

# Chapter 1 - Foundations Solutions

Daniel Bradley\*

September 7, 2020

---

\*Questions from The Art Of Electronics by Paul Horowitz and Winfield Hill

## 1 Exercise 1.1.

### Question

You have a 5k resistor and a 10k resistor. What is their combined resistance: (a) in series and (b) in parallel?

### Solution

a. A circuit in series looks like the following circuit diagram:

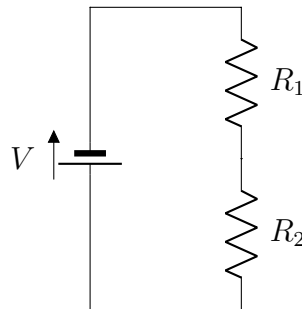


Figure 1: A diagram of two resistors,  $R_1$  and  $R_2$  in series

To calculate the total resistance, we use:

$$R_{combined} = R_1 + R_2$$

As:  $5k = 5000\Omega$ ,  $10k = 10,000\Omega$

$$R_{combined} = 5k\Omega + 10k\Omega = 15k\Omega$$

Final Solution =  $15k\Omega$

b. A circuit in parallel looks like the following circuit diagram:

To calculate the total resistance, we use:

$$\frac{1}{R_{combined}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{combined} = \left( \frac{1}{5k\Omega} + \frac{1}{10k\Omega} \right)^{-1}$$

$$R_{combined} = \frac{3}{10k\Omega}^{-1}$$

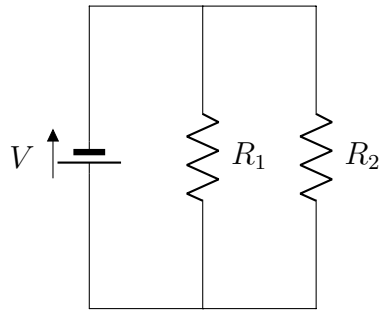


Figure 2: A diagram of two resistors,  $R_1$  and  $R_2$  in parallel

Final Solution =  $3.333k\Omega$

## 2 Exercise 1.2.

### Question

If you place a 1 ohm resistor across a 12 volt car battery, how much power will it dissipate?

### Solution

To calculate the total power dissipation:

$$P = IV = (V \frac{V}{I}) = \frac{V^2}{I}$$

$$P_{dissipated} = \frac{12^2}{1} = 144 \text{ W}$$

Final Solution = 144 W

### 3 Exercise 1.3.

#### Question

Prove the formulas for the series and parallel resistors.

#### Solution

Series Resistor Formula

Using Kirchhoff's Voltage Law (KVL):

$$V_1 + V_2 + V_3 = V_T$$

$$IR_1 + IR_2 + IR_3 = IR_T$$

$$I(R_1 + R_2 + R_3) = IR_T$$

Divide the equation through by "I"

$$R_1 + R_2 + R_3 = R_T$$

Parallel Resistor Formula

Using Kirchhoff's Current Law (KCL):

$$I_1 + I_2 + I_3 = I_T$$

$$\frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \frac{V}{R_T}$$

$$V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = V\left(\frac{1}{R_T}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_T}$$

## 4 Exercise 1.4.

### Question

Show that several resistors in parallel have resistance:

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

### Solution

Given  $\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = R_{1+2}$

We can therefore say -

$$\frac{1}{\frac{1}{R_{1+2}} + \frac{1}{R_3}} = R_{1+2+3}$$

It can then be proved via induction that  $R_{1\dots n}$  of  $n$  resistances  $R_1, R_2, \dots, R_n$  in parallel is:

$$R_{1\dots n} = \frac{1}{\sum_{i=1}^n \frac{1}{R_i}}$$

Now it's trivial to say that when  $n = 1$  the equality holds. Next we say that  $n = x$  is true so that:

$$R_{1\dots x} = \frac{1}{\sum_{i=1}^x \frac{1}{R_i}}$$

Next we can see that this holds for  $n = x + 1$ . Therefore the resistance of  $R_{1\dots x+1}$  for  $x+1$  resistances  $R_1, R_2, \dots, R_{x+1}$  in parallel is equal to the resistance of two resistors  $R_{1\dots x}$  and  $R_{1+x}$  in parallel then:

$$R_{1\dots(k+1)} = \frac{1}{\frac{1}{R_{1\dots x}} + \frac{1}{R_{x+1}}} = \frac{1}{\sum_{i=1}^x \frac{1}{R_i} + \frac{1}{R_{x+1}}} = \frac{1}{\sum_{i=1}^{x+1} \frac{1}{R_i}}$$

which proves the equality holds for  $n = x + 1$ . Finally the resistance of  $n$  resistors in parallel is given by:

$$R_{n\dots 1} = \frac{1}{\sum_{i=1}^n \frac{1}{R_i}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

## 5 Exercise 1.5.

### Question

Show that it is not possible to exceed the power rating of a 1/4 watt resistor of resistance greater than 1k, no matter how you connect it, in a circuit operating from a 15 volt battery.

### Solution

$$\text{Using } P = IV = \frac{V^2}{R}$$

Since the minimum resistance possible is  $1k\Omega$  the maximum power output by the resistor is equal to:

$$\frac{15^2}{1k\Omega} = \frac{225}{1000} = 0.225 \text{ W}$$

This is smaller than the 0.25 W the resistor is rated for and given the inverse nature of the relationship between power output and resistance given a constant voltage, it can be deduced that it is not possible to exceed the power rating.

Or

$$0.225 \text{ W} < 0.25 \text{ W due to } P = \frac{V^2}{R}$$

## 6 Exercise 1.6.

### Question

New York City requires about  $10^{10}$  watts of electrical power, at 115 volts (this is plausible: 10 million people averaging 1 kilowatt each). A heavy power cable might be an inch in diameter. Let's calculate what will happen if we try to supply the power through a cable 1 foot in diameter made of pure copper. Its resistance of  $0.05\mu\Omega$  ( $5 \times 10^{-8}$  ohms) per foot. Calculate (a) the power lost per foot from " $I^2R$  losses," (b) the length of cable of over which you will lose all  $10^{10}$  watts, and (c) how hot the cable will get, if you know the physics involved ( $\sigma = 6 \times 10^{-12} W/K^4 cm^2$ ). If you have done your computations correctly, the result should seem preposterous. What is the solution to this puzzle?

### Solution

a. The total current that will flow through the cable is able to be calculated using:

$$I = \frac{P}{V}$$

$$I = \frac{P}{V} = \frac{10^{10}W}{115V} = 86956521.739 \text{ A} = 8.696 \times 10^7$$

$$(8.696 \times 10^7)^2 * 5 \times 10^{-8} = 3.781 \times 10^8 \frac{W}{ft}$$

$$b. \frac{10^{10}W}{3.781 \times 10^8 W/ft} = 26.45 \text{ ft}$$

$$c. \text{ Using } T = \sqrt[4]{\frac{P}{A\sigma}}$$

Converting ft to cm:  $26.45 \text{ ft} = 806.196 \text{ cm}$

The surface area of the copper cabling will be:

$$\text{Area of the side of a cylinder} = 2\pi * r * h = 2\pi * 15.24 = 95.755744 * 806.196 = 77.197898 \times 10^3$$

$$\text{Area of the end of a cylinder} = \pi * r^2 = \pi * 15.242 = 729.6587699$$



$$\text{Total surface area} = 77.197898 \times 10^3 + (729.6587699 * 2) = 78657.2154 \text{ cm}^2$$

$$T = \sqrt[4]{\frac{10^{10}}{78657.2154 * 6 \times 10^{-12}}} = 12065.00609^\circ\text{C}$$

Of course  $12065.01^\circ\text{C}$  is a crazy temperature, it's over the melting point of copper at  $1085^\circ\text{C}$ . The solution to this problem is to use a material in the wire with a lower resistance, as well as to increase the surface area of the wire by using strategies such as multicore wire and thicker cables.

## 7 Exercise 1.7.

### Question

What will a  $20,000\Omega/V$  meter read, on its 1V scale, when attached to a 1V source with an internal resistance of 10k? What will it read when attached to a 10k-10k voltage divider driven by a "stiff" (zero source resistance) 1V source?

### Solution

As the source internal resistance and meter resistance are in series, they are added together to get  $30000\Omega$

$$\text{Using } I = \frac{V}{R}$$

$I = \frac{1V}{30000\Omega} = 0.000033$  A will flow when the meter is attached to the battery.

Using this current flow we can determine the deflection.

$$\frac{1V}{30000\Omega} * 20000 = 0.666 \text{ V}$$

This solution uses Thévenin's theorem.

$$\text{Thévenin's voltage} = V_{in} * \frac{R_2}{(R_1+R_2)}$$

$$\text{Thévenin's voltage} = 1 * \frac{10000}{(10000+10000)} = 0.5 \text{ V}$$

$$\text{Thévenin's resistance} = \frac{R_1 * R_2}{R_1 + R_2}$$

$$\text{Thévenin's resistance} = \frac{10000 * 10000}{10000 + 10000} = 5000\Omega$$

Therefore when using these values in a Thévenin equivalent circuit:

As the Thévenin's circuit places the Thévenin's resistance and load resistance in series the total resistance is  $25000\Omega$ .

$$\text{Using } I = \frac{V}{R}$$

$$I = \frac{0.5}{25000} = 0.00002 \text{ A}$$

$$0.00002 \text{ A} * 20000\Omega = 0.4 \text{ V}$$

## 8 Exercise 1.8.

### Question

A  $50\mu$  A meter movement has an internal resistance of 5k. What shunt resistance is needed to convert it to a 0-1 A meter? What resistance will convert it to a 0-10 V meter?

### Solution

Using  $V = IR$

$$V = 0.00005 * 5000 = 0.25 \text{ V}$$

Divide the voltage that will be present across the device by the required max current for the shunt resistance:

$$R = \frac{V}{I}$$

$$\frac{0.25}{1} = 0.25\Omega \text{ shunt resistance}$$

To be able to measure 10V, simply use  $V/I = R$  to calculate the total resistance required for full deflection, then adjust the resistor value based on the internal resistance.

$$\frac{10V}{0.00005A} = 200000\Omega$$

$$200000\Omega - 5000\Omega = 195000\Omega$$

## 9 Exercise 1.9.

### Question

The very high internal resistance of digital multimeters, in their voltage measuring ranges can be used to measure extremely low currents (even though the DMM may not offer a low current range explicitly). Suppose, for example, you want to measure the small current that flows through a  $1000M\Omega$  "leakage" resistance (that term is used to describe a small current that ideally should be absent entirely, for example through the insulation of an underground cable). You have available a standard DMM, whose 2V DC range has a  $10M\Omega$  internal resistance, and you have available a DC source of +10V. How can you use what you've got to measure accurately the leakage resistance?

### Solution

Connect the two in series.

Using  $V = IR$

This assumes that the +10VDC source is precise

If the internal DMM resistance in the 2V range is 10M, the voltage you read when the two are in series is:

$$V = 10M * I_{leakage}$$

So you can work out what  $I_{leakage}$  is.

Therefore, you can again use  $V = IR$  to work out that:

$$R_{leakage} = \frac{10V - V_{dmm}}{I_{leakage}}.$$

## 10 Exercise 1.10.

### Question

For the circuit shown in Figure 1.12, with  $V = 30\text{V}$  and  $R_1 = R_2 = 10\text{k}\Omega$ , find (a) the output voltage with no load attached (the open circuit voltage); (b) the output voltage with a  $10\text{k}$  load (treat as a voltage divider, with  $R_2$  and  $R_{load}$  combined into a single resistor); (c) the Thévenin's equivalent circuit; (d) the same as in part (b), but using the Thévenin equivalent circuit [again, you wind up with a voltage divider; the answer should agree with the result in part (b)]; (e) the power dissipated in each of the resistors.

The below diagram, Figure 3 is the diagram shown in figure 1.12 in The Art Of Electronics

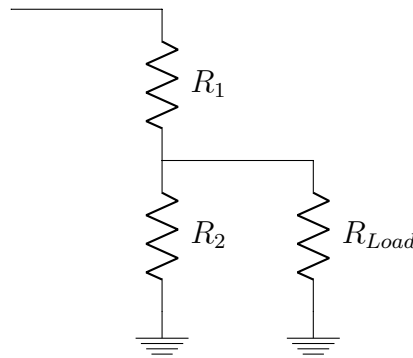


Figure 3: A diagram of a potential divider with  $R_1$  and  $R_2$  and a load of  $R_{Load}$

### Solution

a. Open circuit voltage:

$$V_{out} = V_{in} * \frac{R_2}{R_1 + R_2}$$

$$30 \text{ V} * \frac{10\text{k}\Omega}{10\text{k}\Omega + 10\text{k}\Omega} = 15 \text{ V}$$

b. Output Voltage with 10k load:

Combining load resistance with  $R_2$

$$\left(\frac{1}{10k\Omega} + \frac{1}{10k\Omega}\right)^{-1} = \left(\frac{2}{10}\right)^{-1} = 5k\Omega$$

Using  $5k\Omega$  for  $R_2$

$$30 \text{ V} * \frac{5k\Omega}{10k\Omega + 5k\Omega} = 10 \text{ V}$$

c. Using Figure 4 below as the basis for the Thévenin equivalent circuit

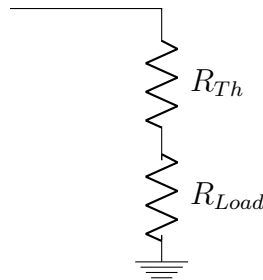


Figure 4: A diagram of a Thévenin equivalent circuit with  $R_{Th}$  representing the potential divider and a load of  $R_{Load}$

$$R_{Th} = \frac{10k\Omega * 10k\Omega}{10k\Omega + 10k\Omega} = 5k\Omega$$

$$V_{Th} = 30 * \frac{10k\Omega}{10k\Omega + 10k\Omega} = 15V$$

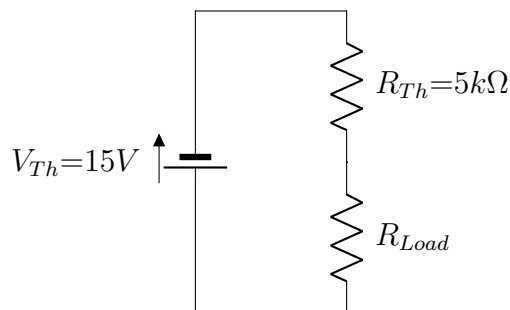


Figure 5: A Thévenin equivalent diagram of for a potential divider

d. Output Voltage with 10k load:

Modelling based on Figure 5 using a potential divider, with  $R_1 = R_{Th}$  and  $R_2 = R_{Load}$

$$15 \text{ V} * \frac{10k\Omega}{5k\Omega + 10k\Omega} = 10 \text{ V}$$

e. Using  $P = \frac{V^2}{R}$