

SECTION REPORTS

MATH 12 // STATISTICS // SPRING 2019

OVERVIEW

This document contains all of the problems that your group must complete through the duration of this class. Submit these problems in “Section Reports”. The content of each section report is detailed below.

After you hand in a section report, you may move onto the next. In the meantime, I will provide comments on your previous section report. You are then asked to rewrite the section report, addressing my comments. You may then hand in the next draft. This process is repeated until I mark it satisfactory. After all section reports in a chapter are marked satisfactory, you may submit section reports for the next chapter.

Each section report should be professional, neat, clear, and written in complete sentences as if you were writing a lab report. You may handwrite the section reports, but if your writing is not neat, at least ensure that the formatting is clean.

An example section report is posted at

<https://danielmichaelcicala.github.io/teaching/example-section-report.pdf>

Note: Many of the statistics used in this document are fabricated for the purpose of teaching statistical concepts.

CHAPTER 1

Chapter 1, Section 1.

- Define a *population* and a *sample*. What is the difference between the two? Create your own example to illustrate the difference.
- Describe the difference between descriptive statistics and inferential statistics. Invent three examples of a situation in which one would use descriptive statistics and another three in which one would use inferential. Justify each example.
- A (fake) study conducted at Norco Community College determined that student who attended class between 90% and 100% of the time usually received A's in the class. Many students who attended between 75% and 90% of the time received a B in the class. Many students who attended less than 75% of classes failed.
 - (a) What *variables* were considered under this study?
 - (b) What are the *data* in this study?
 - (c) Which kind of statistics was used: descriptive or inferential?
 - (d) What is the *population* under study?
 - (e) Was a *sample* collected? If so, from where?
 - (f) Based on the information given, comment on the relationship between the two variables.

Chapter 1, Section 2.

- What is the difference between a *quantitative* variable and *qualitative* variable? Create three examples of each type and justify why your example is correct.
- What is the difference between *discrete* variables and *continuous* variables? Create three examples of each type and justify why your example is correct.
- Imagine you are at the doctors and the nurse is taking your weight. As you step onto the scale, you notice that it only gives weights in hundredths of a pound. For example, the scale will round a weight of 155.246 up to 155.25 and a weight of 123.423 down to 123.42. Is weight a discrete variable? Justify your answer.
- All measuring devices necessarily round. Otherwise, they'd have to display infinitely many decimal places, which is absurd. Can we ever actually determine a piece of continuous data exactly? What is the *boundary* of a number? If I had a thermometer round temperatures to one decimal point, what is the boundary of 34.3°C? If I had a watch that measured seconds, but not fractions of a second, what is the boundary of 45 seconds?
- There are four levels of measurement described in this section. What are they called? Which level would letter grades given in a class fall under? IQ? Weight? Model of car? Describe what is meant by a *true zero*.

Chapter 1, Section 3.

- Define six sampling methods. Create a situation in which each method would be used. Give a negative aspect for each method and describe why it is negative.
- What is a *sampling error*? Can we always determine a sampling error exactly? Why or why not? Give an example of when we can determine a sampling error exactly.
- Define the population considered in the following situations
 - (a) Monmouth University reported that 57% of people with a college degree are homeowners.
 - (b) CNN found that 85% of federal inmates are serving less than 10 years
 - (c) Eating eggs in the morning may contribute to higher cholesterol
 - (d) Indoor cats live, on average, three years longer than outdoor cats.
- Identify the sampling method used
 - (a) The Dean of RCC wants feedback from faculty. So the Dean randomly selects 30 teachers to interview.
 - (b) Between 12:30pm and 12:45pm, every cashier at Target asks their customers whether they live in Riverside County.
 - (c) Every 100th Hersey Kiss made at the factory is weighed to ensure accuracy of the machine.
 - (d) To ensure student satisfaction, the principal at a local high school surveys ten freshmen, ten sophomores, 15 juniors, and 15 seniors about their experience.

CHAPTER 2

Chapter 2, Section 1.

- Write down each name in your group, first and last. Place all of the letters used in your names (count repetitions) into a categorical frequency distribution.
- Choose any sports team. Look up the roster and record the number of every player. Make two grouped frequency distributions for these numbers. One with class width of 5 and another with class width of 10.
- What is the point of class boundaries? How are they different from class limits?
- Look up the average high temperature (in Fahrenheit) of each capital city in North and South America in January of any year. Place the temperatures into a cumulative frequency distribution. Choose your own class width, and describe why you choose that for the class width. Are there any outliers? If yes, justify why you think it's an outlier. If not, say why not.
- List five reasons for constructing frequency distributions

Chapter 2, Section 2.

- Look up the total number of points scored for every Super Bowl (that is, add the points that each team scored). Construct a histogram with class boundaries of your choice. Describe why you chose the class boundaries as you did. Why do you think the shape of the histogram is like it is?
- Look up the age that every Canadian Prime Minister took office. Construct a frequency polygone of the ages.
- Construct two ogives. One for the total number of points scored every Super Bowl and another for the age that every Canadian Prime Minister took office. Compare and contrast how the ogives look relative to the histogram and to the frequency polygone.

Chapter 2, Section 3.

- Define a time series graph and a pie graph. Invent your own example of a situation in which to use a time series graph and another for a pie graph. You can be general.
- Suppose there are 250 people in a movie theater. Of them, 25 are wearing red shirts, 57 are wearing blue shirts, 71 are wearing black shirts, 61 are wearing orange shirts, and 36 are wearing beige shirts. What graph is captures this information better: time series or pie? Construct that graph. *(If you answer a pie graph, be sure to compute the necessary percentages)*
- Carla is using a FitBit to keep track of her steps. She wants to graph her steps each day of the previous week. Which graph is better suited for this purpose: time series or pie? Use your answer to graph the data

Day	Steps
Monday	7,514
Tuesday	8,385
Wednesday	10,014
Thursday	4,598
Friday	12,345

(If you answer a pie graph, be sure to compute the necessary percentages)

CHAPTER 3

Chapter 3, Section 1.

- There are two different formulas to compute a mean. We use the notation \bar{X} to refer to the mean of a *sample*, which is given by the formula

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{\sum X}{n}.$$

We use the notation μ to refer to the mean of a *population*, which is given by the formula

$$\mu = \frac{X_1 + X_2 + \cdots + X_N}{N} = \frac{\sum X}{N}.$$

You can find these in the text. Explain the meaning of the following notations

- (a) X
- (b) $\sum X$
- (c) n
- (d) N
- (e) $X_1, X_2, \dots, X_n, X_N$

What are the difference between these two equations?

- Suppose we are given a bunch of data. If we are told nothing about its source, do we assume it comes from a sample or population? *Hint: the answer to this is contained in a single sentence in the textbook.*
- In your section report for Chapter 2 Section 1, you wrote down the average high temperatures for national capital cities in North and South America. Write all of those same numbers in a table (don't write the city names, just the numbers in any order). What is the average of these temperatures? You don't need to write down the entire equation. Instead describe in a sentence *how* you computed the average. What value is n ? What is the meaning of n in this example?
- Go to the website

<https://weathlygorilla.com/richest-singers-world/>

and make a table containing the net worth of the singers (there's no need to record the names of the singers). From what population are the singers a sample (there are multiple correct answers).

- (a) Is this data discrete or continuous?
- (b) What is the average net worth of the singers? Do you think this average gives a fair representation of the population? Why or why not?
- Place the same dollar amounts into a (grouped) frequency distribution with 10 classes.
 - (a) Based on your answer above, should your frequency distribution contain a column for class limits? Why or why not?
 - (b) What is the mean of this grouped data?
 - (c) Is this the same as the mean of the net worths above? Why or why not?
- Next, write down any 30 numbers between 10,000 and 100,000. Think of these as the net worth of non-famous singers. Combine into a single data set these 30 numbers with the 20 net worths above. What is the mean

of this data set? What is the median? Which is more representative of a person living in the US? Why?

- Define the **mode** of a data set. Create your own example data set (make it have at least 10 numbers) and tell me the mode.
- Clara only eats four foods for dinner: chicken, peanut butter sandwich, spaghetti, or hamburger. She recorded what dinners she ate in a full year: 103 chicken dinners, 49 peanut butter sandwich dinners, 154 spaghetti dinners, and 59 pizza dinners. What is the mode of Clara's dinners? Write another way, using everyday language, of asking "what's the mode of Clara's dinners"?
- Explain in words how GPA is an example of a weighted average. Compute the GPA of a student who receives the following grades

CLASS	CREDITS	GRADE
Stats	3	B+
English	3	A
Music Theory	2	B
History	4	B

Use the standard US system, which is given in the following table

GRADE	SCORE
A	4.0
B+	3.5
B	3.0
C+	2.5
C	2.0
D	1.0
F	0.0

Next, compute the GPA for the same student using the Canadian system:

GRADE	SCORE
A	10.0
B+	9.0
B	8.0
C+	7.0
C	6.0
D	5.0
F	0.0

Chapter 3, Section 2. In the previous section, we introduced measures that try to find the "center" of a data set. Just like any physical object—a plate, a table, whatever—has a center where you can balance it on a super strong finger tip, data has a center too. But since data is more complicated than physical objects, there are different kinds of centers: the mean, the median, the mode.

In this section, we learn different ways to measure how spread out a data set is. These three ways are range, variance, and standard deviation.

- Describe in plain English the terms range, variance, and standard deviation. How are variance and standard deviation related?
- The variance and standard deviation each have two versions: one version for the population and another version for a sample. What notation do we use

for population variance, population standard deviation, sample variance, and sample standard deviation?

- (c) Write down the formula to compute population variance and population standard deviation. Write down two different formulas for sample variance and also for sample standard deviation. One of these two formulas is considered a “short cut”. Why is it shorter than the other?
- (d) Go to the website

<https://www.random.org/integers/>

Generate at least 20 numbers with values between 1 and 250. Write them down. Then find the range, population variance, and population standard deviation for that set of numbers. For the variance and standard deviation, follow the procedure table in the textbook.

- (e) Pick ten of those numbers and compute the sample variance and standard deviation. Again, follow the procedure table.
- (f) Take those 20 numbers from above. Place them into a *grouped* frequency distribution with class width 25. Compute the standard deviation for the grouped frequency distribution. Be sure to follow the procedure table.
- (g) What is the formula for the *coefficient of variation*? What is this statistic (the coefficient of variation) good for? Compute the coefficient of variation for the 20 random integers you used above.
- (h) Write down Chebyshev’s theorem verbatim. Translate this into plain English for when $k=2$. Repeat for $k=3$.
- (i) Here’s a scenario. The mean price of homes in a neighborhood is \$100,000. The standard deviation is \$20,000. Find the price range for which at least 75% of the homes will sell.

Chapter 3, Section 3. So far we learned how to measure the center of a data set and how far away a data point is from the data set. In this section, we learn about another statistic that will help us describe the location of a data point in a data set. Like we can use the yard marker to describe where someone is standing on a football field or to use longitude and latitude to describe a location on the earth, we can use percentiles or the z-score to describe where a data point is in a data set.

- (a) What is the formula used to compute a z-score? Write it in two ways: one with words and the other with symbols. Learn this well. It will be used later in the class.
- (b) What is the point of the z-score?
- (c) Here’s a list of numbers

24 16 52 34 18 74 12 65 31 46

Compute the z-score of 31.

- (d) Here’s another list of numbers

26 26 42 37 28 84 2 35 43 51

Compute the z-score of 37.

- (e) In the context of the two data sets above, which number is higher, 31 or 37? Explain why.
- (f) The first percentile of the integers 0 through 100 is marked by 1. The second percentile of the integers 0 through 1000 is marked by the number 20. What number marks the third percentile of the integers 0 through 10,000?

- (g) One way to measure the quality of health care in a country is by the infant mortality rate. One needs to take care when comparing the infant mortality rate of different countries, because each country measures it in different ways. However, let's compare them anyway. Go to the website

https://en.wikipedia.org/wiki/List_of_countries_by_infant_and_under-five_mortality_rates

Using data on the 35 countries listed in the section titled "Under-five mortality from the World Bank", find the percentile of the United States. Is it an outlier? Why or why not? Use the textbook definition of "outlier" in your explanation.

CHAPTER 4

Chapter 4, Section 1.

- (a) Define the following terms: probability experiment, outcome, and sample space. Given an example of a probability experiment and describe its sample space.
- (b) What has a larger sample space, rolling one die or rolling two dice? Justify your answer.
- (c) Suppose I am playing a casino game with three rounds.
- Round 1: Flip a coin.
 - Round 2: If in round 1, you flipped heads, roll a die. If you flipped tails, flip again.
 - Round 3: If you rolled a die in round 2 and got an even number, roll the die again. If you rolled a die in round 2 and got an odd number, flip the coin. If you flipped heads in round 2, roll a die. If you flipped tails in round 2, flip again.
- Draw a tree diagram representing the sample space for this casino game.
- (d) For the casino game above, is the *event* consisting of the outcome (H,2,3) a simple or compound event? How about the event (T,H,4),(H,3,T),(H,4,1)?
- (e) What is the probability of the event (H,2,3) occurring? The probability for the event (T,H,4),(H,3,T),(H,4,1)?
- (f) Draw a card from an ordinary deck of cards. Compute the probability of drawing
- a heart
 - a red card
 - the 3 of spades
 - an even numbered card.
- (g) Define the complement of an event. If we denote an event by E , then how do we denote the complement of E ? For the casino game above, what is \overline{E} if E is the event

$$E = \{(T, H, 4), (H, 3, T), (H, 4, 1)\}?$$

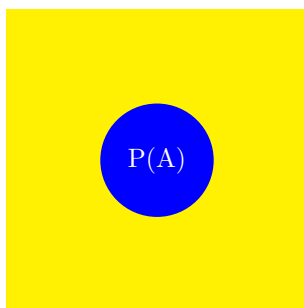
What is $P(\overline{E})$? Compute $P(E) + P(\overline{E})$? Explain why your answer makes sense. Are you using classical or empirical probability in this case?

- (h) In my class, 13 students received an A, 25 received a B, 20 received a C, 9 received a D, and 5 failed. What is the probability that a randomly chosen student received a B? Did you use classical or empirical probability?

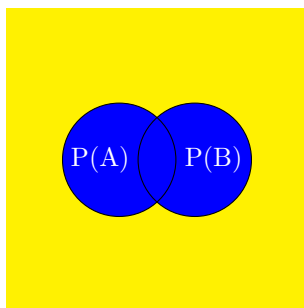
- (i) Think of the law of large numbers. What would you expect to be a more unusual circumstance
- tossing a coin 30 times and getting 28 heads, or
 - tossing a coin 1000 times and getting 990 heads?
- Explain why.

Chapter 4, Section 2.

- (a) Explain the concept of mutually exclusive events. Provide two examples of a pair of events that *are* mutually exclusive and two more examples of a pair of events that *are not* mutually exclusive. Explain why each example is or is not mutually exclusive.
- (b) It is helpful to visualize probabilities using Venn diagrams.



Because the biggest probability is 1, think of the yellow square as having area 1. The circle labeled $P(A)$ refers to an event called 'A' and the area of the circle represents the probability of the event A occurring. If there is another event 'B', then we add another circle to the Venn diagram



What do you think the overlap of the circles represents? Using technical statistics vocabulary, what is the meaning if two circles in a Venn diagram **do not** overlap?

- (c) Write down two “addition rules”: one for adding probabilities of mutually exclusive events and another for adding probabilities of non-mutually exclusive events. Draw two Venn diagrams with two circles each: one representing mutually exclusive events and another that represents non-mutually exclusive events. Explain how the addition rules correspond to their Venn diagram. In particular, explain in why the non-mutually exclusive addition rule has the “ $-P(A \text{ and } B)$ ” term.
- (d) For each of the following, say if the situation describes mutually exclusive or non-mutually exclusive events. Also, compute the probability when asked

- Drawing a face card or a red card from a deck of playing cards. What is the probability of this?
 - From all the musicians with at least one album out since 2010, choosing a musician who won an Oscar or won a Grammy? Don't actually compute the probability, but describe how you'd go about it if you were to, from finding the necessary data to the computations you'd make.
 - Rolling a single die and getting a 4 or a 6. What is the probability of this?
- (e) Draw a Venn diagram with for three events, which we'll call events A , B , and C . Assume that A and B **are** mutually exclusive, A and C **are not** mutually exclusive, and B and C **are not** mutually exclusive. Using intuition developed from part (2), try to write down an equation for what $P(A \text{ or } B \text{ or } C)$ is equal to *for these specific A, B , and C* ?

Chapter 4, Section 3.

- (a) What does it mean that two events are “independent”? Give 4 examples of pairs of events that are independent. Give a brief explanation of why each pair is independent.
- (b) What is the multiplication rule to determine the probability of two independent events both occurring. Write the equation in mathematical notation. Also write an English sentence (with no notation) that describes the equation.
- (c) What does it mean for two events to be “dependent”? Given 4 examples of pairs of events that are dependent. Give a brief explanation of why each pair is dependent.
- (d) Let A and B be two probability events. Explain the notation $P(B|A)$. If A and B are independent, correctly fill in “?” for the equation $P(B|A) = P(?)$. Do not write $? = B|A$. Explain your answer.
- (e) What is the multiplication rule to find the probability of two dependent events both occurring. Write the equation in mathematical notation. Also write an English sentence (with no notation) that describes the equation.
- (f) For each of the following, state whether the given events are independent or dependent. Then calculate the probability of both events occurring.
- A = “flip heads on a coin” and B = “roll a 2 on a die”.
 - There is a bucket filled with four green balls, seven blue balls, and seven red balls. We draw two balls from the bucket. The second draw is done without replacing the first ball. The first draw A = “draw a green ball” and the second draw B = “draw a red ball”.
 - Draw two cards from a deck of cards and after the first draw, replace the card and shuffle the deck before drawing the second card. The first draw is A = “draw a red card” and the second draw is B = “draw the king of spades”.
 - If you steal a car, there is a 85% chance of getting caught (this is a made up stat). Suppose that Jeff in Indianapolis and Cory in Minnesota both steal a car. A = “Jeff gets caught” and B = “Cory gets caught”.
 - We will draw two cards from a deck of cards. After we draw the first card, we do not replace that card before drawing the second card. The

first draw is A = “draw a king” and the second draw is B = “draw another king”.

- (g) What is conditional probability? Describe it. What is a formula for conditional probability? Which formula looks like the conditional probability? (Hint: you’ve seen this formula before) How can you translate one formula to the other.
- (h) Draw a Venn diagram with two overlapping circles. The size of one circle stands for the probability of an event A and the size of the other circle stands for the probability of an event B . What does the size of the overlap of the two circles stand for? Explain the meaning of the conditional probability equation in terms of the size of the areas in your Venn diagram.
- (i) Compute the conditional probability of the following situations.
 - There’s a box containing 45 red balls and 55 green balls. You draw a ball, discard it, then draw another ball. What is the probability of drawing a green ball on the second draw giving that you draw a red ball on the first draw? What is the probability of drawing a green ball given that you drew a green ball on the first draw.
 - A survey asked 50 people whether a hot dog is a sandwich. There were 23 male and 27 female respondents. Of the males, 17 said no and 6 said yes. Of the females, 22 said no and 5 said yes. Given a respondent who answered yes, what’s the probability they are female? Given a male, what’s the probability they answered no?
- (j) For the following questions, compute the probability and also write a sentence or two to explain how your computation works.
 - What is the probability of rolling at least a 4 on a die?
 - What is the probability of drawing from a deck of cards, a card whose value is at least a Jack? Count aces as high.
- (k) Compute this probability of this made up situation in two different ways. Of all new cars sold, 38% have leather interior and the remaining have cloth interior. If 3 customers are randomly selected, what is the probability that *at least* one of them bought leather interior?

Chapter 4, Section 4.

- (a) Describe the fundamental counting rule.
- (b) How many ways can you arrange three letters?
- (c) A car manufacturer makes cars with the following options
 - **color:** black, white, silver, gold
 - **interior:** cloth, leather
 - **engine:** 4-cylinder, 6-cylinder, 8-cylinder
 - **tinting:** front windows, back windows, all windows.

How many different ways cars can be made?

- (d) Compute the following factorials: $6!$, $3!$. Also, compute the following:

$$\frac{6!}{4!}, \quad \frac{7!}{10!}$$

without computing $6!$, $4!$, $7!$, $10!$ individually. Show your work.

- (e) Write down the two permutation rules and the combination rule. For each rule, describe when to use it.

- (f) How many different ways are there to pick a 6 digit combination from the numbers 1,2,3,4,5,6 if you don't repeat numbers? If you can repeat numbers?
- (g) A club wants to select 4 bands from 10. How many ways are there to do this if the order matters? If the order doesn't matter?
- (h) How many permutations of the letters can be made from the word 'MISSISSIPPI'?
- (i) In a group of 5 people, 3 are chosen to give a presentation. How many different ways are there to choose 3 people to present?

CHAPTER 5

Chapter 5, Section 1.

- (a) Define and provide an example of the following terms: random variable, discrete probability distribution. Describe the steps taken to construct a probability distribution.
- (b) Is "flipping a coin" a random variable? Why or why not? If so, construct a probability distribution for it.
- (c) Bic sells pens in packages of 1, packages of 6, packages of 12 and packages of 24. The corner store has two 1-packs, five 6-packs, four 12-packs, and seven 24-packs. Construct the probability distribution for this variable.
- (d) List two requirements that all probability distributions must follow.

Chapter 5, Section 2. The two ways that data is organized in this class is in data sets and probability distributions. A data set is a collection of numbers, like the low temperatures of capital cities in North and South America. A probability distribution is more abstract. For example, the a temperatures of capital cities can be explained in a probability distribution but saying that the probability of a capital city having a high between 70-80 degrees is 35%, between 80-90 degrees in 45%, and over 90 degrees 20%. These numbers are made up, but you get the drift. For data sets, we learn about where the "middle" of the data set is by computing a mean. We learn about how spread apart the data values are by computing the standard deviation. In this section, we take these concepts about data sets and translate them into a more abstract setting of probability distributions.

- (a) One of the most important activities done in mathematics is abstraction. To take a measure (e.g. a mean) and abstract it from one situation to a more abstract situation is difficult. To do it correctly, one must be clear about the most important property of the measure is so that you can translate it into the new context.
What is the formula for the mean of a probability distribution? Which formula on data sets is this most like? Be specific.
- (b) The mean of a probability distribution might not even be possible to achieve. Find the mean of the number of dots that show up when rolling a single die. Why is this impossible to achieve?
- (c) The formula for standard deviation of a data set is

$$\sigma = \sqrt{\frac{\sum (X - \mu)^2}{N}}$$

Rewrite that in the form

$$(1) \quad \sigma = \sqrt{\sum (X - \mu)^2 \cdot \frac{1}{N}}$$

Now, write down the formula for standard deviation in a probability distribution. Compare that to Equation 1. What is the main difference between the two equations? In what way is that difference really not much of a difference at all?

- (d) A box contains ten balls each labeled with a number. The numbers on the balls are

10, 13, 5, 23, 13, 5, 4, 5, 13, 10

Some ancient Greek dude who pissed off Zues has been punished by having to pick a ball at random, writing down the number, then replacing the ball to the box, and repeating this for eternity. What is the standard deviation of the number selected on the balls.

- (e) Write down the formula for Expected Value. How is this different from the mean (consider this may be a trick question)? Try to convince me that calling this measure “Expected Value” is a good name.
- (f) The local hospital is running a lottery. It is selling 5000 tickets at 25 dollars a piece. The prize is a new Kia worth 25,000 dollars. If you buy 2 tickets, what is the expected value of your gain?

Chapter 5, Section 3.

- (a) What is a binomial experiment? What requirements must an experiment satisfy to be considered binomial?
- (b) Which of the following are binomial distributions? If it fails, which requirements does it fail?
- Randomly selecting 25 students and writing their age.
 - Drawing 5 cards from a deck *with* replacement and recording whether they are red or black
 - Drawing 5 cards from a deck *without* replacement and recording whether they are red or black
 - Shooting freethrows at a basketball court.
- (c) Write down the binomial probability formula and explain the meaning of every part of the notation.
- (d) Pick any professional basketball player. What is their freethrow percentage (wikipedia should have this)? If they shot 12 shots in a row, what is the probability of them making exactly 9 shots? (Use their freethrow percentage in your calculation)
- (e) Calculating the mean and standard deviation for this probability distribution is different than for data sets. What are the formulas?
- (f) What is the mean and standard deviation for the experiment above (your basketball player shooting 12 freethrows)?