ECE155: Engineering Design with Embedded Systems Winter 2015

Lecture 12 —Computational Decision-Making

Patrick Lam & Jeff Zarnett

Decision-Making

Let's start with a general process for making decisions.

1. List all choices or courses of action.
2. List all criteria (attributes) that will be used to evaluate the choices.
3. Compare the two lists and remove impractical choices.
4. Evaluate the advantages & disadvantages of each remaining choice according to all of the criteria.

This doesn't really say how to evaluate the advantages and disadvantages systematically, though.

Computational Decision-Making

Our goal is to quantify the decision-making process. We'll do so by assigning a numerical *weight* to each criterion, according to its importance, and a numerical *score* for each choice according to each criterion. Together, these allow us to compute the value of a payoff function for each choice.

Assume that there are m choices and n criteria.

1. Decide on the importance of each criterion, and assign a weight w_j to each criterion. Make sure that $\sum_{j=1}^{n} w_j = 1$ (or 100%). Larger weights are more important.
2. For each choice i and criterion j, assign a score p_{ij} which reflects the extent to which i satisfies j. This should be a number between 0 and 1.

This data will allow us to compute scores s_{ij} and a payoff function f_i for each alternative and to choose the alternative with the largest expected payoff:

$$s_{ij} = p_{ij} w_j; \qquad f_i = \sum_{j=1}^{n} s_{ij} = \sum_{j=1}^{n} p_{ij} w_j$$

(which is just the dot product of the weights and the scores).

Choose the alternative with the largest payoff f_i.

Normalizing scores. As an alternative to choosing scores $p_{ij} \in [0, 1]$, you can instead normalize the scores, as follows.

1. Assign a real number c_{ij} for each alternative i and criterion j. Be sure to use the same units for all of the different alternatives of a single criterion. You can use different units for different criteria.

2. For each criterion, calculate
$$C_j = \max\{|c_{1j}|, |c_{2j}|, \ldots, |c_{mj}|\},$$
so that
$$p_{ij} = \frac{c_{ij}}{C_j}.$$

3. The payoff function is then given by:
$$f_i = \sum_{j=1}^{n} p_{ij} w_j = \sum_{j=1}^{n} \frac{c_{ij}}{C_j} w_j.$$

Example

Let's consider the question: "How should I get to Montreal?"

The possible answers I'll consider are:

- Train
- Personal automobile
- Airplane

In Step 1, we will consider three criteria:

- Cost
- Travel time
- Schedule Flexibility (i.e. when can I leave?)

Let's assign weights to these criteria. Step 2:

- Pretend you have lots of money. Let's estimate $w_1 = 0.2$.
- Time is important, so $w_2 = 0.4$.
- Schedule Flexibility is important; let $w_3 = 0.4$.

(Yes, these are arbitrary. We'll see just how much these arbitrary decisions affect the outcome.)

Now, we need to decide how we'll assign scores. Step 3:

- Cost: dollars (lower = better);
- Time: hours (lower = better);
- Schedule Flexibility: qualitative assessment (higher = better)

All of the scores need to point the same way, so we'll invert our schedule flexibility score before computing.

Let's make up some raw data.

Criterion	Train	Car	Plane	Category Winner
Cost	\$102	\$206	\$191	Train
Time	7.7 hrs	6.5 hrs	3.5 hrs	Plane
Schedule Flexibility	0.5	1.0	0.9	Car

The cost for driving is estimated at \$206 by the calculation of 642km at \$0.321/km, per `http://www.caa.ca/documents/DrivingCostsBrochure-jan09-eng-v3.pdf`. Plane and train costs are based on the ticket purchase prices.

What factors do these data not capture?

Now we can build a decision matrix:

Criterion	Weight	Alternatives								
$(n = 3)$	w_i (%)	Train			Car			Plane		
		c_{1j}	p_{1j}	s_{1j}	c_{2j}	p_{2j}	s_{2j}	c_{3j}	p_{3j}	s_{3j}
Cost	20	102	0.5		206	1.0		191	0.9	
Time	40	7.7	1.0		6.5	0.85		3.5	0.45	
Flexibility	40	0.5	0.5		0.0	0.0		0.1	0.1	
Totals f_j										

Note that, in this matrix, lower is better. It may make sense to make higher better, depending on how your criteria are set up.

Potential Flaws. Did we get an optimal decision? Let's look at threats to validity of the analysis to see how robust our decision is.

- (Not an issue here.) In some cases, your scores might be stuck in a small subrange of the possible range, which affects scores and hence values.
- The values for flexibility are subjective, and the way you assign numbers to the options changes the outcome.
- The cost depends on what you include; for instance, gas alone would be \$72.50, and it would be less if you carpooled.
- The weights are subjective.

Sensitivity Analysis. To investigate the impact of the potential flaws, we can use a *sensitivity analysis*, which is a study of how variations influence the outcome of a mathematical model.

You can do a poor person's sensitivity analysis by performing what-if calculations: simply replace the original value with some other value and repeat the analysis. There can be issues if the scores are not independent.

There are also more complex techniques for sensitivity analysis, which we won't talk about. Let's instead try revising the scores. For instance, the train used to always be late, so you might assign 9 instead of 7.7. (It's mostly on time recently.) On the other hand, you might want to get to the airport slightly earlier than me, so you might also change the time for plane to 5. This gives:

Criterion $(n=3)$	Weight w_i (%)	Alternatives								
		Train			Car			Plane		
		c_{i1}	p_{i1}	s_{i1}	c_{i2}	p_{i2}	s_{i2}	c_{i3}	p_{i3}	s_{i3}
Cost	20	102	0.5		206	1.0		191	0.9	
Time	40	9	1.0		6.5	0.72		5	0.56	
Flexibility	40	0.5	0.5		0.0	0.0		0.1	0.1	
Totals f_j										

Does this change the best decision?

Or, you might be more cost-sensitive and less time-sensitive:

Criterion $(n=3)$	Weight w_i (%)	Alternatives								
		Train			Car			Plane		
		c_{i1}	p_{i1}	s_{i1}	c_{i2}	p_{i2}	s_{i2}	c_{i3}	p_{i3}	s_{i3}
Cost	40	102	0.5		206	1.0		191	0.9	
Time	20	7.7	1.0		6.5	0.85		3.5	0.45	
Flexibility	40	0.5	0.5		0.0	0.0		0.1	0.1	
Totals f_j										

What happens now?

Other references. Previous notes contained an example ("Where should I go to University?") and a further reference to *Introduction to Professional Engineering in Canada* [AAFM08], Example 15.3 (pp. 232–233). You can also see some aspects of computational decision-making if you look at the consideration of Waterloo Region's Rapid Transit alternatives, although there's no actual numbers.

Discussion

Computational decision-making systematically includes diverse factors to arrive at a "best" decision.

Disadvantage. You have to estimate, numerically, the importance (weight) of each criterion and the goodness (score) of each alternative with respect to each criterion. It may be infeasible to come up with good estimates, and estimates may just reflect your pre-conceived notions. It also doesn't give you new alternatives.

Advantage. Using this method, you can ensure that you give each alternative sufficient consideration. You don't necessarily have to pick the alternative with the highest number, because your scores or coefficients might have been imprecise.

Work Term Report Pro Tip. Computational decision-making charts are valuable in your work reports. They provide a consistent way of evaluating alternatives and justifying why you have chosen a particular option as the best.

Further Example

This example came from the Spring 2014 midterm exam. You are a conscientious student who works hard, but you are also a fan of heavy metal bands whose names begin with the letter *M*. You think that you can manage to go to one concert this summer without having a negative impact on your studies. There are three concerts scheduled in your general area, and you have researched the subject and composed the following raw data.

Criterion	**Manowar**	**Megadeth**	**Metallica**
Ticket Cost ($)	100	125	75
Travel Time (h)	1.5	3	3
Show Reviews	8/10	10/10	6/10

You also estimate the following weights:

Criterion	Weight (%)
Ticket Cost	50
Travel Time	20
Show Reviews	30

Complete a computational decision-making chart, and indicate which of the three concerts you plan to attend.

Solution. Show reviews is the only criterion where higher is better, so it must be inverted. Also acceptable is inverting both ticket code and travel time (although not shown here).

Criterion	Weight	Alternatives								
$(n = 3)$	w_i (%)	Manowar			Megadeth			Metallica		
		c_{i1}	p_{i1}	s_{i1}	c_{i2}	p_{i2}	s_{i2}	c_{i3}	p_{i3}	s_{i3}
Ticket Cost	50	100	0.8	40	125	1	50	75	0.6	30
Travel Time	20	1.5	0.5	10	3	1	20	3	1	20
Reviews	30	2	0.2	6	0	0	0	4	0.4	12
Totals f_j				56			70			62

References

[AAFM08] Gordon C. Andrews, J. Dwight Aplevich, Roydon A. Fraser, and Carolyn MacGregor. *Introduction to Professional Engineering in Canada, Third Edition.* Pearson Education Canada, 2008.