Kalman:

$$x_{t+1} = A x_t + B u_{t+1} + \omega_t \quad Q$$

$$y_t = C x_t + v_t \quad R$$

$$x(\bar{\mu}, \bar{\Sigma})$$

$$\begin{cases} u & \text{input through } B. \\ A & \text{matrix of the system.} \\ \omega & \text{perturpation with Covar } Q. \\ y & \text{measurment throught } C. \\ v & \text{noise with Covar } R. \end{cases}$$

$$\bar{\mu}_t = A \mu_{t-1} + B u_{t-1}$$

$$\bar{\Sigma}_t = A \Sigma_{t-1} A^T + Q$$

$$i_t = y_t - C \bar{\mu}_t$$

$$S_t = C \bar{\Sigma}_t C^T + R$$

$$K_t = \bar{\Sigma}_t C^T S_t^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t i_t$$

$$\Sigma_t = (I - KC) \bar{\Sigma}_t$$

$$x = \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \\ \theta \end{bmatrix}$$

$$y = \begin{bmatrix} y_{cam} \\ y_{wheels} \end{bmatrix} = \begin{bmatrix} p_{x\,cam} \\ p_{y\,cam} \\ \theta_{cam} \\ v_{leftw} \\ v_{rightw} \end{bmatrix}$$

$$u = \begin{bmatrix} v_{leftw\,com} \\ v_{rightw\,com} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{2}\cos\theta\,\Delta t & \frac{1}{2}\cos\theta\,\Delta t \\ \frac{1}{2}\sin\theta\,\Delta t & \frac{1}{2}\sin\theta\,\Delta t \\ \frac{1}{2}\cos\theta & \frac{1}{2}\cos\theta \\ \frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta \\ -\frac{\Delta t}{dist} & \frac{\Delta t}{dist} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1/\theta & \frac{dist}{2\Delta t} \\ 0 & 0 & 0 & 1/\theta & \frac{dist}{2\Delta t} \end{bmatrix}$$

Model:

$$x = \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \\ \theta \end{bmatrix}$$

$$\underset{5\times5}{A}\, x =$$

Camera

$$p_{x_t}$$
$$p_{y_y}$$
$$\theta_t$$

$$\frac{V_L + V_R}{2} = V_{str}$$

$$V_x = V_{str} \cdot \cos\theta$$
$$V_y = V_{str} \cdot \sin\theta$$

wheels

$$p_{x_t} = v_{t-1} \cdot \Delta t + p_{x_{t-1}}$$
$$p_{y_t} = v_{t-1} \cdot \Delta t + p_{y_{t-1}}$$
$$\theta_t = \Delta\theta_{t-1} + \theta_{t-1}$$
$$\Delta\theta_{t-1} = \arctan\left(\frac{\Delta v_{t-1} \cdot \Delta t}{dist}\right)$$

linearize

$$\Delta\theta_{t-1} = \frac{\Delta v_{w_{t-1}}}{dist}$$

y measurments:

$$y = \begin{bmatrix} y_{cam} \\ y_{wheels} \end{bmatrix} = \begin{bmatrix} p_{x_{cam}} \\ p_{y_{cam}} \\ \theta_{cam} \\ v_{leftw} \\ v_{rightw} \end{bmatrix}$$

Camera	wheels speed
$p_{x_{cam}}$	v_{leftw}
$p_{y_{cam}}$	v_{rightw}
θ_{cam}	

input:

$$u = \begin{bmatrix} v_{leftwcom} \\ v_{rightwcom} \end{bmatrix}$$

$$\underset{5\times2}{B}\, u = \begin{matrix} \frac{V_L+V_R}{2}\cos\theta \cdot \Delta t \\ \frac{V_L+V_R}{2}\sin\theta \cdot \Delta t \\ \frac{V_L+V_R}{2}\cos\theta \\ \frac{V_L+V_R}{2}\sin\theta \\ \frac{V_R - V_L}{dist} \cdot \Delta t \end{matrix}$$

$$A = \begin{bmatrix} 1 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

camera factor

$$C_f$$

$$Q = (\diagdown)$$

$$R = (\diagdown)$$

$$B = \begin{bmatrix} \frac{1}{2}\cos\theta\,\Delta t & \frac{1}{2}\cos\theta\,\Delta t \\ \frac{1}{2}\sin\theta\,\Delta t & \frac{1}{2}\sin\theta\,\Delta t \\ \frac{1}{2}\cos\theta & \frac{1}{2}\cos\theta \\ \frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta \\ -\frac{\Delta t}{dist} & \frac{\Delta t}{dist} \end{bmatrix}$$

$$\left.\begin{array}{l}\sin\theta \to \theta \\ \cos\theta \to 1-\frac{\theta^2}{2}\end{array}\right\}\text{ everywhere}$$

$$\omega = \frac{V_R - V_L}{dist} = \dot{\theta} = \frac{\theta}{\Delta t}$$

$$(V_L + V_R)\cos\theta = 2v_x$$

$$(V_L + V_R)\sin\theta = 2v_y$$

$$+\begin{cases} V_L + V_R = \frac{2v_y}{\theta} \\ V_R - V_L = \frac{\theta}{\Delta t}\cdot dist \end{cases}$$

$$V_R = \frac{1}{2}\left(\frac{2v_y}{\theta} + \frac{\theta}{\Delta t} dist\right)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1/\theta & \frac{dist}{2\Delta t} \\ 0 & 0 & 0 & 1/\theta & \frac{dist}{2\Delta t} \end{bmatrix}$$

$$\boxed{\begin{array}{l} V_R = \frac{v_y}{\theta} + \frac{dist\,\theta}{2\Delta t} \\ V_L = \frac{v_y}{\theta} - \frac{dist\,\theta}{2\Delta t} \end{array}}$$