

name: Solution

1 (10 points). Determine the truth of the following statements and give an explanation if true or counterexample if false. Assume that a and L are finite numbers.

(a) If $\lim_{x \rightarrow a} f(x) = L$, then $f(a) = L$.

(b) If $\lim_{x \rightarrow a^-} f(x) = L$, then $\lim_{x \rightarrow a^+} f(x) = L$.

(c) The limit $\lim_{x \rightarrow a} (f(x)/g(x))$ does not exist if $g(a) = 0$.

a) α false. Let $f(x) = \begin{cases} x, & x \neq 0 \\ 1, & x = 0 \end{cases}$. Then $\lim_{x \rightarrow 0} f(x) = 0$

but $\alpha f(0) = 1$.

b) α false. Let $f(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$ so $\lim_{x \rightarrow 0^-} f(x) = 0$

$\hat{\alpha}$ $\lim_{x \rightarrow 0^+} f(x) = 1$

c) α false. Let $\frac{f(x)}{g(x)} = \frac{x}{x}$ so $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 0$ but

$\lim_{x \rightarrow 0} g(x) = 0$.