Energy in General Relativity

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$$T^{\mu\nu} = \frac{\partial L}{\partial(\partial_\mu \phi_\alpha)}\partial_\nu \phi_\alpha - \eta^{\mu\nu} L.$$

T^{00} matches the definition of the Hamiltonian:

$$H = \pi_\alpha \dot{\phi}_\alpha - L.$$

Energy in General Relativity

Alternatively, the 'modern' definition:

$$T^{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_{matter})}{\delta g_{\mu\nu}}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}. \quad (1)$$

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This agrees with the Noetherian definition up to a symmetrising improvement term.

Either way, $\partial_\mu T^{\mu\nu} = 0$. Hence

$$E = \int T^{00}\, d^3x$$

is conserved in time.

Energy in General Relativity

For a curved background spacetime, the Noether current approach is less obvious. But the 'modern' definition obeys

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= 0 \\ \Rightarrow \partial_\mu(\sqrt{-g}T^{\mu\nu}) &= 0. \end{aligned}$$

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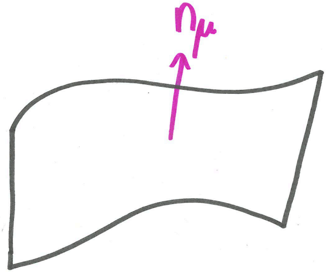
If our spacetime is foliated into 3d spatial Cauchy slices $\Sigma \times \mathbb{R}$,

$$E = \int_\Sigma T^{\mu\nu} n_\mu n_\nu \sqrt{\gamma}\, d^3x.$$

is conserved from one slice to the next by Gauss's Law.



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Equivalence principle $\Rightarrow$ no local definition can work.

Except the equivalence principle is only true up to first-order derivatives of the metric...

Energy in General Relativity

Second derivatives of the metric can be measured locally, e.g. in the theory

$$S_{matter} \quad = \quad \int g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - (m^2 + \xi R)\phi^2\,\sqrt{-g}\,d^4x. \qquad (2)$$

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One might imagine some definition of $T^{\mu\nu}$ in terms of contractions of the Riemann tensor, but no good candidate has been found.

Energy in General Relativity

Misner, Thorne & Wheeler:

> "Anybody who looks for a magic formula for 'local gravitational energy-momentum' is looking for the right answer to the wrong question. Unhappily, enormous time and effort were devoted in the past to trying to 'answer this question' before investigators realised the futility of the enterprise."

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There is room for a non-local definition. In fact, there are several.

Outline

Komar Energy (1959)

ADM Energy (1959)

Wald Energy (1990)

Brown–York Quasilocal Energy (1993)

Conceptual Issues

Komar Energy

For spacetimes with a Killing vector K^μ, we might expect to manipulate it into a conserved quantity.

A Killing vector K^μ satisfies $\pounds_K g_{\mu\nu} = 0$, meaning that it is a 'symmetry' of the metric.

This is equivalent to $\nabla_\mu K_\nu + \nabla_\nu K_\mu = 0$.

Komar Energy

If we define

$$J^\mu = \nabla_\nu \nabla^\mu K^\nu,$$

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Proof:

$$\begin{aligned}
\nabla_\mu J^\mu &= & \nabla_\mu \nabla_\nu \nabla^\mu K^\nu & \\
&= & - \quad \nabla_\mu \nabla_\nu \nabla^\nu K^\mu & \qquad \text{(Killing vector)} \\
&= & - \quad \nabla_\nu \nabla_\mu \nabla^\mu K^\nu & \qquad \text{(relabelling } \mu \leftrightarrow \nu\text{)} \\
\Rightarrow 2\nabla_\mu J^\mu &= & \Big(R^\mu{}_{\mu b a} + R^\nu{}_{a \nu b} \Big) \nabla^a K^b & \\
&= & 0. &
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\Rightarrow 2\nabla_\mu J^\mu &= \quad \left(R^\mu{}_{\mu b a} + R^\nu{}_{a \nu b}\right) \nabla^a K^b \\
&= 0.
\end{aligned}$$

$\Rightarrow$ the Komar charge is conserved:

$$E_K = \frac{1}{8\pi G_N} \int_{\partial\Sigma} \nabla^\mu K^\nu dS_{\mu\nu}.$$

ADM Construction

More powerful notions come from a Hamiltonian reformulation of general relativity.

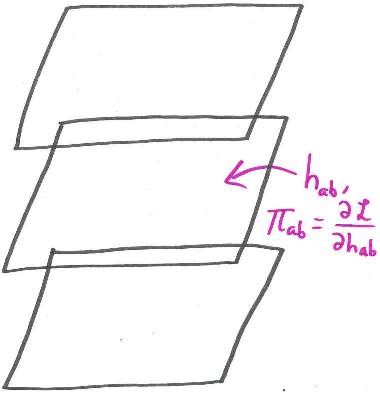
We need to Legendre transform

$$S \sim \int_{\mathcal{M}} R\sqrt{-g}\, d^4x + 2\int_{\partial\mathcal{M}} K\sqrt{h} d^3x$$

where K is the trace of the curvature of the boundary sphere.

ADM Construction

1. Foliate spacetime into 3d spatial slices as $\Sigma \times \mathbb{R}$.



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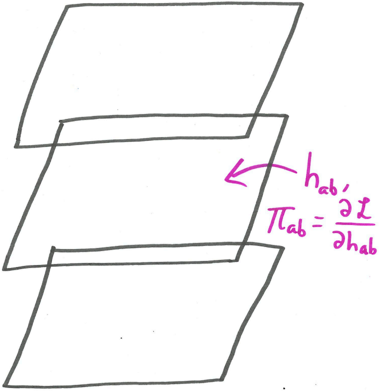
2. Construct canonical (q, p) variables as the metric restricted to each slice, and their conjugate momenta. These variables obey constraints from Einstein's equations.



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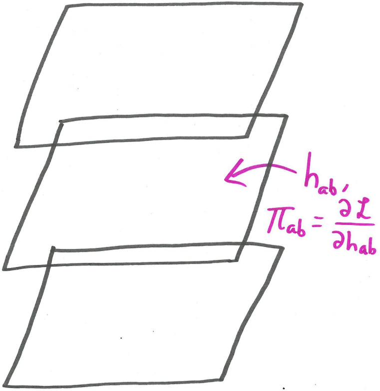
3. Construct the Hamiltonian as

$$H = \pi^{ab}\dot{h}_{ab} - L.$$



ADM Construction

If

- The constraints are obeyed
- Spacetime is asymptotically flat
- The slices are equally spaced at infinity and not tilted at infinity

then

$$H = -\frac{1}{8\pi G_N} \int_{\partial\Sigma} K_S \sqrt{s}\, d^2x.$$

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$$H = -\frac{1}{8\pi G_N}\int_{\partial\Sigma} K_S\,\sqrt{s}\,d^2x.$$

This diverges, so we subtract off Minkowski space:

$$E_{ADM} = -\frac{1}{8\pi G_N}\int_{\partial\Sigma}(K_S - K_{0S})\,\sqrt{s}\,d^2x.$$

Wald Energy

$$\delta L = \delta g_{\mu\nu} \frac{\partial L}{\partial g_{\mu\nu}} + \nabla_\lambda (\delta g_{\mu\nu}) \frac{\partial L}{\partial (\nabla_\lambda g_{\mu\nu})} + ...$$

Wald Energy

$$\delta L = \delta g_{\mu\nu} \frac{\partial L}{\partial g_{\mu\nu}} + \nabla_\lambda(\delta g_{\mu\nu}) \frac{\partial L}{\partial(\nabla_\lambda g_{\mu\nu})} + ...$$

We can express this as

$$\delta L = E^{\mu\nu} + \nabla_\lambda \Theta^\lambda$$

where

$$E^{\mu\nu} = \frac{\partial L}{\partial g_{\mu\nu}} - \nabla_\lambda \frac{\partial L}{\partial(\nabla_\lambda g_{\mu\nu})} + ...,$$

$$\Theta^\lambda[\delta] = \delta g_{\mu\nu} \frac{\partial L}{\partial(\nabla_\lambda g_{\mu\nu})} + ...$$

Wald Energy

$$\boxed{\Theta^{\lambda}[\delta] = \delta g_{\mu\nu} \frac{\partial L}{\partial(\nabla_{\lambda} g_{\mu\nu})} + ...}$$

Now, take a solution of the EOM, so that $\delta L = \nabla_{\lambda} \Theta^{\lambda}$.

Taking second on-shell variations,

$$\delta_{[1}\delta_{2]} L = \nabla_{\lambda}(\delta_{[1}\Theta^{\lambda}_{2]})$$

where $\delta_{[1}\delta_{2]} \equiv \delta_1\delta_2 - \delta_2\delta_1$.

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But $\delta_{[1}\delta_{2]}$ vanishes on a scalar, so

$$\nabla_\lambda(\delta_{[1}\Theta^\lambda_{2]}) = 0.$$

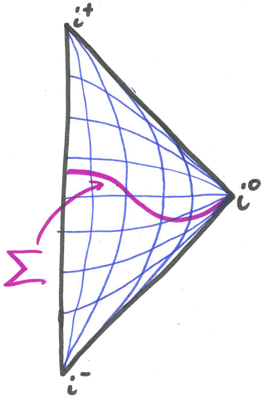
Wald Energy

$$\boxed{\Theta^\lambda[\delta] = \delta g_{\mu\nu}\frac{\partial L}{\partial(\nabla_\lambda g_{\mu\nu})}+\ldots}$$

Hence, the conserved 'Hamiltonian generating δ_η' is

$$\delta_1 H_2 = \int_\Sigma \delta_{[1}\Theta^\lambda_{2]} d\Sigma_\lambda.$$

Conservation means we are independent of the choice of Σ.



Wald Energy

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In GR,

$$\Theta^\lambda = \frac{1}{16\pi G_N} g^{\lambda a} g^{bc} \Big(\nabla_b \delta g_{ac} - \nabla_a \delta g_{bc} \Big).$$

Hence

$$\delta H_\eta = \frac{1}{16\pi G_N} \int_{\partial\Sigma} \left(\begin{array}{l} \delta(\sqrt{\gamma}\ \nabla^{[a} \eta^{b]}) \quad - \\ \sqrt{\gamma}\ \eta^{[a} g^{b]c} g^{de} (\nabla_d \delta g_{ce} - \nabla_c \delta g_{de}) \end{array} \right) dS_{ab}.$$

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This is a boundary term!

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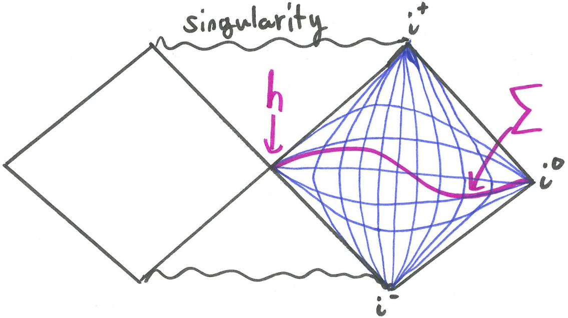
- Wald charges are readily applied to any action. Diffeomorphism-invariant actions give boundary charges.
- Boundary action counterterms (e.g. the GHY term) were not used in the derivation.
- Although I didn't present it as such, this Hamiltonian is embedded in a 'canonical classical mechanics' framework.

‘Covariant Phase Space’ Outline

‘Classical Mechanics’	Covariant Phase Space
Phase space spanned by $\vec{q}$, $\vec{p}$	The space of field configurations ϕ
$\{f, g\} = (\partial_i f)\omega^{ij}(\partial_j g)$ for $\omega = dq^\alpha \wedge dp^\alpha$	$\omega(\phi, \delta_1\phi, \delta_2\phi) \equiv \int_\Sigma \delta_{[1}\Theta^\mu_{2]} d\Sigma_\mu$ maps two $\delta\phi$ to $\mathbb{R}$
$(dH_\eta)_i \equiv \omega_{ij}\eta^j$	$\delta_1 H_2(\phi) \equiv \omega(\phi, \delta_1\phi, \delta_2\phi)$

Wald Approach on a Black Hole Spacetime

On a non-rotating black hole spacetime:



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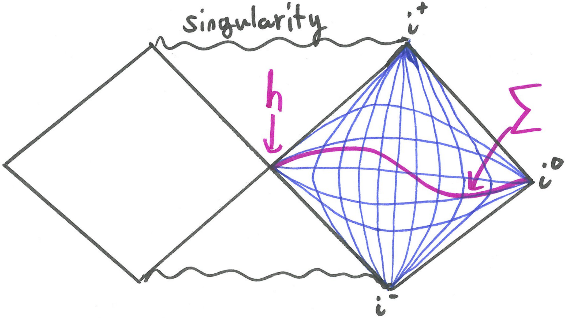

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There are now two boundary contributions to any δH: from h and from i^0.

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$$\delta H_t = \delta E - T\delta S_{Wald}$$

in which

$$T\delta S_{Wald} \equiv \frac{1}{16\pi G_N} \int_h \nabla^{[a} t^{b]} \sqrt{h}\, d^2 h_{ab}.$$

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One can show from our earlier formulas that if the Hamiltonian is generating a Killing vector, then $\delta H = 0$.

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Reminder: What is δ? It's an arbitrary on-shell variation of the fields, i.e. taking us from one black hole solution to a neighbouring one.

Wald Approach on a Black Hole Spacetime

If one takes T to be the Hawking temperature, one can show that in any diffeomorphism-invariant theory,

$$S_{Wald} = -2\pi \int_h \frac{\partial L}{\partial R_{abcd}} n_{ab} n_{cd} \sqrt{-h}\, d^2 S$$

where $n^{ab} = t^{[a} r^{b]}$ is the 'binormal' to the horizon.

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For black holes in GR,

$$S_{Wald} = \frac{A}{4G_N}.$$

Brown–York Quasilocal Energy

In analogy to the Hamilton-Jacobi equation

$$E = -\frac{\partial S_{\text{on-shell}}}{\partial t_{\text{final}}},$$

define a stress-energy tensor living locally on the boundary

$$T^{\mu\nu} \equiv -\frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\text{on-shell}}}{\delta \gamma_{\mu\nu}}$$

and an energy

$$E_{BY} = \int_{\partial\Sigma} T^{\mu\nu} t_\mu t_\nu d^2 S$$

with divergences cured by counterterms.

Energy of Matter + Gravity

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If the black hole formed by two stars of mass $M/2$, conservation $\Rightarrow$

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Thus M_{ADM} counts the combined energy of matter + gravity. To find the purely gravitational energy, subtract off $\int T^{00}$ as described earlier.

Positive Energy Theorem

Schoen and Yau (1979) proved:

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- This is taken to express stability of Minkowski space. What might we have expected its decay to look like?
- With physical spacetime in mind, might quantum effects disrupt the stability?

Why Boundary Terms?

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Why are the energy expressions boundary terms? Early hints of holography in general relativity?

How can a field theory be rich enough to foreshadow theories that supersede it?

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This is precisely the vacuum Casimir energy of the dual $\mathcal{N} = 4$ SU(N) Yang–Mills theory on the boundary at $N \gg 1$!

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Once AdS/CFT was discovered, Wald's approach $\Longrightarrow$ more calculations in gauge theories.

E.G, for low-energy D3-branes in Type IIB String Theory,

$$S_{Wald} = \frac{A}{4G_N}\left(1 + \frac{15}{8}\zeta(3)\frac{l_s^6}{L^6} + ...\right)$$

with AdS radius L.

By AdS/CFT, this is a strong-coupling expansion of the entropy of an equilibrium $\mathcal{N} = 4$ Super–Yang Mills plasma.

Conclusions

- Conserved quantities in GR can be found by noticing a vanishing divergence. But better, use Hamiltonians generating time translations, or Hamilton-Jacobi.
- Variational conservation of H_{Wald} requires boundary-horizon cancellations, giving black hole thermodynamics.
- The gravitational energy is a boundary term $\implies$ hints to holographic description. And the first law is an IR-UV link?