

# Report on Spans of cospans in a topos

by  
Daniel Cicala and Kenny Courser

This is a nice, carefully written paper which presents a new and interesting result. I recommend that it be accepted for TAC.

The authors continue the study of the bicategory of cospans in a topos introduced by the first author in “Spans of cospans” to appear in TAC. Their motivation lies in the theory of complex networks and in order to accommodate the various constructions that they wish to perform, they must take as 2-cells, not the usual morphisms of cospans, but monic spans of cospans.

Their main result is that this gives a symmetric compact closed bicategory. Now, this bicategory is relatively complicated, and proving that any bicategory is symmetric monoidal, let alone compact, is no mean feat. To accomplish this, the authors use a theorem of Shulman’s which constructs a symmetric monoidal bicategory from an isofibrant symmetric monoidal double category. This last structure sounds more complicated but in fact it is much easier to use because the coherences are isomorphisms rather than equivalences. So they enlarge their bicategory to a double category in which the vertical arrows are isomorphisms rather than the identities which are the hallmark of a bicategory. This seemingly minor change allows the coproduct to be a monoidal structure on the objects and then Shulman’s theorem applies. Once this double category has done its job it is summarily discarded. Shulman’s theorem cannot be used to get compactness, so the authors resort to a theorem of Pstragowski for this.

There were a few typos.

- Definition 3.7: monoidal *bicategory*
- Page 15: This comes down to finding *an* isomorphic...
- References [17]: Pstragowski

Perhaps the authors would like to rephrase “first listed author” on page 2, line -7 and page 3, line -2. Both times I read the paper, I thought they were referring to Grandis, before realizing what they meant.