Polinómio de grau 4 $(n=4)$

I

$$g(s) = \sum_{k=0}^{n=4} \binom{s}{k} \Delta^k r_0 = \sum_{k=0}^{n=4} \frac{s!}{k!(s-k)!} \Delta^k r_0$$

$$g(s) = \frac{s!}{0!\,(s-0)!} \Delta^0 r_0 + \frac{s!}{1!\,(s-1)!} \Delta^1 r_0 + \frac{s!}{2!\,(s-2)!} \Delta^2 r_0 + \frac{s!}{3!\,(s-3)!} \Delta^3 r_0 +$$

$$+ \frac{s!}{4!\,(s-4)!} \Delta^4 r_0$$

$$g(s) = \frac{s!}{0!\,(s)!} r(0) + \frac{s(s-1)!}{1!\,(s-1)!} \big(r(1) - r(0)\big) + \frac{s(s-1)(s-2)!}{2!\,(s-2)!} \big(r(2) - 2r(1) + r(0)\big)$$

$$+ \frac{s(s-1)(s-2)(s-3)!}{3!\,(s-3)!} \big(r(3) - 3r(2) + 3r(1) - r(0)\big) +$$

$$+ \frac{s(s-1)(s-2)(s-3)(s-4)!}{4!\,(s-4)!} \big(r(4) - 4r(3) + 6r(2) - 4r(1) + r(0)\big)$$

$$g(s) = r(0) + s\big(r(1) - r(0)\big) + \frac{1}{2} s(s-1)\big(r(2) - 2r(1) + r(0)\big) +$$

$$+ \frac{1}{6} \cdot s(s-1)(s-2)\big(r(3) - 3r(2) + 3r(1) - r(0)\big) +$$

$$+ \frac{1}{24} s(s-1)(s-2)(s-3)\big(r(4) - 4r(3) + 6r(2) - 4r(1) + r(0)\big)$$

II

$$g(s) = \pi(0)\left(1 - s + \frac{1}{2}s(s-1) - \frac{1}{6}s(s-1)(s-2) + \frac{1}{24}s(s-1)(s-2)(s-3)\right)$$

$$+\pi(1)\left(s - s(s-1) + \frac{1}{2}s(s-1)(s-2) - \frac{1}{6}s(s-1)(s-2)(s-3)\right)$$

$$+\pi(2)\left(\frac{1}{2}s(s-1) - \frac{1}{2}s(s-1)(s-2) + \frac{1}{4}s(s-1)(s-2)(s-3)\right)$$

$$+\pi(3)\left(\frac{1}{6}s(s-1)(s-2) - \frac{1}{6}s(s-1)(s-2)(s-3)\right)$$

$$+\pi(4)\left(\frac{1}{24}s(s-1)(s-2)(s-3)\right)$$

$$g(s) = \pi(0)\left(1 - s + \frac{1}{2}s^2 - \frac{1}{2}s - \frac{1}{6}(s^3 - 3s^2 + 2s) + \frac{1}{24}(s^4 - 6s^3 + 11s^2 - 6s)\right)$$

$$+\pi(1)\left(s - (s^2 - s) + \frac{1}{2}(s^3 - 3s^2 + 2s) - \frac{1}{6}(s^4 - 6s^3 + 11s^2 - 6s)\right)$$

$$+\pi(2)\left(\frac{1}{2}(s^2 - s) - \frac{1}{2}(s^3 - 3s^2 + 2s) + \frac{1}{4}(s^4 - 6s^3 + 11s^2 - 6s)\right)$$

$$+\pi(3)\left(\frac{1}{6}(s^3 - 3s^2 + 2s) - \frac{1}{6}(s^4 - 6s^3 + 11s^2 - 6s)\right)$$

$$+\pi(4)\left(\frac{1}{24}(s^4 - 6s^3 + 11s^2 - 6s\right)$$

III

$$g(s) = \pi(0)\left(\frac{1}{24}s^4 - \frac{5}{12}s^3 + \frac{35}{24}s^2 - \frac{50}{24}s + 1\right)$$

$$+\pi(1)\left(-\frac{1}{6}s^4 + \frac{3}{2}s^3 - \frac{26}{6}s^2 + 4s\right)$$

$$+\pi(2)\left(\frac{1}{4}s^4 - 2s^3 + \frac{19}{4}s^2 - 3s\right)$$

$$+\pi(3)\left(-\frac{1}{6}s^4 + \frac{7}{6}s^3 - \frac{14}{6}s^2 + \frac{8}{6}s^2\right)$$

$$+\pi(4)\left(\frac{1}{24}s^4 - \frac{6}{24}s^3 + \frac{11}{24}s^2 - \frac{6}{24}s\right)$$

Abordagem Fechada Polinômio de grau 4 (IV)

$h = \frac{\Delta x}{4}$

$f(x_i) = f(x(s=0)) = g(0)$, $f(x_i + h) = f(x(s=1)) = g(1)$,

$f(x_i + 2h) = f(x(s=2)) = g(2)$, $f(x_i + 3h) = f(x(s=3)) = g(3)$ e

$f(x_i + 4h) = f(x(s=4)) = g(4)$.

$x(s) = x_i + sh$

$$\int_{x_i}^{x_f} f(x)\,dx \approx \int_{x_i}^{x_f} P(x)\,dx = \int_{s_i}^{s_f} P(x(s))\,\frac{dx(s)}{d(s)} = h\int_0^4 P(x(s))\,ds = h\int_0^4 g(s)\,ds$$

Substituindo

$$\int_{x_i}^{x_f} f(x)\,dx \approx h\int_0^4 \Big[g(0)\left(\frac{1}{24}s^4 - \frac{10}{24}s^3 + \frac{35}{24}s^2 - \frac{50}{24}s + 1\right)$$

$$+ g(1)\left(-\frac{1}{6}s^4 + \frac{3}{2}s^3 - \frac{26}{6}s^2 + 4s\right)$$

$$+ g(2)\left(\frac{1}{4}s^4 - 2s^3 + \frac{19}{4}s^2 - 3s\right)$$

$$+ g(3)\left(-\frac{1}{6}s^4 + \frac{7}{6}s^3 - \frac{14}{6}s^2 + \frac{8}{6}s\right)$$

$$+ g(4)\left(\frac{1}{24}s^4 - \frac{6}{24}s^3 + \frac{11}{24}s^2 - \frac{6}{24}s\right)$$

V

$$= h\left[\frac{1}{24} g(0) \int_0^4 (s^4 - 10s^3 + 35s^2 - 30s + 24)\, ds\right.$$

$$+ \frac{1}{6} g(1) \int_0^4 (-s^4 + 9s^3 - 26s^2 + 24s)\, ds$$

$$+ \frac{1}{4} g(2) \int_0^4 (s^4 - 8s^3 + 19s^2 - 12s)\, ds$$

$$+ \frac{1}{6} g(3) \int_0^4 (-s^4 + 7s^3 - 14s^2 + 8s)\, ds$$

$$\left. + \frac{1}{24} g(4) \int_0^4 (s^4 - 6s^3 + 11s^2 - 6s)\, ds \right]$$

$$= h\left(\frac{1}{24} g(0) \cdot \frac{896}{120} + \frac{1}{6} g(1) \frac{1024}{120} + \frac{1}{4} g(2) \cdot \frac{256}{120} + \frac{1}{6} g(3) \frac{1024}{120}\right.$$

$$\left. + \frac{1}{24} g(4) \cdot \frac{896}{120}\right)$$

Simplificando e Substituindo

$$\int_{x_i}^{x_f} f(x)\,dx \approx \frac{2h}{45}\left(7f(x_i) + 32f(x_i+h) + 12f(x_i+2h) + 32f(x_i+3h) + 7f(x_i+4h)\right)$$

Abordagem Aberta Polinômio de Grau 4 (VI)

$$h = \frac{\Delta x}{6}$$

$f(x_0) = f(x_i + h) = f(x(s=0)) = g(0)$, $f(x_i + 2h) = f(x(s=1)) = g(1)$,

$f(x_i + 3h) = f(x(s=2)) = g(2)$, $f(x_i + 4h) = f(x(s=3)) = g(3)$ e

$f(x_i + 5h) = f(x(s=4)) = g(4)$.

$$x(s) = x_i + h + sh$$

$$\int_{x_i}^{x_f} f(x)dx \approx \int_{x_i}^{x_f} p(x)\,dx = \int_{s_i}^{s_f} P(x(s)) \frac{dx(s)}{ds}\,ds = h\int_{-1}^{5} P(x(s))ds = h\int_{-1}^{5} g(s)\,ds$$

Substituindo

$$\int_{x_i}^{x_f} f(x)dx \approx h\int_{-1}^{5}\Big[g(0)\left(\frac{1}{24}s^4 - \frac{10}{24}s^3 + \frac{35}{24}s^2 - \frac{50}{24}s + 1\right)$$

$$+ g(1)\left(-\frac{1}{6}s^4 + \frac{3}{2}s^3 - \frac{26}{6}s^2 + 4s\right)$$

$$+ g(2)\left(\frac{1}{4}s^4 - 2s^3 + \frac{19}{4}s^2 - 3s\right)$$

$$+ g(3)\left(-\frac{1}{6}s^4 + \frac{7}{6}s^3 - \frac{14}{6}s^2 + \frac{8}{6}s\right)$$

$$+ g(4)\left(\frac{1}{24}s^4 - \frac{6}{24}s^3 + \frac{11}{24}s^2 - \frac{6}{24}s\right)$$

$$= h\left[\frac{1}{24}g(0)\int_{-1}^{5}(s^4-10s^3+35s^2-50s+24)\,ds\right.$$

$$+\frac{1}{6}g(1)\int_{-1}^{5}(-s^4+9s^3-26s^2+24s)\,ds$$

$$+\frac{1}{4}g(2)\int_{-1}^{5}(s^4-8s^3+19s^2-12s)\,ds$$

$$+\frac{1}{6}g(3)\int_{-1}^{5}(-s^4+7s^3-14s^2+8s)\,ds$$

$$\left.+\frac{1}{24}g(4)\int_{-1}^{5}(s^4-6s^3+11s^2-6s)\,ds\right]$$

$$= h\left(\frac{1}{24}g(0)\cdot\frac{9504}{120}+\frac{1}{6}g(1)\cdot\frac{3024}{120}+\frac{1}{4}g(2)\cdot\frac{3744}{120}\right.$$

$$\left.+\frac{1}{6}g(3)\cdot\frac{3024}{120}+\frac{1}{24}g(4)\cdot\frac{9504}{120}\right)$$

Simplificando e Substituindo

$$\int_{x_i}^{x_f} f(x)\,dx \approx h\left(\frac{33}{10}f(x_i)+\frac{2}{5}f(x_i+h)+\frac{39}{5}f(x_i+2h)+\frac{21}{5}f(x_i+3h)\right.$$

$$\left.+\frac{33}{10}f(x_i+4h)\right)$$