Chapter 1 - Foundations Solutions

Daniel Bradley[*]

September 7, 2020

[*]Questions from The Art Of Electronics by Paul Horowitz and Winfield Hill

1 Exercise 1.1.

Question

You have a 5k resistor and a 10k resistor. What is their combined resistance: (a) in series and (b) in parallel?

Solution

a. A circuit in series looks like the following circuit diagram:

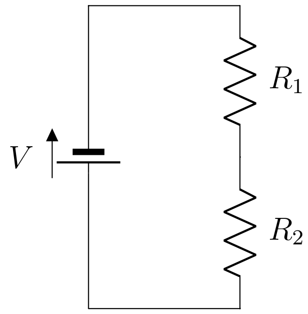

Figure 1: A diagram of two resistors, R_1 and R_2 in series

To calculate the total resistance, we use:

$R_{combined} = R_1 + R_2$

As: 5k = 5000Ω, 10k = $10,000\Omega$

$R_{combined} = 5k\Omega + 10k\Omega = 15k\Omega$

Final Solution = $15k\Omega$

b. A circuit in parallel looks like the following circuit diagram:
To calculate the total resistance, we use:

$\frac{1}{R_{combined}} = \frac{1}{R_1} + \frac{1}{R_2}$

$R_{combined} = (\frac{1}{5k\Omega} + \frac{1}{10k\Omega})^{-1}$

$R_{combined} = \frac{3}{10k\Omega}^{-1}$

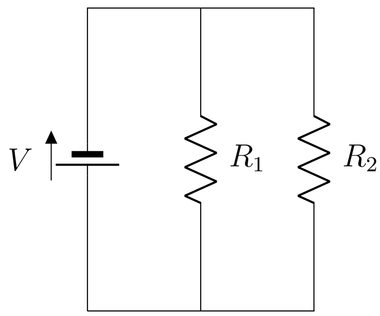

Figure 2: A diagram of two resistors, R_1 and R_2 in parallel

Final Solution = $3.333k\Omega$

2 Exercise 1.2.

Question

If you place a 1 ohm resistor across a 12 volt car battery, how much power will it dissipate?

Solution

To calculate the total power dissipation:

P = IV = (V$\frac{V}{I}$) = $\frac{V^2}{I}$

$P_{dissipated} = \frac{12^2}{1}$ = 144 W

Final Solution = 144 W

3 Exercise 1.3.

Question

Prove the formulas for the series and parallel resistors.

Solution

Series Resistor Formula

Using Kirchhoff's Voltage Law (KVL):

$V_1 + V_2 + V_3 = V_T$

$\mathrm{I}R_1 + \mathrm{I}R_2 + \mathrm{I}R_3 = \mathrm{I}R_T$

$\mathrm{I}(R_1 + R_2 + R_3) = \mathrm{I}R_T$

Divide the equation through by "I"

$R_1 + R_2 + R_3 = R_T$

Parallel Resistor Formula

Using Kirchhoff's Current Law (KCL):

$I_1 + I_2 + I_3 = I_T$

$\frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \frac{V}{R_T}$

$\mathrm{V}(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}) = \mathrm{V}(\frac{1}{R_T})$

$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_T}$

4 Exercise 1.4.

Question

Show that several resistors in parallel have resistance:

$$R = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+...}$$

Solution

Given $\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = R_{1+2}$

We can therefore say -

$$\frac{1}{\frac{1}{R_{1+2}}+\frac{1}{R_3}} = R_{1+2+3}$$

It can then be proved via induction that $R_{1...n}$ of n resistances R_1, R_2,...,R_n in parallel is:

$$R_{1...n} = \frac{1}{\sum_{i=1}^{n}\frac{1}{R_i}}$$

Now it's trivial to say that when n = 1 the equality holds. Next we say that n = x is true so that:

$$R_{1...x} = \frac{1}{\sum_{i=1}^{x}\frac{1}{R_i}}$$

Next we can see that this holds for n = x + 1. Therefore the resistance of $R_{1...x+1}$ for x+1 resistances R_1, R_2,..., R_{x+1} in parallel is equal to the resistance of two resistors $R_{1...x}$ and R_{1+x} in parallel then:

$$R_{1...(k+1)} = \frac{1}{\frac{1}{R_{1...x}}+\frac{1}{R_{x+1}}} = \frac{1}{\sum_{i=1}^{x}\frac{1}{R_i}+\frac{1}{R_{x+1}}} = \frac{1}{\sum_{i=1}^{x+1}\frac{1}{R_i}}$$

which proves the equality holds for n = x + 1. Finally the resistance of n resistors is parallel is given by:

$$R_{n...1} = \frac{1}{\sum_{i=1}^{n}\frac{1}{R_i}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+...+\frac{1}{R_n}}$$

5 Exercise 1.5.

Question

Show that it is not possible to exceed the power rating of a 1/4 watt resistor of resistance greater than 1k, no matter how you connect it, in a circuit operating from a 15 volt battery.

Solution

Using P = IV = $\frac{V^2}{R}$

Since the minimum resistance possible is $1k\Omega$ the maximum power output by the resistor is equal to:

$\frac{15^2}{1k\Omega} = \frac{225}{1000} = 0.225$ W

This is smaller than the 0.25 W the resistor is rated for and given the inverse nature of the relationship between power output and resistance given a constant voltage, it can be deduced that it is not possible to exceed the power rating.

Or

0.225 W $<$ 0.25 W due to P = $\frac{V^2}{R}$

6 Exercise 1.6.

Question

New York City requires about 10^10 watts of electrical power, at 115 volts (this is plausible: 10 million people averaging 1 kilowatt each). A heavy power cable might be an inch in diameter. Let's calculate what will happen if we try to supply the power through a cable 1 foot in diameter made of pure copper. Its resistance of $0.05\mu\Omega$ (5×10^{-8} ohms) per foot. Calculate (a) the power lost per foot from "I^2R losses," (b) the length of cable of over which you will lose all 10^{10} watts, and (c) how hot the cable will get, if you know the physics involved ($\sigma = 6 \times 10^{-12} W/K^4cm^2$). If you have done your computations correctly, the result should seem preposterous. What is the solution to this puzzle?

Solution

a. The total current that will flow through the cable is able to be calculated using:

I = $\frac{P}{V}$

I = $\frac{P}{V} = \frac{10^{10}W}{115V} = 86956521.739$ A $= 8.696 \times 10^7$

$(8.696 \times 10^7)^2 * 5 \times 10^{-8} = 3.781 \times 10^8 \ \frac{W}{ft}$

b. $\frac{10^{10}W}{3.781 \times 10^8 W/ft} = 26.45$ ft

c. Using T = $\sqrt[4]{\frac{P}{A\sigma}}$

Converting ft to cm: 26.45 ft = 806.196 cm

The surface area of the copper cabling will be:

Area of the side of a cylinder = 2π * r * h = 2π * 15.24 = 95.755744 * 806.196 = 77.197898×10^3

Area of the end of a cylinder = π * r^2 = π * 15.242 = 729.6587699

Total surface area = $77.197898 \times 10^3 + (729.6587699 * 2) = 78657.2154$ cm^2

T = $\sqrt[4]{\frac{10^{10}}{78657.2154 * 6 \times 10^{-12}}} = 12065.00609°\mathrm{C}$

Of course 12065.01°C is a crazy temperature, it's over the melting point of copper at 1085°C. The solution to this problem is to use a material in the wire with a lower resistance, as well as to increase the surface area of the wire by using strategies such as multicore wire and thicker cables.

7 Exercise 1.7.

Question

What will a $20,000\Omega/V$ meter read, on its 1V scale, when attached to a 1V source with an internal resistance of 10k? What will it read when attached to a 10k-10k voltage divider driven by a "stiff" (zero source resistance) 1V source?

Solution

As the source internal resistance and meter resistance are in series, they are added together to get 30000Ω

Using I = $\frac{V}{R}$

I = $\frac{1V}{30000\Omega}$ = 0.000033 A will flow when the meter is attached to the battery.

Using this current flow we can determine the defection.

$\frac{1V}{30000\Omega}$ * 20000 = 0.666 V

This solution uses Thévenin's theorem.

Thévenin's voltage = V_{in} * $\frac{R_2}{(R_1+R_2)}$

Thévenin's voltage = 1 * $\frac{10000}{(10000+10000)}$ = 0.5 V

Thévenin's resistance = $\frac{R1*R2}{R1+R2}$

Thévenin's resistance = $\frac{10000*10000}{10000+10000}$ = 5000Ω

Therefore when using these values in a Thévenin equivalent circuit:

As the Thévenin's circuit places the Thévenin's resistance and load resistance in series the total resistance is 25000Ω.

Using I = $\frac{V}{R}$

$\mathrm{I} = \frac{0.5}{25000} = 0.00002$ A

0.00002 A * $20000\Omega = 0.4$ V

8 Exercise 1.8.

Question

A 50μ A meter movement has an internal resistance of 5k. What shunt resistance is needed to convert it to a 0-1 A meter? What resistance will convert it to a 0-10 V meter?

Solution

Using V = IR

V = 0.00005 * 5000 = 0.25 V

Divide the voltage that will be present across the device by the required max current for the shunt resistance:

R = $\frac{V}{I}$

$\frac{0.25}{1}$ = 0.25Ω shunt resistance

To be able to measure 10V, simply use V/I = R to calculate the total resistance required for full deflection, then adjust the resistor value based on the internal resistance.

$\frac{10V}{0.00005A}$ = 200000Ω

200000Ω - 5000Ω = 195000Ω

9 Exercise 1.9.

Question

The very high internal resistance of digital multimeters, in their voltage measuring ranges can be used to measure extremely low currents (even though the DMM may not offer a low current range explicitly). Suppose, for example, you want to measure the small current that flows through a $1000M\Omega$ "leakage" resistance (that term is used to describe a small current that ideally should be absent entirely, for example through the insulation of an underground cable). You have available a standard DMM, whose 2V DC range has a $10M\Omega$ internal resistance, and you have available a DC source of +10V. How can you use what you've got to measure accurately the leakage resistance?

Solution

Connect the two in series.

Using V = IR

This assumes that the +10VDC source is precise

If the internal DMM resistance in the 2V range is 10M, the voltage you read when the two are in series is:

V = 10M * $I_{leakage}$

So you can work out what $I_{leakage}$ is.

Therefore, you can again use V = IR to work out that:

$R_{leakage} = \frac{10V - V_{dmm}}{I_{leakage}}$.

10 Exercise 1.10.

Question

For the circuit shown in Figure 1.12, with V = 30V and $R_1 = R_2 = 10k\Omega$, find (a) the output voltage with no load attached (the open circuit voltage); (b) the output voltage with a 10k load (treat as a voltage divider, with R_2 and R_{load} combined into a single resistor); (c) the Thévenin's equivalent circuit; (d) the same as in part (b), but using the Thévenin equivalent circuit [again, you wind up with a voltage divider; the answer should agree with the result in part (b)]; (e) the power dissipated in each of the resistors.

The below diagram, Figure 3 is the diagram shown in figure 1.12 in The Art Of Electronics

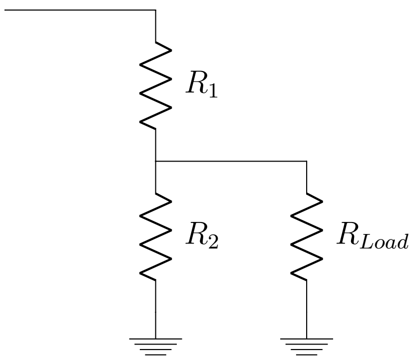

Figure 3: A diagram of a potential divider with R_1 and R_2 and a load of R_{Load}

Solution

a. Open circuit voltage:

$V_{out} = V_{in} * \frac{R_2}{R_1+R_2}$

$30 \text{ V} * \frac{10k\Omega}{10k\Omega+10k\Omega} = 15 \text{ V}$

b. Output Voltage with 10k load:

Combining load resistance with R_2

$(\frac{1}{10k\Omega} + \frac{1}{10k\Omega})^{-1} = (\frac{2}{10})^{-1} = 5k\Omega$

Using $5k\Omega$ for R_2

$30 \text{ V} * \frac{5k\Omega}{10k\Omega+5k\Omega} = 10 \text{ V}$

c. Using Figure 4 below as the basis for the Thévenin equivalent circuit

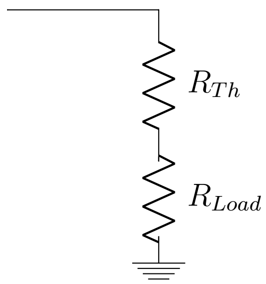

Figure 4: A diagram of a Thévenin equivalent circuit with R_{Th} representing the potential divider and a load of R_{Load}

$R_{Th} = \frac{10k\Omega * 10k\Omega}{10k\Omega+10k\Omega} = 5k\Omega$

$V_{Th} = 30 * \frac{10k\Omega}{10k\Omega+10k\Omega} = 15V$

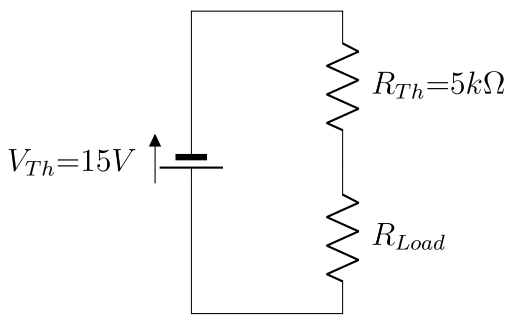

Figure 5: A Thévenin equivalent diagram of for a potential divider

d. Output Voltage with 10k load:

Modelling based on Figure 5 using a potential divider, with $R_1 = R_{Th}$ and $R_2 = R_{Load}$

15 V * $\frac{10k\Omega}{5k\Omega+10k\Omega} = 10$ V

e. Using P = $\frac{V^2}{R}$

$\frac{(10V)^2}{10k\Omega} = 0.01$ W dissipated by load

Using Kirchhoffs Voltage Law and P = $\frac{V^2}{R}$
30 V source minus 10 V dropped across R_2
30 V - 10 V = 20 V across R_1
P = $\frac{(20V)^2}{10k\Omega} = 0.04$ W