name: Solution

exam 1

math1117.02

Fri, 11 Oct

- *justify all answers unless otherwise noted*
- *no notes, phones, calculators, or friends*
- *if you cheat, you will receive a zero on this exam*
- *there are no makeup exams*
- **good luck!**

1. **(2 pts each)** Let $f(x) = x^2 - 1$, $g(x) = x^5$, $h(x) = 1/x$. Evaluate the expressions

(a) $g \circ f(u)$

$g(f(u))$

$= g(u^2 - 1)$

$= (u^2 - 1)^5$

(b) $f \circ h(x)$

$f(h(x))$

$= f(1/x)$

$= \left(\frac{1}{x}\right)^2 - 1$

(c) $g(h(f(x)))$

$g(h(x^2 - 1))$

$= g(1/x^2 - 1)$

$= \left(\frac{1}{x^2 - 1}\right)^5$

2. **(4 pts)** Compute the inverse of $f(x) = e^{3x+8}$

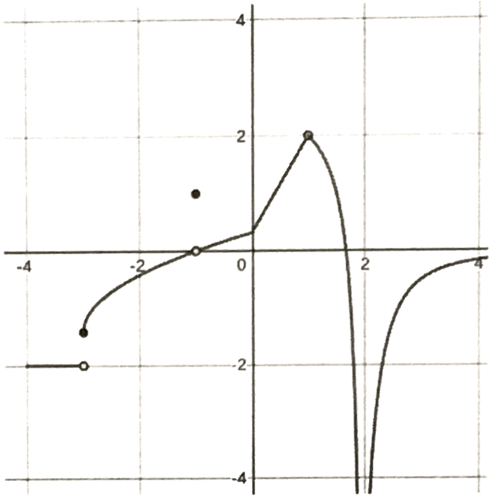
$y = e^{3x+8}$

- $\ln(y) = \ln(e^{3x+8}) = (3x+8) \ln(e) = 3x + 8$

- $\frac{\ln(y) - 8}{3} = x$

- $f^{-1}(x) = \frac{\ln(x) - 8}{3}$

3. (2 pts each) Given this graph of a function f, compute the limits below

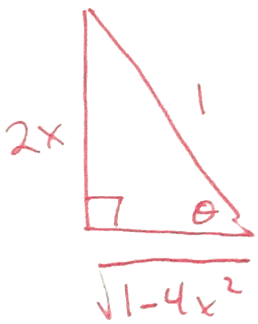


(a) $\lim_{\cancel{x\to -2^-}} f(x)$ $x \to -3^-$ (b) $\lim_{\cancel{x\to -2}} f(x)$ $x \to -3$ (c) $\lim_{x\to -1} f(x)$ (d) $\lim_{x\to 1} f(x)$ (e) $\lim_{x\to 2} f(x)$

-2 DNE 0 2 $-\infty$/DNE

4. (4 pts) Simplify, i.e. find an algebraic expression, for $\cos(\arcsin(2x))$

Let $\theta = \arcsin(2x)$. Then $\sin(\theta) = 2x$.



$\cos(\arcsin(2x)) = \cos(\theta) = \sqrt{1-4x^2}$

5. **(4 pts each)** Respond to two of the three following items. If you answer more than two, specify which you would like me to grade, otherwise I will grade the first two.

(a) Write down the definition for the derivative of a function f.

(b) What is wrong with the following definition of a limit? What should it be?

> "Suppose the function f is defined at $x = a$. If $f(x)$ is arbitrarily close to L for all x sufficiently close to a, we write
> $$\lim_{x \to a} f(x) = L$$
> and say the limit of $f(x)$ as x approaches a equals L."

(c) If a function f is continuous at $x = a$, then what equation must, by definition, be true.

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

b) The first sentence, "... defined at $x=a$", should read "... defined near $x=a$".

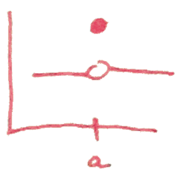
c) $f(a) = \lim_{x \to a} f(x)$.

6. **(4 pts each)** Determine the truth of the following statements and give an explanation if true or counterexample if false. Assume that a and L are finite numbers.

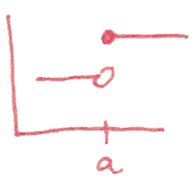
(a) If $\lim_{x \to a} f(x) = L$, then $f(a) = L$.

(b) If $\lim_{x \to a^-} f(x) = L$, then $\lim_{x \to a^+} f(x) = L$.

a) False. Consider the function whose graph is



b) False. Consider the function whose graph is



You do not need to show your work for questions on this page

7. (2 pts) $\lim_{\theta \to \infty} \dfrac{\sin(\theta)}{\theta^2 + 5}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 1/5 (e) None of the above

8. (2 pts) $\lim_{x \to -\infty} \dfrac{x-1}{x+1} + \dfrac{x-2}{x^2-2}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 1 (e) None of the above

9. (2 pts) $\lim_{x \to \infty} -2x^7 + x^2 - \sin(\sqrt{x})$

(a) ∞ (b) $-\infty$ (c) -2 (d) 0 (e) None of the above

10. (2 pts) $\lim_{x \to \infty} \dfrac{15x^3 + 2x^2 - 7}{3x^4 + 3x^3}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 5 (e) None of the above

11. (2 pts) $\lim_{x \to 5} \dfrac{1}{(x-1)(x-5)^3}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 1/4 (e) None of the above

12. (2 pts) $\lim_{x \to -1} \dfrac{x}{|x+1|}$

(a) ∞ (b) $-\infty$ (c) 0 (d) 3 (e) None of the above

13. (4 pts) For a function f and a point a, suppose we know that

$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x).$$

What can we then conclude about $\lim_{x \to a} f(x)$

$\lim_{x \to a} f(x)$ does not exist

14. (4 pts) Is the function

$$f(x) = \begin{cases} x+2, & x \leq 2 \\ x^2, & x > 2 \end{cases}$$

continuous at $x = 2$? Justify your answer.

- $f(2) = 2 + 2 = 4$
- $\left.\begin{array}{l} \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x+2 = 2+2 = 4 \\ \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 = (2)^2 = 4 \end{array}\right\} \lim_{x \to 2} f(x) = 4$

b/c $f(2) = \lim_{x \to 2} f(x)$, the function is continuous at $x = 2$.

15. (4 pts) Use the definition of a derivative to find $g'(x)$ for $g(x) = x^2 + 2x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left((x+h)^2 + 2(x+h)\right) - (x^2 + 2x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h} = \lim_{h \to 0} 2x + h + 2$$

$$= 2x + 2$$

16. (4 pts) For g from the previous problem, what is the equation to the line tangent to g at $x = 2$?

point: $(2, f(2)) = \left(2, (2)^2 + 2(2)\right) = (2, 8)$

slope: $f'(2) = 2(2) + 2 = 6$

$$y - 8 = 6(x - 2).$$

For the questions on this page, be sure to show all of your work.

17. (4 pts) Compute

$$\lim_{x \to \pi} \frac{(x-\pi)^{50} - x + \pi}{x - \pi}$$

$$\lim_{x \to \pi} \frac{(x-\pi)^{50} - (x-\pi)}{x-\pi} = \lim_{x \to \pi} \frac{(x-\pi)((x-\pi)^{49} - 1)}{(x-\pi)}$$

$$= \lim_{x \to \pi} (x-\pi)^{49} - 1 = (\pi-\pi)^{49} - 1 = -1$$

18. (4 pts) Compute

$$\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$$

$$\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25} = \lim_{x \to 25} \frac{(\sqrt{x}-5)}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$= \lim_{x \to 25} \frac{1}{\sqrt{x}+5}$$

$$= \frac{1}{\sqrt{25}+5} = \frac{1}{10}$$