Outline

- Basic concepts of association rule learning
- Apriori algorithm
- FP-Growth Algorithm
- Finding interesting rules

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Two Problems with Association Rule Mining

- Quantity problem
  - Too many rules can be generated
    - Given a dataset, the number of rules generated depends on the support and confidence thresholds.
      - If the support threshold is high, a small number of rules are generated. But some interesting rules are missed.
      - If the support threshold is low, a huge number of rules are generated.
- Quality problem
  - Not all the generated rules are interesting

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Number of Generated Patterns versus Support Threshold (An Example)

Support threshold	0.02	0.01	0.008	0.005	0.003	0.0028	0.0025	0.002	0.001
Num. of rules (conf. thres.=0.5)	2	14	39	88	723	4,556	74,565	4,800,070	$>10^9$
Num. of rules (conf. thres.=0.8)	1	7	17	38	591	4,172	65,615	3,584,339	$>10^9$

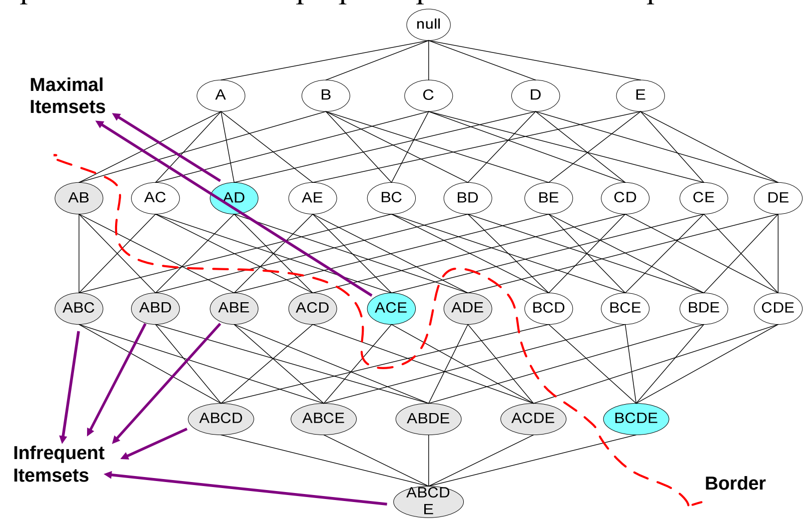
Number of sessions (transactions): 30586
Number of objects (items): 38679

Solutions to the Problems

- Finding only *maximum* or *closed* frequent patterns
  - Other frequent patterns can be generated from them
- Constraint-based data mining
  - Applying constraints in the mining process so the search can be more focused.
- Using interestingness measures to remove or rank rules
  - Remove misleading associations and find correlation rules
  - Prune patterns using other interestingness measures
- Using rule structures
  - Eliminate structurally and semantically redundant rules.
  - Group or summarize related rules

Maximal Frequent Itemset

An itemset *X* is a ***maximal frequent itemset*** in a data set *D* if *X* is frequent and none of the proper super-set of *X* is frequent in *D*.



Maximal Frequent Patterns

- Reducing the # of patterns returned to the user
- Maximal frequent patterns are a *lossy* compression of frequent patterns
  - Given the set of all maximal frequent patterns and their supports in a data set *D*, we can generate all the frequent patterns, *but not their supports*.
- Algorithm for mining maximal frequent itemsets: MaxMiner
  - R. Bayardo. Efficiently mining long patterns from databases. *SIGMOD'98*

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Closed Patterns

- Problem with maximal frequent itemsets:
  - Supports of their subsets are not known – additional DB scans are needed (to get the supports)
- An itemset is ***closed*** if none of its proper supersets has the same support as the itemset

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	2
{A,B,C,D}	2

Closed Frequent Patterns

- An itemset *X* is a ***closed frequent itemset*** in a data set *D* if *X* is both *closed* and *frequent* in *D* with respect to a support threshold.
- Closed frequent itemsets are a *lossless* compression of frequent patterns
  - Reducing the # of patterns returned to the user
  - Given the set of all closed frequent patterns and their supports in a data set *D*, the user can generate all the frequent patterns and their supports.
- Algorithm for finding closed frequent patterns: CLOSET
  - J. Pei, J. Han & R. Mao. “CLOSET: An Efficient Algorithm for Mining Frequent Closed Itemsets", DMKD'00.

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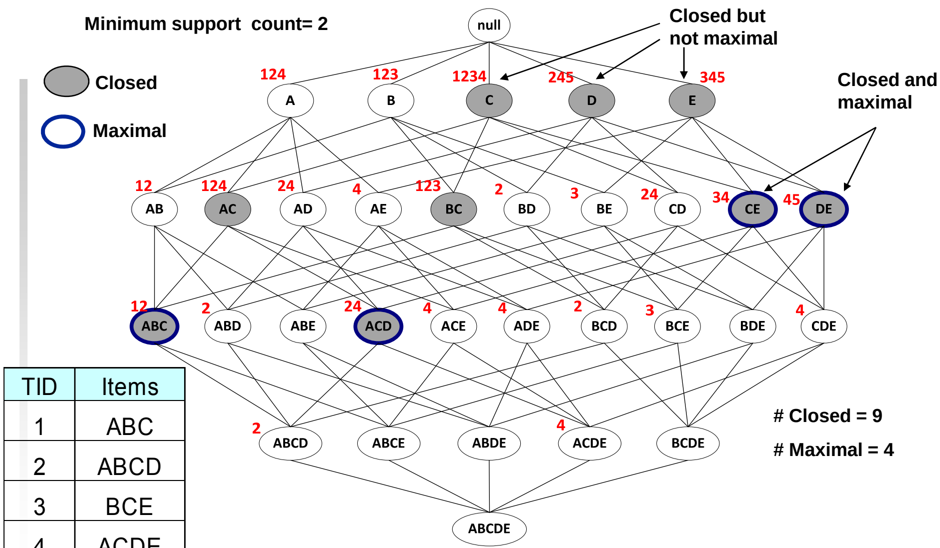
Maximal vs Closed Frequent Itemsets



TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE

Closed Patterns and Max-Patterns

- Exercise. DB = $\{\{a_1, \ldots, a_{100}\}, \{a_1, \ldots, a_{50}\}\}$
  - Min_sup_count = 1.
- What is the set of closed frequent itemsets?
  - $\{a_1, \ldots, a_{100}\}$: 1
  - $\{a_1, \ldots, a_{50}\}$: 2
- What is the set of maximal frequent itemsets?
  - $\{a_1, \ldots, a_{100}\}$: 1
- What is the set of all frequent itemsets?
  - !!

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Solutions to the Problems

- Finding only *maximum* or *closed* frequent patterns
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- *Constraint-based data mining*
  - *Applying constraints in the mining process so the search can be more focused.*
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Constrain-based Frequent Pattern Mining

- Mining frequent patterns with constraint C
  - find all patterns satisfying not only min_sup, but also constraint C
- Examples of Constraints
  - *?* → *a particular product*
  - *a particular product* → *?*
  - small sales (price < \$10) triggers big sales (sum > \$200)

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Constrain-based Frequent Pattern Mining (Cont'd)

- A naïve solution
  - Testing frequent patterns on C as a post-processing process
- Some constraints can be incorporated into the mining process to improve the efficiency
- More efficient approaches
  - Analyze the properties of constraints comprehensively
  - Push the constraint as deeply as possible inside the frequent pattern mining
  - Example: find all frequent itemsets containing item "b"

Types of Constraints

- Anti-monotonic constraints
  - An itemset S satisfies the constraint, so does any of its subset (That is, S violates the constraint, so does any of its superset).
- Monotonic constraints
  - An itemset S satisfies the constraint, so does any of its superset
- Examples
  - Sum of the prices of items in S ≤ 100 is anti-monotone
  - Maximum price in S ≤ 15 is anti-monotone
  - Sum of the prices of items in S ≥ 100 is monotone
  - Minimum price in S ≤ 15 is monotone

How to Use Antimonotonic or Monotonic Constraints in Mining

- Antimonotonic constraints
  - In Apriori:
    - Use it to prune candidates in each iteration
  - In FP-growth
    - Use it to stop growing a pattern
- Monotonic constraints
  - If an itemset satisfies a monotonic constraint, no need to check its supersets on the constraint
    - Only checks their support

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Types of Constraints (Cont'd)

- Convertible constraints
  - Some constraints are not anti-monotonic or monotonic
  - But can be converted to anti-monotonic or monotonic by properly ordering items
- Example of convertible constraint:
  - Average price of the items in $S \geq 25$
  - Order items in price-descending order
    - <a, f, g, d, b, h, c, e>
  - If an itemset afb violates C
    - So does afbh, afb*
    - It becomes anti-monotone!

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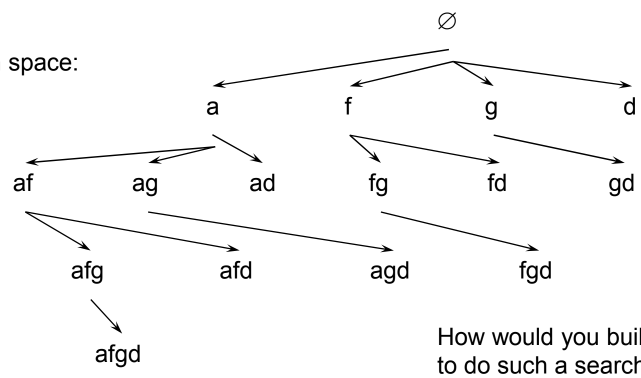
Example of Convertible Constraints

- Convertible constraint:
  - Average price of the items in S $\geq$ 25
- Price-descending order of items: a, f, g, d

Search space:



How would you build an FP-tree to do such a search?

Solutions to the Problems

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Misleading Association Rules

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

- *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading
  - The overall percentage of students eating cereal is 75% which is higher than 66.7%
- *play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence

Association ≠ Correlation

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Interestingness Measure: Correlation

- Correlation
  - If $P(A|B)>P(A)$, A and B are *positively correlated.*

    Note: $P(A \mid B) > P(A) \Leftrightarrow P(B \mid A) > P(B) \Leftrightarrow P(A\ B) > P(A)P(B)$
  - If $P(A|B)<P(A)$, A and B are *negatively correlated.*

    Note: $P(A \mid B) < P(A) \Leftrightarrow P(B \mid A) < P(B) \Leftrightarrow P(A\ B) < P(A)P(B)$
  - If $P(A|B)=P(A)$, A and B are *independent.*

    Note: $P(A \mid B) = P(A) \Leftrightarrow P(B \mid A) = P(B) \Leftrightarrow P(A\ B) = P(A)P(B)$
- A measure of correlation (called lift)

$$corr_{A,B} = \frac{P(AB)}{P(A)P(B)}$$

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Pruning Misleading Rules (Keep Correlation Rules)

- A measure of correlation (lift) for rule A→ B

$$lift(A \rightarrow B) = \frac{P(AB)}{P(A)P(B)}$$

- Rules whose lift ≤ 1 is ***misleading***, which should be removed
  - E.g. the following rule:

    play basketball ⇒ *eat cereal* [40%, 66.7%]

    should be removed because its lift is 0.89

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Many interestingness measures for A→ B

symbol	measure	range	formula
ϕ	ϕ-coefficient	-1…1	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
Q	Yule's Q	-1 …1	$\frac{P(A,B)P(\overline{A},\overline{B})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{A},\overline{B})+P(A,\overline{B})P(\overline{A},B)}$
Y	Yule's Y	-1 …1	$\frac{\sqrt{P(A,B)P(\overline{A},\overline{B})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{A},\overline{B})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}}$
k	Cohen's	-1 …1	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
PS	Piatetsky-Shapiro's	-0.25 …0.25	$P(A,B)-P(A)P(B)$
F	Certainty factor	-1 …1	$\max(\frac{P(B\mid A)-P(B)}{1-P(B)}, \frac{P(A\mid B)-P(A)}{1-P(A)})$
AV	added value	-0.5 …1	$\max(P(B\mid A)-P(B), P(A\mid B)-P(A))$
K	Klosgen's Q	-0.33 …0.38	$\sqrt{P(A,B)}\max(P(B\mid A)-P(B), P(A\mid B)-P(A))$
g	Goodman-kruskal's	0 …1	$\frac{\Sigma_j \max_k P(A_j,B_k)+\Sigma_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-max_k P(B_k)}$
M	Mutual Information	0 …1	$\frac{\Sigma_i \Sigma_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\Sigma_i P(A_i)\log P(A_i)\log P(A_i), -\Sigma_i P(B_i)\log P(B_i)\log P(B_i))}$
J	J-Measure	0 …1	$\max(P(A,B)\log(\frac{P(B\mid A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}\mid A)}{P(\overline{B})}))$ $P(A,B)\log(\frac{P(A\mid B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}\mid B)}{P(\overline{A})})$
G	Gini index	0 …1	$\max(P(A)[P(B\mid A)^2+P(\overline{B}\mid A)^2]+P(\overline{A}[P(B\mid\overline{A})^2+P(\overline{B}\mid\overline{A})^2]-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\mid B)^2+P(\overline{A}\mid B)^2]+P(\overline{B}[P(A\mid\overline{B})^2+P(\overline{A}\mid\overline{B})^2]-P(A)^2-P(\overline{A})^2)$
s	support	0 …1	$P(A,B)$
c	confidence	0 …1	$max(P(B\mid A), P(A\mid B))$
L	Laplace	0 …1	$\max(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2})$
IS	Cosine	0 …1	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
γ	coherence(Jaccard)	0 …1	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
α	all_confidence	0 …1	$\frac{P(A,B)}{\max(P(A),P(B))}$
o	odds ratio	0 …∞	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(\overline{A},B)P(A,\overline{B})}$
V	Conviction	0.5 …∞	$\max(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})})$
λ	lift	0 …∞	$\frac{P(A,B)}{P(A)P(B)}$
S	Collective strength	0 …∞	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})}\times\frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
χ^2	χ^2	0 …∞	$\Sigma_i\frac{(P(A_i)-E_i)^2}{E_i}$

Pruning rules with interestingness measure

- Choose a measure in your belief to assess the significance of a rule A→ B.
- Rank the rules according to their interestingness value.
- Remove rules with small interestingness values

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Pruning Redundant Rules

- **Pruning Rule 1:** If there are two rules of the form $A \rightarrow C$ and $A \wedge B \rightarrow C$, and the interestingness value of rule $A \wedge B \rightarrow C$ is not significantly better than rule $A \rightarrow C$, then rule $A \wedge B \rightarrow C$ is redundant and should be pruned.

- **Pruning Rule 2:** If there are two rules of the form $A \rightarrow C_1$ and $A \rightarrow C_1 \wedge C_2$, and the interestingness value of rule $A \rightarrow C_1$ is not significantly better than rule $A \rightarrow C_1 \wedge C_2$, then rule $A \rightarrow C_1$ is redundant and should be pruned.

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Summarizing and Grouping Association Rules

- Toivonen et al. (KDD'95)
  - Compute a subset of rules, called a structural rule cover, to reduce the number of rules and further grouped the rules in the cover using clustering
- Cristofor and Simovici (2002)
  - Define another type of rule cover, called informative cover, to group and summarize related rules.
- Khan, An and Huang (ICDM'03)
  - Proposed two algorithms
    - Objective grouping of rules according to the rule structure
    - Subjective grouping of rules according to the semantic relationship among items.

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Related Topics

- Mining high utility patterns
  - Consider
    - the quantity $q(i, T_j)$ of an item i in a transaction T_j
    - the value (e.g., price $p(i)$) of an item i
  - Utility of an item i in a transaction T_j:

$$u(i, T_j) = q(i, T_j) \times p(i)$$

  - Utility of an itemset X in a transaction T_j:

$$(\quad)$$

  - Utility of an itemset X in a dataset D:

$$(\quad)$$

  - *High utility pattern*: itemsets whose utility in the dataset is no less than a minimum utility threshold

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Related Topics

- Mining high utility patterns (*cont'd*)
  - Challenge: utility does not have the downward closure property. That is,

    The utility of a subset /superset of a set S may be smaller or larger than the utility of S

  - This means we cannot use Apriori or FP-growth to find high utility patterns directly since
    - the two algorithms use the downward closure property of support to cut down the search space
  - Solution: use an upper bound of utility with downward closure property to generate candidates first, and then scan DB to find high utility patterns from the set of candidates

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Related Topics

- Mining frequent patterns over data streams
  - A continuous flow of data generated often at high-speed in a dynamic, time-changing environment
  - Memory is limited to hold all the data
  - Processing time may be limited by the rate of arrival of instances
  - One scan of data set is required for online mining
  - Pattern changes over time
    - Incremental learning
    - Change detection
    - etc

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Related Topics

- Contrast pattern mining
  - Finding patterns and models contrasting two or more classes or conditions
  - Contrasting groups:
    - Objects at different time periods
    - Objects at different spatial locations
    - Objects across different classes.
  - Measures for measuring the difference
    - Frequent/infrequent
    - Frequency ratio
    - Odds ratio, etc.
  - A challenge: need to find infrequent itemsets in a group.

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Next Class

- Sequential pattern mining (papers on the supplementary reading list)

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