

# Comprehensive Cost Minimization for Bus Charging

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*Abstract*—**TODO: add abstract**

*Index Terms*—**TODO: Add Keywords**

## I. INTRODUCTION

Recent calls for a reduced carbon footprint have pushed transit authorities to adopt battery electric buses (BEBs). Conversion from diesel and CNG to BEB reduces environmental impact [21] as BEBs provide zero emissions and access to renewable energy [14].

These benefits are possible because BEB draw power from electrical infrastructure. The loads introduced by charging are substantial and can exceed the grid capacity [17][6][3], requiring prohibitively expensive upgrades. The cost of upgrading is reflected in the billing structure used by power providers and can make large-scale charging undesirable for consumers.

One approach to reducing charge costs is to defer upgrades by efficiently managing when and at what rates buses charge. However, developing charge plans must consider a number of factors. All buses must maintain a minimum charge level while adhering to route schedules. When charging, batteries must have sufficient charge time and share a limited number of chargers. All charging must also be done while other services draw power from the grid, acting as uncontrolled loads. The focus of this work is to find an optimal charge schedule which meets these requirements and minimizes the cost of grid use in the presence of uncontrolled loads. This problem is referred to hereafter as the ‘charge problem’.

The remainder of this paper is organized as follows: Section II describes related work and Section III outlines a graph-based framework for modeling the operations environment. Section IV extends the content of III to account for differences between day and night operations, and Section V incorporates the problem constraints involving battery charge dynamics. Sections VI and VII translate the rate schedule used for billing and into an objective function to minimize. Finally, Sections VIII, IX, and X briefly describe the optimization software used to solve the resulting mixed integer linear program developed in previous sections, presents results, and describes future work.

## II. LITERATURE REVIEW

This section summarizes prior work related to the charge problem and includes discussion on battery charging and managing runtime costs. The final subsection discusses the contributions of this paper, and how they relate to prior methods.

### A. Battery Charging

The refueling process for BEBs takes longer than the process for diesel and CNG buses [18]. A diesel or CNG engine can refuel in several minutes but an electric bus may require up to several hours, making the extended charge time a primary deterrent in converting to BEBs.

To circumvent long refuel times, [20] and [11] propose an approach which replaces batteries when the state of charge is low. The exchange would replace the current battery with one that was fully charged and return spent batteries to recharge afterward. Exchanging batteries would reduce down time, but is non-trivial because the replacement requires specialized tools and/or automation.

Another alternative is to inductively charge buses as they move about. Dynamic charging would simplify logistics because it eliminates the need to stop and recharge. Both [2] and [12] propose methods that inductively charge BEBs using specialized hardware in the road. A technique developed by [5] provides a planning framework for charger placement that maximizes the benefits of dynamic charging. Unfortunately, inductive charging requires infrastructure which may not be available and is costly and disruptive.

Refueling BEBs in a charge station allows buses to charge in the traditional sense and only requires an intelligent charge schedule. Following a charge schedule requires minimal infrastructure and utilizes native charge ports in the BEBs with no need for additional tools or automation. Planning algorithms use foreknowledge of the runtime environment and battery dynamics to identify when, and to which buses chargers should connect. Planning algorithms discussed in this review are considered on a scale from ‘reactive’ to ‘global’, where reactive methods focus only on the present, and global techniques use knowledge about the operating environment to form a plan.

Because reactive planning tend to focus on presential circumstances, they require minimal knowledge of the operational environment, making them extremely versatile. Methods of this type are both computationally efficient and adapt to many use cases. One such example is illustrated in [4], which splits the total power draw between the grid and an external battery to regulate the instantaneous load.

Reactive algorithms can be enhanced by encoding details for future events to improve decision making. If only event details within a finite horizon are used, the algorithm becomes a hybrid, containing elements from both reactive and global techniques. For example, [1] describes a technique for optimizing a charging schedule up to a scheduling horizon. Changing the horizon adjusts both the scope and computational complexity of the solution. In stochastic environments, a smaller window would be beneficial as charge schedules must be frequently

recomputed, whereas in more stable circumstances, a larger window size would yield higher performance.

When an algorithm includes information over all time periods, it becomes global. Because global algorithms assume complete foreknowledge of future events, they provide globally optimal plans and give the highest performance. The authors of [10] provide a technique which assumes foreknowledge of the current grid use. The grid schedule is encoded in the algorithm to inform optimal charging periods. Both [19] and [7] assume that bus schedules are known a priori and use this knowledge to stagger charge times and meet operational constraints.

### B. Cost Optimization

In addition to charge constraints, This paper focuses on minimizing the cost associated with charging and minimizes over energy charges for on and off-peak, power (or demand) charges for on and off-peak, and facilities charges [16]. Prior work has dealt with charge costs in various ways.

The authors in [8] propose a method to forecast power use. Work done by [15] propose a method which reduces the demand charge by using power forecasts [8] to plan charge times. When forecasting is not possible, both [13] and [4] propose methods that decreases power demand by observing the load and drawing additional power from an on-site battery. Additionally, [7] minimized over on/off peak energy as part of their work.

### C. Contributions

This work develops a comprehensive charge schedule planning framework which extends the planner proposed by [19] to include multi-rate charging, uncontrolled loads, night/day charging, and a realistic rate schedule in [16]. Our method formulates the bus charge problem as a Mixed Integer Linear Program (MILP) and is unique because the objective function is the cost for the transit authority and includes charges for on-peak and off-peak energy use, on-peak and off-peak demand, and overall, or facilities, demand. The proposed framework handles contention for charging resources in a globally optimal manner which guarantees charger availability even when chargers are scarce.

Prior work has also made assumptions for night time charge behavior. This work eliminates the need for such by including night charging in the charge schedule. The modeling of night and day charging also includes their respective operational constraints such as charge rates, bus availability, and the number of available chargers.

Our work also seeks to understand how variable rate, as compared to single rate, charging affects the cost optimality. necessary? with the addition of variable charge rates and a more accurate representation of battery charging dynamics.

The final contribution is recognizing that our framework enables transit authorities and power providers to come together and find a mutually beneficial solution that predicts montly costs for transit authorities and infrastructure demand for power providers.

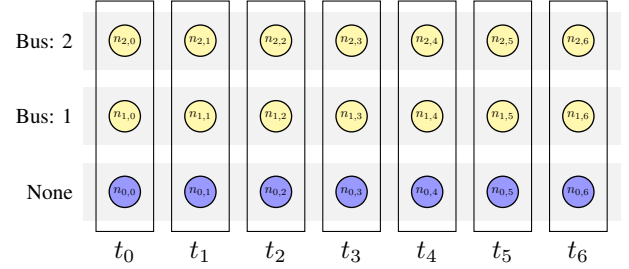


Fig. 1: Graph showing buses and timesteps

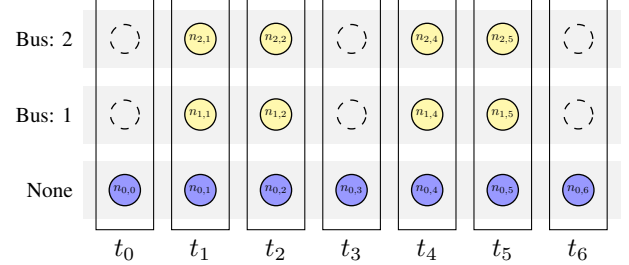


Fig. 2: Bus availability represented in a graph

## III. GRAPH BASED PROBLEM FORMULATION

This section formulates the charge problem as a graph. The first subsection describes the intuition behind this graph-based approach and the second represents the graph as a series of equality and inequality constraints in a Mixed Integer Linear Program (MILP).

### A. Graph Formulation

A solution to the bus charge problem must reveal both *when* and *to which* bus a charger should connect, suggesting a model with two dimensions. The first dimension characterizes time and is given discretely in a left to right fashion. The second dimension encodes the charger state and extends vertically as shown in figure 1. The charger may occupy one of several states. It may be connected to one of the  $N$  buses, or it may be unconnected, giving a total of  $N + 1$  charge states. The 2-D representation is encoded as a grid and a node,  $n_{i,j}$ , can be placed wherever a charge state and time index intersect. These nodes are used to represent a charger in the  $i^{\text{th}}$  charge state during the  $j^{\text{th}}$  time index (see figure 1). For example,  $n_{1,0}$  from figure 1 represents a state where a charger is connected to Bus 1 at  $t_0$ .

However, the presence of nodes at every state-time intersection suggests that buses can always connect. This does not reflect reality because buses cannot connect when they are away from the station. Nodes are therefore only present in the grid when a corresponding bus can connect to a charger and are absent when a bus leaves the station. Consider the two bus scenario from figure 1 where buses 1 and 2 provide transit services at  $t_0$ ,  $t_3$ , and  $t_6$ . The schedule is encoded by removing  $n_{1,0}$ ,  $n_{2,0}$ ,  $n_{1,3}$ ,  $n_{2,3}$ ,  $n_{1,6}$ , and  $n_{2,6}$  to reflect the grid shown in figure 2.

The state of a charger at any time is represented by occupying a corresponding node. Changes in charger state

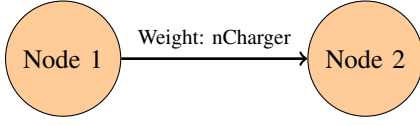


Fig. 3: Node to Node Connection

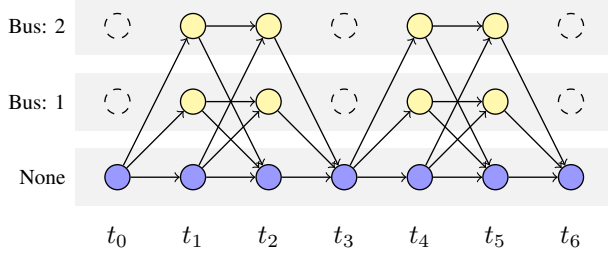


Fig. 4: Graph-based model of the complete decision-space

and time, are therefore reflected by a transition from one node to the next. These transitions are called edges (see figure 3) and represent decisions to connect to, charge, not charge, or disconnect from buses. An edge's decision is determined by the nodes on either end. Consider the edge from  $n_{0,0}$  to  $n_{0,1}$  in figure 5. This edge represents a no-charge decision because the nodes on both ends represent a disconnected charge state. Chargers cannot charge while disconnected, so the edge decision is no-charge. Similarly, the edge between  $n_{1,1}$  and  $n_{1,2}$  indicates a to-charge decision as both  $n_{1,1}$  and  $n_{1,2}$  represent states where a charger is connected. Both to-charge and no-charge decisions are represented by *horizontal* transitions in the graph and only reflect the passing of time as no changes to the physical hardware are made.

Conversely, diagonal transitions imply physical hardware changes because they represent decisions where chargers connect to or disconnect from a bus. One such example from figure 5 includes the edge from  $n_{0,0}$  to  $n_{1,1}$ . The state represented by  $n_{0,0}$  is disconnected. This edge represents an interval where a charger is disconnected at  $t_0$  and connected at  $t_1$ , implying a 'to-connect' decision. The same logic applies in reverse for the edge between  $n_{1,2}$  and  $n_{0,3}$ . Hence, the bus charge problem can be described in terms of nodes and edges where nodes represent bus availability and edges encode all possible charge decisions.

A charge schedule can be thought of as a list of charge decisions that govern charge behavior. Because decisions are represented by edges in the graph, a schedule is also represented by a sequence of connected edges that form a path through the graph. If an edge is selected, or active, it is considered part of the path. Active and inactive edges are respectively represented edge weights equal to 1, and 0.

A graph with binary edge weights can only represent a plan for one charger. This representation can be expanded to represent an arbitrary number of chargers by using integer valued weights, where each weight gives the number of chargers in the transition.

Consider a three-charger scenario using the graph in figure 4. A solution where one charger is connected to Bus 1 from  $t_1$  to  $t_2$  and to Bus 2 from  $t_4$  to  $t_5$  would be expressed

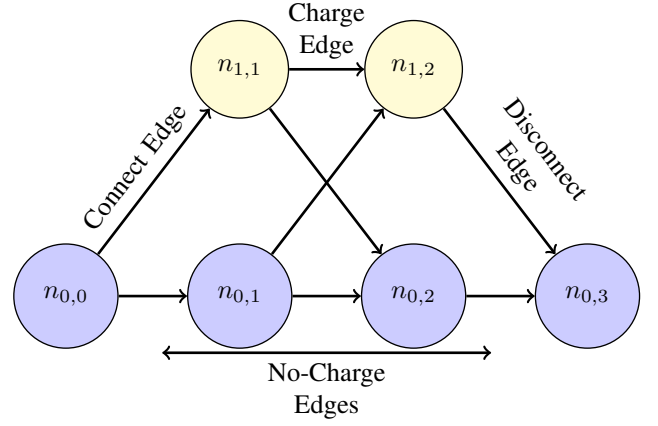


Fig. 5: Connect, Disconnect, and Charge Edges

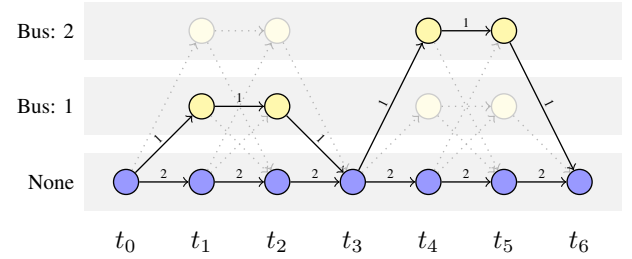


Fig. 6: One solution to a 2-bus 3-charger scenario

by assigning unit weights to the appropriate connect, charge, and disconnect edges. The second charger remains idle as illustrated by the active edges along the bottom row of charger states (see figure 6).

In summary, the graph encodes bus availability with nodes, decisions with edges, and schedules with edge weights. Solving the bus charge problem becomes a matter of finding the optimal set of edge weights, where optimal is used to denote the most cost effective charge plan.

### B. Graph Constraints

Finding the optimal charge schedule can be expressed as an optimization problem, where the graph is encoded through equality/inequality constraints for a Mixed Integer Linear Program of the form

$$\begin{aligned} \min_{\mathbf{y}} \quad & \mathbf{r}^T \mathbf{y} \text{ subject to} \\ & F\mathbf{y} = \mathbf{f}, \quad Q\mathbf{y} \leq \mathbf{q} \end{aligned} \quad (1)$$

where the equality and inequality constraints are encoded in  $F$ ,  $\mathbf{f}$ ,  $Q$  and  $\mathbf{q}$ . The variable  $\mathbf{y}$  is a vector containing the elements of the solution and is of the form

$$\mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \mathbf{d} \\ \mathbf{g} \\ \mathbf{e} \\ \mathbf{p} \\ \hat{p} \\ \hat{p}_{\text{off-peak}} \\ \hat{p}_{\text{on-peak}} \end{bmatrix}, \quad (2)$$

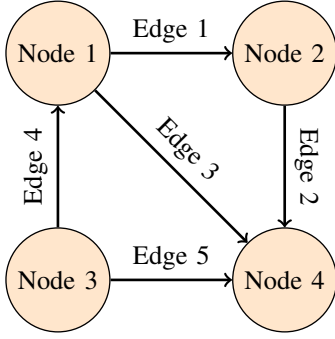


Fig. 7: A generic directed graph consisting of nodes and edges

where each element of  $\mathbf{y}$  is defined later in this paper.

This subsection formulates two sets of constraints. The first encodes the graph, enforces conservation of chargers, and defines the number of chargers through a set of net-flow constraints. The second prevents the charger from reconnecting shortly after disconnecting, and allows only a single charger to connect at a time by enforcing group-flow constraints.

1) *Net-Flow Constraints:*

$$A\mathbf{x} = \mathbf{c}_f, \quad (3)$$

where  $A$  is the graph incidence matrix,  $\mathbf{x}$  is the  $n\text{Edge} \times 1$  vector of edge weights and corresponds to  $\mathbf{x}$  in equation 2, and  $\mathbf{c}_f$  is  $n\text{Node} \times 1$  and equals the difference between incoming and outgoing edge weights, or *net-flow*.

An incidence matrix organizes relationships between nodes and edges by describing which edges leave and enter which nodes. The matrix  $A$  is an  $n\text{Node} \times n\text{Edge}$  matrix and expresses incoming connections between the  $i^{\text{th}}$  node and  $j^{\text{th}}$  edge as  $A_{i,j} = 1$ . Similarly, outgoing connections are given with  $A_{i,j} = -1$ , and no connection with  $A_{i,j} = 0$ . For example, the graph in figure 7 is represented as:

$$\begin{bmatrix} -1 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad (4)$$

The difference between the number of chargers entering and leaving, or the *net-flow*, can be expressed in terms of  $A$  as seen in equation 3. Because the number of chargers must be conserved, the number of incoming and outgoing chargers must be equal. This is expressed in linear form as  $a_i^T \mathbf{x} = 0$ , where  $a_i$  is the  $i^{\text{th}}$  row of  $A$ . The only exceptions occur at *source* and *sink* nodes.

A source node represents the beginning state for all chargers. Because edges originate here, there are no incoming edges and the net-flow will be minus the number of chargers. This is described in linear form as  $a_i^T \mathbf{x} = -n\text{Charger}$ .

Sink nodes represent the final state, where all edges terminate (see figure 8). Because sinks have no outgoing edges, they maintain a positive net-flow equal to the number of chargers and is expressed as  $a_i^T \mathbf{x} = n\text{Charger}$ .

Therefore, the *flow constraints* require that  $\mathbf{c}_f$  be equal to zero for all non-source and non-sink nodes as seen in equation 5.

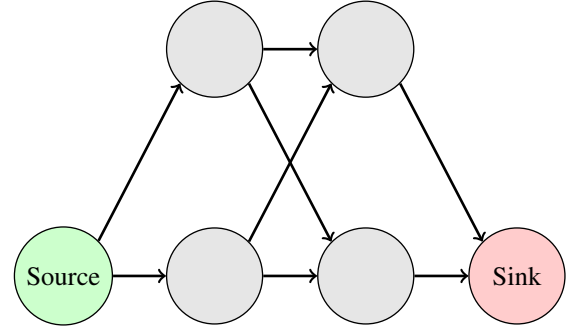


Fig. 8: Network flow illustrating sources and sinks

$$A\mathbf{x} = \begin{bmatrix} 0 \\ \vdots \\ -n\text{Charger} \\ \vdots \\ 0 \\ n\text{Charger} \\ \vdots \\ 0 \end{bmatrix}. \quad (5)$$

Equation 5 can be formulated in terms of  $\mathbf{y}$  by appropriately zero-padding  $A$  such that

$$\begin{aligned} \mathbf{c}_f &= \begin{bmatrix} A & 0 \end{bmatrix} \mathbf{y} \\ &= \tilde{A} \mathbf{y} \end{aligned} \quad (6)$$

2) *Group-flow Constraints:* Another flow type, known as group flow, can be used to regulate the number of chargers entering a set of nodes. This is desired for two reasons: It avoids multiple reconnects during a charge period, and limits the number of connecting chargers to one.

Define a charge group as the set of all nodes for a given bus corresponding to one station visit as shown in figure 9. The *group flow* is the number of chargers that enter a group and is represented as the sum of all incoming edge weights (see figure 10).

Denote the  $n\text{Group} \times n\text{Edge}$  group incidence matrix as  $B$ , where  $B_{i,j}$  is 1 if the  $j^{\text{th}}$  edge enters the  $i^{\text{th}}$  group and 0 otherwise. For example, the group incidence matrix corresponding to the graph in figure 11 contains 1 in the 7<sup>th</sup> and 10<sup>th</sup> columns for Group 1, and the 12<sup>th</sup> and 15<sup>th</sup> columns for group 2 as given in equation 7.

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (7)$$

Let  $\mathbf{x}$  be the edge weights as before and  $\mathbf{c}_g$  be an  $n\text{Group} \times 1$  vector where the  $i^{\text{th}}$  element gives the group flow for group  $i$ . The group flow is then computed as

$$B\mathbf{x} = \mathbf{c}_g \quad (8)$$

But the group flow is required to be one at most to avoid thrashing. This is expressed by the inequality given in equation

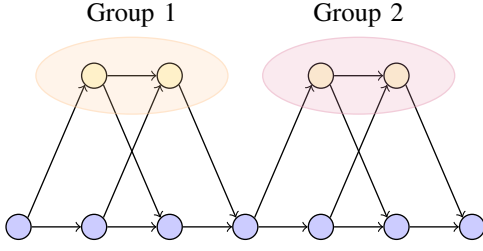


Fig. 9: Example of groups in a network flow graph

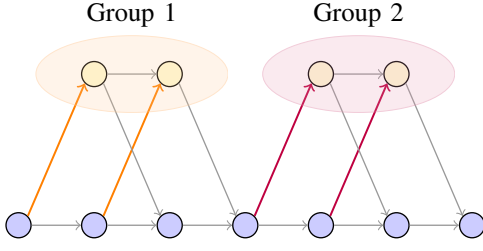


Fig. 10: Incoming Group Edges

9.

$$Bx \leq \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}. \quad (9)$$

Similarly to equation 6, equation 9 can also be expressed in terms of  $\mathbf{y}$  with appropriate zero padding as

$$\begin{bmatrix} B & 0 \end{bmatrix} \mathbf{y} = \mathbf{1} \\ \tilde{B}\mathbf{y} = \mathbf{1}. \quad (10)$$

### C. Section Summary

In summary, the bus charge problem can be formulated as a graph with nodes and edges, where charge plans are encoded as a path. The charge problem aims to find a feasible path which minimizes the cost of power. Feasibility is defined through a set of net-flow and group-flow constraints. Net-flow constraints are encoded through an adjacency matrix and enforce both the conservation and total number of chargers. The group-flow constraints prevent charge thrashing and regulate the number of simultaneous charger-to-bus connections.

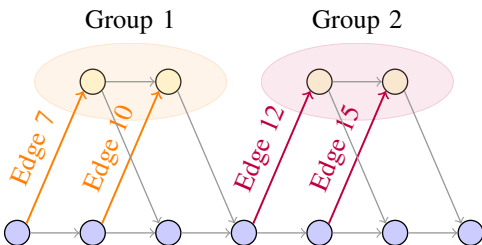
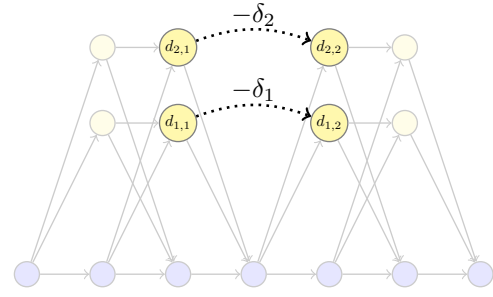


Fig. 11: Connect edge example for groups

Fig. 12: Relationship between exit nodes (left) and entrance nodes (right) as  $\delta$ 

## IV. BATTERY STATE OF CHARGE

Battery state of charge (SOC) plays a role in the bus charge problem. Battery charge levels decrease as a bus traverses a route. Solutions to the bus charge problem account for routes and require that SOC values remain above a minimum threshold.

A SOC thresholding constraint requires that battery charge levels be modeled. The  $k^{\text{th}}$  SOC for bus  $i$  is denoted  $d_{i,k}$ , where  $k$  is the *node index*. Node indices are time agnostic such that  $d_{i,k+1}$  represents the SOC bus node following  $d_{i,k}$  as seen in figure 13. The set of all  $d_{i,k}$  can be organized as the vector  $\mathbf{d}$  from equation 2.

Because no charging is performed while on route,  $d_{i,k}$  will be lowest when buses enter the charge station. Let  $d_{i,k+1}$  be the charge level for bus  $i$  as it enters the charge station, and  $\delta_i$  represent the power discharged while on-route. The entrance SOC can be expressed as

$$d_{i,k+1} = d_{i,k} - \delta_i, \quad (11)$$

where  $d_{i,k}$  is the previous departure SOC for bus  $i$ . Consider the example in figure 12, where buses 1 and 2 leave the station at  $t_2$  and enter at  $t_4$ . The corresponding change in SOC is given as  $d_{1,2} = d_{1,1} - \delta_1$  and  $d_{2,2} = d_{2,1} - \delta_2$  for buses 1 and 2 respectively. The constraints from equation 11 can be expressed in linear standard form as

$$\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} d_{i,k} \\ d_{i,k+1} \end{bmatrix} = \delta_i. \quad (12)$$

Equation 12 can be expressed in terms of  $\mathbf{y}$  with appropriate zero padding and expanded to account for the decrease in SOC for all buses outside the station. The expanded constraint is given as

$$\begin{bmatrix} 0 & \dots & -1_{d_{i,k}} & 0 & \dots & 1_{d_{i,k+1}} \end{bmatrix} \mathbf{y} = \mathbf{d}_\delta \\ D_\delta \mathbf{y} = \mathbf{d}_\delta, \quad (13)$$

where  $-1_{d_{i,k}}$  and  $1_{d_{i,k+1}}$  represent  $-1$  and  $1$  in locations corresponding to  $d_{i,k}$  and  $d_{i,k+1}$  respectively. Similar notation will be used throughout this paper as a means to imply a corresponding index for other variables.

Time periods between entrance and exit nodes represent time spent in the charge station and have the potential to charge the battery. An edge over which charging occurs is referred to as  $x_{i,k}$ , where  $k$  gives the index of the edge's outgoing node, and  $i$  refers to the bus. When a charger occupies  $x_{i,k}$ , the



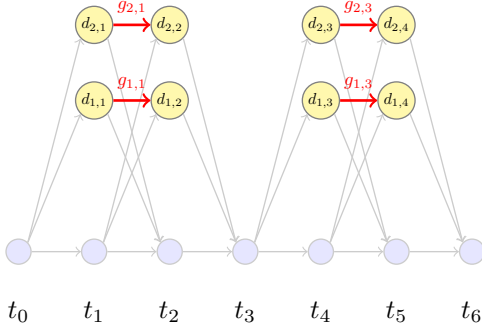


Fig. 13: Depiction of which edges increase SOC for the single rate case

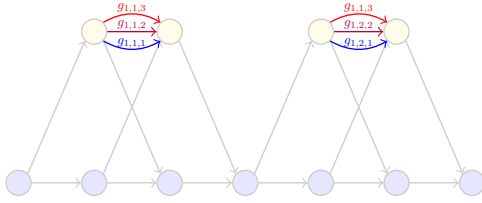


Fig. 14: Multi-Rate Charging

resulting increase, or *gain*, in battery charge is denoted  $g_{i,k}$ , where  $i$  and  $k$  mirror the edge indices (see figure 13).

The value for  $g_{i,k}$  is computed using a single charge rate. Multiple charge rates can be encoded by connecting bus nodes with multiple edges, denoted  $x_{i,k,l}$ , where each edge has a distinct charge rate and gain denoted  $g_{i,k,l}$  (see figure 14). Having multiple charge rates gives the option for fast charging when necessary, and slow charging when possible to preserve battery health and decrease the electrical load [9].

The rate is selected by setting  $x_{i,k,l} = 1$ . All gains associated with unselected rates are equal to zero. Gains that correspond to selected rates are computed using the Constant Current Constant Voltage (CCCV) model as derived in [19] which gives:

$$d_{i,k+1} = \bar{a}_l d_{i,k} - \bar{b}_l M, \quad (14)$$

where  $\bar{a}_l \sim (0, 1]$ , depends on the charge rate and is experimentally determined,  $M$  is the battery charge capacity in kWh, and  $\bar{b}_l = \bar{a}_l - 1$ . Equation 14 is used to show that

$$\begin{aligned} d_{i,k+1} &= \bar{a}_l d_{i,k} - \bar{b}_l M \\ d_{i,k+1} - d_{i,k} &= \bar{a}_l d_{i,k} - \bar{b}_l M - d_{i,k}, \end{aligned} \quad (15)$$

but the gain is equal to the difference in  $d_{i,k+1}$  and  $d_{i,k}$  such that  $g_{i,k,l} = d_{i,k+1} - d_{i,k}$ . So

$$\begin{aligned} g_{i,k,l} &= \bar{a}_l d_{i,k} - \bar{b}_l M - d_{i,k} \\ g_{i,k,l} &= (\bar{a}_l - 1) d_{i,k} - \bar{b}_l M. \end{aligned} \quad (16)$$

Therefore,

$$\begin{cases} g_{i,k,l} = d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M & x_{i,k,l} = 1 \\ g_{i,k,l} = 0 & x_{i,k,l} = 0 \end{cases}. \quad (17)$$

The conditions given in equation 17 can be rewritten as

$$\begin{aligned} &\begin{cases} g_{i,k,l} \leq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M \\ g_{i,k,l} \geq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M \end{cases} & x_{i,k,l} = 1 \\ &\begin{cases} g_{i,k,l} \leq 0 \\ g_{i,k,l} \geq 0 \end{cases} & x_{i,k,l} = 0 \end{aligned} \quad (18) \\ \Rightarrow &\begin{aligned} g_{i,k,l} &\leq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M - M(1 - x_{i,k,l}) \\ g_{i,k,l} &\geq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M \\ g_{i,k,l} &\leq 0 + Mx_{i,k,l} \\ g_{i,k,l} &\geq 0, \end{aligned} \end{aligned}$$

where  $M$  is the battery capacity. The results of equation 18 obtain a switching effect. When  $x_{i,k,l} = 1$ , equation 18 becomes

$$\begin{aligned} &\left. \begin{aligned} g_{i,k,l} &\leq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M \\ g_{i,k,l} &\geq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M \end{aligned} \right\} \text{Active} \\ &\left. \begin{aligned} g_{i,k,l} &\leq M \\ g_{i,k,l} &\geq 0 \end{aligned} \right\} \text{Inactive} \end{aligned} \quad (19)$$

The active constraints imply equality for  $g_{i,k,l} = (\bar{a}_l - 1)d_{i,k} - \bar{b}_l M$ . The inactive constraints imply that  $g_{i,k,l}$  is greater than zero and less than the battery capacity, which are trivially satisfied. When  $x_{i,k,l} = 0$ , equation 18 becomes

$$\begin{aligned} &\left. \begin{aligned} g_{i,k,l} &\leq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M - M \\ g_{i,k,l} &\geq d_{i,k}(\bar{a}_l - 1) - \bar{b}_l M \end{aligned} \right\} \text{Inactive} \\ &\left. \begin{aligned} g_{i,k,l} &\leq 0 \\ g_{i,k,l} &\geq 0 \end{aligned} \right\} \text{Active} \end{aligned} \quad (20)$$

Where the inactive constraints are again trivially satisfied, and the active imply equality for  $g_{i,k,l} = 0$ .

Equation 18 can be expressed in standard form as

$$\begin{aligned} -g_{i,k,l} + d_{i,k}(\bar{a}_l - 1) + x_{i,k,l} &\leq M(\bar{b}_l + 1) \\ g_{i,k,l} - d_{i,k}(\bar{a}_l - 1) &\leq -\bar{b}_l M \\ g_{i,k,l} - Mx_{i,k,l} &\leq 0 \\ -g_{i,k,l} &\leq 0 \end{aligned} \quad (21)$$

and in matrix form as

$$\begin{bmatrix} -1 & \bar{a}_l - 1 & 1 \\ 1 & 1 - \bar{a}_l & 0 \\ 1 & 0 & -M \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{i,k,l} \\ d_{i,k} \\ x_{i,k,l} \end{bmatrix} \leq \begin{bmatrix} M(\bar{b}_l + 1) \\ -\bar{b}_l M \\ 0 \\ 0 \end{bmatrix}. \quad (22)$$

Equation 22 can be expanded to include constraints for all  $g_{i,k,l}$ . Because each value for  $g_{i,k,l}$ ,  $d_{i,k}$ , and  $x_{i,k,l}$  is an element of  $\mathbf{y}$ , the constraints from equation 22 can be written as

$$\mathbf{G}\mathbf{y} \leq \mathbf{b}_g. \quad (23)$$

The value of  $d_{i,k}$  can be expressed as

$$d_{i,k+1} = d_{i,k} + \sum_l g_{i,k,l} \quad (24)$$

or

$$d_{i,k+1} - d_{i,k} - \sum_l g_{i,k,l} = 0 \quad (25)$$

because a non-zero element of  $g_{i,k,l}$  is only present for one corresponding  $l$ . This relationship is described in terms of an equality constraints such that

$$\begin{bmatrix} 1 & -1 & \dots & -1 \end{bmatrix} \begin{bmatrix} d_{i,k+1} \\ d_{i,k} \\ g_{i,k,1} \\ \dots \\ g_{i,k,l} \end{bmatrix} = 0. \quad (26)$$

Equation 26 can be appropriately zero padded such that

$$\begin{bmatrix} 1_{d_{i,k+1}} & -1_{d_{i,k}} & \dots & -1_{g_{i,k,l}} \end{bmatrix} \mathbf{y} = 0. \quad (27)$$

and expanded to define the values for all  $d_{i,k} \ni k > 0$  as

$$D_d \mathbf{y} = \mathbf{0}. \quad (28)$$

The values for  $d_{i,0}$  are defined with initial SOC conditions with additional equality constraints, denoted  $\mathbf{d}_0$  such that

$$\begin{bmatrix} 1_{d_{1,0}} & 0 & 0 & \dots & 0 \\ 0 & \dots & 1_{d_{2,0}} & 0 & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1_{d_{i,0}} \end{bmatrix} \mathbf{y} = \mathbf{d}_0, \quad (29)$$

or

$$D_0 \mathbf{y} = \mathbf{d}_0. \quad (30)$$

Once all values for  $d_{i,k}$  are computed, they must be constrained to remain above a threshold  $\tau$ . The SOC thresholding constraint can be expressed as an inequality constraint such that

$$\begin{aligned} d_{i,k} &\geq \tau \\ \Rightarrow -d_{i,k} &\leq -\tau \\ \Rightarrow \begin{bmatrix} 0 & \dots & -1_{d_{i,k}} & \dots & 0 \end{bmatrix} \mathbf{y} &\leq -\tau \end{aligned} \quad (31)$$

Equation 31 can be expanded to a matrix  $D_\tau$ , where each  $d_{i,k}$  contains a corresponding constraint row such that

$$\begin{aligned} D_\tau \mathbf{y} &\leq \begin{bmatrix} -\tau \\ \vdots \\ -\tau \end{bmatrix} \\ &\leq \mathbf{d}_\tau \end{aligned} \quad (32)$$

In summary, the minimum SOC for all feasible charge plans must exceed a given threshold. SOC values are computed while the bus is in the charge station. SOC values are updated when a bus enters by subtracting the discharged energy from the previous SOC estimate. SOC values are updated for in-station periods by adding the charge gains as given in equation 24. Gains are computed using a switching constraint which sets them to zero when not charging, otherwise they follow the CCCV model as derived in equation 16. Initial SOC values are handled with the equality constraint given in equation 30 and the SOC is constrained to remain above the threshold  $\tau$  in equation 32. All constraints for  $d$  can be concatenated such that

$$\begin{bmatrix} D_0 \\ D_\delta \\ D_d \end{bmatrix} \mathbf{y} = \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_\delta \\ \mathbf{0} \end{bmatrix}, \quad \begin{bmatrix} D_g \\ D_\tau \end{bmatrix} \mathbf{y} \leq \begin{bmatrix} \mathbf{d}_g \\ \mathbf{d}_\tau \end{bmatrix} \quad (33)$$

and expressed as

$$D_{eq} \mathbf{y} = \mathbf{d}_{eq}, \quad D_{ineq} \mathbf{y} \leq \mathbf{d}_{ineq}. \quad (34)$$

## V. MULTI-GRAPH ADDITIONS

An additional contribution this work offers is the expansion to night vs day charging. Day and night operations differ in two aspects: number of chargers, and bus availability. During the day, the buses can charge on chargers in the charge station. The number of chargers in the station are limited, causing contention between buses. At night, each bus docks in a holding stall with one charger per stall, eliminating charger contention.

Bus availability also changes because buses do not leave their stalls at night. This simplifies the charge problem because buses are always available for charging.

Equation 5 in section III-B1 describes the net-flow constraints which constrain the number of chargers in the source and sink nodes. Because the number of chargers are different from night to day, a separate graph is used at each transition as shown in figure 15.

Each graph is connected by equating the appropriate SOC values. Consider the multi-graph formulation given in figure 16. The morning graph is related to the day graph because  $d_{1,1}$  and  $d_{2,1}$  represent the same SOC values as  $d_{1,2}$  and  $d_{2,2}$  respectively. The same applies for the day and night graph, where  $d_{1,5}$  and  $d_{2,5}$  represent the SOC values for  $d_{1,6}$  and  $d_{2,6}$ . This equality relationship can be expressed as an equality constraint where

$$\mathbf{d}_{\text{graph } 1} - \mathbf{d}_{\text{graph } 2} = \mathbf{0} \quad (35)$$

or by

$$D_{\text{multi-graph}} \mathbf{y} = \mathbf{0}, \quad (36)$$

where  $D_{\text{multi-graph}}$  is an  $n_{\text{Bus}} \times n_{\text{Var}}$  matrix such that

$$D_{\text{multi-graph}} \mathbf{y} = \mathbf{d}_{\text{graph } 1} - \mathbf{d}_{\text{graph } 2}. \quad (37)$$

Because all SOC values  $d$  are contained in  $\mathbf{y}$ , forming the matrix  $D$  amounts to placing 1 and  $-1$  at the indices corresponding to  $d_{\text{graph } 1}$  and  $d_{\text{graph } 2}$  respectively and zero otherwise.

## VI. OBJECTIVE FUNCTION

The objective function in this work models the rate schedule used in [16], where the cost is modeled as the monthly charge a transit authority might receive from the power provider. The objective function includes charges for energy, power, and facility use and implements both on and off-peak rates.

The objective function also includes effects and costs of uncontrolled loads. Uncontrolled loads might include the effects of patrons charging personal electric vehicles, electric trains passing through, CNG stations, etc. The loads used in this work were recorded at the UTA intermodal hub station in Salt Lake City as the average power between time intervals.

### A. Energy

Energy cost is assessed per Kilowatt-hour of energy consumed and includes energy consumed by uncontrolled loads and bus chargers. Let  $\mathbf{p}$  be the average external power used at each timestep, where  $\mathbf{p}_i$  is the average power draw between

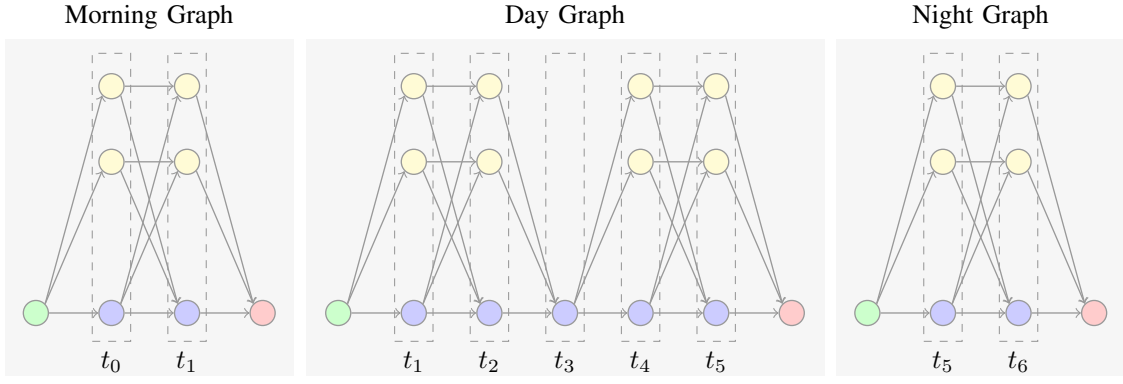


Fig. 15: Night and Day Graphs

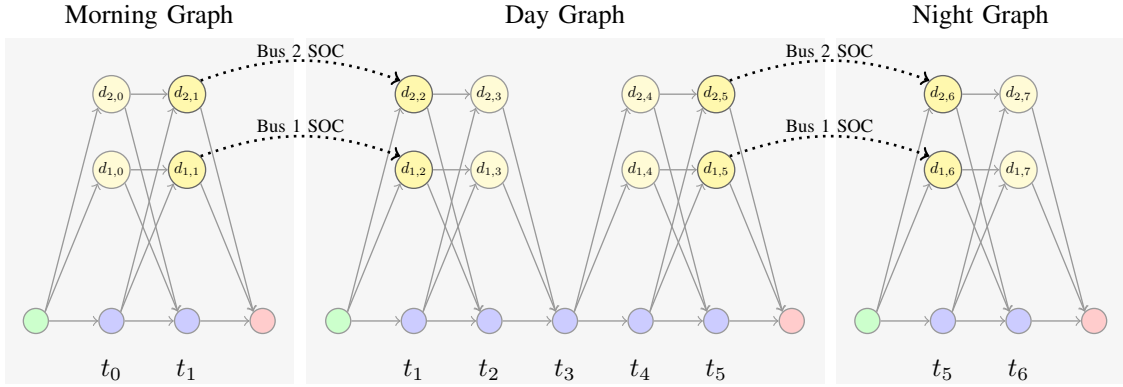


Fig. 16: Bus SOC between Night and Day Graphs

$t_j$  and  $t_{j+1}$ . The energy consumed by external loads from  $t_j$  to  $t_{j+1}$  is computed as

$$e_j^l = \mathbf{p}_i \cdot \Delta_t, \quad (38)$$

where  $\Delta_t$  is the change in time from  $t_j$  to  $t_{j+1}$  in hours. The energy consumed by bus chargers for the same interval is computed as

$$e_j^b = \sum_{k \in t} g_{i,k,l}, \quad (39)$$

where  $k \in t$  represents all values for  $g$  that took place between  $t_i$  and  $t_{i+1}$ . The total energy is computed as

$$e_j = e_j^l + e_j^b \quad (40)$$

The equation for 40 can be written in standard form as

$$e_j - \sum_{k \in t} g_{i,k,l} = p_i \cdot \Delta_t \quad (41)$$

$$\begin{bmatrix} 1_{e_j} & -1_{g_1} & \dots & -1_{g_n} \end{bmatrix} \begin{bmatrix} e_j \\ g_1 \\ \vdots \\ g_n \end{bmatrix} = p_i \cdot \Delta_t$$

Because power providers charge different rates for the total power consumed during the respective on and off-peak hours, equation 41 be modified to reflect the energy consumed in arbitrary time periods. Let  $T$  be a set of  $t_j$ , or just  $j$ , which will later be used to denote on and off-peak periods as  $T_{\text{on}}$

and  $T_{\text{off}}$ . Equation 41 can be expanded to compute the total energy consumed in  $T$  as

$$e_T - \sum_{k \in T} g_{j,k,l} = \left( \sum_{j \in T} p_j \right) \cdot \Delta_t \quad (42)$$

$$\begin{bmatrix} 1_{e_T} & -1_{g_1} & \dots & -1_{g_n} \end{bmatrix} \begin{bmatrix} e_T \\ g_1 \\ \vdots \\ g_n \end{bmatrix} = e_T^{\text{load}}$$

For multiple time periods, the constraint can be expanded in matrix form, where row  $i$  corresponds to the periods of time in  $T_i$ . Furthermore, by including the values for each  $e_{T_i}$  in  $\mathbf{y}$  and zero-padding appropriately, the expanded form of equation 42 can be written as

$$E\mathbf{y} = \mathbf{e}^{\text{load}}, \quad (43)$$

where row  $i$  in  $E$  reflects equation 42 for the time intervals in  $T_i$ , and  $\mathbf{e}_i^{\text{load}}$  contains the energy consumed by uncontrolled loads during  $T_i$ .

### B. Power

Power costs are computed for the maximum average power draw, where the average is computed over a 15 minute sliding window. The average power can be computed as the energy in the window divided by the window length in hours. In this case, a 15 minute window equates to a quarter hour. Let  $\bar{p}_j$



be the average power from  $j - 15$  to  $j$ . Equation 42 can be adapted to compute the average power as

$$\bar{p}_j - \left( \sum_{k \in T_j} \frac{1}{4} g_{i,k,l} \right) = \left( \sum_{i \in T_j} p_i \right) \cdot \frac{\Delta_t}{4}$$

$$\begin{bmatrix} 1_{\bar{p}_j} & -\frac{1}{4}g_1 & \dots & -\frac{1}{4}g_n \end{bmatrix} \begin{bmatrix} e_j \\ g_1 \\ \vdots \\ g_n \end{bmatrix} = \frac{p_T \cdot \Delta_t}{4}. \quad (44)$$

Equation 44 can further be expanded and zero padded to compute the average power at each time,  $t_j$  by applying equation 44 to the corresponding window as

$$P\mathbf{y} = \mathbf{p}. \quad (45)$$

The maximum average power, denoted  $\hat{p}$ , is greater than or equal to each average power computed in equation 45. This yields an additional set of inequality constraints

$$\begin{bmatrix} -1_{\hat{p}} & 1_{\bar{p}_0} & 0 & \dots & 0 \\ -1_{\hat{p}} & 0 & 1_{\bar{p}_1} & \dots & 0 \\ -1_{\hat{p}} & 0 & 0 & \dots & 1_{\bar{p}_j} \end{bmatrix} \mathbf{y} \leq \mathbf{0} \quad (46)$$

$$P_{\max} \mathbf{y} \leq \mathbf{0}.$$

Because the max average power is minimized in the objective function, the value for  $\hat{p}_{\max}$  will be forced down to the value of the greatest average power computed in equation 45, and accurately reflect the maximum average power.

### C. On/Off Peak Rates

Power providers divide each day into on and off-peak periods during which different rates are applied for both energy and power costs. Let  $H$  and  $L$  be the respective sets of all time indices in on and off peak periods respectively. The cost of energy during on-peak hours can be expressed as

$$c_{\text{energy}_H} = \left( \sum_{j \in H} e_j \right) r_{e_{\text{on}}} \quad (47)$$

$$= [r_{e_1} \ 0 \ \dots \ 0 \ r_{e_4} \ \dots \ 0] \mathbf{y}$$

$$= \mathbf{r}_{e_{\text{on}}}^T \mathbf{y},$$

where  $\mathbf{r}_{e_{\text{on}}}^{\text{on}}$  contains the value of  $r_{e_{\text{on}}}^{\text{on}}$  at the index corresponding to  $e_j$  in  $\mathbf{y} \ \forall j \in H$ . A similar formulation can be used to describe the cost of energy consumed during off-peak hours.

An on-peak rate also applies to charges for power. Equation 46 can be adapted to only include rows that correspond to average power values during on-peak hours such that

$$\begin{bmatrix} -1_{\hat{p}_{\text{on}}} & 1_{\bar{p}_0} & 0 & \dots & 0 \\ -1_{\hat{p}_{\text{on}}} & 0 & 1_{\bar{p}_1} & \dots & 0 \\ -1_{\hat{p}_{\text{on}}} & 0 & 0 & \dots & 1_{\bar{p}_j} \end{bmatrix} \mathbf{y} \leq \mathbf{0} \quad (48)$$

$$P_{\text{on}} \mathbf{y} \leq \mathbf{0}.$$

Similarly, the off-peak max average power can be computed as

$$\begin{bmatrix} -1_{\hat{p}_{\text{off}}} & 1_{\bar{p}_0} & 0 & \dots & 0 \\ -1_{\hat{p}_{\text{off}}} & 0 & 1_{\bar{p}_1} & \dots & 0 \\ -1_{\hat{p}_{\text{off}}} & 0 & 0 & \dots & 1_{\bar{p}_j} \end{bmatrix} \mathbf{y} \leq \mathbf{0} \quad (49)$$

$$P_{\text{off}} \mathbf{y} \leq \mathbf{0},$$

where each row corresponds to  $\bar{p}_j \ \forall j \in L$ .

Many power providers include a facilities charge. The facilities charge is assessed per kW of the maximum average power in *both* on and off-peak intervals. The total max average power is calculated using equation 46.

The total power cost can be computed as the sum of the on-peak, off-peak, and facilities charges as

$$c_{\text{power}} = [r_{\hat{p}_{\text{on}}} \ 0 \ \dots \ 0 \ r_{\hat{p}_{\text{off}}} \ 0 \ \dots \ 0 \ r_{\hat{p}_{\text{facilities}}}] \mathbf{y}$$

$$= \mathbf{r}_{\hat{p}}^T \mathbf{y} \quad (50)$$

### D. Objective Function

The objective function combines the cost of energy and power, where the on-peak and off-peak energy is combined as

$$c_{\text{energy}} = \mathbf{r}_{e_{\text{on}}}^T \mathbf{y} + \mathbf{r}_{e_{\text{off}}}^T \mathbf{y}$$

$$= (\mathbf{r}_{e_{\text{on}}} + \mathbf{r}_{e_{\text{off}}})^T \mathbf{y} \quad (51)$$

$$= \mathbf{r}_e^T \mathbf{y}.$$

The combined expression is given as

$$c_{\text{total}} = c_{\text{power}} + c_{\text{energy}}$$

$$= \mathbf{r}_e^T \mathbf{y} + \mathbf{r}_{\hat{p}}^T \mathbf{y} \quad (52)$$

$$= (\mathbf{r}_e + \mathbf{r}_{\hat{p}})^T \mathbf{y}$$

$$= \mathbf{r}^T \mathbf{y}.$$

Equation 52 is used as the objective function in a mixed integer linear program of the form

$$\min_{\mathbf{y}} \mathbf{r}^T \mathbf{y} \text{ subject to} \quad (53)$$

$$C_{\text{eq}} \mathbf{y} = \mathbf{c}_{\text{eq}}, \ C_{\text{ineq}} \mathbf{y} \leq \mathbf{c}_{\text{ineq}},$$

Where  $C_{\text{eq}}$ ,  $\mathbf{c}_{\text{eq}}$ ,  $C_{\text{ineq}}$ , and  $\mathbf{c}_{\text{ineq}}$  are formed by stacking the equality and inequality constraints from equations 6, 10, 34, 45, 46, 48, and 49.

$$\min_{\mathbf{y}} \mathbf{r}^T \mathbf{y} \text{ subject to}$$

$$\begin{bmatrix} \tilde{A} \\ D_{\text{eq}} \\ P \end{bmatrix} \mathbf{y} = \begin{bmatrix} \mathbf{c}_f \\ \mathbf{d}_{\text{eq}} \\ \mathbf{p} \end{bmatrix}, \quad \begin{bmatrix} \tilde{B} \\ D_{\text{ineq}} \\ P_{\max} \\ P_{\text{on}} \\ P_{\text{off}} \end{bmatrix} \mathbf{y} \leq \begin{bmatrix} \mathbf{1} \\ \mathbf{d}_{\text{ineq}} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (54)$$

## VII. RESULTS

This section contains analysis and results of the planning framework and is subdivided into three subsections: uncontested results, contested results, and multi-rate comparisons.

### A. Uncontested Results

This section compares the charge schedule associated with equation 54 with a schedule developed by a baseline algorithm. The baseline adheres to all the constraints given previously, but maximizes the objective

$$\max_{\mathbf{y}} \mathbf{1}^T \mathbf{B} \mathbf{y}, \quad (55)$$

which incentivizes bus charging when possible. An initial cost comparison is given for an uncontested scenario where the

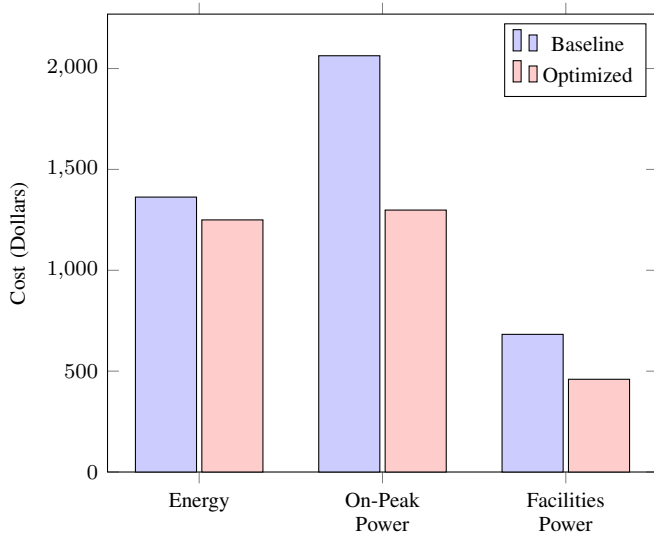


Fig. 17: Cost Comparison between optimized and baseline algorithms

number of chargers is equal to the number of buses. The cost breakdown is given in figure 17.

Note how the energy charge is equivalent, which indicates that similar amounts of energy were consumed in both scenarios. Despite the similarity in energy consumption, however, facilities and on-peak power chargers are significantly larger for the unoptimized schedule.

Figure 18 compares the average power use between the optimized and baseline algorithms. There are two primary differences between the two. The first is that the optimized schedule avoids charging during on-peak hours. The second is how the optimized charge schedule regulates each charge event to spread the power draw over larger, intermittent periods of time. Avoiding the on-peak time window and minimizing average power draw resulted in the cost disparities shown in figure 17.

Figure 19 separates the power draw into power from external loads and power used by buses and shows again how bus charging was optimized to avoid on-peak hours and spread power use over time.

### B. Contested Results

An optimized schedule can also be produced in resource contested environments. Contention occurs when the number of chargers is scarce in comparison to the number of buses and can require that buses charge in disoptimal circumstances. For example, if charging resources are saturated during off-peak hours, other buses might be forced to charge in the on-peak window.

Figure 20 shows the monthly cost from a scenario with one charger and multiple buses. Note the minimal cost increase per bus. Each subsequent bus incurs a minimal additional cost, showing that charge plans also minimize charge in the presence of contention.

Figure 21 demonstrates how cost is minimized in the presence of contention. Figure 21 compares loads between a

5 and 11 bus scenario, where only one charger is given and the uncontrolled loads are equivalent. In the 5 bus scenario, loads are distributed amongst off-peak hours, resulting in an optimized cost. The 11 bus scenario requires significantly more power and is forced to charge during on-peak hours. Note however that average power use is kept relatively low and seldom increases the maximum average power for on-peak use. Both scenarios also make ample use of night charging, where the number of chargers is the same as the number of buses.

### C. Multi-Rate Comparison

Figure 22 shows the comparison between multi and single-rate charging. Both scenarios include 5 buses and 1 charger. As shown in figure 22, the cost difference is marginal at best. The cost of the multi-rate scenario is \$3006.94 and the cost of the single-rate scenario is \$3007.77. The small cost decrease is also consistent with larger scenarios as shown in figure 23.

Upon further observation, maximum charge edges in multi-rate scenarios are used more frequently than their low-charge counterparts as shown in figure 24. This would help explain the similarities in cost. If the highest rate is almost always selected, then the resulting plan would resemble a single-rate schedule.

Cost is also based on the average instantaneous power. Because both single and multi-rate plans charge buses in relatively small time periods the average power use is kept relatively low (see figures 21, and 20). There are also many circumstances when a bus's charge time is limited, resulting in high charge rates.

## VIII. FUTURE WORK

Future work should extend to preserving battery health. The algorithms presented in this paper were primarily focused on minimizing average power draw and avoiding on-peak time periods. High instantaneous power use, as observed in this work, can reduce the lifespan of the battery. A battery-centric scenario would lend itself well to a multi-rate charging framework.

Additionally, this work included only discrete charge rates. Future work might generalize to continuous charge selection, and real-time deployment. Because this work presents a planning framework that gives a global solution for large stretches of time, it is computationally expensive. During real-time operations, additional work might include algorithms to handle deviations in a stochastic environment, where the objective is to get back to the global plan.

The global plan could also be improved by optimising in a more robust manner, where areas of high uncertainty for either bus schedules or uncontrolled loads are generally avoided.

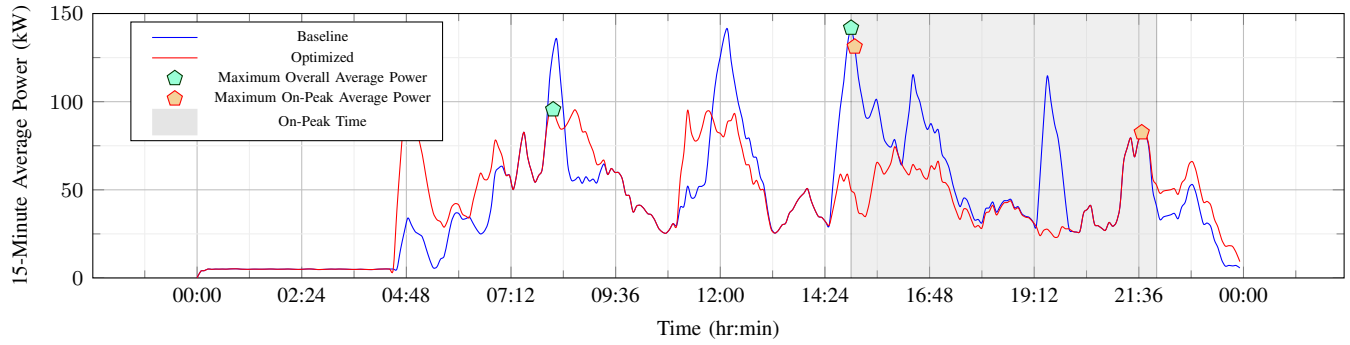


Fig. 18: 15-Minute Average Power for one day

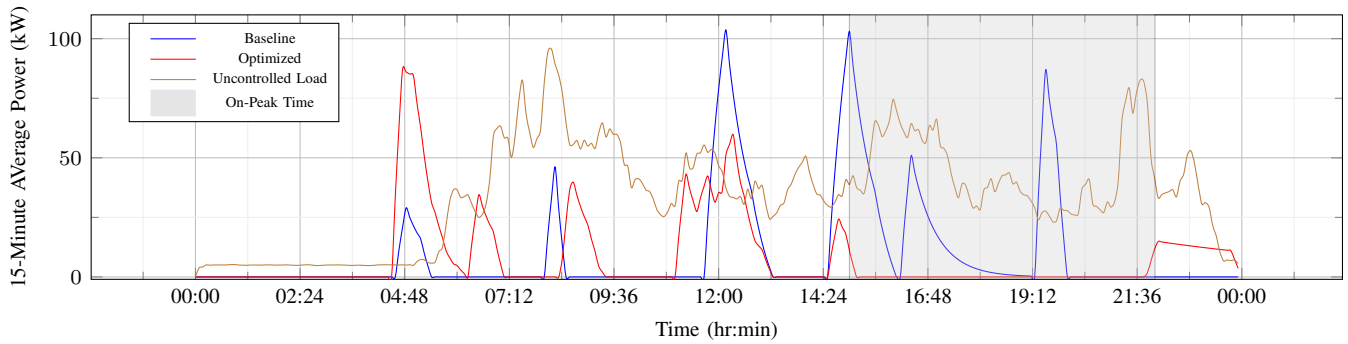


Fig. 19: Comparison between uncontrolled and bus loads

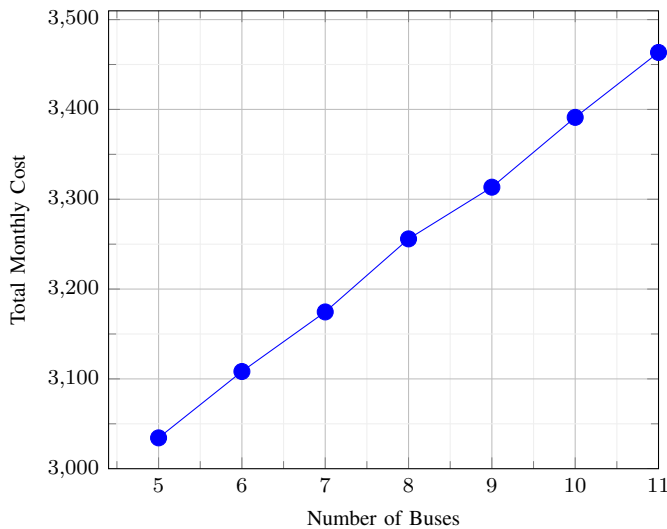


Fig. 20: Results for several single charger scenarios.

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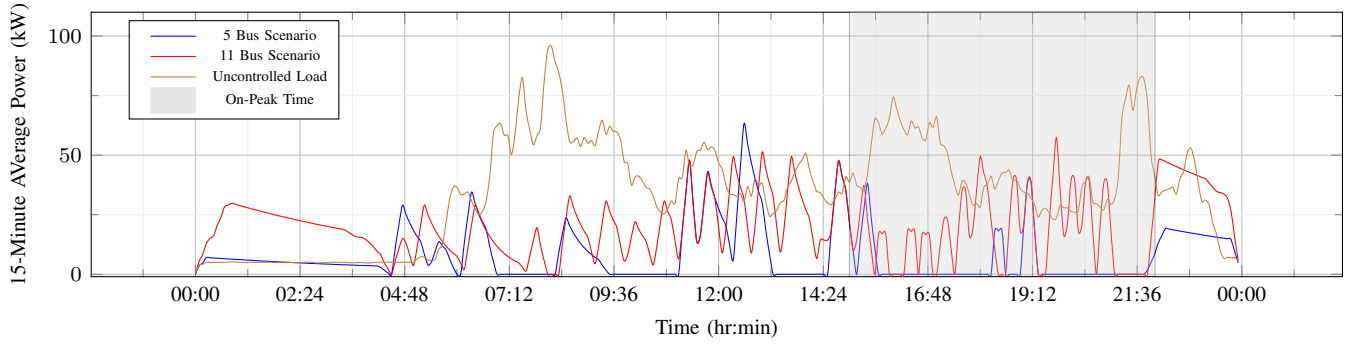


Fig. 21: Comparison of the loads for a 5 and 11 bus scenario with one overhead charger

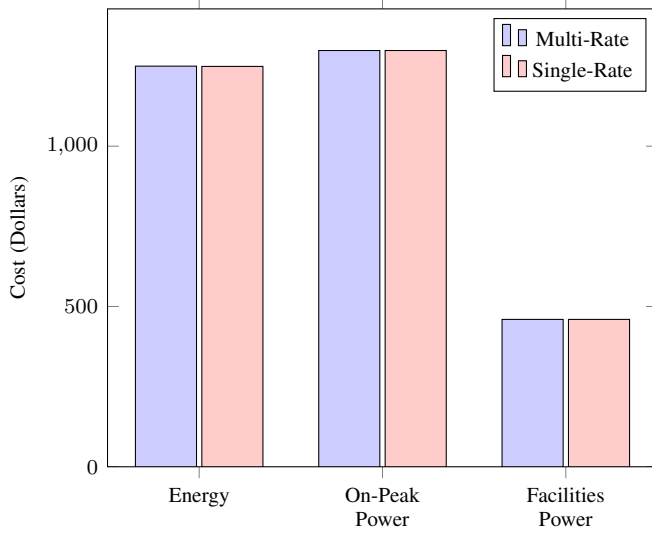


Fig. 22: Cost Comparison between a multi-rate and single-rate charge schedule

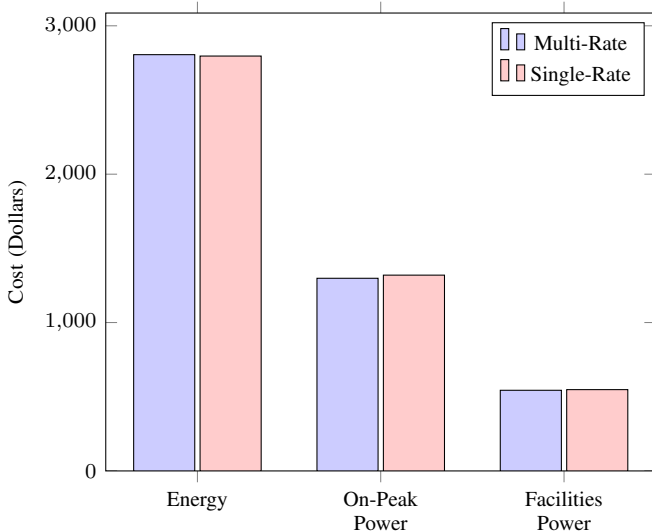


Fig. 23: Cost Comparison between multi and single rate charging for a 35 bus 6 charger scenario

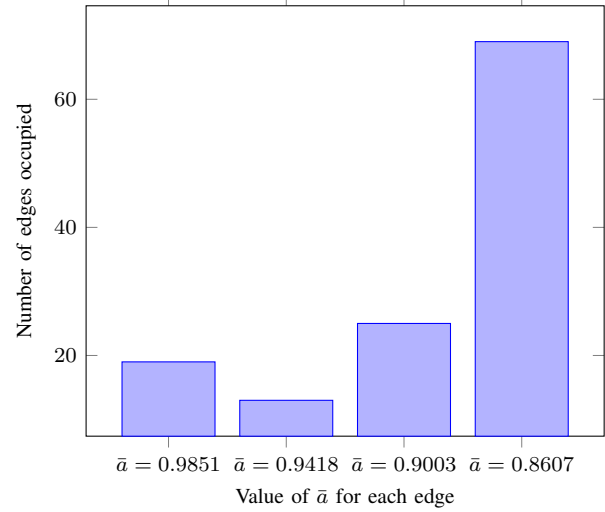


Fig. 24: Histogram of charge rates

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