1 Rewriting

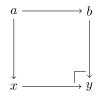
Definition 1.1. A category with pullbacks is **adhesive** if pushouts along monics exist and are *Van Kampen*.

Theorem 1.2. Topoi are adhesive.

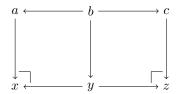
Corollary 1.3. Let $L \dashv R : \mathbf{A} \to \mathbf{X}$ be an adjunction between topoi. The category L is adhesive.

Definition 1.4. For **C** an adhesive category, an **C-rewrite rule** (often called a production) is a span $a \leftarrow b \rightarrow c$ inside **C**. When both legs of the span are monic, we say the rewrite rule is **linear**.

Definition 1.5. Given composable arrows $a \to b \to y$ we say that an arrow $a \to x$ is a **pushout complement** if it fits into a pushout diagram



Definition 1.6. Given a **C**-rewrite rule $a \leftarrow b \rightarrow c$ and a **C**-arrow $a \rightarrow x$ such that $b \rightarrow a \rightarrow x$ has a pushout complement, a **derived (linear) rewrite rule** is the bottom row of the induced double pushout diagram



Definition 1.7. A (linear) grammar consists of an adhesive category \mathbf{A} and a set of (linear) \mathbf{A} -rewrite rules. Observe that \mathbf{A} -rewrite rules are actually arrows in $\mathbf{Span}(\mathbf{A})$. Given a grammar Γ , the subcategory $\mathcal{L}(\Gamma)$ of $\mathbf{Span}(\mathbf{A})$ generated by the set of rewrites derived from Γ is called a language.

Lemma 1.8. Let $(\mathbf{A}, \otimes_{\mathbf{A}}, I_{\mathbf{A}})$ be a symmetric monoidal category with pullbacks and $(\mathbf{X}, \otimes_{\mathbf{X}}, I_{\mathbf{X}})$ be a symmetric monoidal topos. Let $L \dashv R \colon \mathbf{A} \to \mathbf{X}$ be an adjunction where L preserves pullbacks.

Fix a grammar Γ in the topos L. The generated language $\mathcal{L}(\Gamma)$ is a sub-bicategory of L.