

name: _____

1. (4 points each) For the following functions, determine the location and value of the absolute extreme values on the given interval, if they exist.

(a) $f(x) = \frac{x}{(x^2+3)^2}$ on $[-2, 2]$

(b) $f(x) = x^4 - 4x^3 + 4x^2$ on $[-1, 3]$

$$\begin{aligned} f'(x) &= \frac{(x^2+3)^2 - x(2(x^2+3) \cdot 2x)}{(x^2+3)^4} \\ &= \frac{(x^2+3) - 4x^2}{(x^2+3)^3} \\ &= \frac{-3(x^2-1)}{(x^2+3)^3} \end{aligned}$$

Setting $f'(x) = 0$:

$$0 = \frac{-3(x^2-1)}{(x^2+3)^3} \Rightarrow x = \pm 1$$

Also $f'(x)$ is not undefined anywhere, so critical points are $-1, 1$.

$$\left. \begin{aligned} f(-2) &= -\frac{2}{49}, & f(1) &= \frac{1}{16} \\ f(-1) &= -\frac{1}{16}, & f(2) &= \frac{2}{49} \end{aligned} \right\} \begin{aligned} &\bullet \text{ Absolute max @ } (1, \frac{1}{16}) \\ &\bullet \text{ Absolute min @ } (-1, -\frac{1}{16}) \end{aligned}$$