Complex Analysis

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Chapter 1

The Gamma Function

1.1 Definitions

Definition (Gamma Function)**.** *We define the* ***Gamma Function*** *as*

$$\Gamma(z) := \int_0^\infty x^{z-1}e^{-x}dx.$$

if $\Re(z) > 0$.

1.2 Properties

Proposition 1.2.1.

$$(\forall z \in \mathbb{C}, \Re(z) > 0), \Gamma(z+1) = z\Gamma(z).$$

Proposition 1.2.2.

$$(\forall n \in \mathbb{N}), \Gamma(n) = (n-1)!.$$

1.3 Particular Values of the Gamma Function

$$\Gamma(1) = 1$$
$$\Gamma(\frac{1}{2}) = \sqrt{\pi}$$
$$\Gamma(2) = 1$$

Chapter 2

Analytic Functions

2.1 Holomorphic Functions

Definition (Holomorphic)**.** *Let f be a function from $\mathbb{C}^n$ to $\mathbb{C}$. We say that f is **holomorphic** if $\forall x \in \mathrm{dom}(f)$, f is complex differentiable in a neighborhood $\mathcal{N}(x)$.*