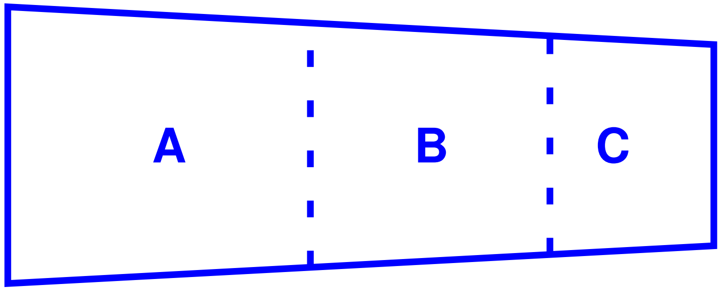
## - - - Signal Processing Basics - - -

- **Systems**
  - interact with signals, possibly modifying them.
  - physical entities, such as a resonant cavity, or computer-based entities, such as a digital filter.




$$h[\cdot] = \{h_0, h_1, ..., h_{N-1}\}$$

- **Discrete-time Convolution**
  - the way a signal ($x[\cdot]$) interacts with a system ($h[\cdot]$)
  - $y_n = h[\cdot] * x[\cdot] = x[\cdot] * h[\cdot] = \sum_k h_k \cdot x_{n-k}$
- **Linear-Time Invariant (LTI) Systems**
  - Linear: outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs
  - Time-invariant: output does not depend on *when* an input was applied

- - - Signal Processing Basics - - -

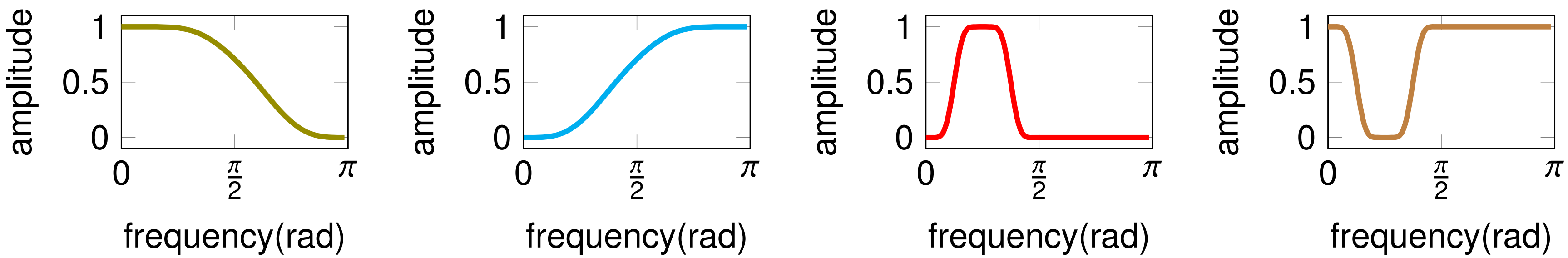
- **Practical Procedure for Convolution**:

| | | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | signal $s[\cdot]$ |
| | | 4 | 5 | filter $h[\cdot]$ |
| | 5 | 10 | 15 | |
| 4 | 8 | 12 | + | |
| 4 | 13 | 22 | 15 | resulting signal $y[\cdot]$ |

- Try it yourself, by hand, with different signals to get some hands on experience!
- Concluding, in time-domain, a signal interacts with a system by means of convolution.

## - - - Signal Processing Basics - - -

- For us in this course, the main type of system is a **filter**. Essentially, a filter allows for specific frequency intervals to pass through, retaining the others.
- Filters are classified as follows:
  - filter type: finite impulse response (FIR) and infinite impulse response (IIR)
  - filter function: low-pass, high-pass, band-pass and band-stop
  - additional details, such as filter order, filter frequency response style, and so on.



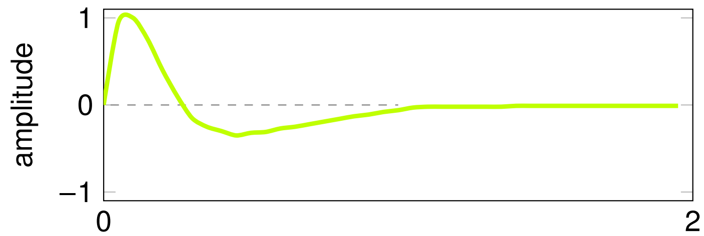

(a) typical low-pass filter (b) typical high-pass filter (c) typical band-pass filter (d) typical band-stop filter

We will take a look on how to project filters soon, however, to do that, we need first comment more on Fourier analysis, sampling and quantization.

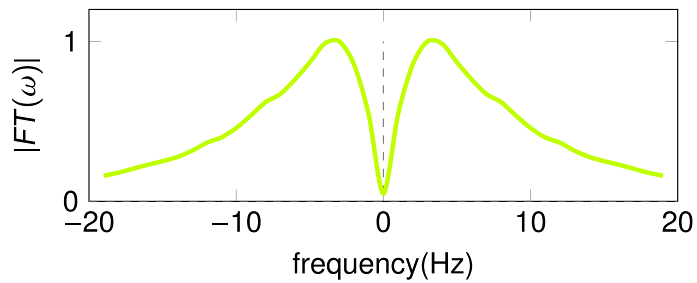
## - - - Frequency-domain Processing - - -

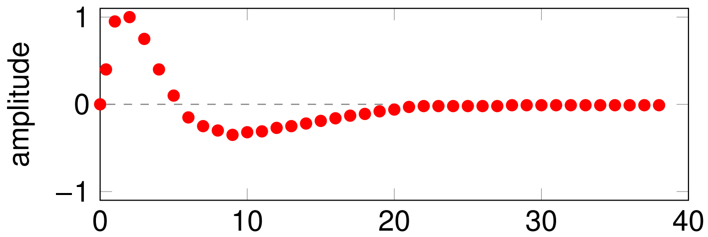
- **Fourier Analysis**: any arbitrary function can be expressed as a sum of sinusoidal, i.e., “pure”, functions

  **Frequency Domain**: Fourier analysis provides different results depending on the signal type, i.e., continuous- or discrete-time

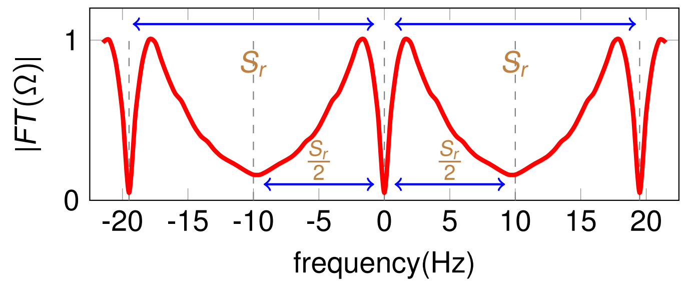



real continuous-time signal x(t) where time is denoted in seconds



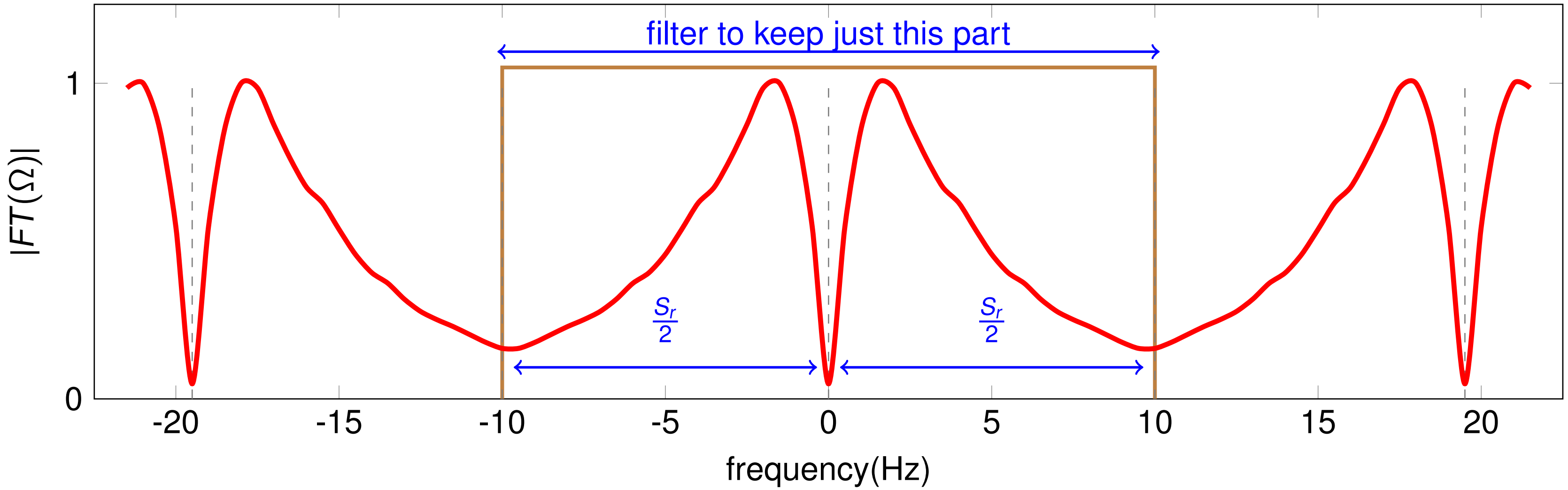




real discrete-time signal *x* □*n*□ sampled at *Sr* samples per second



## - - - Signal Processing Basics - - -

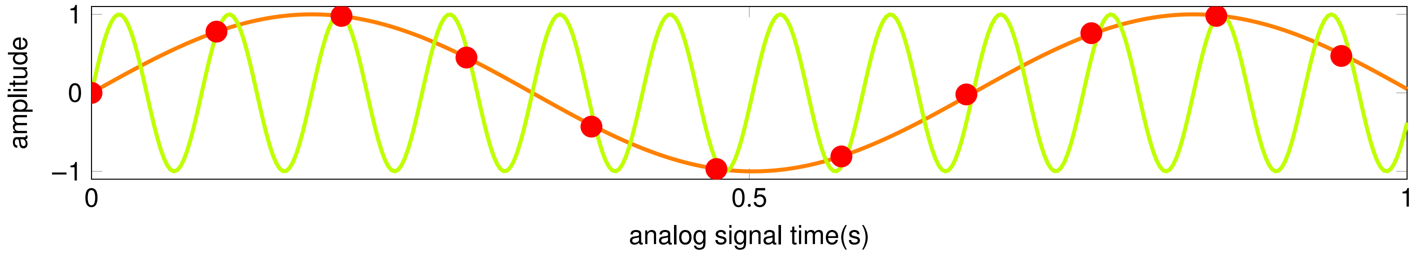
- to reconstruct the analog signal from its digital version, we have to filter the latter, keeping only the frequencies from $-\frac{S_r}{2}$ to $\frac{S_r}{2}$.



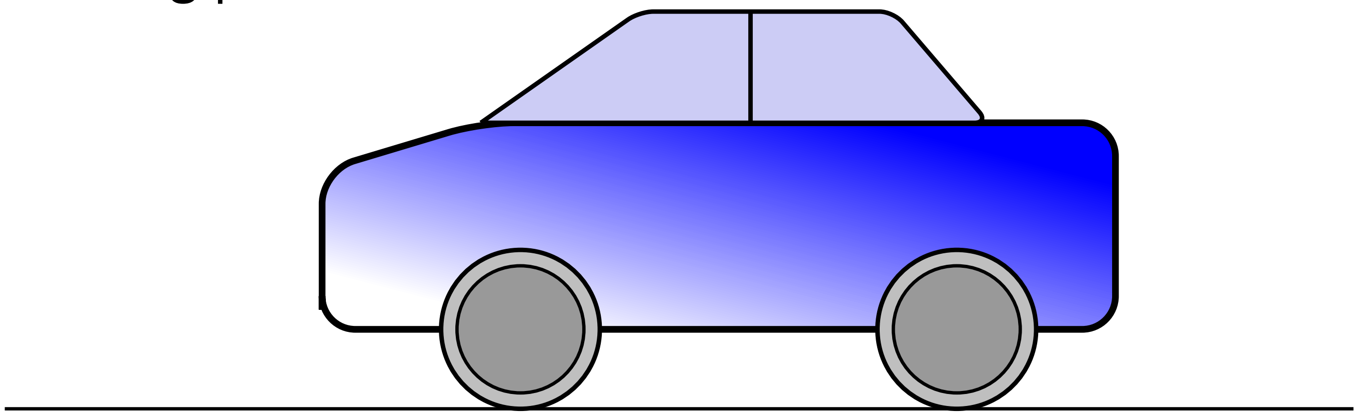

- to reconstruct the original signal from its sampled version, $S_r$ **should be at least twice the highest signal frequency component**
- **Suggested Activity**: by using our WhatsApp group, comment on Fourier analysis. Why is it important? Why is sampling of fundamental importance in signal processing?

- - - Signal Processing Basics - - -

- **Aliasing**

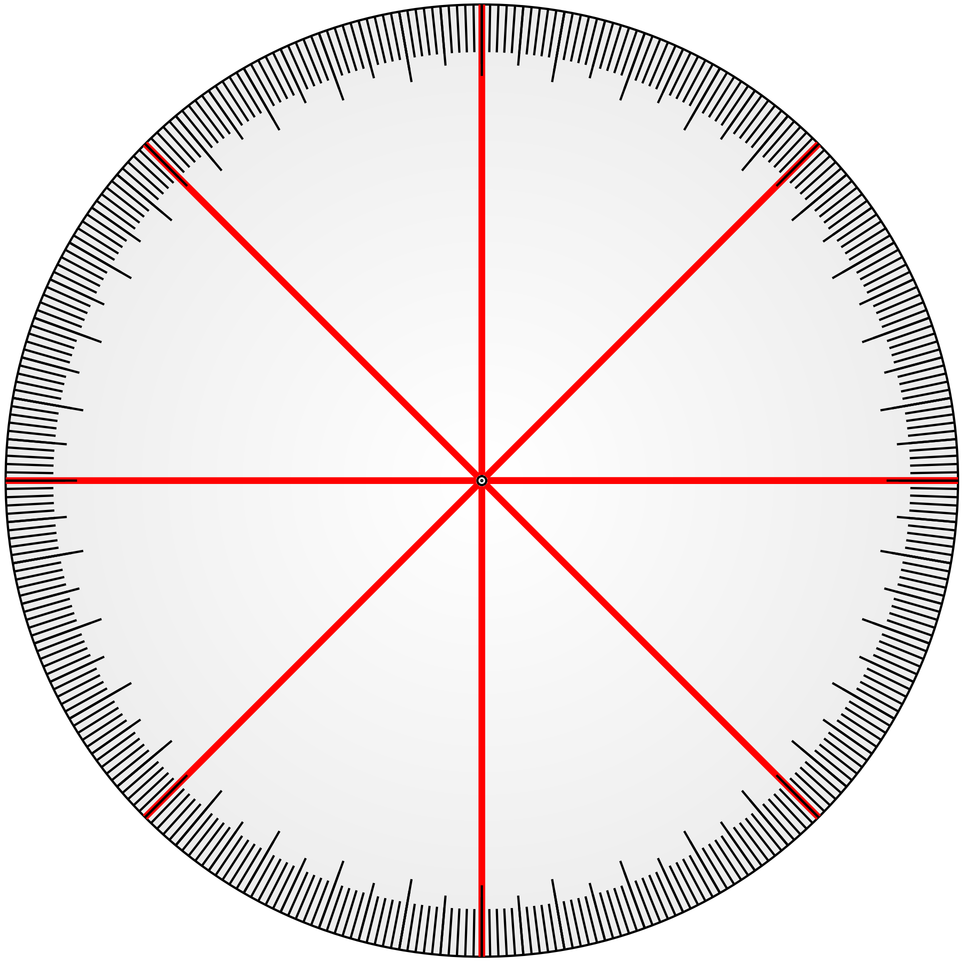



- **Digital Signals: car wheel example**
  - when we look at a car moving fast from left to right, we have the wrong impression its wheels are rotating counterclockwise instead of clockwise. Why?
  - **aliasing** phenomenon



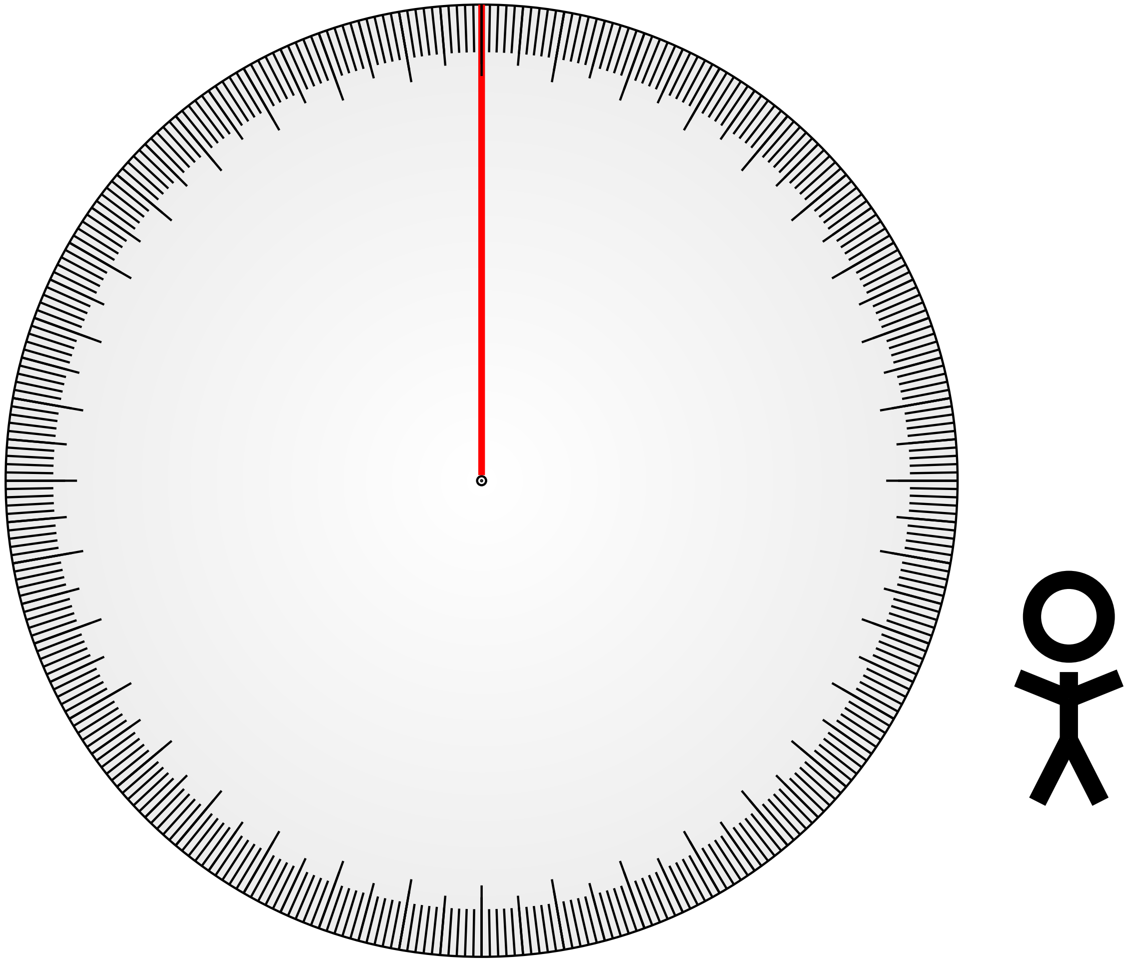
- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (in red, we have the wheel rims or “spokes”... they are easily seen when the car is not moving...)



- - - Signal Processing Basics - - -

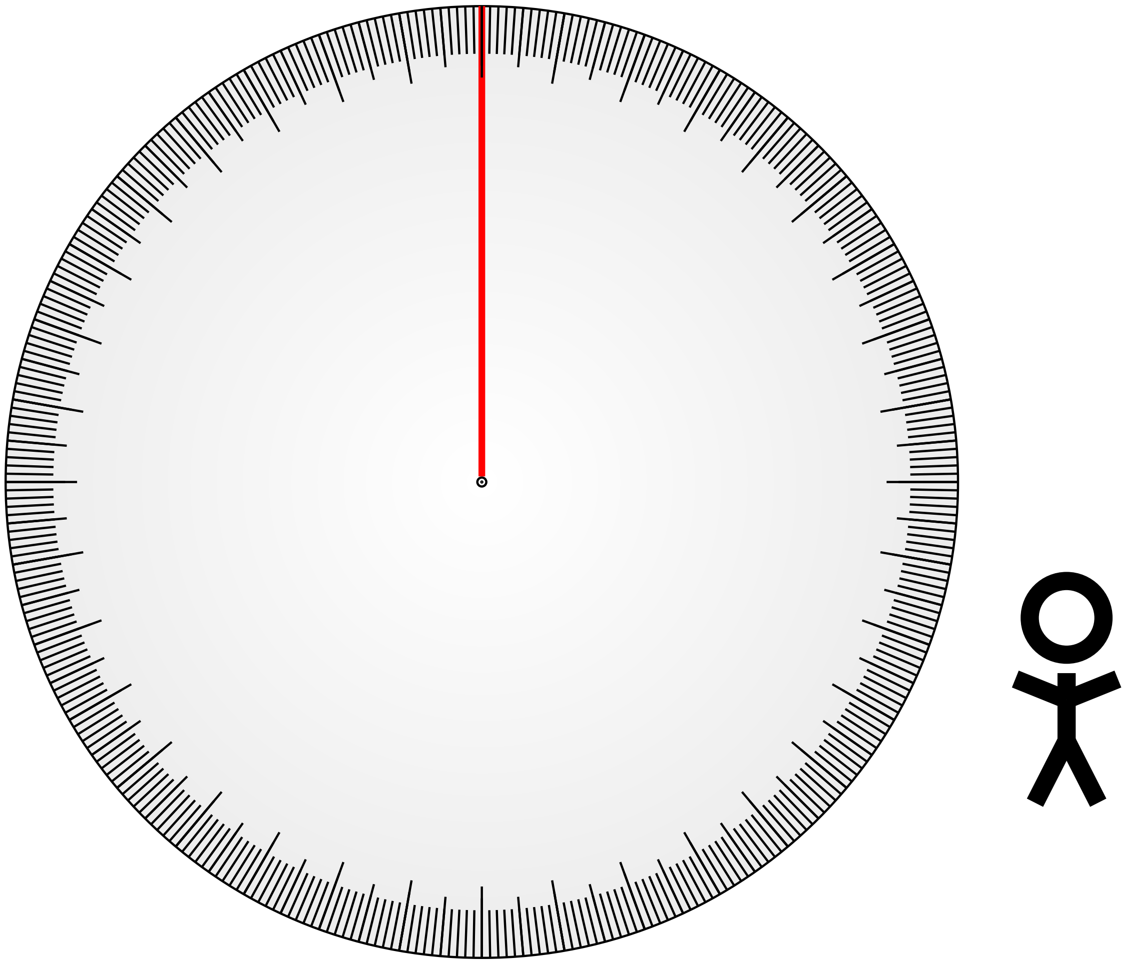
- **Aliasing: the car wheel example** (to ease the analysis in movement, we keep only one wheel rim)



- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

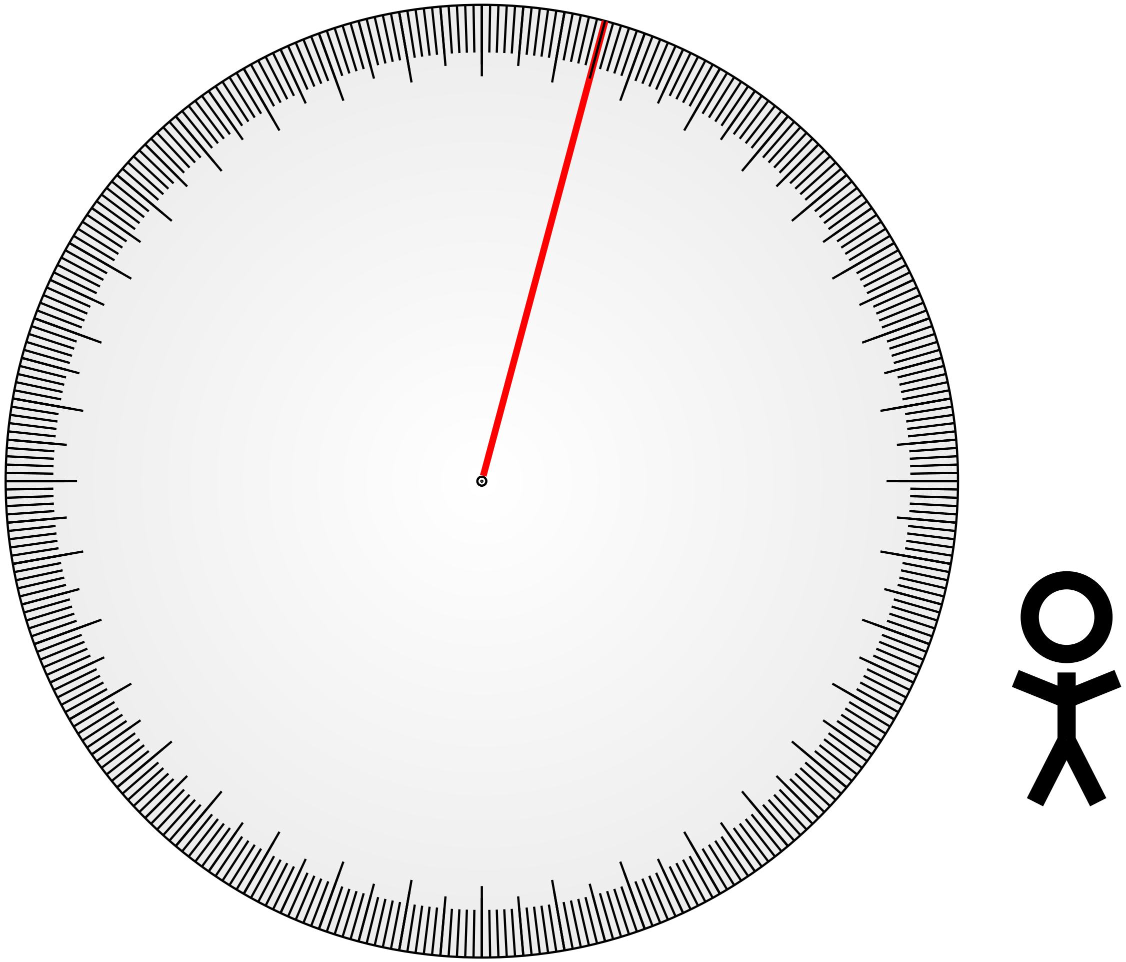
FIRST time



- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

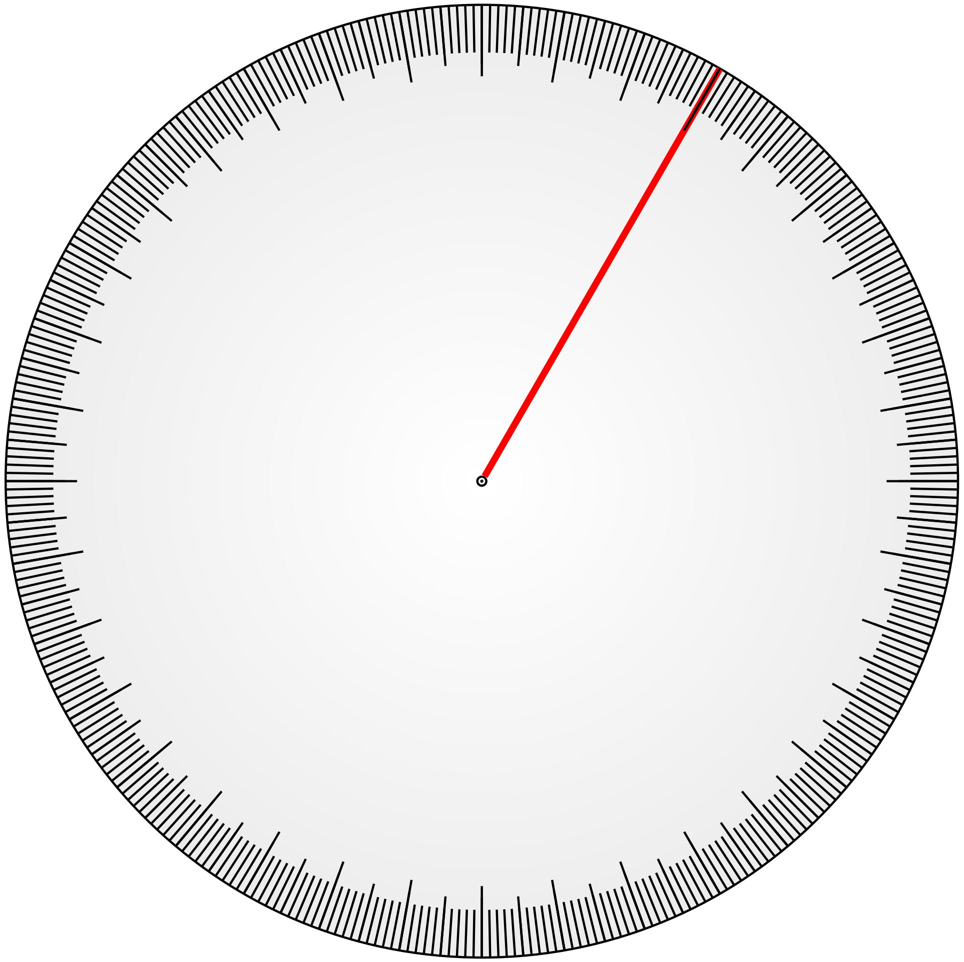
SECOND time

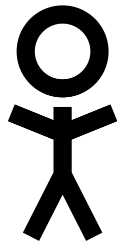


- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

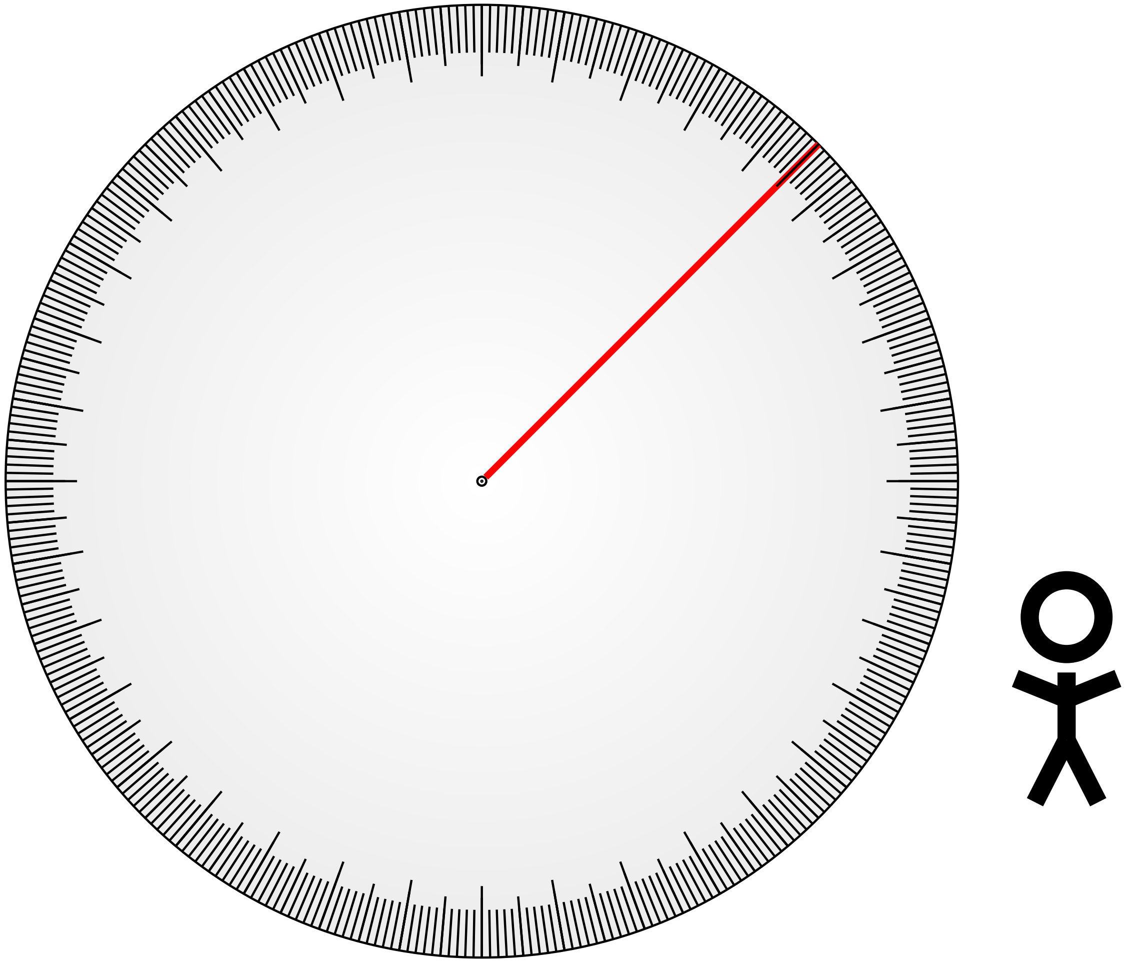
THIRD time





- - - Signal Processing Basics - - -

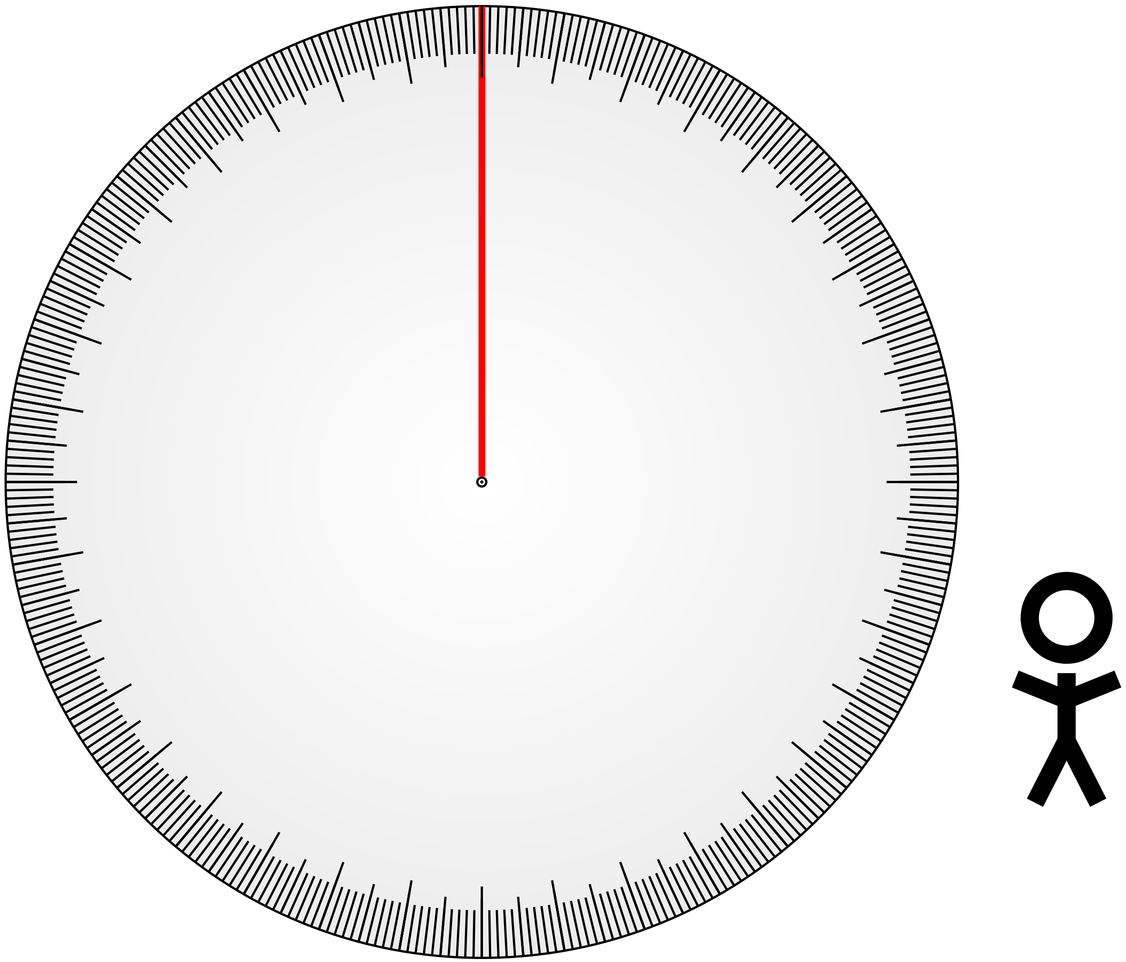
- **Aliasing: the car wheel example** (human eyes work as a sampling machine...) FOURTH time and so on...



- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

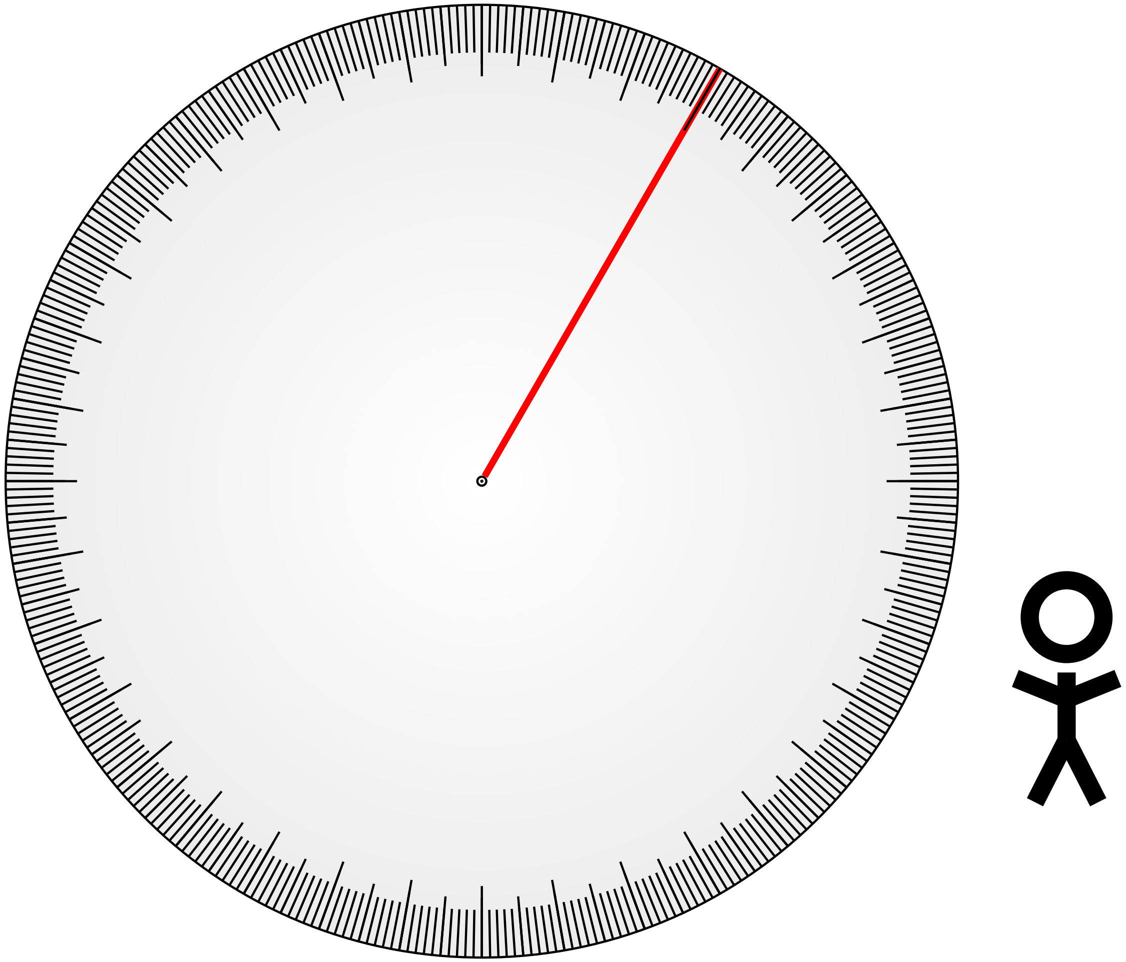
FIRST time (faster)



- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

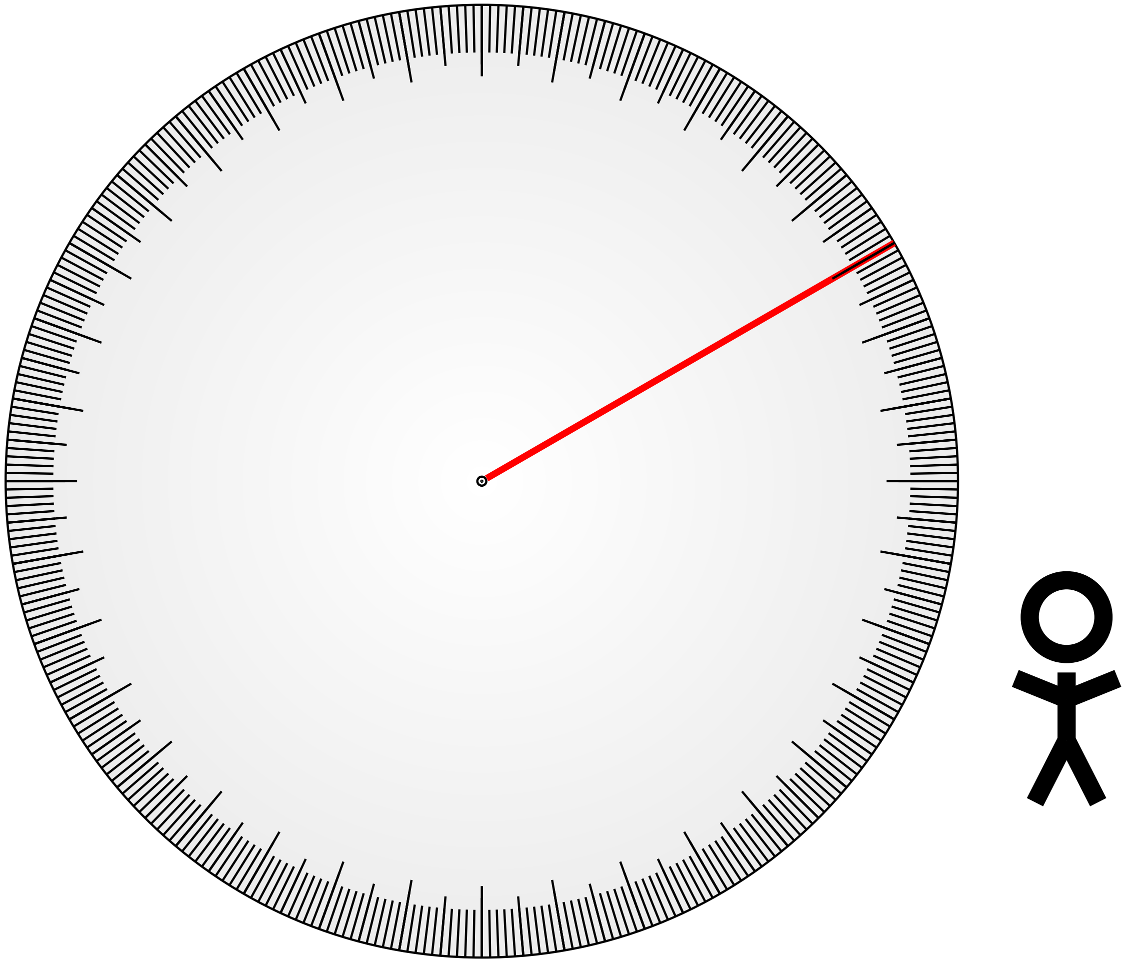
SECOND time (faster)



- - - Signal Processing Basics - - -

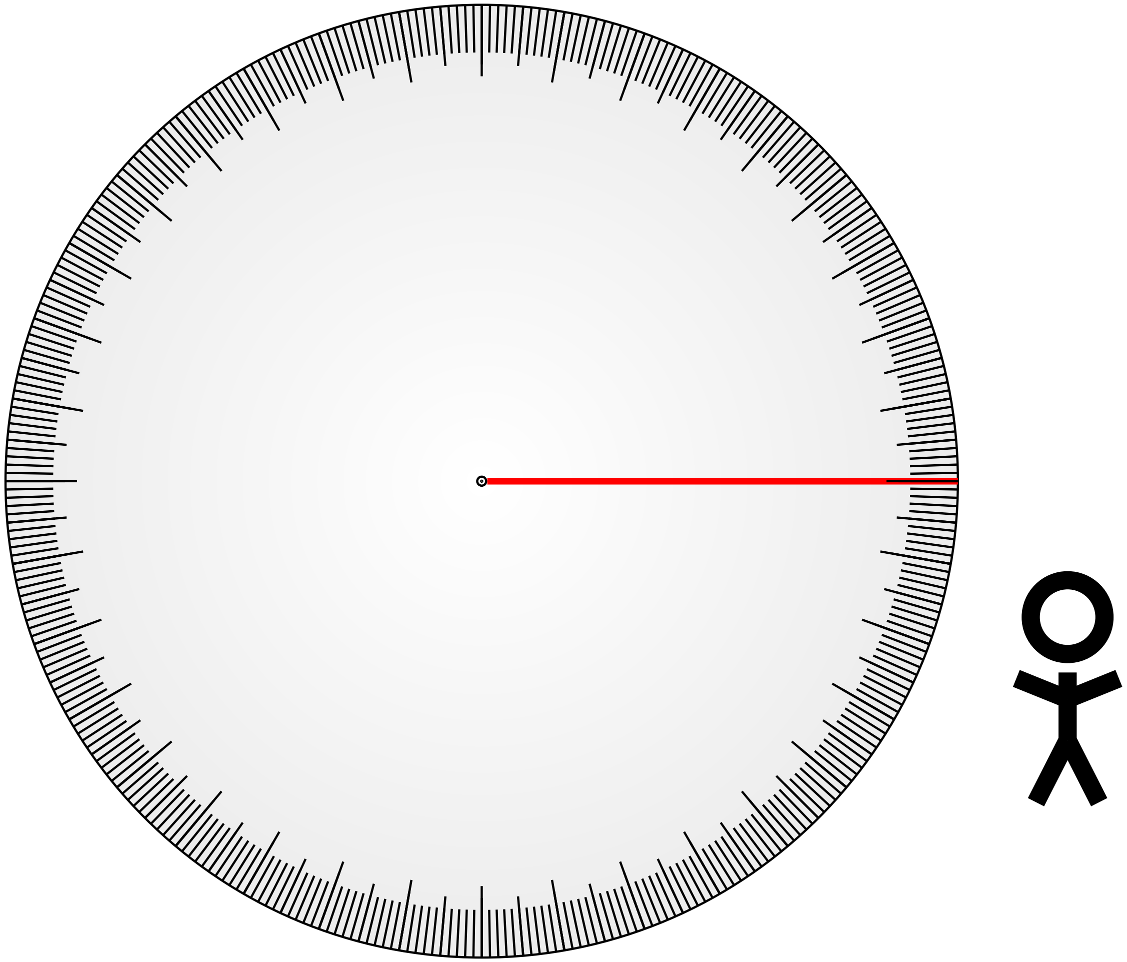
- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

THIRD time (faster)



- - - Signal Processing Basics - - -

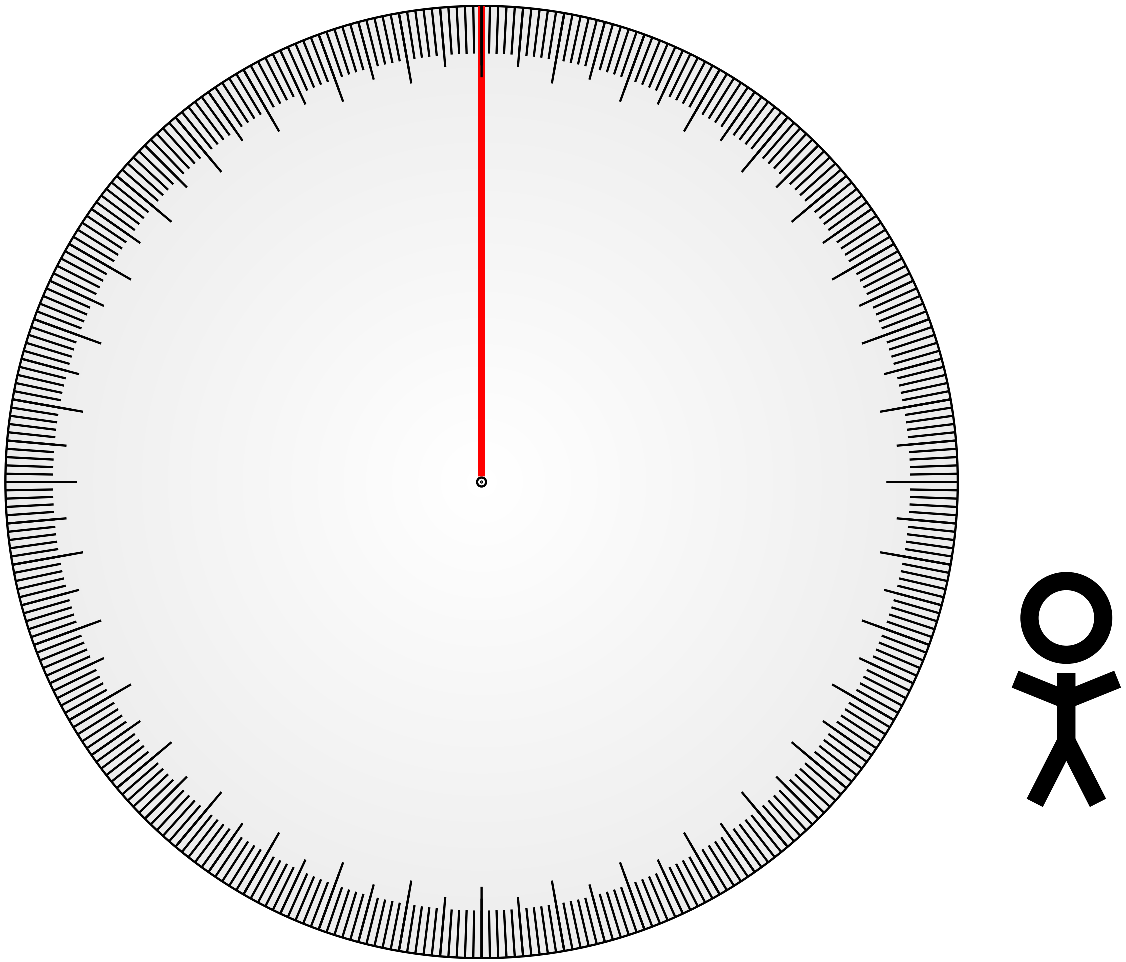
- **Aliasing: the car wheel example** (human eyes work as a sampling machine...) FOURTH time and so on... (faster)



- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

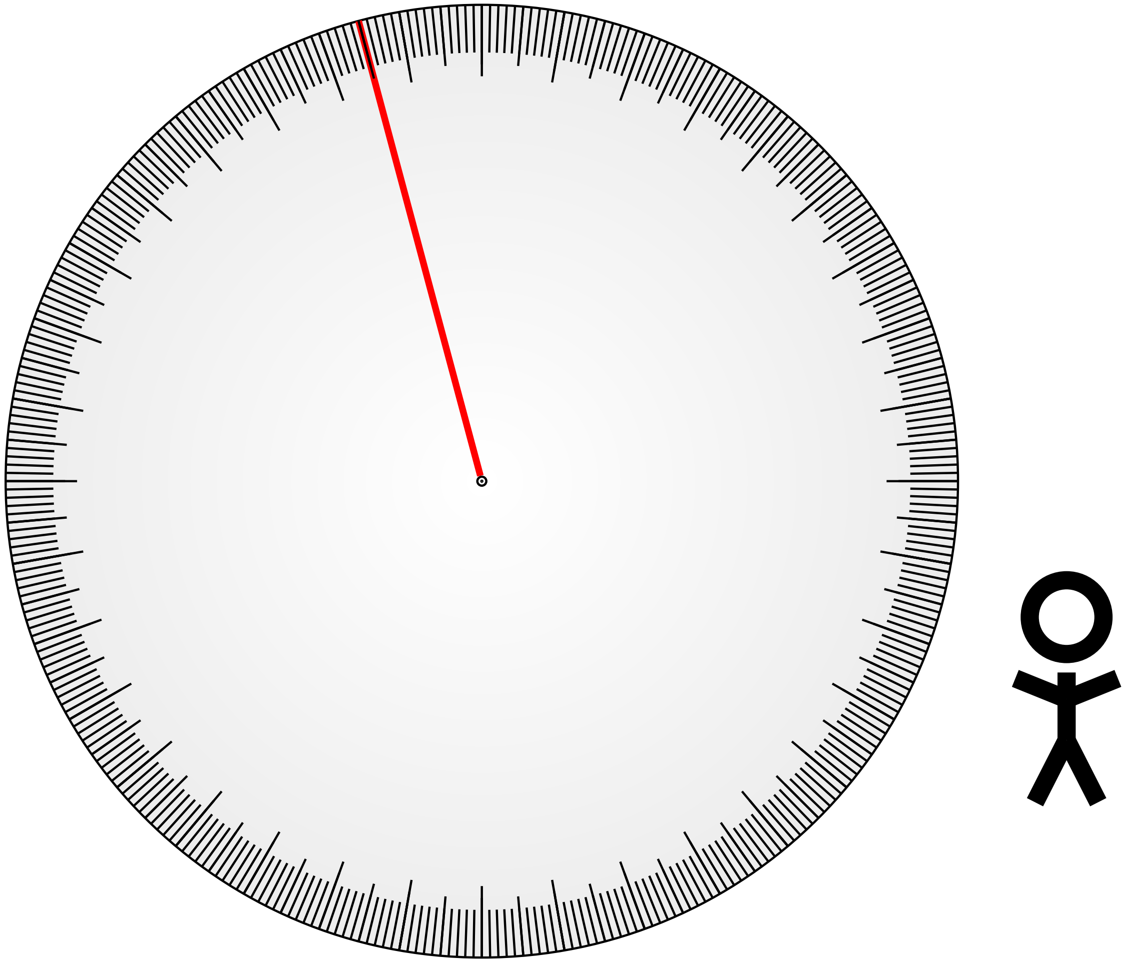
FIRST time (much faster)



- - - Signal Processing Basics - - -

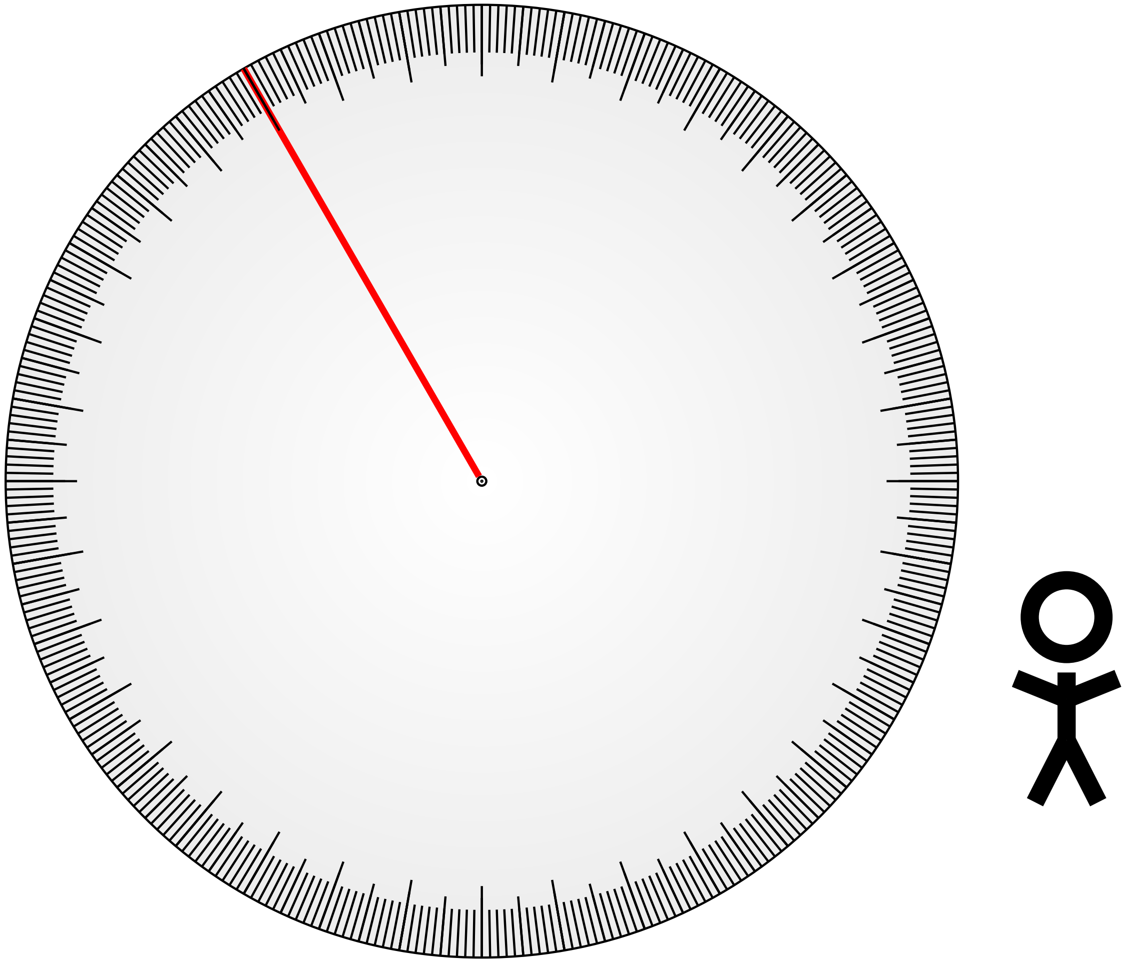
- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)

SECOND time (much faster)



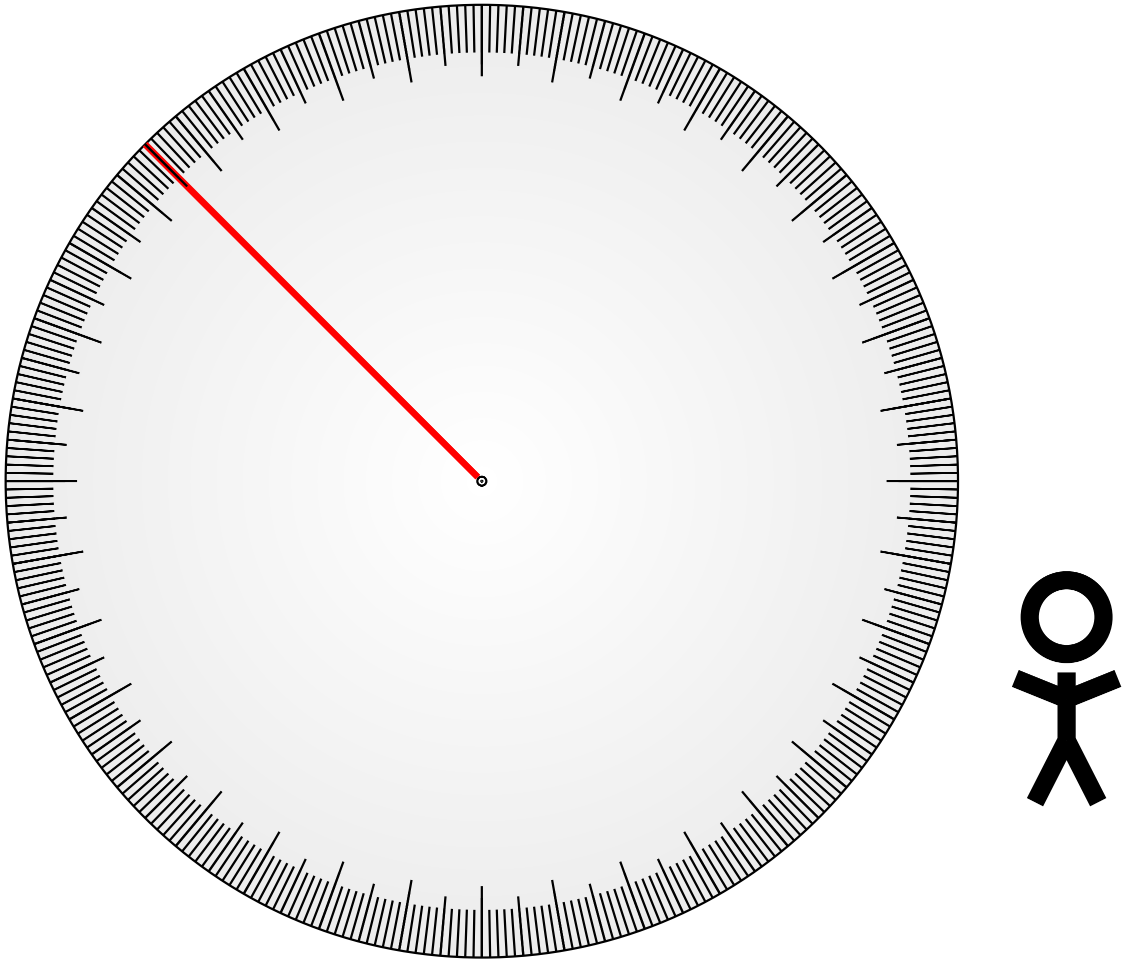
- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...) THIRD time (much faster)



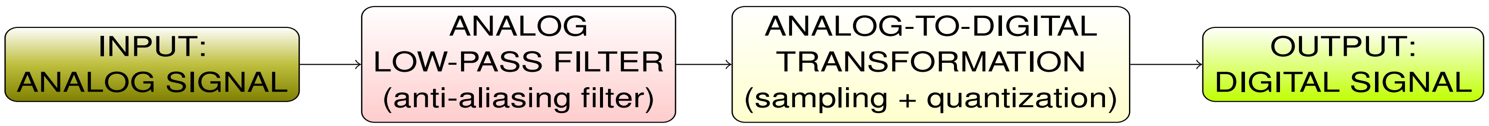
- - - Signal Processing Basics - - -

- **Aliasing: the car wheel example** (human eyes work as a sampling machine...)    FOURTH time and so on... (much faster)



- - - Signal Processing Basics - - -

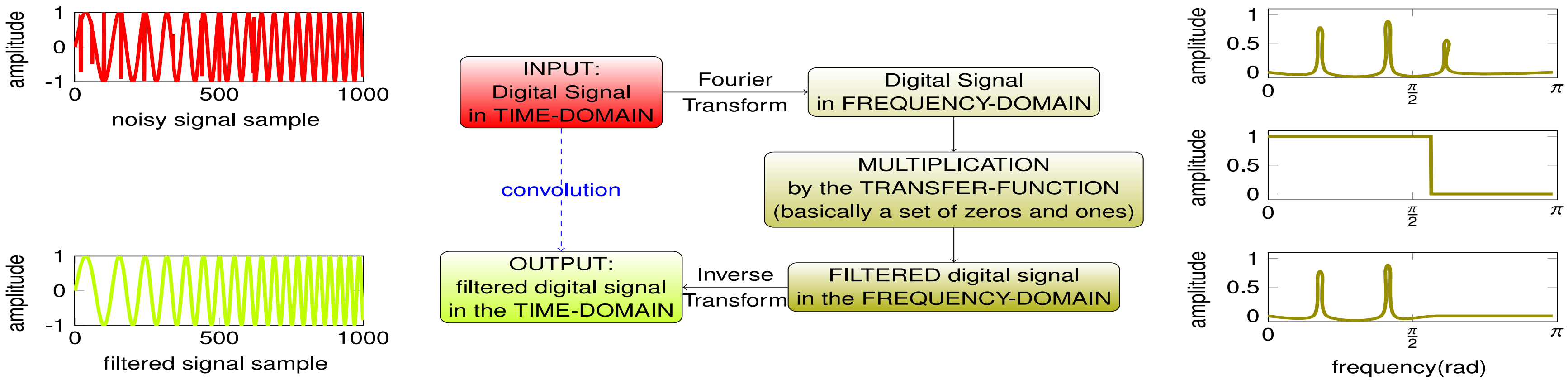
- **Reflection**: by using our WhatsApp group, discuss the concepts of convolution and sampling
- **Important Conclusion** [Sampling Theorem or Nyquist's Theorem]: a digital signal sampled at $X$ samples per second contains frequencies up to $\frac{X}{2}$ Hz coming from its analog version.
- The structure to convert an analog signal to its digital version is as follows:




- **Today's Short Test (ST2)**:
  1. define two discrete-time signals, i.e., $a[\cdot]$ and $b[\cdot]$, being the former two-sample long and the latter four-sample long. Then, obtain the resulting signal $y[\cdot] = a[\cdot] * b[\cdot]$.
  2. convolve $y[\cdot]$, just obtained, with itself.
  3. find a general equation to calculate the length of any resulting convolved signal from the lengths of both the input signals.

## - - - Signal Processing Basics - - -

- **Basics on Filtering**: frequency-domain versus time-domain




- Let $x[n]$ be the input time-domain signal in such a way that Fourier($x[n]$) = $X[\omega]$ is the frequency domain representation of $x[n]$;
- Let $H[\omega]$ be the frequency domain filter, that is basically a set of 0s and 1s;
- Let $Y[\omega]$ be the filtered signal in frequency domain;
  - Then, $Y[\omega] = X[\omega] \cdot H[\omega]$, i.e., the filtered signal in the frequency domain, is just the element-wise multiplication of the arrays $X[\omega]$ and $H[\omega]$;