EXAM 2
MATH1117.07
2019-11-25

name: *Solution*

Total points = 38

1. Consider the function $f(x) = \dfrac{x-7}{x+5}$. *(4 points each)*

(a) Describe the intervals where f in increasing, where f is decreasing, and list the local extrema.

$$f'(x) = \frac{(x+5)-(x-7)}{(x+5)^2} = \frac{12}{(x+5)^2}$$

critical point @ $x=-5$.

-5

$f'(10) > 0$ | $f'(0) > 0$

- f is increasing on $(-\infty,-5)$ & $(-5,\infty)$.
- there are no local extrema

(b) Describe the intervals where f is concave up and those where f is concave down.

$$f''(x) = \frac{-36}{(x+5)^3}$$

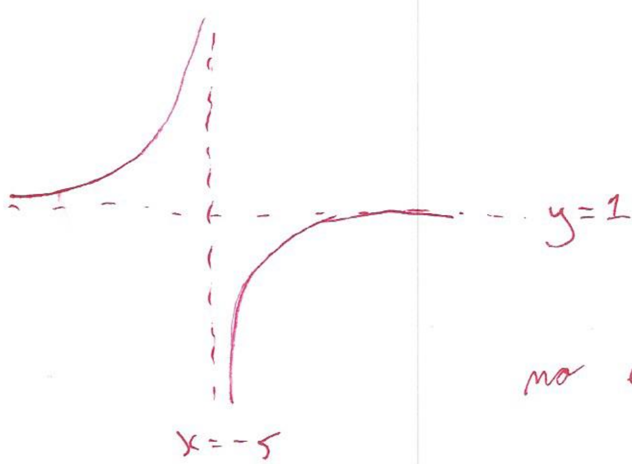
-5

$f''(-10) > 0$ | $f''(0) < 0$

f is concave up on $(-\infty,-5)$

f is concave down on $(-5,\infty)$

(c) Graph the function of f. Label all asymptotes and extrema.



no extrema.

2. Compute the derivatives for the following functions: *(4 points each)*

(a) $f(x) = \dfrac{3e^x - 3x}{2\sin(x) + x^2}$

$$f'(x) = \frac{(3e^x - 3)(2\sin(x) + x^2) - (3e^x - 3x)(2\cos(x) + 2x)}{(2\sin(x) + x^2)^2}$$

(b) $f(x) = \arcsin(\ln(x^2))$

$$f'(x) = \frac{1}{\sqrt{1 - (\ln(x^2))^2}} \cdot \frac{1}{x^2} \cdot 2x$$

3. Without finding the inverse, evaluate the derivative of the inverse of the function $f(x) = 2e^{3x}$ at the point $(2, 0)$. *(4 points)*

$(f^{-1})'(2) = \frac{1}{f'(0)}$ & $f'(0) = 6e^{3(0)} = 6$, so $(f^{-1})'(2) = \frac{1}{6}$

4. Which of the following are required to be true of a function f for the Mean Value Theorem to hold on an interval $[a, b]$: (fill in all that apply) *(2 points)*

- ● f in continuous on (a, b)
- ○ f has an inverse on (a, b)
- ● f is differentiable on $[a, b]$
- ○ $f(a) = f(b)$

For the function and interval

$$f(x) = x^2 + 3x + 5; \ [2, 4]$$

find the points guaranteed to exist by the Mean Value Theorem *(4 points)*

M.V.T. states $f'(x) = \frac{f(4) - f(2)}{4-2}$ has a solution on $(2,4)$.

$$2x + 3 = \frac{(4^2 + 3\cdot 4 + 5) - (2^2 + 3\cdot 2 + 5)}{4-2}$$

$$2x + 3 = \frac{1}{2}(16 + 12 + 5 - 4 - 6 - 5)$$

$$2x + 3 = \frac{18}{2}$$

$$x = 3.$$

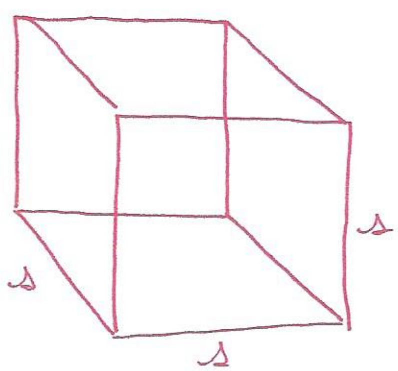
5. Solve $x^2y + 7\cos(y) = 2y^{2/3}$ for $\frac{dy}{dx}$. *(4 points)*

$$2xy + x^2 y' - 7\overset{\sin}{\cancel{\cos}}(y)\cdot y' = \frac{4}{3}y^{-1/3}\cdot y'$$

$$2xy = \frac{4}{3}y^{-1/3}y' + 7\sin(y)\cdot y' - x^2 y'$$

$$\frac{2xy}{\frac{4}{3}y^{-1/3} + 7\sin(y) - x^2} = y'$$

6. The sides of a cube decreases in length at a rate of 2 meters per second. At what rate is the volume of the cube changing when the sides are 4 metes long? *(4 points)*



$$V = s^3$$

$$\frac{dV}{dt} = 3s^2\frac{ds}{dt} \qquad \left(s = 4,\ \frac{ds}{dt} = -2\right)$$

$$= 3\cdot 4^2\cdot(-2)$$

$$= -96$$