Math 46 — Week 2 Discussion

The difficulty from the question

$$y' = \frac{2y + x}{y}$$

stems from a typo. The equation should be

$$y' = \frac{2y - x}{y}.$$

<u>SOL</u>

Substitute $y = xv$ & $y' = v + xv'$ to get

$$v + xv' = \frac{2xv - x}{xv} = \frac{2v - 1}{v}$$

$$\Rightarrow xv' = \frac{2v - 1 - v^2}{v} = -\frac{(v-1)^2}{v}$$

$$\Rightarrow -\frac{v}{(v-1)^2} v' = \frac{1}{x} \quad . \quad \text{Let } u = v - 1, \frac{du}{dx} = \frac{dv}{dx}$$

$$\Rightarrow -\frac{u+1}{u^2} \frac{du}{dx} = \frac{1}{x} \quad \Rightarrow \quad \int -\frac{1}{u} + \frac{1}{u^2} \frac{du}{dx} dx = \int \frac{1}{x} dx$$

$$\Rightarrow -\ln|u| + \frac{1}{u} = \ln|x| + c$$

$$\Rightarrow -\ln|v-1| + \frac{1}{v-1} = \ln|x| + c \Rightarrow \boxed{-\ln\left|\frac{y}{x}-1\right| + \frac{1}{\frac{y}{x}+1} = \ln(x) + c}$$