Enumeration

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Chapter 1

Generating Functions

1.1 Ordinary Generating Functions

Definition (Ordinary Generating Function)**.** *Let a_n be a sequence. We define the* ***ordinary generating function*** *of a_n, denoted by $G(a_n; x)$, to be a formal power series given by*

$$G(a_n; x) := \sum_{n=0}^{\infty} a_n x^n.$$

Example 1.1.1 (Constant Sequence)**.**

$$G(1; x) = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}.$$

Example 1.1.2 (Geometric Sequence)**.**

$$\forall a \in \mathbb{R}, \quad G(a^n; x) = \sum_{n=0}^{\infty} a^n x^n = \frac{1}{1-ax}.$$

Example 1.1.3 (Square Numbers)**.**

$$G(n^2; x) = \sum_{n=0}^{\infty} n^2 x^n = \frac{x(x+1)}{(1-x)^3}.$$

Example 1.1.4 (Binomial)**.**

$$\forall k \in \mathbb{Z}_+, \quad G(\binom{n+k}{k}; x) = \sum_{n=0}^{\infty} \binom{n+k}{k} x^n = \frac{1}{(1-x)^{k+1}}.$$