

name: solution

1. For the following two equations, use implicit differentiation to find $\frac{dy}{dx}$. Then find the slope of the curve at the given point. (4 points each)

(a) $y^2 + 3x = 8$; $(1, \sqrt{5})$

(b) $(x+y)^{2/3} = y$; $(4, 4)$

$$\begin{aligned} \text{(a)} \quad \frac{d}{dx}(y^2 + 3x) &= \frac{d}{dx}(8) & @ (1, \sqrt{5}) \quad \frac{dy}{dx} &= -\frac{3}{2\sqrt{5}} \\ 2y \frac{dy}{dx} + 3 &= 0 \\ \frac{dy}{dx} &= -\frac{3}{2y} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{d}{dx}((x+y)^{2/3}) &= \frac{d}{dx}(y) & @ (4, 4) \\ \frac{2}{3}(x+y)^{-1/3} \frac{d}{dx}(x+y) &= \frac{dy}{dx} \\ \frac{2}{3}(x+y)^{-1/3} (1 + \frac{dy}{dx}) &= \frac{dy}{dx} \\ \frac{2}{3}(x+y)^{-1/3} &= \frac{dy}{dx} (1 - \frac{2}{3}(x+y)^{-1/3}) \\ \frac{dy}{dx} &= \frac{\frac{2}{3}(x+y)^{-1/3}}{1 - \frac{2}{3}(x+y)^{-1/3}} \\ & \text{but } \frac{2}{3}(4+4)^{-1/3} = \frac{2}{3\sqrt[3]{8}} = \frac{1}{3} \\ \text{so } \frac{dy}{dx} &= \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2} \end{aligned}$$