Model Based Adaptive Systems
Control Theory Models

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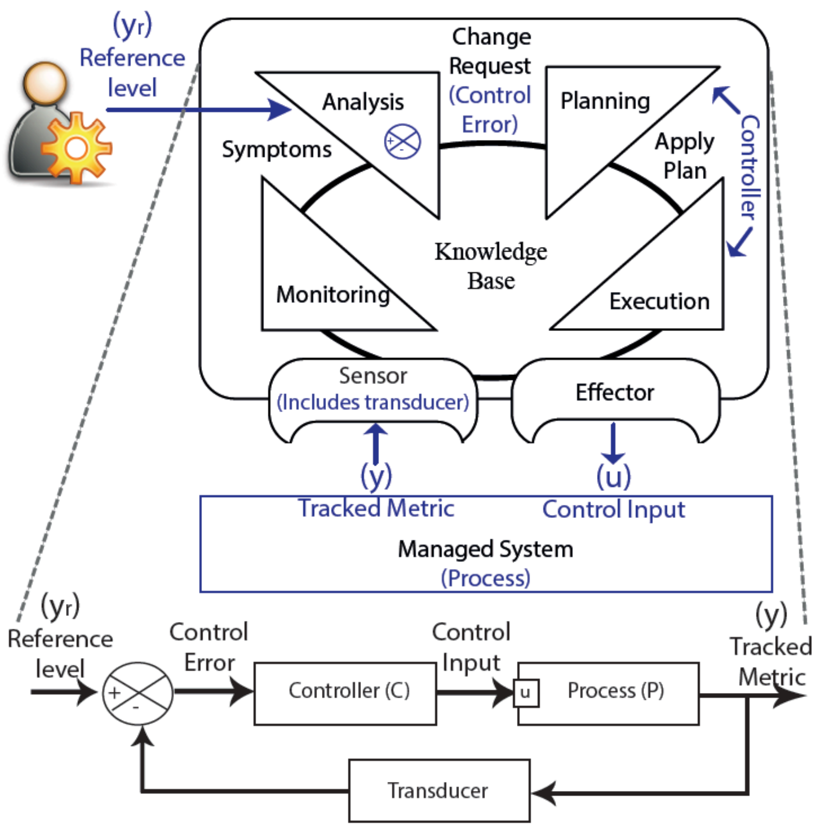
Models

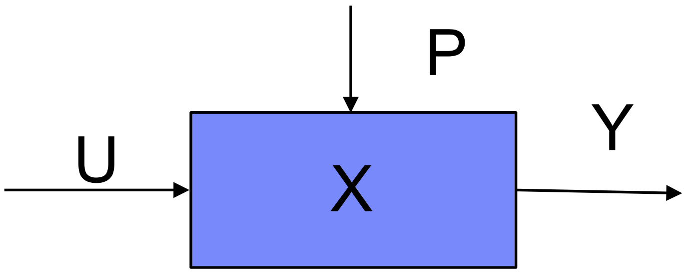
- **Queuing Network Models**
- **Control Theoretic Models**
- **Machine Learning Models**
  - Regression models
  - Neural networks models

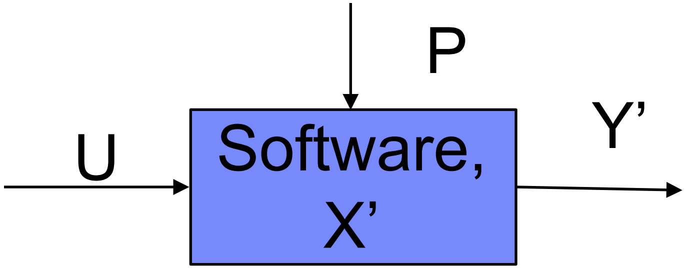
Controllers and Self-Adaptive Managers



The Role of Models

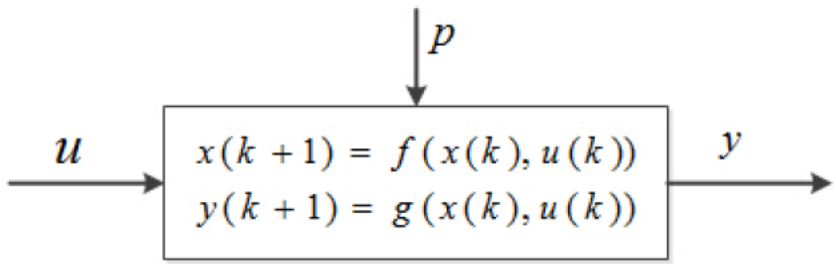
- **Y= F(U, P, X)**
  - Y outputs: response time, etc
  - P disturbances: number of users, arrival rate, etc..
  - X states: utilization, queue length, etc
- **F a (non-linear) function**
- **An autonomic manager, periodically**
  - Will find U that gives the desired Y, given the P and X
  - Will execute a plan that implements U
- **An expert can use the model to design the controller**
  - See ki, kp, kd tuning for PID controller





Control Theory Models**

See Lecture 11(Readings): “What Can Control Theory Teach Us About Assurances in Self-Adaptive Software Systems?”



k: time instant
k+1: next time interval
x: state, p: disturabance, y: output

Control Theory Linear Models

In many cases, the non-linear model can be simplified or approximated with a linear one, such as the one below. In this model we assumed an additive external perturbation:

$$x(k+1) = A * x(k) + B * u(k) + F * p(k) \tag{3}$$

$$y(k) = C * x(k) + D * u(k) \tag{4}$$

where A, B, C, D, F are constant matrices that can be determined experimentally for a specific deployments and under particular perturbations[3].

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Example: the linear model for an web server

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \quad (5)$$

$$y(k) = [C_1 \ 0] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (6)$$

- **Consider a web server**
- **x1: server utilization**
- **x2: memory utilization**
- **u1: no of http connections**
- **u2: keep alive interval**
- **y: response time**

Properties of Control Systems: Stability

- **Stability often means that for any bounded input over any amount of time, the output will also be bounded. This is known as bounded input- bounded-output (BIBO) stability.**
- **BIBO stable → output cannot "blow up" (i.e., become infinite) if the input remains finite**

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Properties of Control Systems: Controllability

- **Main issues in the analysis of a system before deciding on a control strategy to be applied**
- **Controllability**
  - Guiding or forcing the system into a particular state with appropriate control signals
  - State is not controllable ← no signal will ever be able to control the state
  - With A, B from the model, S

$$S = [B\ AB\ A^2B, ..., A^{n-1}B] \tag{10}$$

has the rank n, where n is the number of state variables.

Properties of Control Systems: Observability

- **Main issues in the analysis of a system before deciding on a control strategy to be applied**
- **Observability**
  - Possibility of "observing", through output measurements, the state of a system
  - State is not observable ← the controller will never be able to determine the behaviour of an unobservable state and hence cannot use it to stabilize the system.
  - A, C from the model, O has to be invertible

$$O = [C \, CA \, CA^2, ..., CA^{n-1}]$$

has the rank n, where n is the number of the state variables.

Properties of Control Systems: Robustness

- A control system must always have some robustness property
- The properties of a robust controller do not change much if applied to a system slightly different from the mathematical one used for its synthesis.
- This specification is important: no real physical system truly behaves like the series of differential equations used to represent it mathematically
- Typically a simpler mathematical model is chosen in order to simplify calculations, otherwise the true system dynamics can be so complicated that a complete model is impossible.

How are the control theory models used?

- **Model used to study**
  - Controlability
  - Observability
  - Stability
  - Robustness
- **Models used to design the controller such a way that the qualities of the control are met (overshoot, stability, etc..)**
  - With a model, you can quickly tune a PID controller
  - There are “recipes/ procedures” for designing controllers
    - Linear Quadratic
    - Look-ahead, etc…

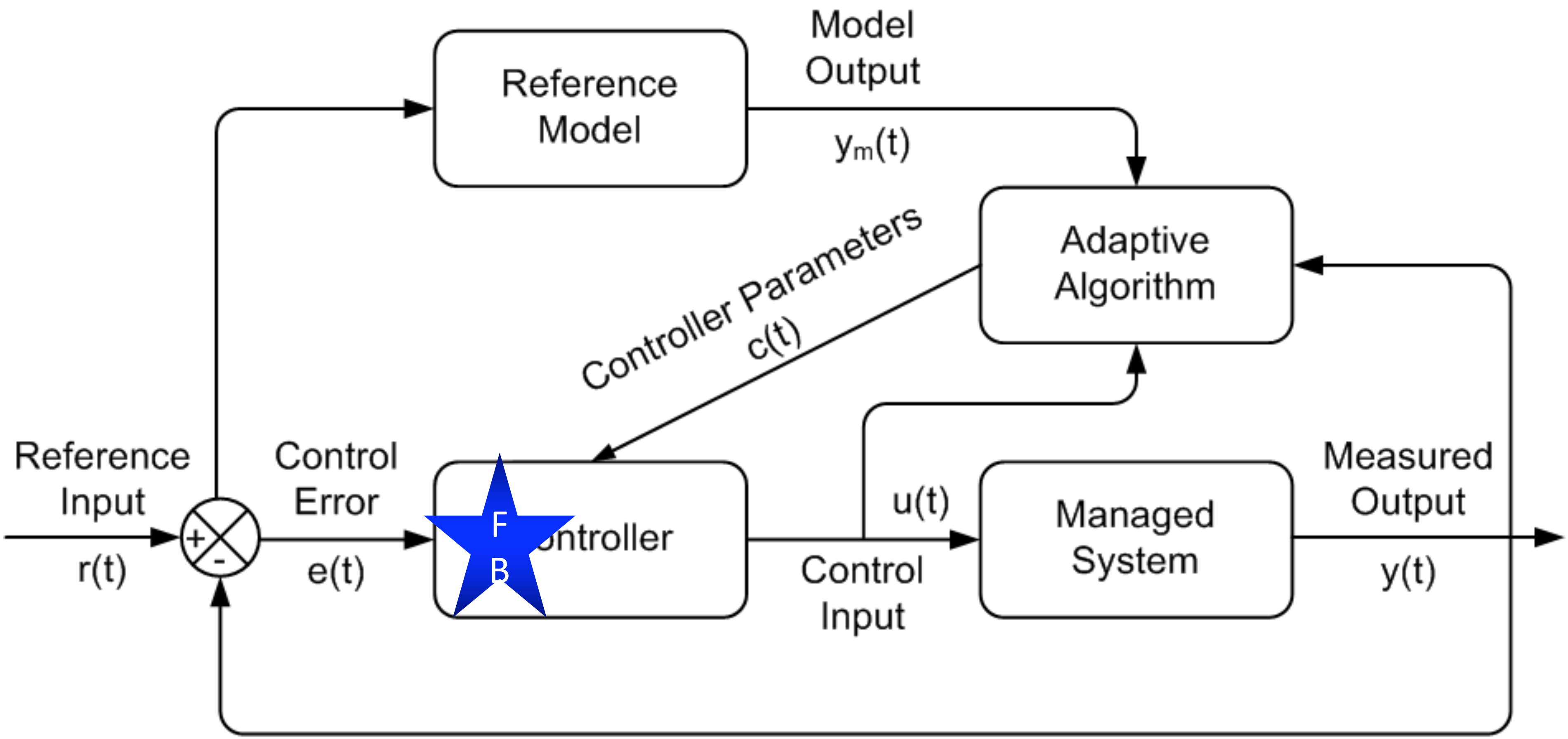
Properties of Control Systems: System Identification

- **The process of determining the equations that govern the model's dynamics is called system identification**
- **This can be done off-line: for example, executing a series of measures from which to calculate an approximated mathematical model, typically its transfer function or matrix**
- **Such identification from the output, however, cannot take account of unobservable dynamics**
- **Even assuming that a "complete" model is used in designing the controller, all the parameters included in these equations (called "nominal parameters") are never known with absolute precision; the control system will have to behave correctly even when connected to physical system with true parameter values away from nominal.**

Model Reference Adaptive Controllers—MRAC

- **Also referred to as Model Reference Adaptive System (MRAS)**
- **Closed loop controller with parameters that can be updated to change the response of the system**
- **The output of the system is compared to a desired response from a reference model (e.g., simulation model)**
- **The control parameters are updated based on this error**
- **The goal is for the parameters to converge to ideal values that cause the managed system response to match the response of the reference model.**

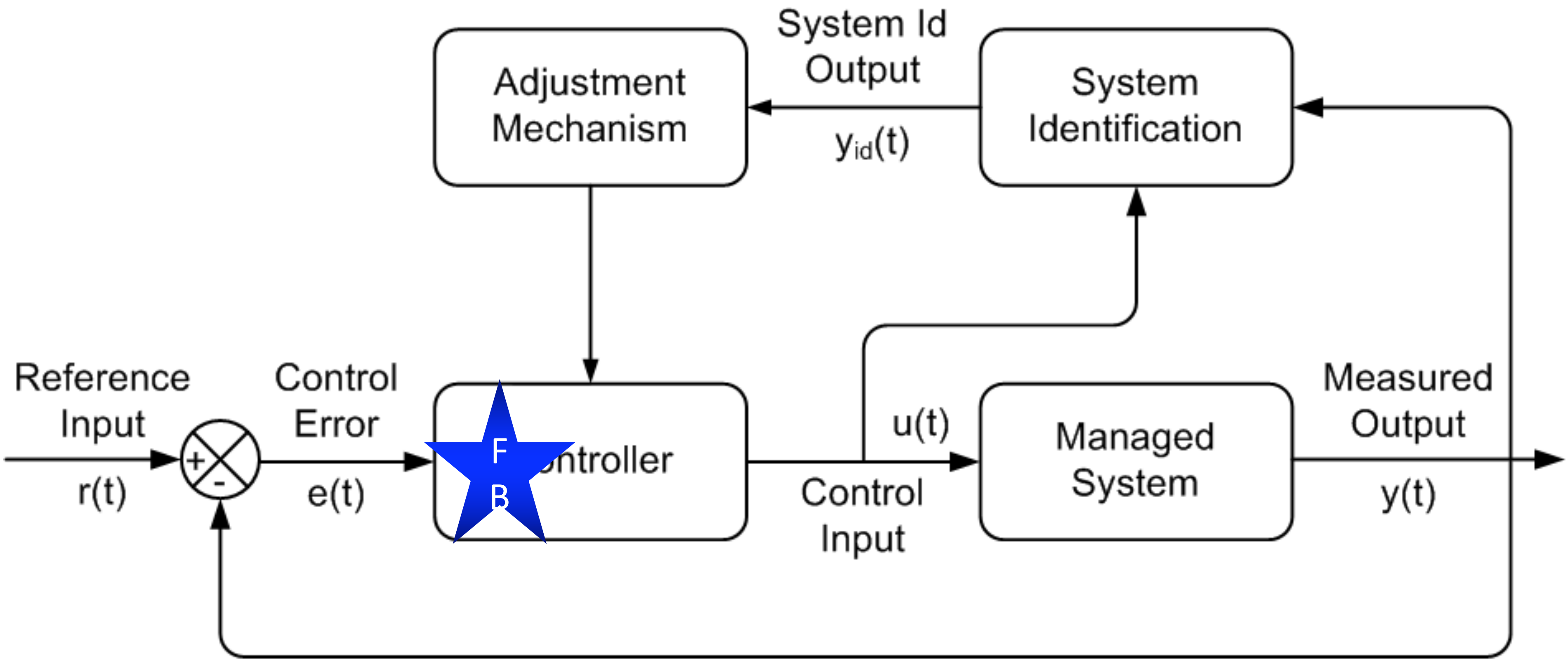
Model Reference Adaptive Controllers—MRAC



Model Identification Adaptive Controllers—MIAC

- **Perform system identification while system is running to modify the control laws**
  - Create model structure and perform parameter estimation using the Least Squares method
- **Cautious adaptive controllers**
  - Use current system identification to modify control law, allowing for system identification uncertainty
- **Certainty equivalent adaptive controllers**
  - Take current system identification to be the true system, assume no uncertainty
  - Nonparametric adaptive controllers
  - Parametric adaptive controllers

Model Identification Adaptive Controllers—MIAC



MIAC versus MRAC

- **In the MRAC approach, the reference model is static (i.e., given or pre-computed and not changed at run-time)**
- **In the MIAC approach, the reference model is changed at run-time using system identification methods**
- **The goal of both approaches is to adjust the control laws in the controller**

Your turn..

- **Compare QNMs with Control Theory Models**
  - What are the advantages and disadvantages of each one?