A modeling approach to forecasting data with reporting delay

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Motivation

Monitoring the spread of illnesses through a surveillance system is essential!

A surveillance system aims to:

$$\text{observe} \longrightarrow \text{predict} \longrightarrow \begin{array}{c}\text{increase knowledge}\\ \text{and}\\ \text{minimize damage}\end{array}$$

and should present:

- sensitivity
- specificity
- timeliness

Lack of timeliness may be due to:

- laboratory confirmation
- logistical problems
- infrastructure difficulties

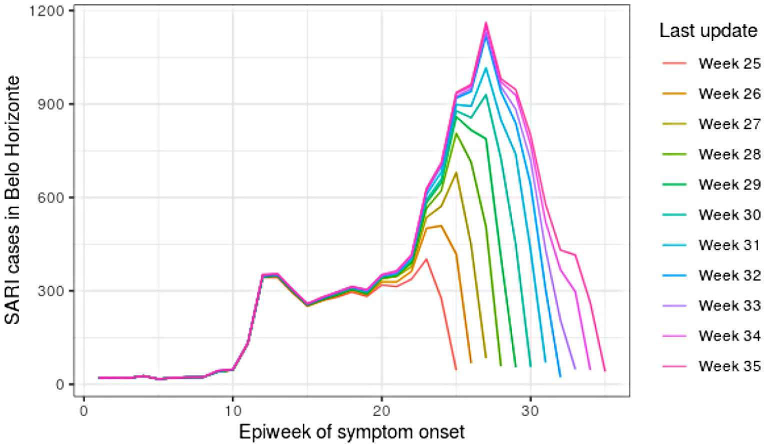
The difference between the reported and the true disease incidence varies according to the reporting delays

This is a problem where the observable data will eventually become available

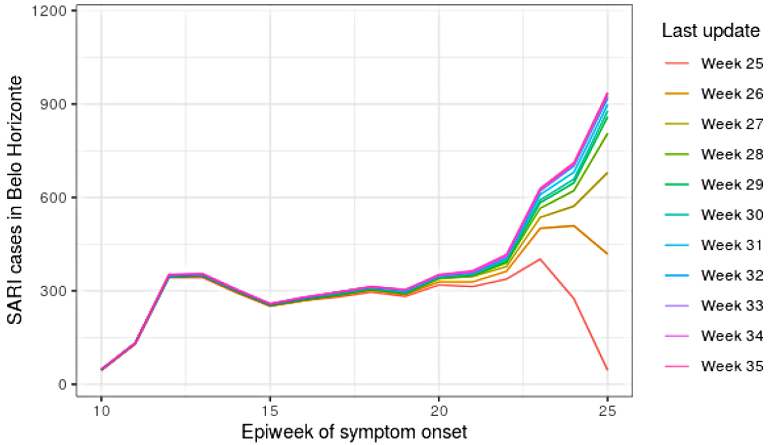
! observed data $\neq$ truth

The reporting delay problem

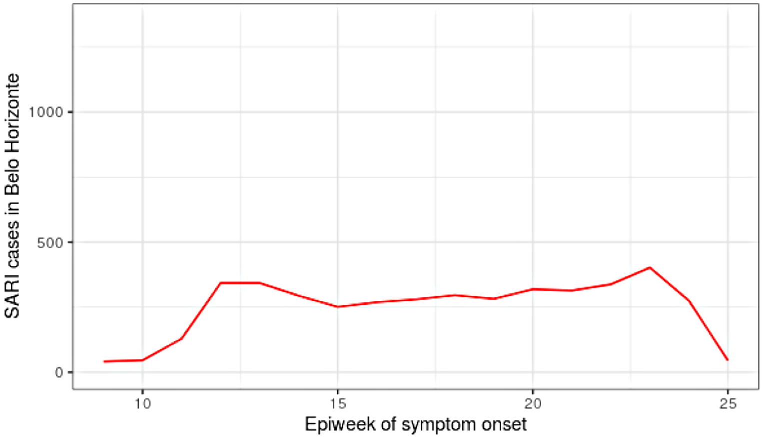
SARI cases in Belo Horizonte, Brazil by reporting date



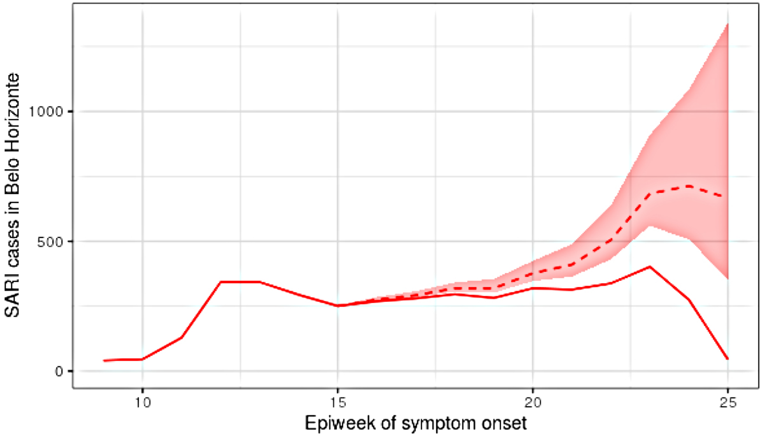
SARI cases in Belo Horizonte by reporting date up to week 25



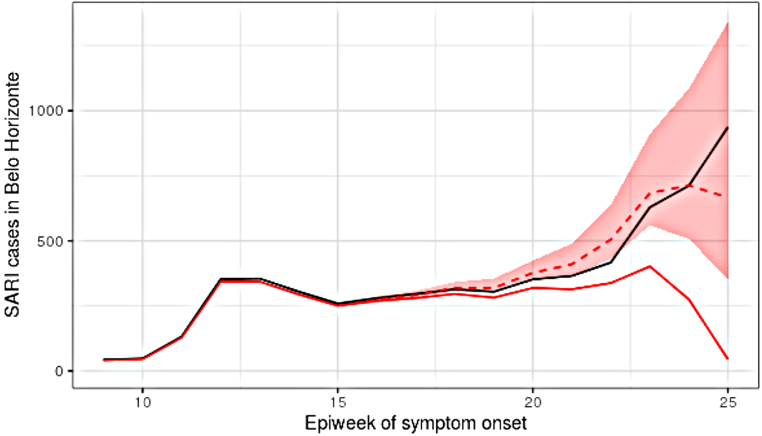
Suppose today is 20/6/2020 (week 25)



We aim to predict the occur-but-not-yet-reported cases



and in the future, compare with what actually happened



Now + Forecasting = Nowcasting!

There are different approaches in the literature:

- Regression-like approach using proxies (Ginsberg et al., 2009)
- Model historic delay to correct present data (Brookmeyer and Damiano, 1989)
- Bayesian hierarchical modeling approach (Bastos et al., 2019)

In these models, there is no specific component relating to the disease dynamics that collaborate toward forecasting

We aim to propose a Bayesian model that enables nowcasting and forecasting!

- T: current time (today)
- t: time index, varying in $\{1, 2, \ldots, T + H\}$
- D: maximum relevant delay
- d: delay index, varying in $\{k, k + 1, \ldots, D\}$
- $n_{t,d}$: number of events occurred at time t recorded after d units of time
- $N_t = \sum_{d=k}^{D} n_{t,d}$: total number of events occurred at time t

! when there is concomitant available information, set $k = 0$

$t \backslash d$	0	1	$\cdots$	$D-1$	D	N
1	$n_{1,0}$	$n_{1,1}$	$\cdots$	$n_{1,D-1}$	$n_{1,D}$	N_1
2	$n_{2,0}$	$n_{2,1}$	$\cdots$	$n_{2,D-1}$	$n_{2,D}$	N_2
3	$n_{3,0}$	$n_{3,1}$	$\cdots$	$n_{3,D-1}$	$n_{3,D}$	N_2
$\vdots$			$\cdots$			$\vdots$
$T-D$	$n_{T-D,0}$	$n_{T-D,1}$	$\cdots$	$n_{T-D,D-1}$	$n_{T-D,D}$	N_{T-D}
$T-D+1$	$n_{T-D+1,0}$	$n_{T-D+1,1}$	$\cdots$	$n_{T-D+1,D-1}$	$n_{T-D+1,D}$	N_{T-D+1}
$\vdots$			$\cdots$			$\vdots$
$T-1$	$n_{T-1,0}$	$n_{T-1,1}$	$\cdots$	$n_{T-1,D-1}$	$n_{T-1,D}$	N_{T-1}
T	$n_{T,0}$	$n_{T,1}$	$\cdots$	$n_{T,D-1}$	$n_{T,D}$	N_T
$T+1$	$n_{T+1,0}$	$n_{T+1,1}$	$\cdots$	$n_{T+1,D-1}$	$n_{T+1,D}$	N_{T+1}
$T+2$	$n_{T+2,0}$	$n_{T+2,1}$	$\cdots$	$n_{T+2,D-1}$	$n_{T+2,D}$	N_{T+2}
$\vdots$			$\cdots$			$\vdots$
$T+H$	$n_{T+H,0}$	$n_{T+H,1}$	$\cdots$	$n_{T+H,D-1}$	$n_{T+H,D}$	N_{T+H}

Tabela 1: Data structure in a reporting delay problem.

$t \backslash d$	k	$k+1$	$\cdots$	$D-1$	D	N
1	$n_{1,k}$	$n_{1,k+1}$	$\cdots$	$n_{1,D-1}$	$n_{1,D}$	N_1
2	$n_{2,k}$	$n_{2,k+1}$	$\cdots$	$n_{2,D-1}$	$n_{2,D}$	N_2
3	$n_{3,k}$	$n_{3,k+1}$	$\cdots$	$n_{3,D-1}$	$n_{3,D}$	N_2
$\vdots$			$\cdots$			$\vdots$
$T-D+k$	$n_{T-D+k,k}$	$n_{T-D+k,k+1}$	$\cdots$	$n_{T-D+k,D-1}$	$n_{T-D+k,D}$	N_{T-D+k}
$T-D+k+1$	$n_{T-D+k+1,k}$	$n_{T-D+k+1,k+1}$	$\cdots$	$n_{T-D+k+1,D-1}$	$n_{T-D+k+1,D}$	$N_{T-D+k+1}$
$\vdots$			$\cdots$			$\vdots$
$T-1$	$n_{T-1,k}$	$n_{T-1,k+1}$	$\cdots$	$n_{T-1,D-1}$	$n_{T-1,D}$	N_{T-1}
T	$n_{T,k}$	$n_{T,k+1}$	$\cdots$	$n_{T,D-1}$	$n_{T,D}$	N_T
$T+1$	$n_{T+1,k}$	$n_{T+1,k+1}$	$\cdots$	$n_{T+1,D-1}$	$n_{T+1,D}$	N_{T+1}
$T+2$	$n_{T+2,k}$	$n_{T+2,k+1}$	$\cdots$	$n_{T+2,D-1}$	$n_{T+2,D}$	N_{T+2}
$\vdots$			$\cdots$			$\vdots$
$T+H$	$n_{T+H,k}$	$n_{T+H,k+1}$	$\cdots$	$n_{T+H,D-1}$	$n_{T+H,D}$	N_{T+H}

Tabela 2: Data structure in a reporting delay problem, with first delay k.

We assume the following structure for N_t

$$\begin{aligned} N_t &\sim NegBin(\theta_t, \phi) \\ \theta_t &= \frac{a_\theta c_\theta f_\theta \exp(-c_\theta t)}{[b_\theta + \exp(-c_\theta t)]^{f_\theta + 1}} \end{aligned}$$

for $t = 1, \ldots, T + H$, such that

$$E[N_t] = \theta_t \quad \text{and} \quad Var[N_t] = \theta_t \left(1 + \frac{\theta_t}{\phi}\right)$$

! when $\phi \to \infty$, the Negative Binomial reduces to the Poisson

! for $t > T - D + k$, N_t is a function of unobserved quantities

We assume the following structure for $n_{t,d}$

$$\begin{aligned} n_{t,d} &\sim NegBin(\lambda_{t,d}, \sigma) \\ \log(\lambda_{t,d}) &= \alpha_t + \beta_d \end{aligned}$$

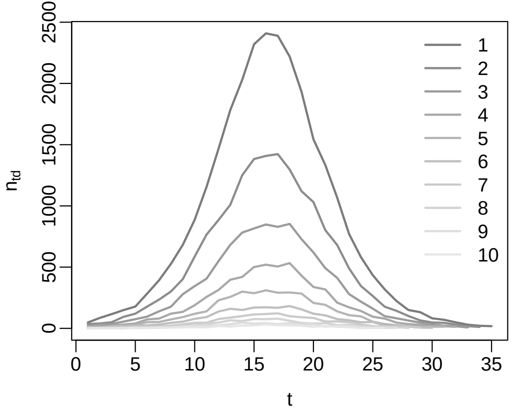
for $t = 1, \ldots, T+H$, $d = k+1, \ldots, D$, where

$$\exp(\alpha_t) = \frac{a_\alpha c_\alpha f_\alpha \exp(-c_\alpha t)}{[b_\alpha + \exp(-c_\alpha t)]^{f_\alpha + 1}} \quad \text{and} \quad \beta_d = \gamma d$$

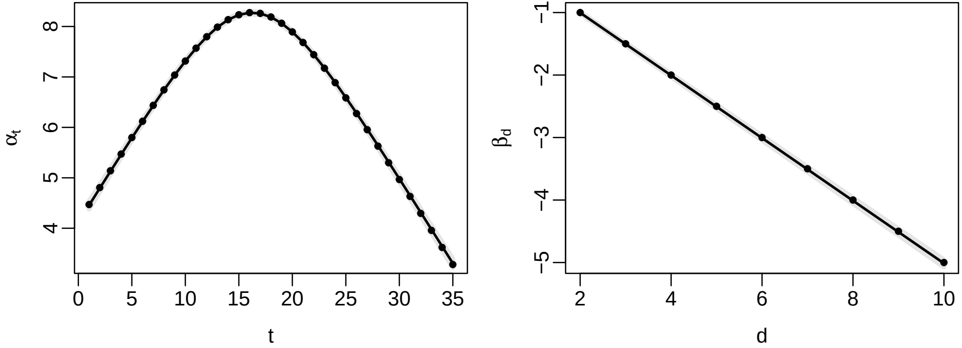
! we do not specify a distribution for $n_{t,k}$ since $n_{t,d} = N_t - \sum_{d=k+1}^{D} n_{t,d}$

! θ_t must be greater than $\sum_{d=k+1}^{D} \lambda_{t,d}$

To assess the effectiveness of the proposed model, artificial data was generated with $\phi \to \infty$ and $\sigma \to \infty$



The results accurately recovered the real parameters' values



t \ d	2	3	4	5	6	7	8	9	10	N_t
27									5 2 (0;6)	567 589 (542;642)
28								4 3 (0;7)	3 2 (0;5)	406 425 (383;468)
29							5 4 (0;8)	4 2 (0;5)	2 1 (0;4)	311 306 (273;343)
30						3 4 (1;9)	6 2 (0;6)	1 1 (0;4)	1 1 (0;3)	209 219 (190;249)
31					8 5 (1;10)	2 3 (0;7)	5 2 (0;5)	2 1 (0;3)	0 0 (0;3)	185 157 (130;182)
32				9 6 (2;11)	3 3 (0;8)	1 2 (0;6)	1 1 (0;4)	0 1 (0;3)	1 0 (0;2)	124 112 (92;133)
33			9 7 (3;13)	8 4 (1;9)	2 2 (0;6)	6 1 (0;4)	1 1 (0;3)	1 0 (0;2)	0 0 (0;2)	83 79 (62;98)
34		8 8 (3;15)	4 5 (2;10)	4 3 (0;7)	2 2 (0;5)	1 1 (0;3)	2 0 (0;3)	0 0 (0;2)	0 0 (0;2)	54 58 (44;73)
35	8 10 (4;16)	3 6 (2;11)	5 4 (0;8)	2 2 (0;5)	2 1 (0;4)	1 1 (0;3)	0 0 (0;2)	0 0 (0;2)	0 0 (0;2)	39 40 (29;55)

Tabela 3: True values (upper number), posterior median (lower number), and respective 95% credibility interval (in parenthesis) for the non-observed counts.

We are proposing a promising model to nowcast and forecast

The model can be used in real-time decision-making as well as in making short-term and long-term predictions

The example shows the model's ability to recover the parameters accurately and nowcast the unobserved values

Future work: extend this model to accommodate more waves

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Thank You!