1. *(4 points each)* **This is a multi-part question.** Suppose that $P(A) = 0.6$, $P(B) = 0.5$, and $P(B|A) = 0.8$.

(a) Find the probability that A and B both occur.

(b) Draw a Venn diagram that explains this calculation.

a) $P(A \& B) = P(A) \cdot P(B|A)$

$= (0.6) \cdot (0.8)$

$= 0.48$

b)

2. *(2 points each)* **This is a multi-part question.** Find the proportion of observations from a standard Normal distribution for each of the following events:

(a) $Z = -1.43$

(b) $Z \geq 1.22$

(c) $-2.12 \leq Z < 0.23$

a) $P(Z = -1.43) = 0$

b) $P(Z \geq 1.22) = 1 - P(Z < 1.22)$

$= 1 - 0.8888$

$= 0.1112$

c) $P(-2.12 \leq Z < 0.23) = P(Z < 0.23) - P(Z < -2.12)$

$= 0.5910 - 0.0170$

$= 0.5740.$

3. *(4 points each)* **This is a multi-part question.** The SAT and ACT are two different tests that measure "college readiness". The distribution of SAT scores are approximately normal with a mean score of 1135 and standard deviation 210. The distribution of ACT scores are also approximately normal with a mean score of 23.5 and standard deviation 6.3. Answer the following two questions

(a) Maru scored 1355 on the SAT and Gary scored a 29 on the ACT. Whose has the higher score?

(b) What score would you need to get on the SAT to place you higher than 80% of all test takers? *(Note: Round as you wish, but state how you did.*

a)

Maru

$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{1355 - 1135}{210}$$

$$= 1.05$$

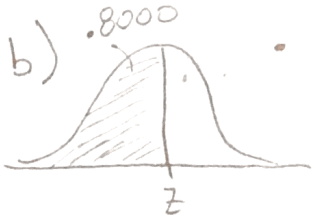
Gary

$$z = \frac{x - \mu}{\sigma} = \frac{29 - 23.5}{6.3}$$

$$= 0.87$$

Maru scored higher

b) .8000



- For what z is $P(Z < z) = 0.8000$?

$z = 0.84$.

- $0.84 = \frac{x - 1135}{210} \Rightarrow x = 1311.4$

A score of 1311.4 will place you higher than 80% of test takers.

4. *(6 points)* A random variable X has distribution

X	-5	-3	0	3	5
$P(X)$	0.15	0.20	0.40	0.05	0.20

Find the mean and standard deviation X.

$$\text{mean} = (-5)(.15) + (-3)(.20) + (0)(.40) + (3)(.05) + (5)(.20)$$

$$= -0.2$$

$$\text{standard deviation} = \Big[(-5-(-0.2))^2(.15) + (-3-(-0.2))^2(.20) + (0-(-.2))^2(.4) + (3-(-0.2))^2(.05) + (5-(-.2))^2(.20)\Big]^{1/2}$$

$$= 3.50$$

5. *(4 points each)* **This is a multi-part question.** In a recent survey, 24% of students admitted to cheating on a test. Of those surveyed, 15% are male and admitted to cheating. Assume that 47% of students are male.

(a) What is the probability that a male never cheated?

(b) If we choose, at random, a person who has cheated, what is the probability they are female?

	male	female	totals
cheat	(.15)	.09	(.24)
don't cheat	.32	.44	.76
totals	(.47)	.53	1

The circled numbers are known

a) $.47 - .15 = .32$

b) $P(\text{female} \mid \text{cheated}) = \frac{.09}{.24} = 0.375$

6. *(2 points each)* **This is a multi-part question.** Find the value z of a standard Normal variable Z that satisfies each of the following conditions:

(a) 87.7% of observations fall below z

(b) 82.89% of observations fall above z

a) .8770

$z = 1.16$

b) .8289

There's $1 - .8289 = 0.1711$ to the left of z, so

$z = -0.95$

7. *(2 points)* **This is a multiple choice question.** Which of the following is the standard deviation for the standard Normal distribution? Choose from the following:

(a) 0

(b) 1

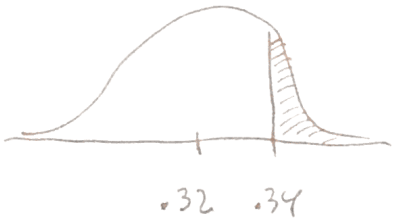
(c) $1/\sqrt{n}$ where n is the sample size

(d) something else

8. *(4 points)* A study showed the amount of time that a teenager can hold their breath is Normally distributed with a mean of 0.32 minutes and a standard deviation of 0.12 minutes. What is the probability that the mean duration of 100 teenagers holding their breath exceeds 0.34 minutes?

$\bar{X}$ = average time a sample of 100 teens can hold their breath

$P(\bar{X} > 0.34)$ $\qquad$ $P(Z > 1.67)$



$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$z = \frac{.34 - .32}{(.12/\sqrt{100})}$

$= 1.67$

$P(\bar{X} > 0.34) = P(Z > 1.67) = 1 - P(Z < 1.67)$

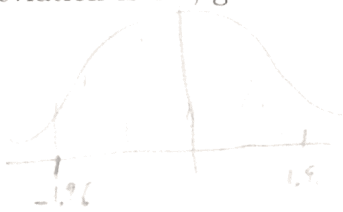
$= 1 - .9525 = .0475$

9. *(2 points)* **This is a multiple choice question.** Suppose you rolled a fair die 1000 times and never rolled a six. Which of the following is true of the 1001st roll?

(a) $P(\text{roll a } 6) < 1/6$ *(b)* $P(\text{roll a } 6) = 1/6$ *(c)* $P(\text{roll a } 6) > 1/6$ Justify your answer.

Rolling a die is independent from previous rolls.

10. *(4 points)* A medical student studying pain collected data on 50 hospital patients who were asked to rate their pain on a scale of 1–10, with 10 indicating extreme pain. The student found the mean of the sample was a rating of 6.8. Assuming the population standard deviation is 1.7, give the 95% confidence interval for the population mean pain rating.



Confidence Interval: $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

where $\bar{x} = 6.8$, $z^* = 1.96$, $\sigma = 1.7$, $n = 50$.

Plugging these values in gives

$$6.8 \pm (1.96)\frac{1.7}{\sqrt{50}}.$$

This corresponds to the interval

$$(6.34, 7.27)$$