name: Solution

exam 1

math1117.07

Fri, 11 Oct

- *justify all answers unless otherwise noted*
- *no notes, phones, calculators, or friends*
- *if you cheat, you will receive a zero on this exam*
- *there are no makeup exams*
- **good luck!**

1. (2 pts each) Let $f(x) = x^3 - 2$, $g(x) = x^4$, $h(x) = \frac{1}{3x}$. Evaluate the expressions

(a) $g \circ f(u)$

$g(f(u))$

$= g(x^3 - 2)$

$= (x^3 - 2)^4$

(b) $f \circ h(x)$

$f(h(x))$.

$= f(1/3x)$

$= \left(\frac{1}{3x}\right)^3 - 2$

(c) $g(h(f(x)))$

$g(h(f(x)))$

$= g(h(x^3 - 2)))$

$= g(1/2(x^3-2))$

$= \left(\frac{1}{2(x^3-2)}\right)^4$

2. (4 pts) Compute the inverse of $f(x) = e^{5x+7}$

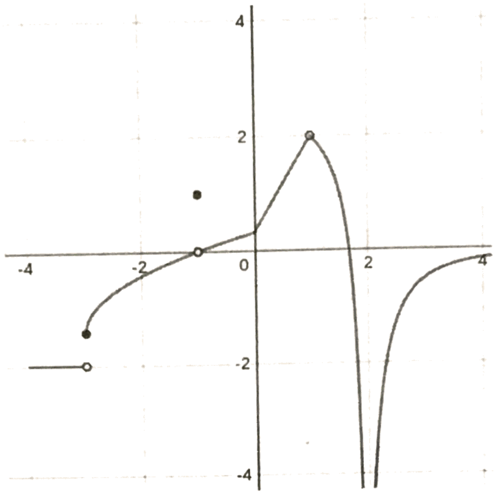
$y = e^{5x+7}$

$\ln(y) = \ln\left(e^{5x+7}\right) = (5x+7)\ln(e) = 5x + 7$

$\frac{\ln(y) - 7}{5} = x$

$f^{-1}(x) = \frac{\ln(x) - 7}{5}$

3. (2 pts each) Given this graph of a function f, compute the limits below



(a) $\lim\limits_{\cancel{x \to -2^-}} f(x)$ $x \to -3^-$

(b) $\lim\limits_{\cancel{x \to -2}} f(x)$ $x \to -3$

(c) $\lim\limits_{x \to -1} f(x)$

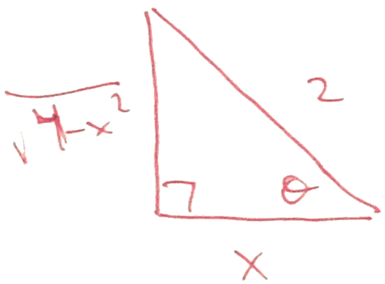
(d) $\lim\limits_{x \to 1} f(x)$

(e) $\lim\limits_{x \to 2} f(x)$

-2 DNE 0 2 $-\infty$

4. (4 pts) Simplify, i.e. find an algebraic expression, for $\sin(\arccos(\frac{x}{2}))$.

Let $\theta = \arccos(x/2)$, so $\cos(\theta) = x/2$.



$$\sin(\arccos(\tfrac{x}{2})) = \sin(\theta) = \frac{\sqrt{4-x^2}}{2}$$

5. (4 pts each) Respond to two of the three following items. If you answer more than two, specify which you would like me to grade, otherwise I will grade the first two.

(a) Write down the definition for the derivative of a function f.

(b) What is wrong with the following definition of a limit? What should it be?

> "Suppose the function f is defined at $x = a$. If $f(x)$ is arbitrarily close to L for all x sufficiently close to a, we write
> $$\lim_{x \to a} f(x) = L$$
> and say the limit of $f(x)$ as x approaches a equals L."

(c) If a function f is continuous at $x = a$, then what equation must, by definition, be true.

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

b) The sentence "... is defined at $x=a$" should read "... is defined near a"

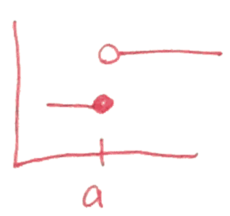
c) $f(a) = \lim_{x \to a} f(x)$

6. (4 pts each) Determine the truth of the following statements and give an explanation if true or counterexample if false. Assume that a and L are finite numbers.

(a) If $\lim_{x \to a^-} f(x) = L$, then $\lim_{x \to a^+} f(x) = L$.

(b) The limit $\lim_{x \to a} (f(x)/g(x))$ does not exist if $g(a) = 0$.

a) False. Consider the function whose graph is



b) False. Consider the function $\lim_{x \to 1} \frac{x-1}{x-1}$. The limit is 1 but $x-1 \to 0$.

You do not need to show your work for questions on this page

7. (2 pts) $\lim_{\theta \to \infty} \frac{\sin(\theta)}{\theta^3 - 3}$

(a) ∞ (b) $-\infty$ (c) 0 (d) $-1/3$ (e) None of the above

8. (2 pts) $\lim_{x \to \infty} \frac{x+4}{x-4} + \frac{x-3}{x^2-9}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 1 (e) None of the above

9. (2 pts) $\lim_{x \to \infty} x^2 + 4x - \cos(\sqrt{2x})$

(a) ∞ (b) $-\infty$ (c) 0 (d) 1 (e) None of the above

10. (2 pts) $\lim_{x \to \infty} \frac{24x^3 + 12x^2 - 4}{3x^4 + 9x^3}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 8 (e) None of the above

11. (2 pts) $\lim_{x \to 2} \frac{1}{(x-3)(x-2)^3}$

(a) ∞ (b) $-\infty$ (c) 0 (d) -1 (e) None of the above

12. (2 pts) $\lim_{x \to 1} \frac{x}{|x-1|}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 3 (e) None of the above

13. (4 pts) For a function f and a point a, suppose we know that

$$\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x).$$

What can we then conclude about $\lim_{x\to a} f(x)$

$\lim_{x\to a} f(x)$ does not exist

14. (4 pts) Is the function

$$f(x) = \begin{cases} 4x+4, & x \leq 2 \\ 3x^2, & x > 2 \end{cases}$$

continuous at $x = 2$? Justify your answer.

$f(2) = 4(2) + 4 = 12.$

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} 4x+4 = 4(2)+4 = 12 \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} 3x^2 = 3(2)^2 = 12 \end{aligned}\right\} \lim_{x\to 2} f(x) = 12$$

Since $f(2) = \lim_{x\to 2} f(x)$, the function is continuous at $x = 2$.

15. (4 pts) Use the definition of a derivative to find $g'(x)$ for $g(x) = 2x^2 + 3x$.

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h\to 0} \frac{\left(2(x+h)^2 + 3(x+h)\right) - (2x^2+3x)}{h}$$

$$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+3x+3h-2x^2-3x}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2+3h}{h} = \lim_{h\to 0} 4x+2h+3 = 4x+3$$

16. (4 pts) For g from the previous problem, what is the equation to the line tangent to g at $x = 2$?

point : $(2, g(2)) = (2, 2(2)^2 + 3(2))$

$= (2, 14)$

slope : $f'(2) = 4(2) + 3 = 11$

$$y - 14 = 11(x-2)$$

For the questions on this page, be sure to show all of your work.

17. (4 pts) Compute

$$\lim_{x\to e} \frac{(x-e)^{50} - x + e}{x - e}$$

$$\lim_{x\to e} \frac{(x-e)^{50} - (x-e)}{x-e} = \lim_{x\to e} \frac{(x-e)((x-e)^{49} - 1)}{(x-e)}$$

$$= \lim_{x\to e} (x-e)^{49} - 1 = (e-e)^{49} - 1 = -1$$

18. (4 pts) Compute

$$\lim_{x\to 36} \frac{\sqrt{x} - 6}{x - 36}$$

$$\lim_{x\to 36} \frac{\sqrt{x} - 6}{x - 36} = \lim_{x\to 36} \frac{\sqrt{x} - 6}{(\sqrt{x} - 6)(\sqrt{x} + 6)} = \lim_{x\to 36} \frac{1}{\sqrt{x} + 6}$$

$$= \frac{1}{\sqrt{36} + 6} = \frac{1}{12}$$