DEPARTMENT OF AEROSPACE ENGINEERING
FACULTY OF ENGINEERING, ARCHITECTURE AND SCIENCE

AER 715 AVIONICS AND SYSTEMS

Laboratory 3:
Flight Control - Estimation of Model Parameters and Simulation

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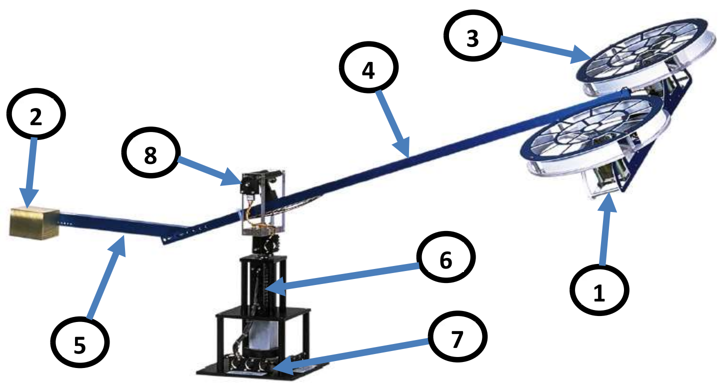
1. Lab Instructions

- SAFETY FIRST – DO NOT PUT YOUR FINGERS OR ANY LOOSE ITEMS IN THE SERVOMOTOR GEARS.
- This lab is to be done **in groups of no more than three students**.
- Download the lab manual, worksheet, and files from D2L and save them on the Desktop in a folder called LAB3.
- Read the instructions in the laboratory manual carefully and follow the specified procedures.
- Answer all questions in the provided worksheet.
- At the end of the lab, submit one lab worksheet along with the standard Ryerson Aerospace Assignment/Laboratory Cover Sheet. Each student must attend the laboratory and sign the Cover Sheet in order to receive a mark.

2. Estimation of Model Parameters and Simulation in Flight Control

2.1 Introduction

The three degrees-of-freedom laboratory helicopter (3-DOF helicopter) to be used in this lab is shown in Figure 1. The pair of 3-blade propellers is each driven by a brushed DC motor. The helicopter body is located on one end, with a brass counterweight on the other. They are connected with an arm that is attached to a pivot point. The counterweight is used to offset the required thrust to lift the helicopter body. The helicopter is free to elevate, pitch, and travel. For this lab the pitch axis has been fixed mechanically. As shown in Figure 1, the helicopter's body angles are measured using encoders. The encoder signals pass through the slip ring to a data acquisition board. The motor power from the amplifier also passes through the slip ring to the motors. Figure 2 depicts the system schematic of the 3-DOF helicopter and its required devices.



No.	Part	No.	Part
1	Helicopter Body	5	Counterweight Arm
2	Counterweight	6	Slip Ring(s) and Pedestal
3	Fan Cage	7	Power and Sensor Signal Connections
4	Helicopter Arm	8	Elevation Encoder

Figure 1 3-DOF Helicopter

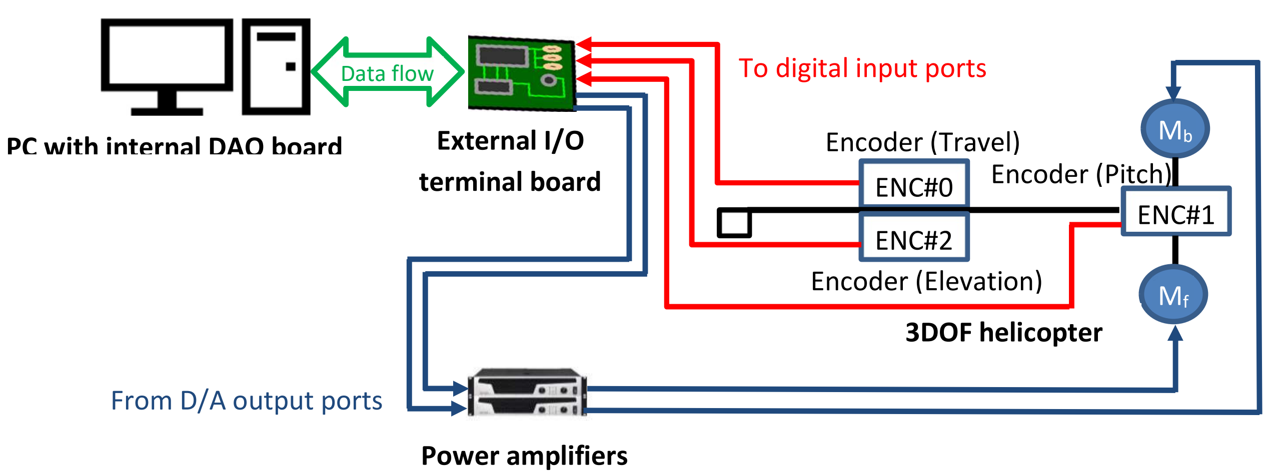


Figure 2 System Schematic Diagram of the 3-DOF Helicopter

2.2 Purpose

The objective of this lab is to model and simulate the elevation dynamics of the 3-DOF Helicopter system. You will characterize the system dynamics analytically first and then experimentally. More specifically, you will study the analytical model of the elevation dynamics first and then estimate the model parameters using experimental data.

Once you have obtained the model and parameters of the helicopter you will be able to simulate the helicopter and visualize its performance using a virtual model.

At the end of this laboratory, you should understand the following:

- How to mathematically model the elevation dynamics of the 3-DOF helicopter
- How to use your model to simulate the system in Simulink

2.3 Apparatus

To complete this lab, the following hardware is required:

- Quanser 3-DOF Helicopter
- Quanser UPM-2405 or VoltPAQ-X2 Power Module
- Quanser Q4 data acquisition and control board
- PC equipped with the necessary software including MATLAB/Simulink

2.4 Parameters of the 3-DOF Helicopter

The physical parameters of the four 3-DOF helicopters in the laboratory are given in Table 1, and parameters of the system components and joint limits are provided in Table 2. Use the parameters of your helicopter for the modeling and simulations.

Table 1: 3-DOF Helicopter Physical Parameters

Symbol	MATLAB	Description	Unit	Value			
				Heli 1	**Heli 2**	**Heli 3**	**Heli 4**
M_h	Mh	Mass of Heli Body	[kg]	1.442	1.422	1.464	1.450
M_c	Mc	Mass of CW	[kg]	1.914	1.916	1.919	1.918
L_a	La	Distance from Pivot to Helicopter body centre	[in]	25.75			
L_b	Lb	Distance from Pivot to counterweight centre	[in]	18.125		18.5	
L_h	Lh	Distance from pitch axis to rotor center	[in]	6.985	6.932	6.995	6.933
J_e	Je	Moment of Inertia	[kg-m^2]	TBD	TBD	TBD	TBD
D_e	De	Viscous Damping	[N-m-s/rad]	TBD			
K_e	Ke	Spring Constant	[N-m/rad]	TBD			
F_t	Ft	Lift Force @ SLF	[N]	TBD	TBD	TBD	TBD

Table 2: 3-DOF Component Parameters and Joint Limits

Symbol	MATLAB	Description	Unit	Value			
				Heli 1	Heli 2	Heli 3	Heli 4
K_f	Kf	Motor-Prop Force Constant	[N/V]	0.140			
K_{rt}	Krt	Motor-Prop Torque Constant	[N.m/V]	0.0036	0.0032	0.0038	0.0027
ε		Elevation Range	[Degrees]	[~-26 to ~30]			
ε_o		Elevation Start	[Degrees]	-25.75			
λ		Travel Range	[Degrees]	0 to 360			
g	g	Gravity constant	[m/s^2]	9.81			
	KE_CNT	Encoder Resolution	[counts/rev]	-4096			
	KE_RAD	Encoder Resolution	[rad/count]	1.5340E-2			
	K_CABLE	Amplifier Gain	[V/V]	3		5	

2.5 Dynamic Model

The 3-DOF helicopter in the elevation axis can be modeled as a lever balancing on a fulcrum or pivot as shown in Figure 3.

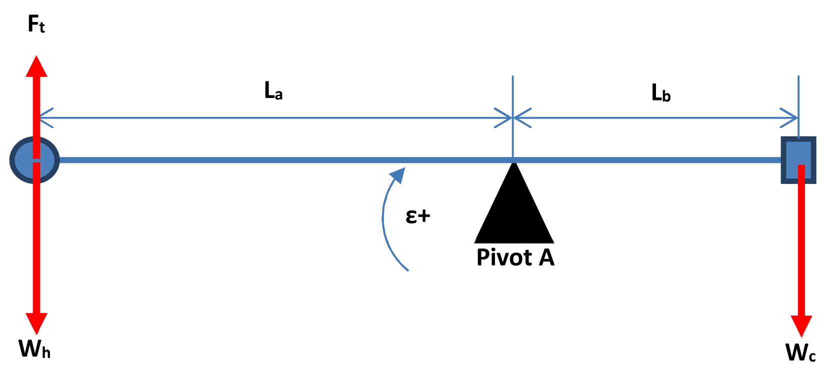


Figure 3 Known Forces Acting on the Helicopter

Using D'Alembert's principle, we can derive the equation of motion by summing all applied torques and the inertial forces as follows:

$$\sum_A T_i = L_a F_t - L_a W_h \cos\varepsilon + L_b W_c \cos\varepsilon = J_e \ddot{\varepsilon} \qquad \textbf{Equation 1}$$

where W_α, α=c,h are the weights of the helicopter components as indicated in Figure 3, J_e is the total moment of inertia, and F_t is the total thrust produced by the propellers. We are neglecting any drag or damping forces.

Through inspection, we can see that the differential equation is non-linear since the variable ε is operated on by a trigonometric function. In order to use the classical control techniques that were covered in AER 509, we first need to linearize the system about the position $\varepsilon = 0$. We use small angle theorem and approximate $\cos(\varepsilon) \approx 1$ for small ε. Substituting the cosine approximation into Eq. 1 and rearranging we get:

$$L_a F_t + T_g = J_e \ddot{\varepsilon} \qquad \textbf{Equation 2}$$

where $T_g = L_b W_c - L_a W_h$.

2.6 Pre-Lab Assignment

This lab involves simulating the elevation dynamics of the 3-DOF helicopter. You will use both analytical and experimental means to determine the dynamic characteristics of the helicopter elevation. **Note:** For the following tables, you only need to complete the information for one of the four helicopters you are assigned for.

<table>
<tr><th>STEP</th><th>DESCRIPTION/TASK</th></tr>
<tr><td>1</td><td>Using the data in Tables 1 and 2, and Figure 3, determine the moment of inertia of the elevation axis J_e in the base units [kg-m^2] for the helicopter that you use.

<table><tr><th>Helicopter No.</th><th>Inertia [kg-m^2]</th></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr></table>
Hint: Consider the helicopter body and counterweight as point masses. Neglect the weights of the lever arm, but add 5% to your final value to compensate for the assumption.</td></tr>
<tr><td>2</td><td>Using the data in Tables 1 and 2, and Figure 3, determine the lift force F_t required to achieve steady level flight (ε = 0 deg) for the helicopter that you use.

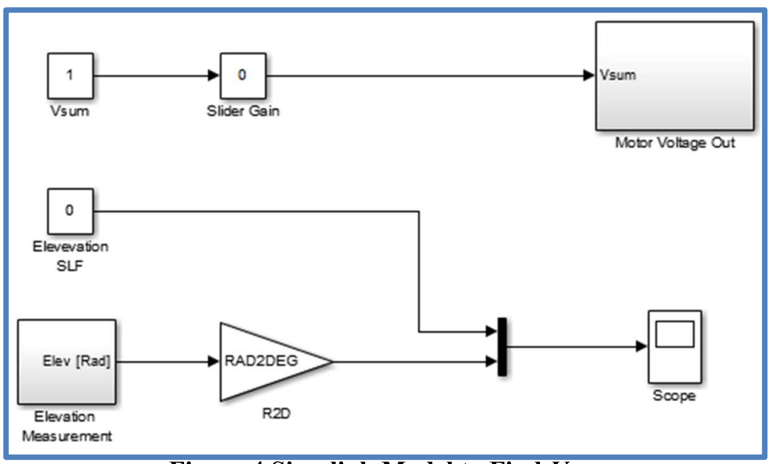
<table><tr><th>Helicopter No.</th><th>F_t [N]</th></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr></table></td></tr>
<tr><td>3</td><td>Given $\mathrm{F}_t = K_f \mathrm{V}_{sum}$, solve Equation 2 for the V_{sum} required to keep the helicopter at steady level flight (ε = 0 deg). Solve for the helicopter that you use.

<table><tr><th>Helicopter No.</th><th>V_{sum} [V]</th></tr><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr></table></td></tr>
</table>

3. Lab Work

3.1 Part A: Establishing the Step Input Voltage in Elevation (SLF) Control

In this section we will determine the <u>actual</u> V_{sum} required to keep the helicopter at SLF. This V_{sum} will establish our reference step input that will help us experimentally determine the model of the system. **NOTE:** If you choose to work remotely from the lab, you will have to use **helicopter #1 (heli = 1)** from now on for the remainder of this course as the experimental data of only helicopter #1 will be provided. Those who are present in the lab can collect your own voltage and elevation data.

STEP	DESCRIPTION/TASK
1	Download the files for Lab 3 from D2L and put them in a Lab 3 folder on your desktop.
2	Start "**MATLAB**" and change the working directory to Lab3.
3	Before you begin, you must edit the m-File called "**<u>Heli_Setup.m</u>**". Open up the script file and modify the value of the variable **heli** which is the number of the helicopter station (**see the sticker on your monitor**) you have been assigned to by the GA. Leave $V_{sum} = 0$. Once you have set the value of heli (e.g. **heli = 1**) in the M-file script, run the script.
3	In Simulink, open a model called "**<u>Heli_Voltage_Set.slx</u>**" as shown in Figure 4. This will allow you to adjust V_{sum} to determine the voltage required to keep the helicopter at SLF.  **Figure 4 Simulink Model to Find V_{sum}**
4	Now, turn on the amplifiers by deactivating the Emergency Stop (pull-up **OR** turn counterclockwise – see sticker on the emergency stop).
5	You can now build the system using the **QUARC >> Build** menu. You will see the model compiled into real-time C++ code (this may take a minute).
6	Next, open up the **Scope** block as you will use this to monitor the test. Also open up the **Slider Gain** block.
7	When you start the test, slowly ramp up the voltage using the Slider Gain. Your goal is to get the helicopter to oscillate around the zero angle elevation (the yellow line in your scope). Select **QUARC >> Start** to begin the experiment.
8	Increment the voltage by **1 volt** at a time and keep adjusting V_{sum} until the helicopter is oscillating about $\varepsilon = 0$. You can fine tune the voltage by changing the middle number of the Slider Gain window. Once you are happy with the result, record V_{sum} and move onto Part B. **The $\varepsilon = 0$ level will be indicated by a yellow line in the Scope.** Lower your helicopter back down to the table slowly using the **Slider Gain**.

	Note: *Because we are operating the helicopter as an open-loop control system, it cannot reject disturbances, so you will not be able to keep the helicopter perfectly still. Expect some oscillation about the yellow line akin to a pendulum swinging.*

3.2 Part B: Implementation of the Plant Model for Open-loop Elevation (SLF) Control

In the first part of this lab we will gather some experimental data so that we can extract a linear model of the elevation dynamics of the 3-DOF Helicopter.

<table>
<tr><th>STEP</th><th>DESCRIPTION/TASK</th></tr>
<tr><td>1</td><td>Before you begin, you must change V_{sum} to the value you found in Part A. You can do this directly in “<u>Heli_Setup.m</u>” or in the MATLAB command window.

If you changed the value in the script, you must run the script again.</td></tr>
<tr><td>2</td><td>In Simulink, open a model called “<u>Heli_Open_Loop.slx</u>”. When executed, this file will run a test and store the data in the MATLAB workspace.

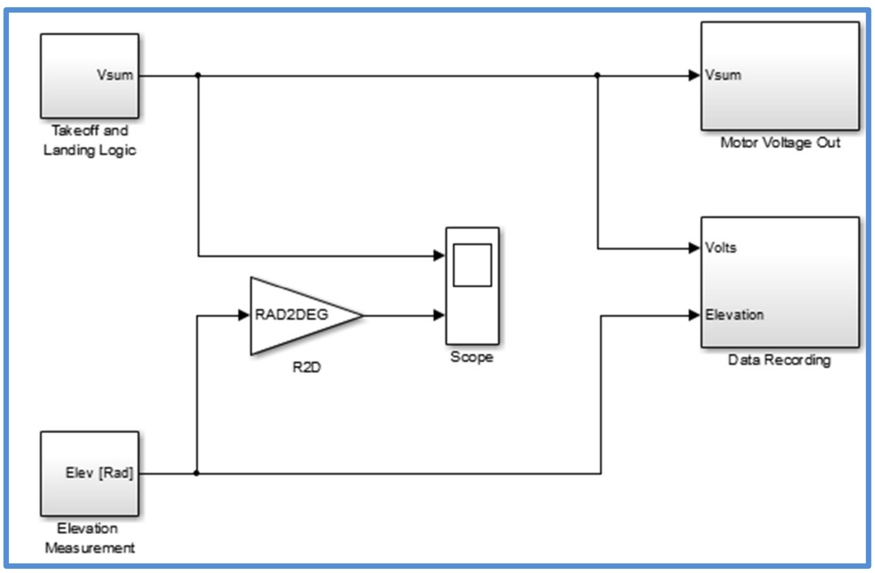


Figure 5 The 3DOF Helicopter Open-loop Step Test Simulink Model</td></tr>
<tr><td>3</td><td>You can now build the system using the QUARC >> Build menu. You will see the model compiled into real-time C++ code (this may take a minute).
In the meantime, turn on the amplifiers, by deactivating the Emergency Stop.</td></tr>
<tr><td>4</td><td>Next, open up the Scope block as you will use this to monitor the test.</td></tr>
<tr><td>5</td><td>When you start the test, the helicopter will take off and overshoot the SLF position and will oscillate around the approximate zero elevation angle based on the V_{sum} you established in Part A. After 25 seconds, the helicopter will switch to a landing mode, where the voltage applied to the motors will gradually decrease. After 60 seconds, the program will stop.

Select QUARC >> Start to begin the experiment. Your plant should now be responding and oscillating up and down.</td></tr>
</table>

6	Repeat the test, but rename the filename to "**ElevationData2**" and the variable name to "**elev2**" in the "**To File**" Block located in the Data Recording block.
7	Repeat the test one more time, but rename the filename to "**ElevationData3**" and the variable name to "**elev3**" in the "**To File**" Block located in the Data Recording block.
8	E-mail or save the test data to a USB key to complete the Lab.

3.3 Part C: Post Lab

STEP	DESCRIPTION/TASK
1	In a **<u>new</u>** script, type the following: %--- % AER 715 Avionics and Systems % Lab 3 – "Lab Title" % Your Full Name(s) & SID(s) %--- % %% Introduction % Type your introduction in this section % %% Post Lab Exercises – % Put your exercises in this section % %% Conclusion % Write your lab conclusion for the WHOLE lab in this section.
2	**Create a new section called Question 1 using the %% command.** Using the data in Tables 1 and 2, and Figure 3, determine the polar moment of inertia of the elevation axis J_e in the base units [kg-m^2] for the helicopter that you use. ***Hint:*** Consider the helicopter body and counterweight as point masses. Neglect the weights of the lever arm, but **add 5% to your final value to compensate for the assumption**.
3	**Create a new section called Question 2 using the %% command.** Using the data in Tables 1 and 2, and Figure 3, determine the lift force $\boldsymbol{F_t}$ required to achieve steady level flight (ε = 0 deg) for your helicopter.
4	**Create a new section called Question 3 using the %% command.** Given $\mathrm{F}_t = K_f \mathrm{V}_{sum}$, solve Equation 2 for the V_{sum} required to keep the helicopter at steady level flight (ε = 0 deg). How does this value compare to the one you found experimentally? If there is a discrepancy, explain why?
5	**Create a new section called Question 4 using the %% command.** Take the Laplace transform of Equation 2 to get the open loop transfer function $G_{elev}(s) = \frac{E(s)}{V_{sum}(s)}$. Use $F_t = K_f V_{sum}$ to approximate the lift force.

	Create a transfer function called **<u>G#_elev1</u>** using the ***tf*** command for the helicopter you used in the lab. Replace the **#** symbol above and all the function names mentioned in the later sections with your helicopter station number (e.g. **G1_elev1**). **Hint:** In order to isolate the output angle $E(s)$, you must ignore T_g. As you may remember from AER 509, we have to assume that the system's initial conditions are zero, which is a limitation of this approach.
6	**Create a new section called Question 5 using the %% command.** Download the experimental data from D2L. Import it to MATLAB and do the following: • Extract the time and put it in a variable ***time***. • Create **volts1** and **elev1** variables for test 1. Repeat for test data 2 and 3. • On a single graph, plot the **relevant** experimental elevation vs time data. • Create three ***iddata*** objects and then combine them with the ***merge*** command into one data set. Next, use the ***tfest*** command to estimate a continuous time transfer function from the data. Perform a fit using 2 and 3 poles respectively and create transfer functions for each called G#_elev2 and G#_elev3 respectively. Remember to also specify the number of zeroes exactly or ***tfest*** command will use as many as needed to produce the best fit. **Note: Use only the <u>relevant data</u> from the tests for your estimates otherwise you estimated transfer function will be inaccurate.** **Hint:** You can use **tfdata** with the "**v**" option to extract the numerator and denominator polynomials from the transfer function.
7	**Create a new section called Question 6 using the %% command.** Find the poles of G#_elev2 and G#_elev3. Based on the poles of G#_elev3, comment on whether or not it is acceptable to consider this system to be second order? Also, what system dynamics would the real pole be associated with in the actual helicopter?
8	**Create a new section called Question 7 using the %% command.** Use the ***step*** command to compare G#_elev1, G#_elev2, and G#_elev3. Insert a legend into the plot. Make comments on the plot in the script file as to why each transfer function behaves the way it does. Also, in G#_elev2, where might the second and third coefficients come from?
9	**Create a new section called Question 8 using the %% command.** Why did we need to find the V_{sum} that would make the helicopter hover at $\varepsilon = 0$? Why does the helicopter travel around as we apply power to the motors? Explain.
10	Publish your script and submit along with the standard Aerospace cover sheet. Include a copy of your pre-lab. Keep a copy of your transfer functions as we will use them in the next lab.