EXAM 2
MATH1117.02
2019-11-25

name: Solution

1. Consider the function $f(x) = \dfrac{x+3}{x-4}$. *(4 points each)*

(a) Describe the intervals where f in increasing, where f is decreasing, and list the local extrema.

$$f'(x) = \frac{(x-4)-(x+3)}{(x-4)^2}$$

$$= \frac{-7}{(x-4)^2}$$

This gives critical point $x=4$

	4
$f'(0)<0$	$f'(5)<0$

Therefore f is decreasing on $(-\infty,4)$ & $(4,\infty)$

There are no local extrema.

(b) Describe the intervals where f is concave up and those where f is concave down.

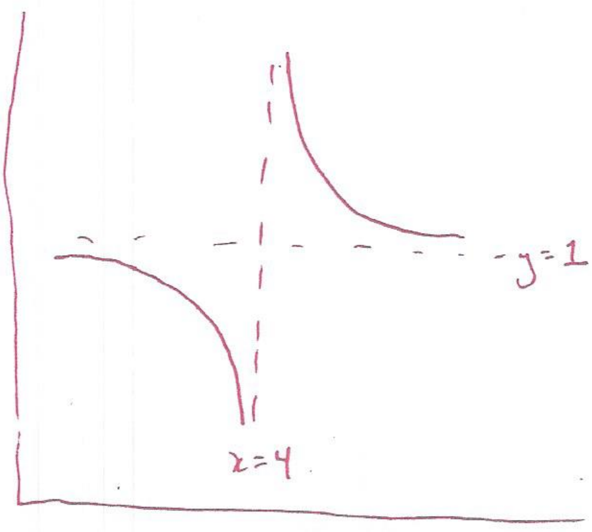
$$f''(x) = \frac{14}{(x-4)^3}$$

	4
$f''(0)<0$	$f''(5)>0$

f is concave down on $(-\infty,4)$ & concave up on $(4,\infty)$.

(c) Graph the function of f. Label all asymptotes and extrema.

- Vertical asymptote @ $x=4$
- Horizontal asymptote @ $y = \lim_{x\to\infty} \frac{x+3}{x-4} = 1$
- There are no extrema



2. Compute the derivatives for the following functions: *(4 points each)*

(a) $f(x) = \dfrac{2e^x - 1}{2\cos(x) + 1}$

$$f'(x) = \frac{(2e^x)(2\cos(x) + 1) - (2e^x - 1)(-2\sin(x))}{(2\cos(x) + 1)^2}$$

(b) $f(x) = \arcsin(\ln(3x))$

$$f'(x) = \frac{1}{\sqrt{1 - (\ln(3x))^2}} \cdot \frac{1}{3x} \cdot 3$$

3. Without finding the inverse, evaluate the derivative of the inverse of the function $f(x) = 3e^{7x}$ at the point $(3, 0)$. *(4 points)*

Use $(f^{-1})'(3) = \frac{1}{f'(0)}$.

$f'(0) = 21e^{7(0)} = 21$.

$(f^{-1})'(3) = \frac{1}{21}$.

4. Which of the following are required to be true of a function f for the Mean Value Theorem to hold on an interval $[a, b]$: (fill in all that apply) *(2 points)*

- ● f in continuous on (a, b)
- ○ f has an inverse on (a, b)
- ● f is differentiable on $[a, b]$
- ○ $f(a) = f(b)$

For the function and interval

$$f(x) = x^2 + 2x + 4;\ [0, 2]$$

find the points guaranteed to exist by the Mean Value Theorem *(4 points)*

The mean value theorem states there's a solution to the equation $f'(x) = \frac{f(2)-f(0)}{2-0}$ in $[0,2]$. That is:

$$2x + 2 = \frac{(4+4+4)-(0+0+4)}{2-0}$$

$$2x + 2 = 4$$

$$2x = 2$$

$$x = 1.$$

5. Solve $x^2 + 7\tan(y) = \sqrt{2y}$ for $\frac{dy}{dx}$. *(4 points)*

$$\frac{d}{dx}\left(x^2 + 7\tan(y)\right) = \frac{d}{dx}\left((2y)^{1/2}\right)$$

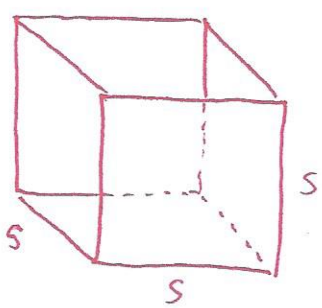
$$\Rightarrow \quad 2x + 7\sec^2(y)\cdot y' = \tfrac{1}{2}(2y)^{-1/2}\, 2y'$$

$$\Rightarrow \quad 2x = \tfrac{1}{2}(2y)^{-1/2}\cdot 2y' - 7\sec^2(y)\,y'$$

$$\Rightarrow \quad 2x = \left(\tfrac{1}{2}(2y)^{-1/2}\cdot 2 - 7\sec^2(y)\right) y'$$

$$y' = \frac{2x}{\tfrac{1}{2}(2y)^{-1/2}\cdot 2 - 7\sec^2(y)}$$

6. The sides of a cube decreases in length at a rate of 3 meters per second. At what rate is the volume of the cube changing when the sides are 2 meters long? *(4 points)*



$$V = s^3$$

$$\frac{dV}{dt} = 3s^2\cdot\frac{ds}{dt} \quad \text{where } s = 2 \text{ ; } \frac{ds}{dt} = -3$$

$$\frac{dV}{dt} = 3(2)^2\cdot(-3) = -36$$