CS1010S Programming Methodology

Lecture 11

Memoization, Dynamic Programming & Exception Handling

7 Nov 2018

Practical Exam

- Saturday 17 Nov, 12 to 6 pm
- Open Book
 - But no electronic media
- No Internet Access
 - Except for Coursemology
 - No Facebook login!
- 3 Locations
 - COM1 Labs
 - AS6 Labs
 - I³ Building 3rd floor

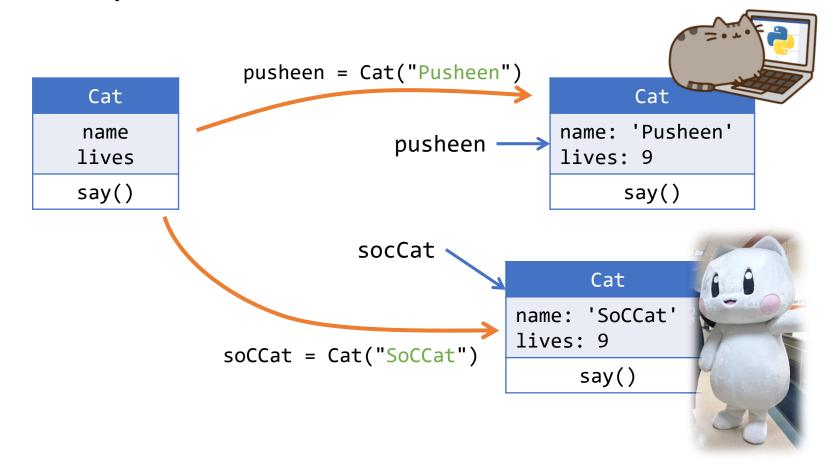
Calendar of Events

Mon	Tue	Wed	Thu	Fri	Sat	Sun
		Lecture 11	Recitat	ion 10		
Tutorial 10		Lecture 12			Practical Exam	
Reading Week			Makeup PE			
	Final Assessment					

Recap: OOP Concepts

Class vs Instance

- Class is a template, which is used to create instances



Recap: Class

- Contains
 - 1. Fields/Properties
 - 2. Methods/Functions

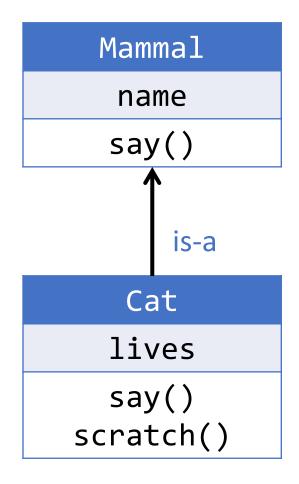
Cat

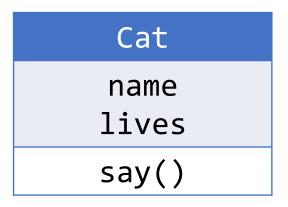
name
lives

say()

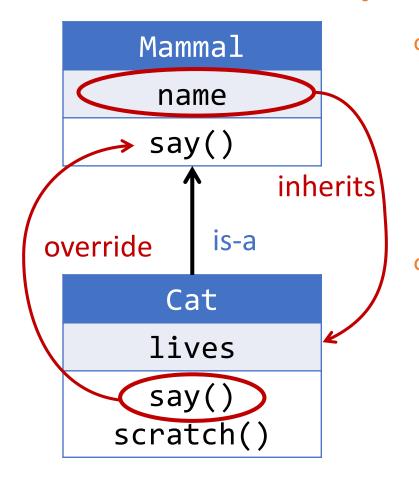
```
class Cat:
    def __init__(self, name):
         self.name = name
         self.lives = 9
    def say(self):
         print("meow")
                  _init___ method
                 is called on creation
pusheen = Cat("Pusheen")
pusheen.say()
Meow
```

Recap: Inheritance





Recap: Inheritance



```
class Mammal:
    def __init__(self, name):
        self.name = name
    def say(self):
        print(self.name + " say")
class Cat(Mammal):
    def __init__(self, name):
        super().__init__(name)
        self.lives = 9
    def say(self):
        print("meow")
    def scratch(self):
        print(self.name,
              "scratch!")
```

Recap: Inheritance

```
Mammal
       name
       say()
                inherits
           is-a
override
        Cat
       lives
       say
    scratch()
```

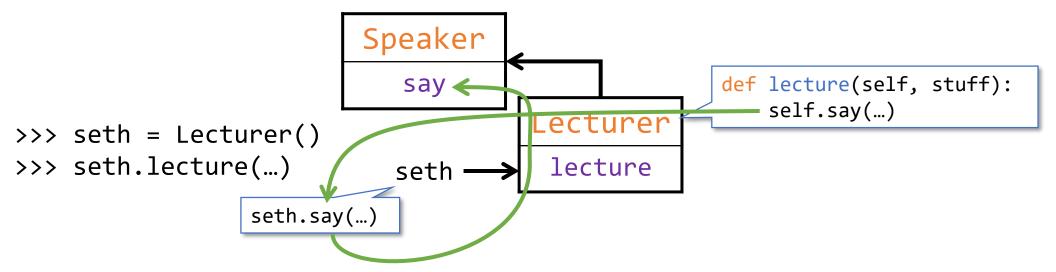
```
>>> pusheen = Cat("Pusheen")
>>> pusheen.say()
meow
>>> pusheen.scratch()
Pusheen scratch!
>>> pusheen.lives
9
```

Recap: super()

```
Mammal
                          super() refers to superclass
                          self refers to instance
   name
  say()
                          class Cat(Mammal):
                 super()
      is-a
                             def say(self):
   Cat
                                 super().say()
  lives
                                 print("meow")
  say()
scratch()
                          >>> pusheen.say()
                          Pusheen say
                          meow
```

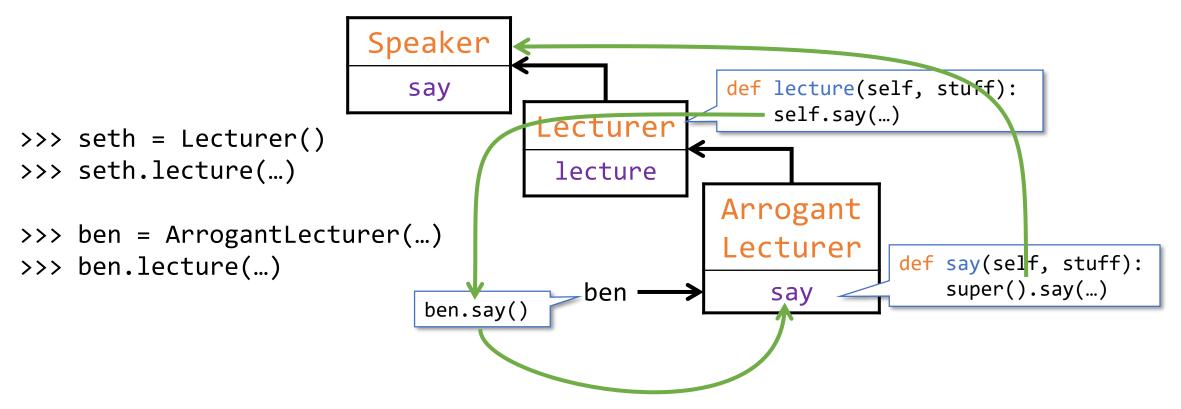
Polymorphism

- Which superclass does super() resolve to?
 - Depends on the type of the object/instance



Polymorphism

- Which superclass does super() resolve to?
 - Depends on the type of the object/instance

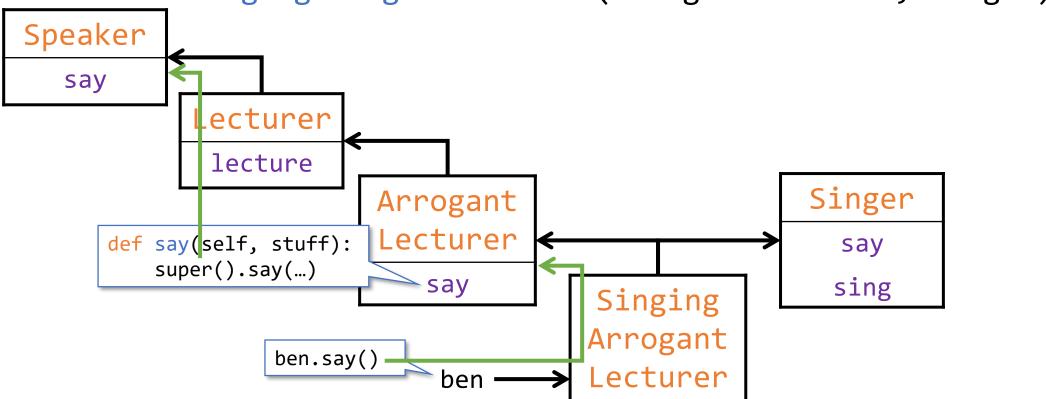


Lecturer.lecture has different behaviour

Multiple Inheritance

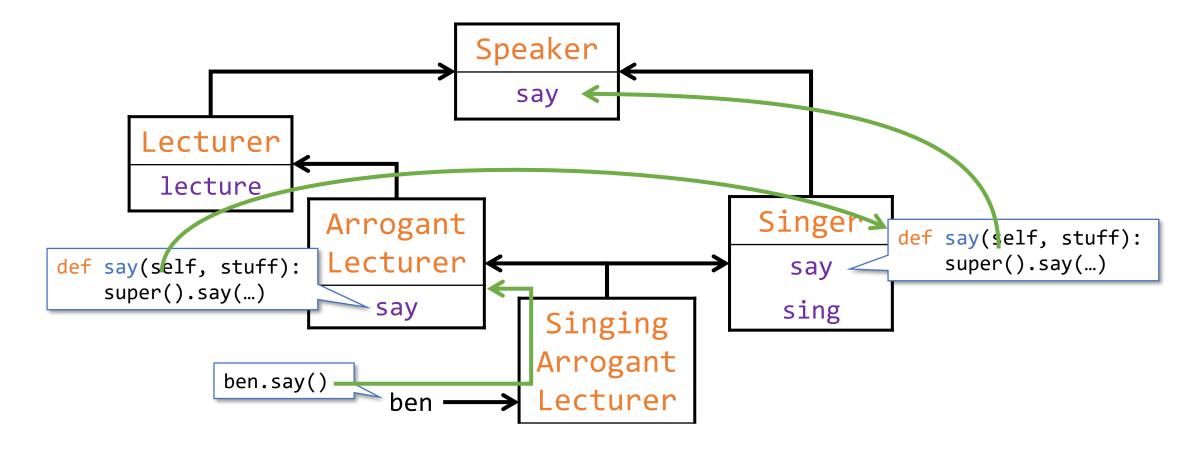
- Which superclass does super() resolve to?
 - Order of declaration

class SingingArrogantLecturer(ArrogantLecturer, Singer)



Diamond Hierachy

- Intuition: All subclasses' method must be called before superclass'
 - Allows polymorphism to occur naturally



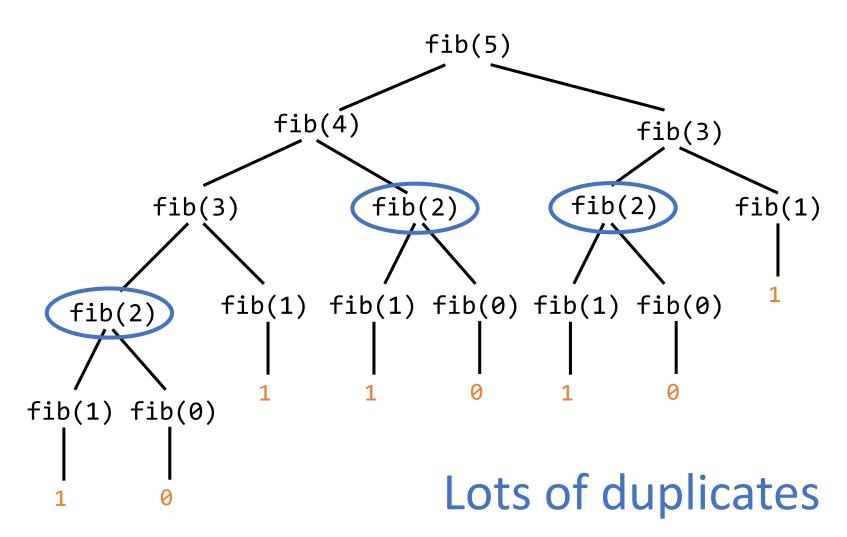
Today's Agenda

- Memoization
- Dynamic Programming
- Exception Handling

Recall: Fibonacci

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
Time complexity =
                          O(\Phi^n) (exponential!)
How can we do better?
```

Computing Fibonacci



What's the obvious way to do better?

Remember what you had earlier computed!

Memoization

Notice the spelling, NOT memorization

Simple Idea!!

Function records, in a table, values that have previously been computed.

A memoized function:

- Maintains a table in which values of previous calls are stored
- Use the arguments that produced the values as keys

- When the memoized function is called, check table to see if the value exists:
 - If so, return value.
 - Otherwise, compute new value in the ordinary way and store this in the table.

Implementing Memoization

```
memoize_table = {}
def memoize(f, name):
    if name not in memoize_table:
        memoize_table[name] = {}
    table = memoize table[name]
    def helper(*args):
        if args in table:
            return table[args]
        else:
            result = f(*args)
            table[args] = result
            return result
    return helper
```

Name to store in reference table

Fibonacci with Memoization

 Now that we have memoize, the obvious thing to do is:

```
memo_fib = memoize(fib, "fib")
```

What's the time complexity now?

Still exponential!

HUH??

Fibonacci with Memoization

 Now that we have memoize, the obvious thing to do is:

```
memo_fib = memoize(fib, "fib")
```

• Problem: recursive step in fib will call fib instead of memo fib.

Doing it Right!

```
def memo_fib(n):
    def helper(n):
        if n == 0:
            return 0
        elif n == 1:
            return 1
        else:
            return memo_fib(n-1) +
                    memo fib(n-2)
    return memoize(helper, "memo fib")(n)
                                      O(n) (linear)!
What's the time complexity now?
```

Doing it Right!

```
def memo_fib(n):
    def helper(n):
        if n == 0:
            return 0
        elif n == 1:
            return 1
        else:
            return memo_fib(n-1) +
                   memo_fib(n-2)
    return memoize(helper, "memo_fib")(n)
```

Each fib(n) is computed only once!

Doing it Right!

```
def memo fib(n):
    def helper(n):
        if n == 0:
             return 0
        elif n == 1:
             return 1
        else:
             return memo_fib(n-1) +
                    memo_fib(n-2)
    return memoize(helper, "memo_fib")(n)
Efficiency of table lookup is important: Table lookup should be O(1), i.e. hash table.
   What happens to time complexity if table lookup is not constant, say O(n)?
```

Another Example: C_k^n

```
def choose(n, k):
    if k > n:
        return 0
    elif k=0 or k==n:
        return 1
    else:
        return choose(n-1, k) +
               choose(n-1, k-1)
```

Why is the recursion true?

Remember Count-Change?

- Consider one of the elements x. x is either chosen or it is not.
- Then number of ways is sum of:
 - Not chosen. Ways to choose k elements out of remaining n-1 elements; and
 - Chosen. Ways to choose k-1 elements out of remaining n-1 elements

Another Example: C_k^n

What is the order of growth? How can we speed up the computation?

Memoization!

Memoized Choose

```
def memo choose(n, k):
                                       Don't need to use
    def helper(n, k):
                                       memoize function.
        if k > n:
                                         Can just use a
             return 0
                                          dictionary!
        elif k=0 or k==n:
            return 1
        else:
            return memo_choose(n-1, k) +
                    memo choose(n-1, k-1)
    return memoize(helper, "choose")(n, k)
```



Suppose we are at a chocolate candy store and want to assemble a kilogram box of chocolates.

Some of the chocolates (such as the caramels) at this store are absolutely the best, and others are only so-so.

Chocolate Packing

We rate each one on a scale of 1 to 10, with 10 being the highest.

Each piece of chocolate has a weight; for example, a caramel-flavoured chocolate weighs 13 grams.

How do we put together the best box weighing at most 1 kilogram?

Abstract Data Type for Chocolates

```
def make chocolate(desc, weight, value):
    return (desc, weight, value)
def get_description(choc):
    return choc[0]
def get weight(choc):
    return choc[1]
def get_value(choc):
    return choc[2]
```

Here are the Chocolates

```
shirks chocolates =
  (make chocolate('caramel dark', 13, 10),
  make_chocolate('caramel milk', 13, 3),
  make_chocolate('cherry dark', 21, 3),
  make_chocolate('cherry milk', 21, 1),
  make chocolate('mint dark', 7, 3),
  make chocolate('mint milk', 7, 2),
  make chocolate('cashew-cluster dark', 8, 6),
  make chocolate('cashew-cluster milk', 8, 4),
  make chocolate('maple-cream dark', 14, 1),
  make chocolate('maple-cream milk', 14, 1))
```

Implementing a Box

```
def make_box(list_of_choc, weight, value):
    return (list of choc, weight, value)
def make empty box():
    return make box((), 0, 0)
def box chocolates(box):
    return box[0]
def box weight(box):
    return box[1]
def box value(box):
    return box[2]
```

Implementing a Box

```
def add to box(choc, box):
  return make box(
          box chocolates(box) + (choc,),
          box weight(box) + get_weight(choc),
          box value(box) + get value(choc))
def better box(box1, box2):
    if box_value(box1) > box_value(box2):
        return box1
    else:
        return box2
```

How to Solve this Problem?

- Enumerate all the possible boxes (constrained by weight limit)
- Compute value of each packing
- Pick box with highest value

Simple Solution

```
def pick(chocs, weight_limit):
    if chocs==() or weight_limit==0:
        return make empty box()
    elif get_weight(chocs[0]) > weight_limit: # 1st too heavy
        return pick(chocs[1:], weight_limit)
    else:
        # none of 1st kind
        box1 = pick(chocs[1:], weight_limit)
        # at least one of 1st kind
        box2 = add_to_box(chocs[0],
                          pick(chocs,
                               weight_limit -
                               get_weight(chocs[0])))
        return better_box(box1, box2)
```

Simple Solution

- What is the order of growth?
 - Exponential! $O(2^n)$

Again, a lot of repeat computations.

Think memoization!

Memoized Version

```
def memo_pick(chocs, weight_limit):
    def helper(chocs, weight limit):
        if chocs==() or weight limit==0:
            return make_empty_box()
        elif get_weight(chocs[0]) > weight_limit:
            return memo_pick(chocs[1:], weight_limit)
        else:
            box1 = memo_pick(chocs[1:], weight_limit)
            box2 = add_to_box(chocs[0],
                              memo pick(chocs,
                                        weight_limit -
                                       get weight(chocs[0])))
            return better box(box1, box2)
    return memoize(helper, "pick")(chocs, weight_limit)
```

Recap: Memoization

- Two Steps:
 - 1. Write function to perform required computation
 - 2. Add wrapper that stores the result of the computation (O(1) | lookup | table)

 When you wrap your function, just make sure that he recursive calls go to the wrapped version and not the raw form.

Homework

Re-factor the chocolate packing code into OOP format.

Design Pattern: Wrapper (also called Decorator)

- Memoization as an idea is simply to remember the stuff you have done before so that you don't do the same thing twice
- However, the method that we used to implement memoization is also an important concept

Design Pattern: Wrapper (also called Decorator)

- Design Pattern: Wrapper (also known as Decorator)
- Key idea is that you add an extra layer to introduce additional functionality and use the original function to do "old work"

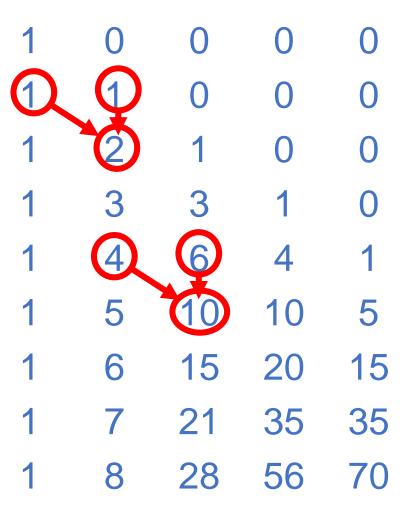
Revisting memo_choose

- Considermemo_choose(9, 4)
- The following is the table of values at the end of the computation:

```
#f #f
             #f
      #f
          #f
             #f
          #f
             #f
   3 3 1
             #f
   5 10 10
#f
   6 15 20 15
   #f 21 35 35
#f
   #f #f 56 70
#f
```

Recall Pascal's Triangle

 If we were to fill up the table, we expect



Dynamic Programming

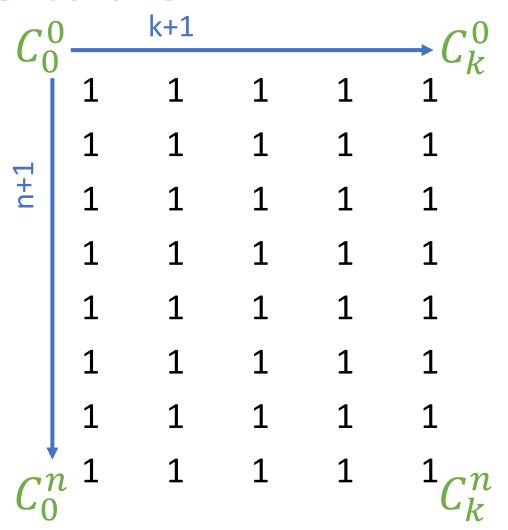
- Idea: why don't we compute choose by filling up this table from the bottom?
- Fancy name for this simple idea Dynamic Programming :-)
- What is the order of growth then? O(n) or more accurately O(nk)

Dynamic Programming: choose

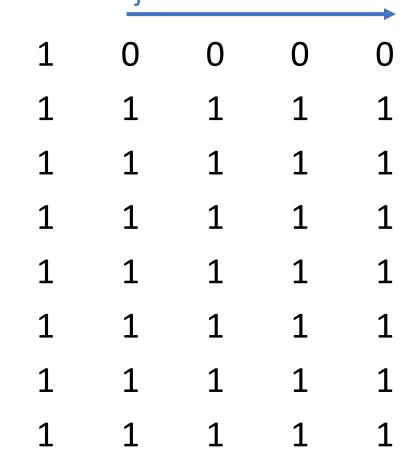
```
def dp_choose(n, k):
    row = [1] * (k+1)
    table = []
    for i in range(n+1):
        table.append(row.copy())
                                               Fill first row
    for j in range(1, k+1):
        table[0][j] = 0
    for i in range(1, n+1):
        for j in range(1, k+1):
            table[i][j] = table[i-1][j-1]
                           + table[i-1][j]
    return table[n][k]
                                       return answer
```

```
1 1 1 1 1 1
```

```
row = [1] * (k+1)
table = []
for i in range(n+1):
   table.append(
          row.copy())
```



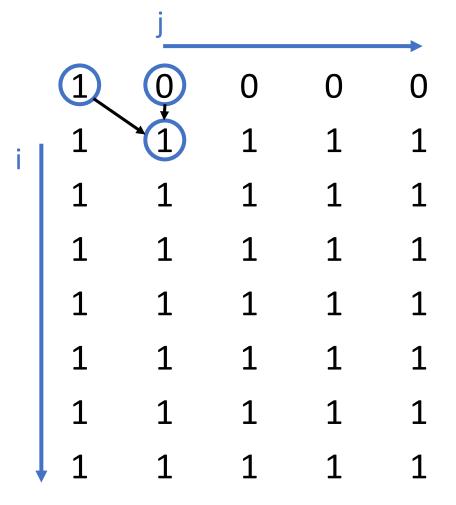
```
for j in range(1,k+1):
    table[0][j] = 0
```

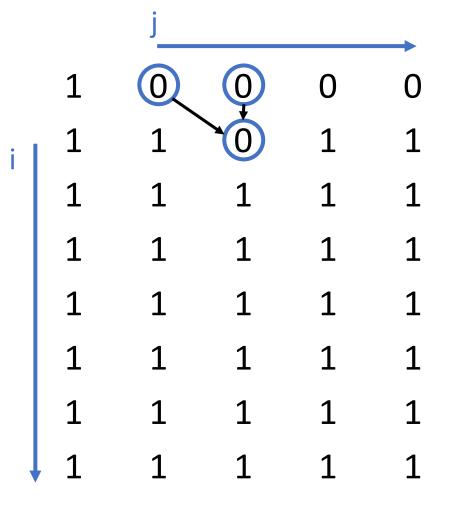


```
for j in range(1,k+1):
    table[0][j] = 0
```

1	0	0	0	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

```
for i in range(1,n+1):
    for j in
        range(1,k+1):
             table[i][j] =
        table[i-1][j-1] +
        table[i-1][j]
```

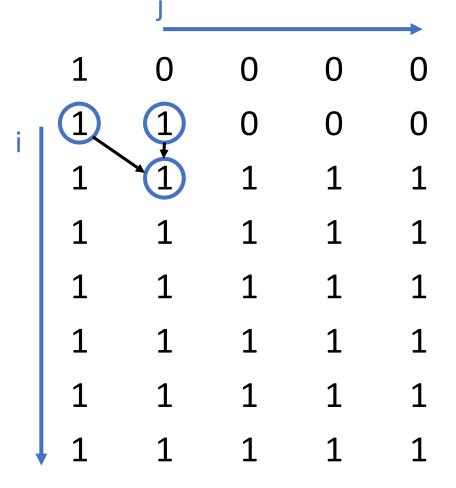




```
for i in range(1,n+1):
                                                        0
  for j in
    range(1,k+1):
            table[i][j] =
    table[i-1][j-1] +
    table[i-1][j]
```

```
for i in range(1,n+1):
                                              0
  for j in
    range(1,k+1):
            table[i][j] =
    table[i-1][j-1] +
    table[i-1][j]
```

```
for i in range(1,n+1):
    for j in
        range(1,k+1):
            table[i][j] =
    table[i-1][j-1]
        + table[i-1][j]
```



```
for i in range(1,n+1):
                                              0
                                                        0
  for j in
                                                        0
    range(1,k+1):
            table[i][j] =
    table[i-1][j-1] +
    table[i-1][j]
```

Fast Forward

1	0	0	0	0
1	1	0	0	0
1	2	1	0	0
1	3	3	1	0
1	4	6	4	1
1	5	10	10	5
1	6	15	20	15
1	7	21	35	(35)

Can we adopt a dynamic programming approach for solving the chocolate packing problem?

YES (of course)!

How?

Let's take another look at the chocolate packing problem...

Remember the table dynamic programming is about filling up some table efficiently so that the total time complexity is $O(size\ of\ table)$

What does the chocolate packing table look like?

- Table of list of chocolates vs weight limit
- Each table entry is the optimal choice for a given list and weight limit.

What does the chocolate packing table look like?

- Observation: If we have no chocolates, answer is, easy: Empty set ().
- Otherwise, the answer for a list of x types of chocolates is the better between:
 - 1. Additional chocolate + Optimal choice with remaining x types of chocolates and reduced weight limit
 - 2. Optimal choice with remaining x-1 types of chocolates and current weight limit.

Idea: build a table from the smaller cases to larger cases

Weight Limit	()		
0	()		
1	()		
2	()		
3	()		
4	()		
5	()		
6	()		
7	()		
8	()		
	()		

Weight Limit	()	((A 4 2))	
0	()		
1	()		
2	()	() +A	
3	()	0	
4	()	((A))	
5	()	(A)	
6	()	(<u>A</u>)+A	
7	0	(4)	
8	()	(A,A)	
:	()	:	

Weight Limit	()	((A 4 2))	((A 4 2) (B 5 3))
0	()	()	
1	()	()	
2	()	()	Cannot
3	()	()	♦add B
4	()	(A)	(A)
5	()	(A)	(B)
6	()	(A)	
7	()	(A)	
8	()	(A,A)	
:	()	:	

Weight Limit	()	((A 4 2))	((A 4 2) (B 5 3))	
0	()	()		
1	()	()		
2	()	()		
3	()	()		
4	()	(A)	(A) +B	
5	()	(A)		
6	()	(A)	+B	
7	()	(A)	В	
8	()	(A,A)		
:	()	:		

Weight Limit	()	((A 4 2))	((A 4 2) (B 5 3))	
0	()	()	()	
1	()	()	()	
2	()	()	()	
3	()	()	(1)	
4	()	(A)	(4)	
5	()	(A)	В	
6	()	(A)	В	
7	()	(A)	+B	
8	()	(A,A)	(A,A)	
:	()		:	

Challenge of the Day: Write dp_pick_chocolates over the weekend. ©

Prime Numbers

- In recitation, we defined a function is_prime to check whether a number is prime
- But what if we wanted to list ALL of the numbers that are prime, in the interval [0, ..., n]?

Prime Numbers: Naïve Soln

```
def is_prime(n): \# O(n^{0.5})
    if n==0 or n==1:
        return False
    elif n == 2:
        return True
    for i in range(2, int(sqrt(n))+1):
        if n % i == 0:
             return False
    return True
def naive_prime(n): # O(n<sup>1.5</sup>)
    return [is prime(i) for i in range(n+1)]
```

Prime Numbers

Idea: why don't we compute the primes by filling up a table from the bottom?

Prime Numbers: DP

```
def dp prime(n):
    bitmap = [True for i in range(n+1)]
    bitmap[0] = False # 0 is not prime
    bitmap[1] = False # 1 is not prime
   for i in range(2,n):
        if bitmap[i] == True:
            for j in range(2*i,n+1,i):
                bitmap[j] = False
    return bitmap
```

Prime Numbers: DP

How does it work?

Errors and Exceptions

Errors and Exceptions

- Until now error messages haven't been more than mentioned, but you have probably seen some
- Two kinds of errors (in Python):
 - 1. syntax errors
 - 2. exceptions

Syntax Errors

```
>>> while True print('Hello world')
SyntaxError: invalid syntax
```

Exceptions

- Errors detected during execution are called exceptions
- Examples:
 - ZeroDivisonError,
 - NameError,
 - TypeError

ZeroDivisionError

```
>>> 10 * (1/0)
Traceback (most recent call last):
   File "<pyshell#3>", line 1, in <module>
      10 * (1/0)
ZeroDivisionError: division by zero
```

NameError

```
>>> 4 + spam*3
Traceback (most recent call last):
   File "<pyshell#4>", line 1, in <module>
      4 + spam*3
NameError: name 'spam' is not defined
```

TypeError

ValueError

```
>>> int('one')
Traceback (most recent call last):
   File "<pyshell#2>", line 1, in <module>
        int('one')
ValueError: invalid literal for int() with base 10:
'one'
```

Handling Exceptions

The simplest way to catch and handle exceptions is with a try-except block:

```
x, y = 5, 0
try:
    z = x/y
except ZeroDivisionError:
    print("divide by zero")
```

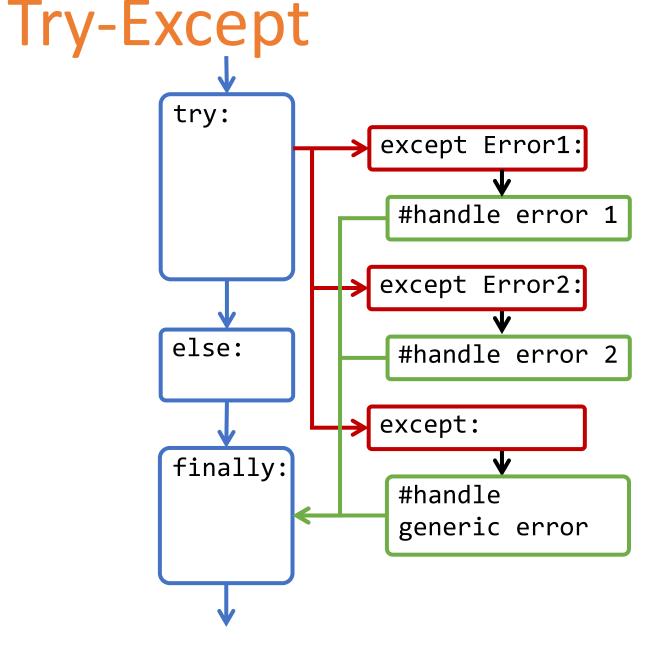
Try-Except (How it works I)

- The try clause is executed
- If an exception occurred, skip the rest of the try clause, to a matching except clause
- If no exception occurs, the except clause is skipped (go to the else clause, if it exists)
- The finally clause is always executed before leaving the try statement, whether an exception has occurred or not.

Try-Except

- A try clause may have more than 1 except clause, to specify handlers for different exception.
- At most one handler will be executed.
- Similar with if-elif-else
- finally will always be executed

try: # statements except Error1: # handle error 1 except Error2: # handle error 2 except: # wildcard # handle generic error else: # no error raised finally: # always executed



Try-Except Example

```
def divide test(x, y):
    try:
      result = x / y
    except ZeroDivisionError:
      print("division by zero!")
    else:
      print("result is", result)
    finally:
      print("executing finally clause")
```

Try-Except Blocks

```
>>> divide_test(2, 1)
result is 2.0
executing finally clause
>>> divide test(2, 0)
division by zero!
executing finally clause
>>> divide test("2", "1")
executing finally clause
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
  File "<stdin>", line 3, in divide
TypeError: unsupported operand type(s) for /: 'str' and 'str'
```

```
def divide test(x, y):
    try:
      result = x / y
    except ZeroDivisionError:
      print("division by zero!")
    else:
      print("result is", result)
    finally:
      print("executing finally
             clause")
```

Raising Exceptions

The raise statement allows the programmer to force a specific exception to occur:

```
>>> raise NameError('HiThere')
Traceback (most recent call last):
   File "<stdin>", line 1, in ?
NameError: HiThere
```

Exception Types

- Built-in Exceptions: https://docs.python.org/3/library/exceptions.html
- User-defined Exceptions

User-defined Exceptions I

```
class MyError(Exception):
    def __init__(self, value):
        self.value = value
    def __str__(self):
        return repr(self.value)
```

User-defined Exceptions II

```
try:
    raise MyError(2*2)
except MyError as e:
    print('Exception value:', e.value)
Exception value: 4
raise MyError('oops!')
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
main .MyError: 'oops!'
```

Why use Exceptions?

In the good old days of C, many procedures returned special ints for special conditions, i.e. -1

Why use Exceptions?

- But Exceptions are better because:
 - More natural
 - More easily extensible
 - Nested Exceptions for flexibility

Summary

- Memoization dramatically reduces computation.
 - Once a value is computed, it is remembered in a table (along with the argument that produced it).
 - The next time the procedure is called with the same argument, the value is simply retrieved from the table.
- memo_fib takes time = O(n)
- memo_choose takes ?? time?

Memoization vs Dynamic Programming

- Sometimes DP requires more computations
- DP requires the programmer to know exactly which entries need to be computed
- For smart programmer, DP can however be made more space efficient for some problems, i.e. limited history recurrences like Fibonacci