

name: Solution

1 (10 points). Use limits to find the derivative to the function

$$f(x) = \frac{x}{x+2}$$

and evaluate $f'(-1)$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{x+2}{x+2} \left(\frac{x+h}{x+h+2} \right) - \left(\frac{x+h+2}{x+h+2} \right) \left(\frac{x}{x+2} \right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+2)(x+h) - (x+h+2)(x)}{h(x+2)(x+h+2)} = \lim_{h \rightarrow 0} \frac{x^2 + xh + 2x + 2h - x^2 - xh - 2x - 2h}{h(x+2)(x+h+2)} \\ &= \lim_{h \rightarrow 0} \frac{2h}{h(x+2)(x+h+2)} = \lim_{h \rightarrow 0} \frac{2}{(x+2)(x+h+2)} = \frac{2}{(x+2)^2} \end{aligned}$$

$$f'(-1) = \frac{2}{(-1+2)^2} = \frac{2}{(1)^2} = 2.$$