

name: Solution

1 (10 points). Determine the truth of the following statements and give an explanation if true or counterexample if false. Assume that a and L are finite numbers.

- (a) If $\lim_{x \rightarrow a} f(x) = L$, then $f(a) = L$.
(b) If $\lim_{x \rightarrow a^-} f(x) = L$, then $\lim_{x \rightarrow a^+} f(x) = L$.
(c) If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = L$, then $f(a) = g(a)$.

(a) γ False. $\lim_{x \rightarrow 1} \frac{x-1}{x-1} = 1$ but $\frac{x-1}{x-1}$ is undefined at $x=1$.

(b) γ False. $f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$ has
 $\lim_{x \rightarrow 0^-} f(x) = 0$ but $\lim_{x \rightarrow 0^+} f(x) = 1$.

(c) γ False. $f(x) = \begin{cases} x/x, & x \neq 0 \\ 1, & x = 0 \end{cases}$ $g(x) = \begin{cases} x/x, & x \neq 0 \\ 2, & x = 0 \end{cases}$

Then $\lim_{x \rightarrow 0} f(x) = 1 = \lim_{x \rightarrow 0} g(x)$ but

$f(0) = 1 \neq g(0) = 2$.