YORK UNIVERSITÉ UNIVERSITY

ASRL Adaptive Systems Research Lab

Model Based Adaptive Systems

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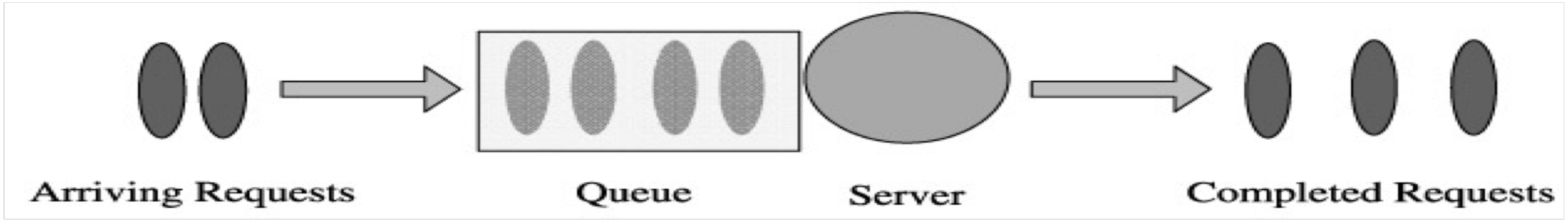
Performance Modeling

- Queuing theory
- Little’s Law
- Queing Networks

Queuing – Basic concepts

- **Queuing is not developed specifically for software performance**
  - Manufacturing
  - Customer facing offices
  - Networks
  - ..wherever there is line of things, people..
- **It helps you better understand performance**
  - Find the cause of a performance problem
  - Find the solution to a performance problem

Notations



Symbol	Semantics
S	Service time
V	Number of visits to the server
*D=V*S*	Service demand
R	Response time
R′	Residence time
X	Throughput
λ	Arrival rate
U	Utilization
W	Wait time
N	Total queue length (waiting and/or being serviced)

Examples of Queues

-CPU

-DISK

-Network card

-a critical section

-a semaphore

-a threading pool

Little’s Law

- **$N_i=X_i*R_i$; created by John Little (1961)**
  - where i is the queue; R_i is the response time; X_i is the throughput of queue;
- **Since $R_i=(W_i+S_i)$, waiting time plus service time**
- **Then $N_i= X_i*W_i+ X_i*S_i$**
- **$N_{i\_wait}= X_i*W_i$; requests waiting**
- **$N_{i\_service}= X_i*S_i$; requests being served**
- **Quiz:**
  - If the customers arrive 10 per hour and leave the bank office after 0.5 hours then there how many customers are in the bank office?
    - N=?
  - Further, if the service time per customer is 0.2 hours, how many customers are served at one time? How many are waiting
  - $N_{i\_service}$=? N_{i_wait}=?

Fundamental Laws: Utilization Law

- **Relates the utilization, mean throughput (X) and service time or demand (D)**
- **$U_i = X_i * D_i$; where i is the queue; D_i is the Demand(service time to complete a request at queue i); X_i is the throughput of queue i;**
  - Proof: if B is the total time the server i is busy over time T, and N is the number of customers, then U=B/T=(B/N)*(N/T)=D*X;.
- **Notes**
  - Utilization law applies to any subsystem (hardware and software)
  - It is assumed that the arrivals equals the completions(no customers/requests are lost inside the system)
  - No assumptions made on the arrival or service time distributions
- **We can write the utilization for each class of customers c: If Xc is the throughput of customers in class c and Dk,c is the demand of customers c at queue k, then**
  - Uk,c=Xc* Dk,c
- **Then the total utilization of queue k is the sum of per class utilizations**

$$Uk = \sum_{c=1..C} Uk,c$$

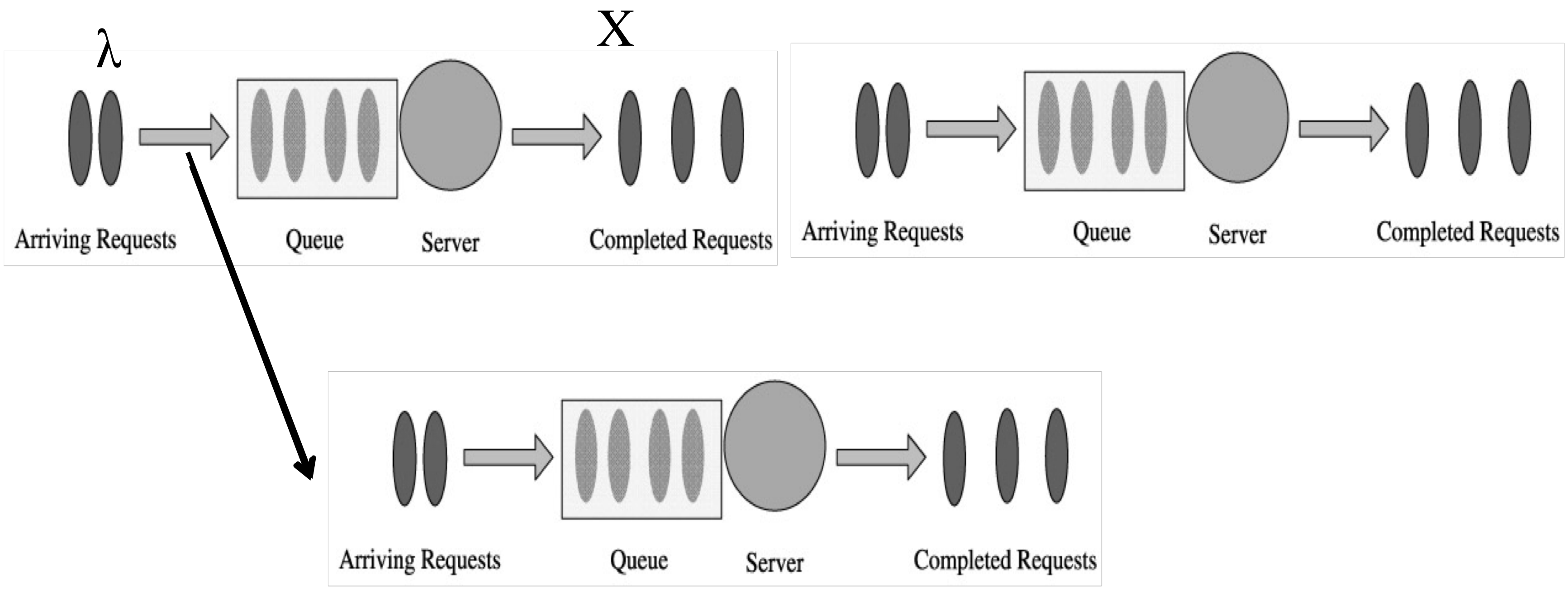
Utilization Analysis

- **We can use the utilization law to check if the resources exceeds (or a close to) their capacity.**
  - Reminder: Utilization cannot exceed 100%. Rule of thumb: Utilization should be well below 100%, for example, it should be less than 60%.
  - If we know the throughput we want to support in the system and the demand Dk at each resource k, then we can compute the utilization of each resource
  - If the utilization is too high, then we either reduce the throughput or make Dk smaller
- **Consider a 2-tier application, with a database and web server. The queuing centres are the DISKs and CPUs at each server.**
  - The demands are:
    - Dweb-cpu = 10ms, Dweb-disk=8ms;
    - Ddb-cpu = 12ms, Dweb-disk=4ms;
  - The application needs to support a throughput of 100 transactions/seconds (the throughput is the same at each device, that is called *the forced flow law*) .
  - What is the utilization of each device?
  - What is the maximum throughput we can support?
- **Utilization can be applied to software components as well**
  - Assume the webserver container (that runs the servlets) has a Response time of R=1second
  - Its utilization is Us=Rs*X=100.
  - Note that the utilization is greater than 1, that is because the container runs threads, therefore a container can be multiplied as needed
  - The meaning is that there are 100 threads running on average

Your turn

- **Assuming you have a running system described by the simple models**
  - Little formula
  - Utilization law
- **How would you use that to create autonomic system**
- **Y=?**
- **U=?**
- **What would be the effectors (those parameters, variables, etc..) you can change?**

Queues: there is a network of queues

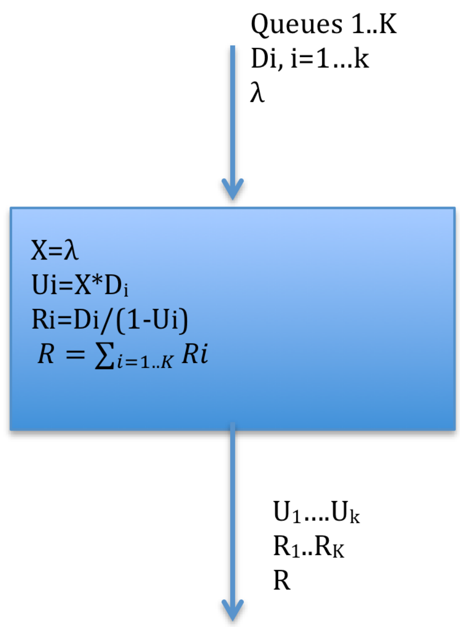


Queuing Network Models

- **Approximations of the real system**
  - Approximate the user interaction, the structure and the behavior of the system
  - Information about the sequence of operations is not included in the model
  - The system is a set of queues and customers' requests flow from queue to queue with fixed probabilities
- **Open Queuing Network Models**
  - Consider the system has an infinite number of customers that arrive with a specific rate, usually called lambda (measured in requests per second)
  - Used for large systems where the response time does not affect the arrival rate (requests per second)
- **Closed Models**
  - Consider the system has N customers and each customer has a think time Z
  - Used for system in which the arrival rate depends on the response time
- **Both closed and open models can distinguish between classes of customers; typically a class is specified by an index *c***
  - In the simplest case, a "class" is a use case(scenario)

Open Models

- Very simple and fast formulas
- Inputs: Queues and demands in queues; arrival rate (req/sec)
- Outputs: overall response time; throughput and per server utilization
- Note that Xi=X (forced flow law): the customers requests go through all queues and are not lost.



Open Models for Multiple Classes

- Very simple and fast formulas
- Inputs: Queues and demands in queues; arrival rate (req/sec) for each class of request
- Outputs: overall response time per each class; throughput and per server utilization per each class of request
- You can implement this in Excel
  - Keep in mind U<1, therefore, (1-Uk) should not be 0 or negative

Queues 1..K
Classes 1..C
Demands: Dk,c k=1…K; c=1..c
Arrival rates λc, c=1..C

$Xc=\lambda c$
$Uk,c=Xc*D_{k,c}$

$$Uk = \sum_{C=1..C} Uk,c$$

$Rk,c=Dk,c/(1-Uk)$

$$Rc = \sum_{k=1..K} Rk,c$$

$U_1....U_k$
$R_1..R_K$
Xc, Rc

Exercise

- **Assume the Tutorial 3-4 application**
  - 1 web server (Queue 1)
  - 1 database server (Queue 2)
  - Two classes (scenarios)
    - Browse
    - Search

lambda_1	lambda_2	D_App, search	D_App, browse	D_DB, Search	D_DB, browse
0.15	0.1	0.1	0.15	0.2	0.25

  - Can you write the model for this application?
    - What is the response time for each scenario
    - Where is the bottleneck
    - What is the max througpuy?

Data for a a 3 tier app, 2 classes, 2 queuing centres (application server and database server)

lambda_1	lambda_2	D11	D12	D21	D22	U11	U12	U21	U22	U1	U2	R11	R12	R21	R22	R1	R2
0.15	0.1	0.1	0.15	0.2	0.25	0.015	0.015	0.03	0.025	0.03	0.055	0.103092784	0.154639175	0.211640212	0.264550265	0.314732995	0.41918944
0.3	0.2	0.1	0.15	0.2	0.25	0.03	0.03	0.06	0.05	0.06	0.11	0.106382979	0.159574468	0.224719101	0.280898876	0.33110208	0.440473344
0.45	0.3	0.1	0.15	0.2	0.25	0.045	0.045	0.09	0.075	0.09	0.165	0.10989011	0.164835165	0.239520958	0.299401198	0.349411068	0.464236362
0.6	0.4	0.1	0.15	0.2	0.25	0.06	0.06	0.12	0.1	0.12	0.22	0.113636364	0.170454545	0.256410256	0.320512821	0.37004662	0.490967366
0.75	0.5	0.1	0.15	0.2	0.25	0.075	0.075	0.15	0.125	0.15	0.275	0.117647059	0.176470588	0.275862069	0.344827586	0.393509128	0.521298174
0.9	0.6	0.1	0.15	0.2	0.25	0.09	0.09	0.18	0.15	0.18	0.33	0.12195122	0.182926829	0.298507463	0.373134328	0.420458682	0.556061158
1.05	0.7	0.1	0.15	0.2	0.25	0.105	0.105	0.21	0.175	0.21	0.385	0.126582278	0.189873418	0.325203252	0.406504065	0.451785531	0.596377483
1.2	0.8	0.1	0.15	0.2	0.25	0.12	0.12	0.24	0.2	0.24	0.44	0.131578947	0.197368421	0.357142857	0.446428571	0.488721805	0.643796992
1.35	0.9	0.1	0.15	0.2	0.25	0.135	0.135	0.27	0.225	0.27	0.495	0.136986301	0.205479452	0.396039604	0.495049505	0.533025905	0.700528957
1.5	1	0.1	0.15	0.2	0.25	0.15	0.15	0.3	0.25	0.3	0.55	0.142857143	0.214285714	0.444444444	0.555555556	0.587301587	0.76984127
1.65	1.1	0.1	0.15	0.2	0.25	0.165	0.165	0.33	0.275	0.33	0.605	0.149253731	0.223880597	0.506329114	0.632911392	0.655582845	0.856791989
1.8	1.2	0.1	0.15	0.2	0.25	0.18	0.18	0.36	0.3	0.36	0.66	0.15625	0.234375	0.588235294	0.735294118	0.744485294	0.969669118
1.95	1.3	0.1	0.15	0.2	0.25	0.195	0.195	0.39	0.325	0.39	0.715	0.163934426	0.245901639	0.701754386	0.877192982	0.865688812	1.123094622
2.1	1.4	0.1	0.15	0.2	0.25	0.21	0.21	0.42	0.35	0.42	0.77	0.172413793	0.25862069	0.869565217	1.086956522	1.04197901	1.345577211
2.25	1.5	0.1	0.15	0.2	0.25	0.225	0.225	0.45	0.375	0.45	0.825	0.181818182	0.272727273	1.142857143	1.428571429	1.324675325	1.701298701
2.4	1.6	0.1	0.15	0.2	0.25	0.24	0.24	0.48	0.4	0.48	0.88	0.192307692	0.288461538	1.666666667	2.083333333	1.858974359	2.371794872
2.55	1.7	0.1	0.15	0.2	0.25	0.255	0.255	0.51	0.425	0.51	0.935	0.204081633	0.306122449	3.076923077	3.846153846	3.28100471	4.152276295
2.7	1.8	0.1	0.15	0.2	0.25	0.27	0.27	0.54	0.45	0.54	0.99	0.217391304	0.326086957	20	25	20.2173913	25.32608696

Q: what device is the bottleneck? What is the max arrival rate you can accommodate above?

Exercise: Extend the model to 3 classes: use demands and arrival rates to be half of class 1. What do you notice with regard to Utilization and Response time?

Summary

- **Performance metrics**
  - Utilization, response time, throughput
- **Scalability depends on how fast ONE transaction is, that is depends on D (demand); minimizing D makes an application more scalable;**