Analytic Number Theory

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Chapter 1

Additive Number Theory

1.1 Additive Basis

Definition (Additive Basis)**.** *We define an* ***additive basis*** S*, to be a set of natural numbers such that* $\exists k \in \mathbb{N}$ *such that every natural number can be expressed as a sum of* k *or fewer elements of* S*. That is, the sumset of* k *copies of* S *consists of all natural numbers.*

Definition (Order of an Additive Basis)**.** *We define the* ***order*** *of* S *to be* k*.*

1.2 Lagrange's Four-Square Theorem

Theorem 1 (Lagrange's Four-Square Theorem)**.** *Every natural number can be represented as the sum of 4 integer squares. That is, the squares form an additive basis of order 4.*