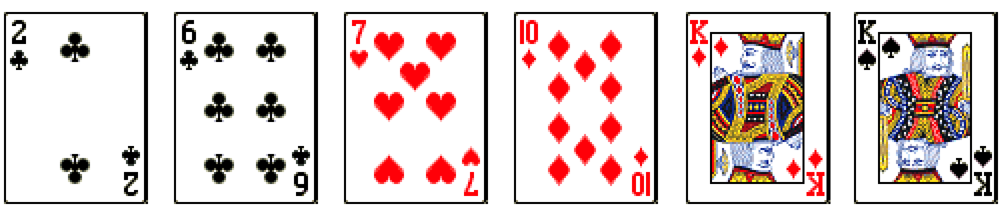
Smallest



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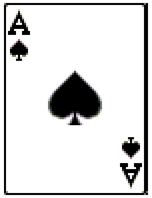
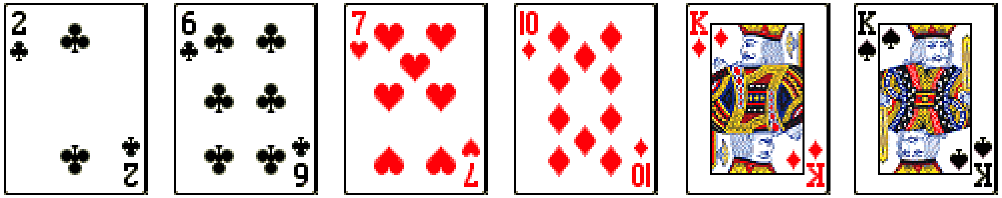
Repeat

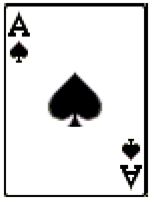
Unsorted



Sorted



Smallest



Obvious Way

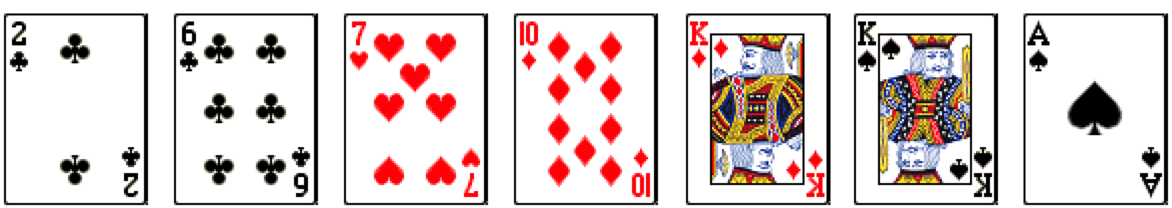
Find the smallest card not in hand (SCNIH), and put it at the end of your hand. Repeat.

Done

Unsorted

Sorted

Smallest



There is actually a name for this:

Selection Sort!

Let’s Implement it!

```
a = [4,12,3,1,11]
sort = []

while a:   # a is not []
    smallest = a[0]
    for element in a:
        if element < smallest:
            smallest = element
    a.remove(smallest)
    sort.append(smallest)
    print(a)
```

Output

```
[4, 12, 3, 11]
[4, 12, 11]
[12, 11]
[12]
[]
print(a)
[]

print(sort)
[3, 4, 11, 12]
```

Order of Growth?

- Time:
  - Worst $O(n^2)$
  - Average $O(n^2)$
  - Best $O(n^2)$

- Space: $\cancel{O(n)}$ $O(1)$

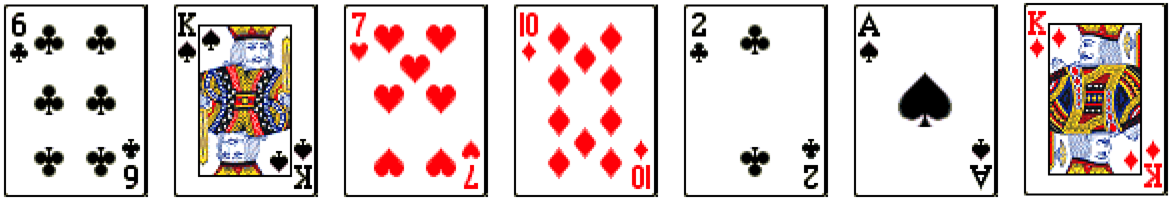
Let’s try something else…

suppose you have a friend

Doing it with a friend

- Split cards into two halves and sort. Combine halves afterwards. Repeat with each half.

Split into halves



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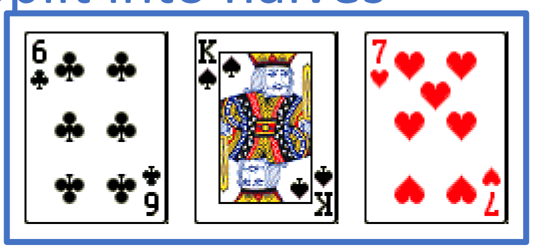

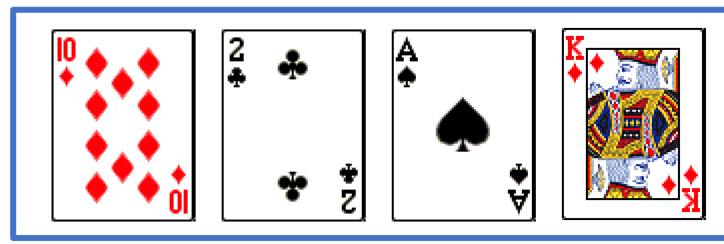

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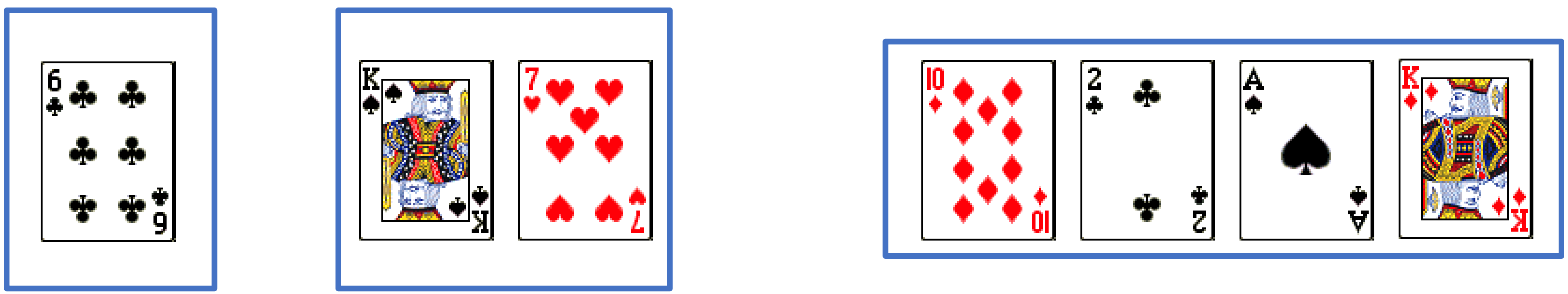

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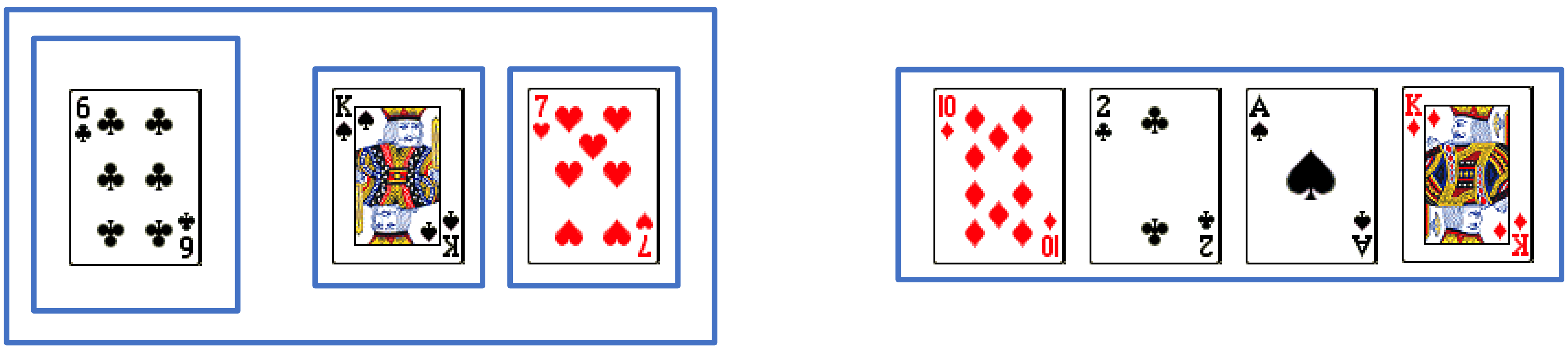

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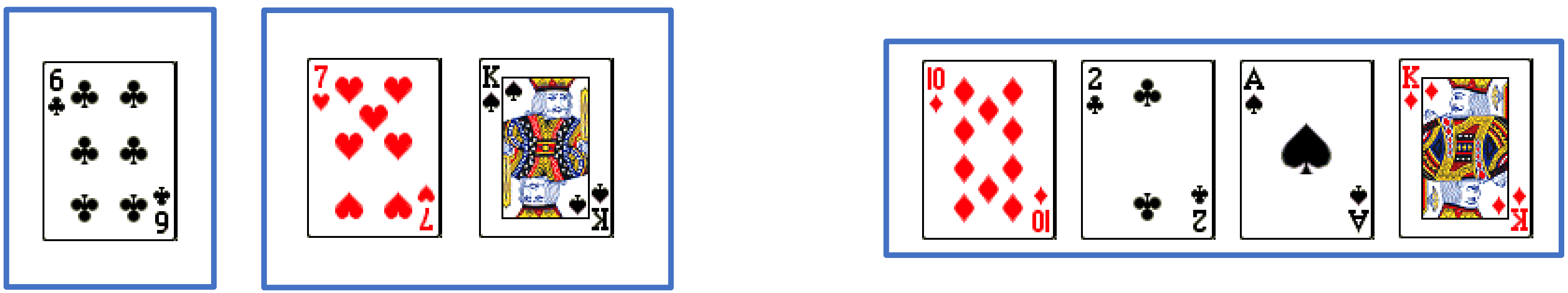

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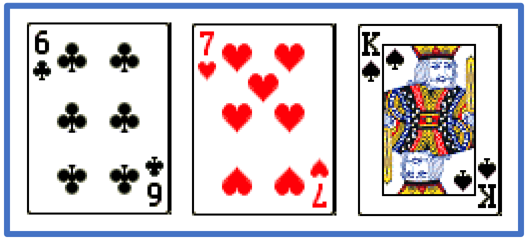

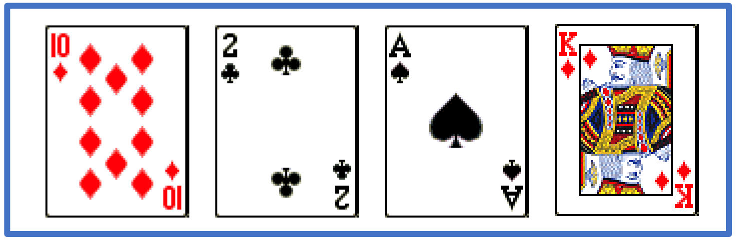

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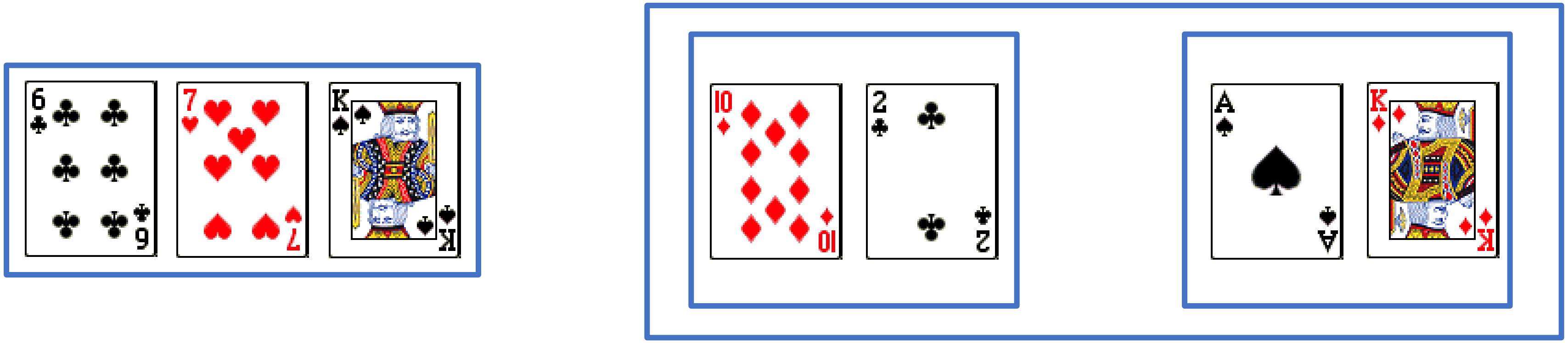

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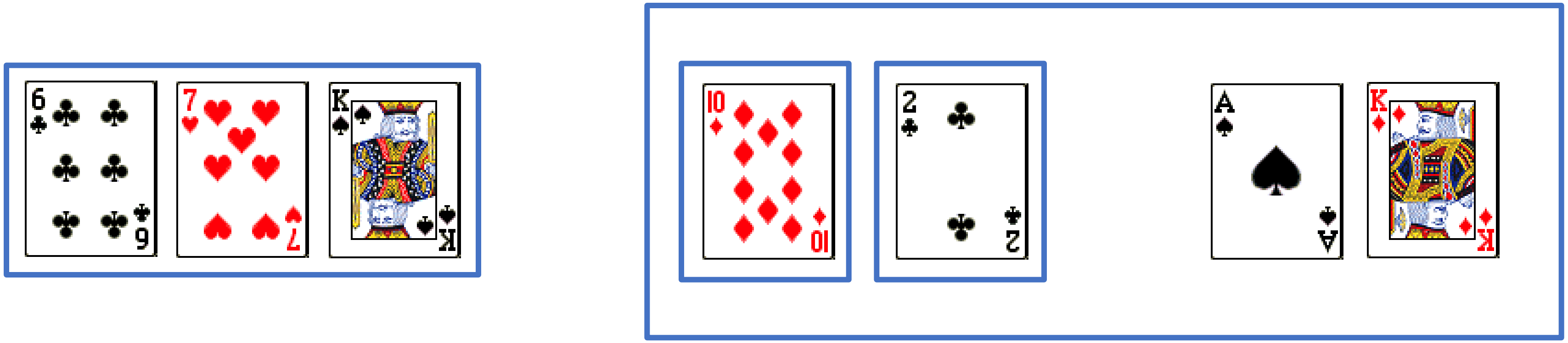

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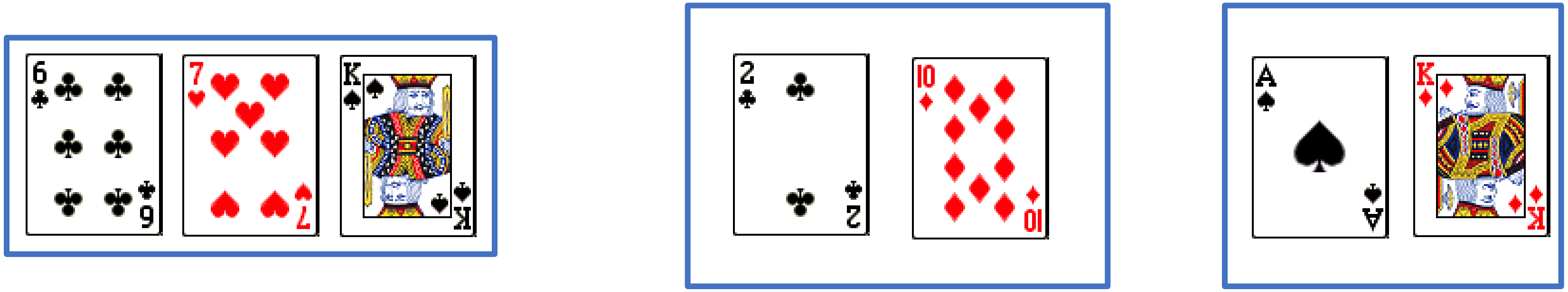

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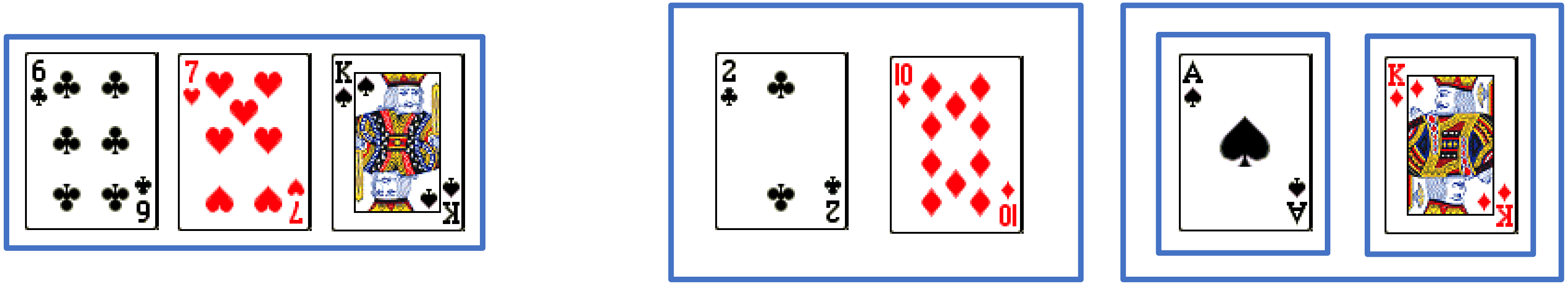

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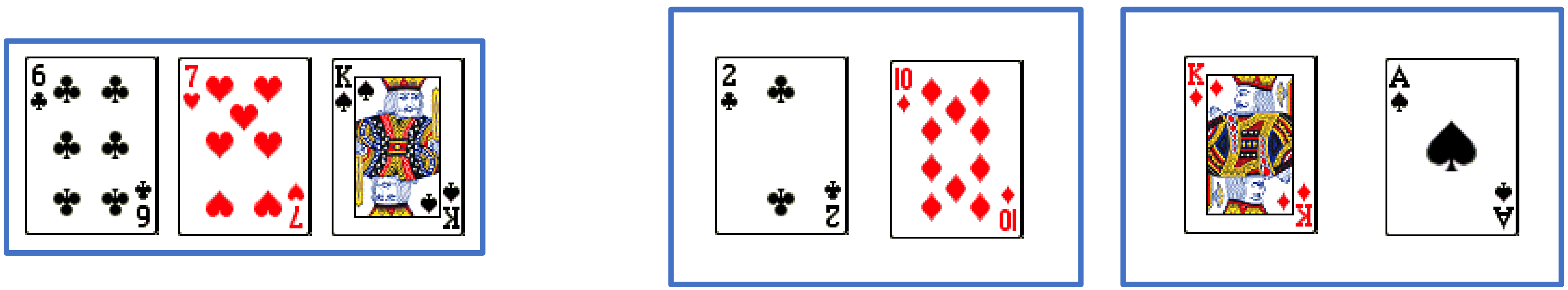

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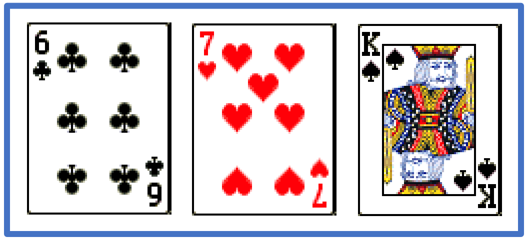

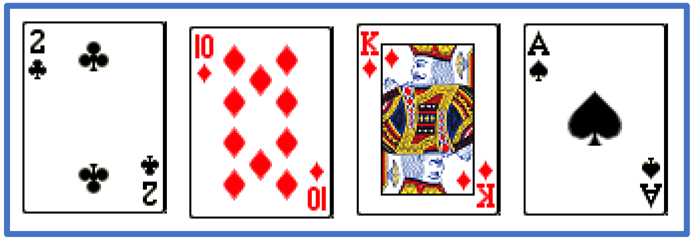

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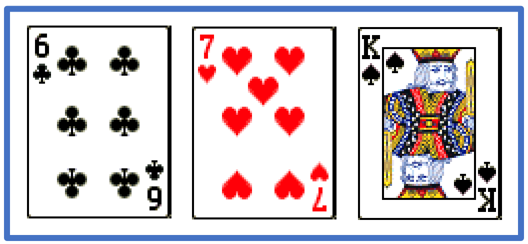
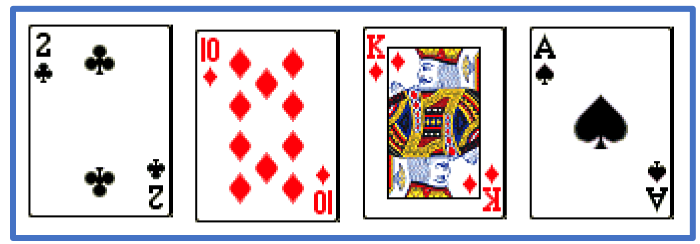


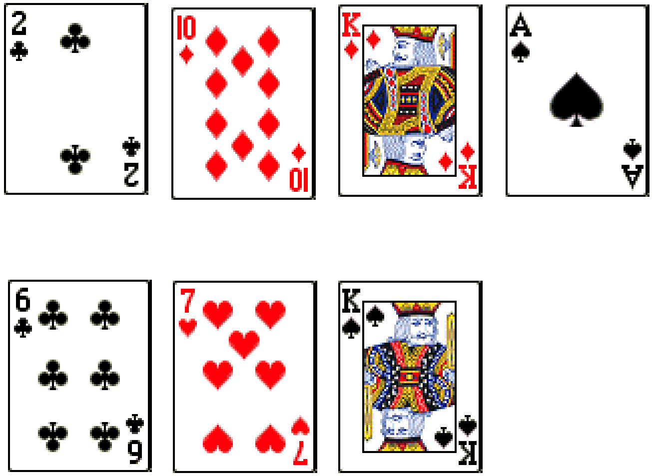


How to combine the 2 sorted halves?

Doing it with a friend

- Split cards into two halves and sort. Combine halves afterwards. Repeat with each half.

Compare first elements



Doing it with a friend

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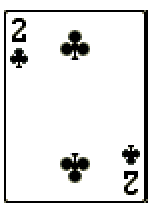
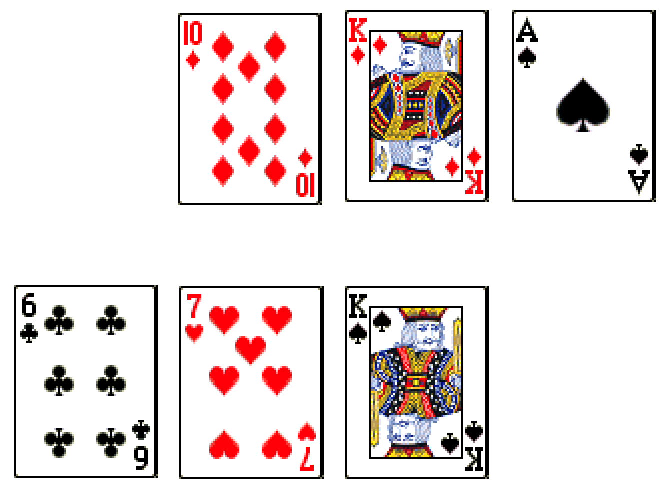
Compare first elements





Doing it with a friend

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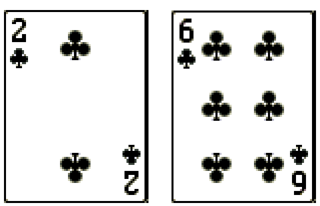
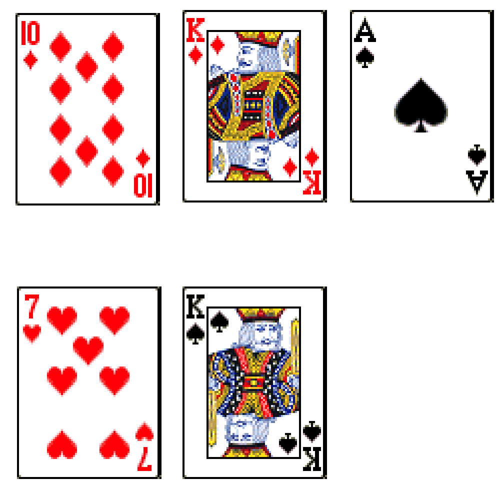
Compare first elements

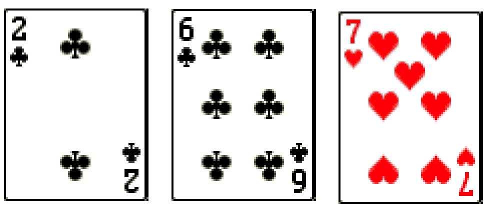


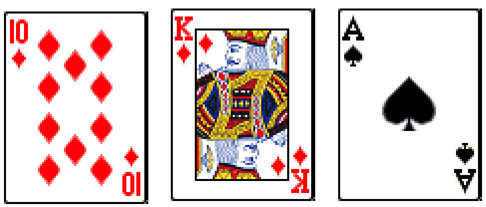


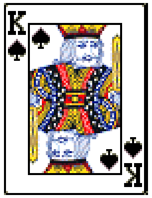
Doing it with a friend

- Split cards into two halves and sort. Combine halves afterwards. Repeat with each half.

Compare first elements



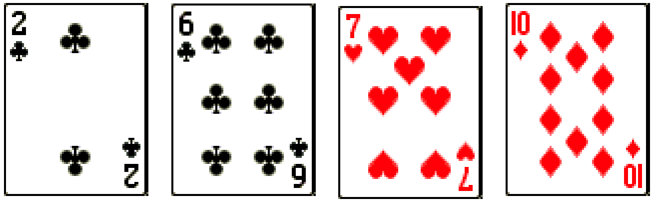


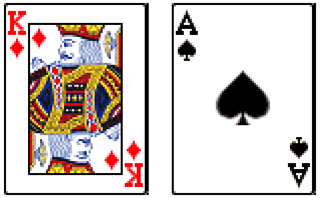


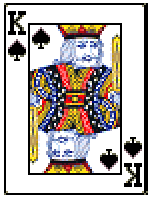
Doing it with a friend

- Split cards into two halves and sort. Combine halves afterwards. Repeat with each half.

Compare first elements







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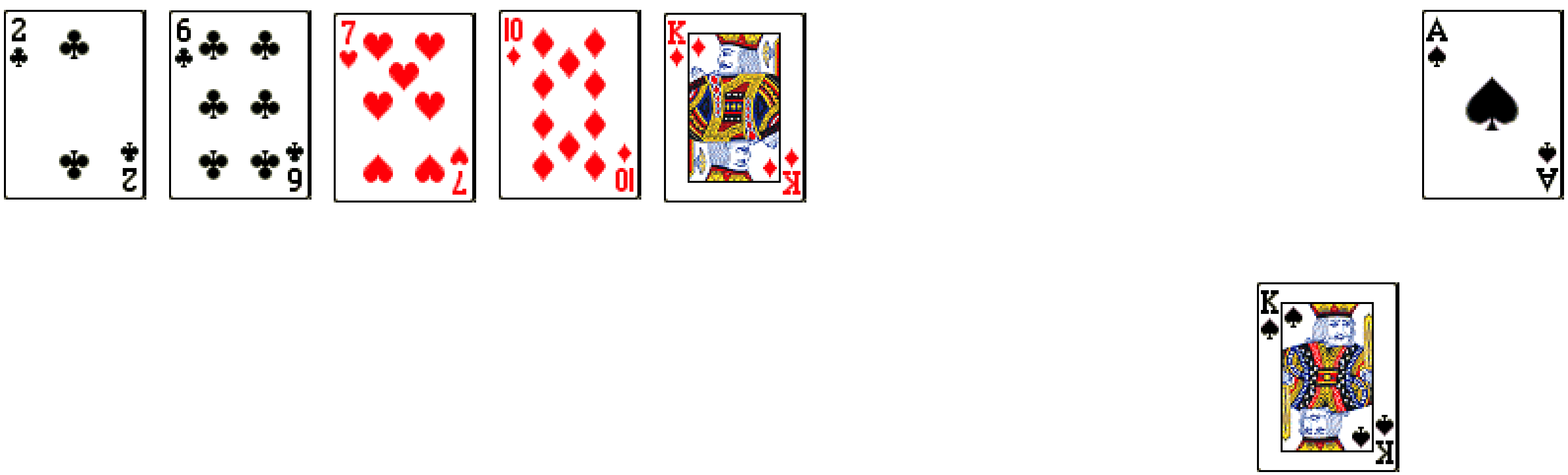
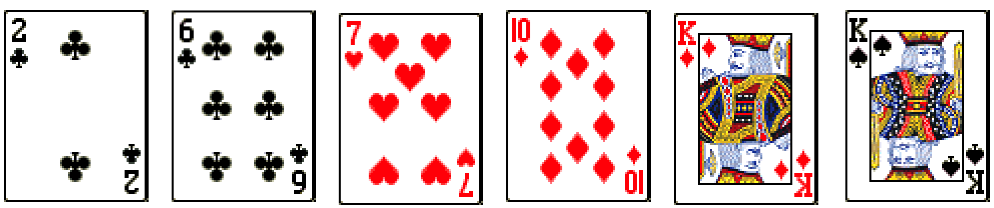
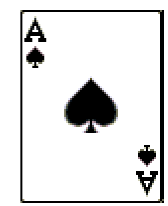


Doing it with a friend

- Split cards into two halves and sort. Combine halves afterwards. Repeat with each half.

Compare first elements

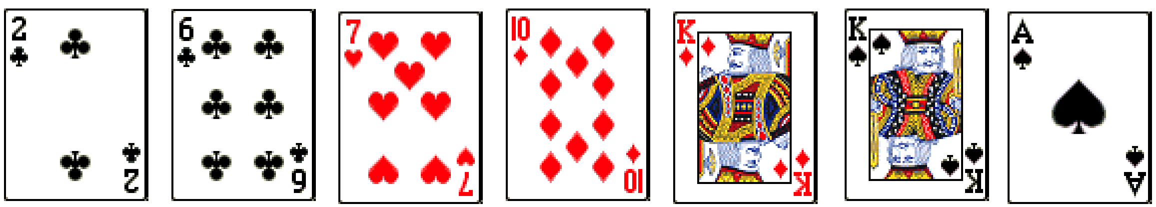




Doing it with a friend

- Split cards into two halves and sort. Combine halves afterwards. Repeat with each half.

Compare first elements

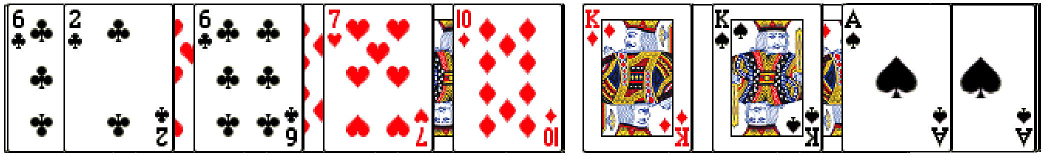


There is also a name for this:

Merge Sort!

Let’s Implement It!

First observation: RECURSION!

- Base case: n< 2, return lst
- Otherwise: 
  - Divide list into two
  - Sort each of them
  - Merge!

Merge Sort

```
def merge_sort(lst):
    if len(lst) < 2:  # Base case!
        return lst
    mid = len(lst) // 2
    left = merge_sort(lst[:mid])  #sort left
    right = merge_sort(lst[mid:]) #sort right
    return merge(left, right)
```

How to merge?

How to merge?

- Compare first element
- Take the smaller of the two
- Repeat until no more elements

Merging

```
def merge(left, right):
    results = []
    while left and right:
        if left[0] < right[0]:
            results.append(left.pop(0))
        else:
            results.append(right.pop(0))
    results.extend(left)
    results.extend(right)
    return results
```

Order of Growth?

- Time: Worst $O(n\log n)$
  Average $O(n\log n)$
  Best $O(n\log n)$

- Space: $O(n)$

No need to memorize

Sort Properties

In-place: uses a small, constant amount of extra storage space, i.e., $O(1)$ space

Selection Sort: No (Possible)

Merge Sort: No (Possible)

Sort Properties

Stability: maintains the relative order of items with equal keys (i.e., values)

Selection Sort: Yes (maybe)

Merge Sort: Yes

How Many Ways to Sort?

Too many. ☺

How Many Sort Must You Learn?

None (sort of) ☺

`lst.sort()`

No need to remember the time/space complexity

list.sort

- `sort(*, key=None, reverse=None)`
  - This method sorts the list in place, using only < comparisons
  - Exceptions are not suppressed - if any comparison operations fail, the entire sort operation will fail (and the list will likely be left in a partially modified state).
  - key specifies a function of one argument that is used to extract a comparison key from each list element
  - reverse is a boolean value. If set to True, then the list elements are sorted as if each comparison were reversed.
  - This method modifies (mutates) the sequence in place.
  - The sort() method is guaranteed to be stable

Example

```
>>> a = [4, 32, 3, 34, 7, 31, 2, 1]

>>> a.sort()
>>> print(a)
[1, 2, 3, 4, 7, 31, 32, 34]

>>> a.sort(key=lambda x: x%5)
>>> print(a)
[1, 31, 2, 7, 32, 3, 4, 34]
```

Sorting Records

Mostly not entirely too useful to sort values. Typically, we sort records using a key.

Example

```
students = [
    ('john', 'A', 15),
    ('jane', 'B', 10),
    ('ben', 'C', 8),
    ('simon', 'A', 21),
    ('dave', 'B', 12)]
```

Example

```
students.sort()
print(students)
[('ben', 'C', 8),
 ('dave', 'B', 12),
 ('jane', 'B', 10),
 ('john', 'A', 15),
 ('simon', 'A', 21)]
```

Example

```
students.sort(
    key=lambda x: x[2],
    reverse=True)
print(students)
[('simon', 'A', 21),
 ('john', 'A', 15),
 ('dave', 'B', 12),
 ('jane', 'B', 10),
 ('ben', 'C', 8)]
```

Example

```
students.sort(key=lambda x: x[1])
print(students)
[('simon', 'A', 21),
 ('john', 'A', 15),
 ('dave', 'B', 12),
 ('jane', 'B', 10),
 ('ben', 'C', 8)]
```

Example

```
students = [
    ('john', 'A', 15),
    ('jane', 'B', 10),
    ('ben', 'C', 8),
    ('simon', 'A', 21),
    ('dave', 'B', 12)]
```

Example

```
students.sort(key=lambda x: x[1])
print(students)
[('john', 'A', 15),
 ('simon', 'A', 21),
 ('jane', 'B', 10),
 ('dave', 'B', 12),
 ('ben', 'C', 8)]
```

Scipy

Numpy

Summary

- Python Lists are mutable data structures
- Searching
  - Linear Search
  - Binary Search: Divide-and-conquer
- Sorting
  - Selection Sort
  - Merge Sort: Divide-and-conquer + recursion
  - Properties: In-place & Stability