

name: Solution

1. (4 points each) For the following functions, determine the location and value of the absolute extreme values on the given interval, if they exist.

(a) $f(x) = \frac{x}{(x^2 + 3)^2}$ on $[-2, 2]$

(b) $f(x) = x^4 - 4x^3 + 4x^2$ on $[-1, 3]$

a) Find critical points

$$\begin{aligned} f'(x) &= \frac{(x^2 + 3)^2 - x(2(x^2 + 3) \cdot 2x)}{(x^2 + 3)^4} \\ &= \frac{(x^2 + 3) - 4x^2}{(x^2 + 3)^3} \\ &= \frac{-3(x^2 - 1)}{(x^2 + 3)^3} \end{aligned}$$

C.P. @ $x = \pm 1$

Test @ $x = \pm 1, \pm 2$

$f(-1) = \frac{-1}{(1+3)^2} = -\frac{1}{16} \leftarrow \text{Min}$

$f(1) = \frac{1}{(1+3)^2} = \frac{1}{16} \leftarrow \text{Max}$

$f(-2) = \frac{-2}{(4+3)^2} = -\frac{2}{49}$

$f(2) = \frac{2}{(4+3)^2} = \frac{2}{49}$

b) Find critical points

$$\begin{aligned} f'(x) &= 4x^3 - 12x^2 + 8x \\ &= 4x(x^2 - 3x + 2) \\ &= 4x(x-1)(x-2) \end{aligned}$$

C.P. @ $x = 0, 1, 2$

Test @ $x = -1, 0, 1, 2, 3$

$f(-1) = 1 + 4 + 4 = 9 \leftarrow \text{Max}$

$f(0) = 0$

$f(1) = 1 - 4 + 4 = 1$

$f(2) = 16 - 32 + 16 = 0$

$f(3) = 81 - 108 + 36 = -9 \leftarrow \text{Min}$