A Continuous Approach to Minimize Cost for Charging Electric Bus Fleets

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***Abstract*—** temp

***Index Terms*—Battery Electric Buses, Cost Minimization, Multi-Rate Charging, Mixed Integer Linear Program**

I. Introduction

II. Literature Review

— Notes to you

— Specific changes

??: Not sure what the underlined / circled phrase means

← not clear

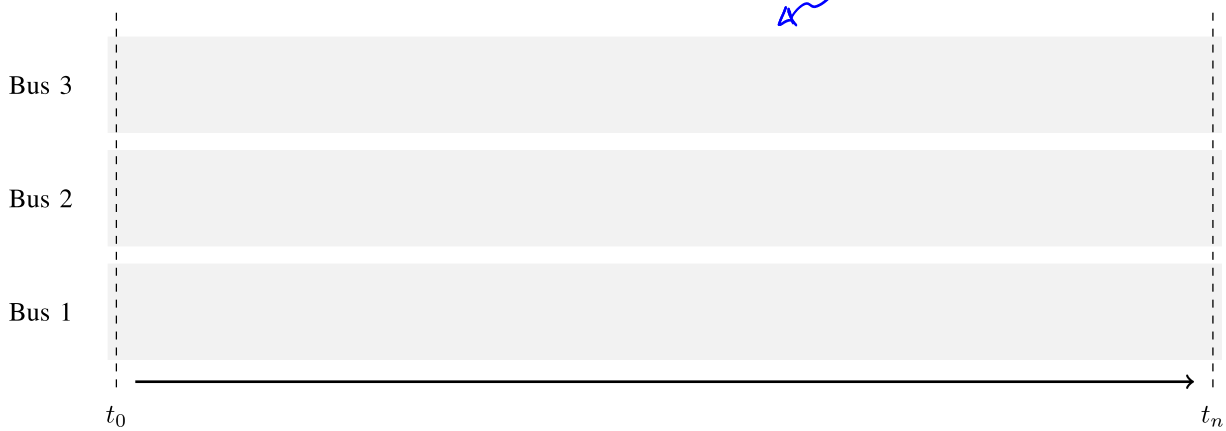

Fig. 1: Description of the bus and time axis

To maintain continuity, you should probably name this section to be descriptive of the charge queuing which it discusses.

III. Continuous Problem Formulation

The charge scheduling framework described in this paper is formulated as a constrained optimization problem that can be solved as a Mixed Integer Linear Program (MILP) of the form

$$\begin{aligned}&\min_{\mathbf{y}} \mathbf{y}^T \mathbf{g} \text{ subject to} \\ &\tilde{A}\mathbf{y} = \tilde{\mathbf{b}},\ A\mathbf{y} \le \mathbf{b},\end{aligned} \tag{1}$$

where $\mathbf{y}$, $\tilde{A}$, A, and $\mathbf{g}$ represent the solution vector, equality and inequality constraints, and cost vector respectively. In this paper, $\mathbf{y}$ is comprised of several variables, and is expressed as

Why is σ not bold?

$$\mathbf{y} = \begin{bmatrix} \sigma \\ \mathbf{c} \\ \mathbf{s} \\ \mathbf{h} \\ \mathbf{k} \\ \mathbf{r} \\ \mathbf{g} \\ \mathbf{p} \\ q_{\text{on}} \\ q_{\text{all}} \end{bmatrix}, \tag{2}$$

← Would be good to have a table defining notation so reader doesn't have to wait.

where σ, $\mathbf{c}$, $\mathbf{s}$, $\mathbf{h}$, $\mathbf{k}$, $\mathbf{r}$, $\mathbf{p}$, q_{on}, and q_{all} will be developed through the course of this paper.

The cost function in equation 1 ~~models~~ will be designed to model a realistic billing structure used by reference to rocky mountain power schedule 8 and minimises the cost even in the presence of uncontrolled loads. Additionally, the constraints are designed to encorporate bus schedules, limit bus state of charges, and include a linear charge model calibrated on data from the Utah Transit Authority.

← (1) Doesn't look like it yet, so you may not want to say it is.

A. Setup

What about charge duration?

Solving the bus charge problem requires knowing when, and on which charger a bus must charge, indicating a solution with two dimensions. The first dimension represents time continuously from left to right, and the second describes the buses as shown in Fig. 1.

I would recommend discussing more of the rectangle packing problem to motivate the "two dimensions"

Each bus follows a schedule made up of a series of arrival and departure times, where the bus's n^{th} stop begins at arrival time a_n and terminates at departure time d_n (see Fig. 2).

Is this the correct grammar

The next section immediately uses a double notation. You should introduce that here (i.e. the i^{th} bus's n^{th} arrival as a_{in})

A bus can be assigned to charge anytime the bus is in the station. The charge start time during the n^{th} stop is denoted c_n and the stop-charge time is denoted s_n as shown in Fig. 2.

B. Constraints

The relationship between the arrival, departure, and charge intervals for the i^{th} bus at the j^{th} stop can be expressed as a set of inequality constraints such that

You need to specify what variables are optimization variables

$$\begin{cases} a_{ij} < c_{ij} \\ c_{ij} < s_{ij} \\ s_{ij} < d_{ij} \end{cases} \tag{3}$$

what is this?

Which can be expressed in standard form as

$$\begin{aligned} -c_{ij} &< -a_{ij} \\ c_{ij} - s_{ij} &< 0 \\ s_{ij} &< d_{ij} \end{aligned} \tag{4}$$

and finally,

$$\begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{ij} \\ s_{ij} \end{bmatrix} \le \begin{bmatrix} -a_{ij} \\ 0 \\ d_{ij} \end{bmatrix}. \tag{5}$$

Equation 5 can be expressed in terms of $\mathbf{y}$ such that

not defined

$$\begin{gathered}\begin{bmatrix} -1_c & 0 & \dots & 0 \\ 1_c & 0 & \dots & -1_d \\ 0 & 0 & \dots & 1_d \end{bmatrix} \mathbf{y} \le \begin{bmatrix} -a \\ 0 \\ d \end{bmatrix} \quad \forall i, j \\ A_1 \mathbf{y} \le \mathbf{b}_1, \end{gathered} \tag{6}$$

not defined

Perhaps this should be a_i: vector of all arrival times

In (2), you specify vectors in bold. You should use bold for a and d

i, j do not appear in this equation

where A_1 and $\mathbf{b}_1$ stack the constraints given in equation 5 for all i, j.

Need to specify optimization variables

A charge plan can be formulated by reserving time slots at chargers when buses need to charge (see Fig. 3). Note how there are several decisions that go into the charge schedule, namely when and to which charger a bus will connect. ← and how long to connect

← what is meant by this?

The variables for time have already been discussed in ~~Equation~~ 5 but variables for which charger have not been given. ?? Let σ_{ijk} be a binary variable that is 1 when bus i charges

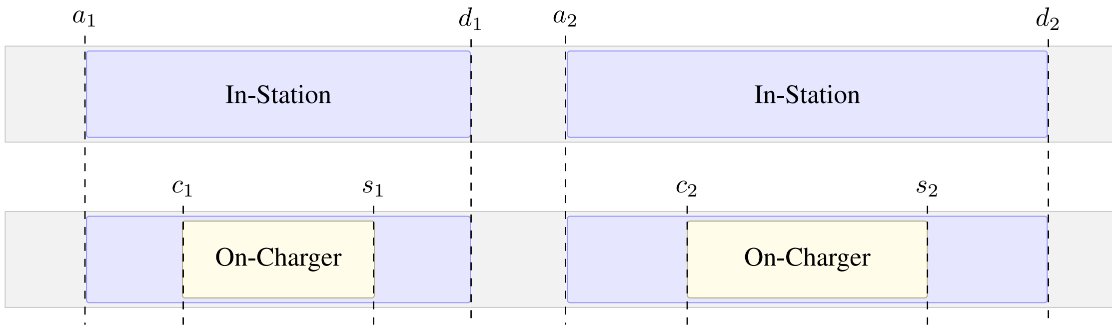

Fig. 2: Bus Charging

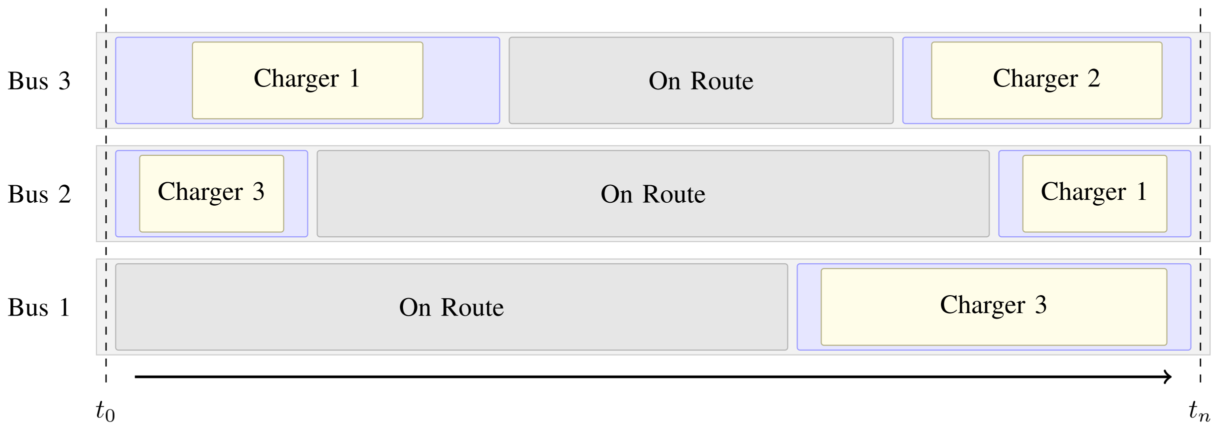

Fig. 3: Reserving time slots on chargers

Need to describe how σ_{ijk} relates to σ

during the j^{th} stop at charger k. Because a bus can only charge at one charger at a time, we also constrain σ such that

$$\sum_k \sigma_{ijk} \leq 1 \ \forall i, j \tag{7}$$

or in standard form as

not defined

$$\begin{aligned} \begin{bmatrix} 1_{ij1} & 0 & \dots & 0 & 1_{ijk} \end{bmatrix} \mathbf{y} \leq \mathbf{1} \ \forall i, j \\ A_2 \mathbf{y} \leq \mathbf{b}_2. \end{aligned} \tag{8}$$

The variable σ_{ijk} is used in several scenarios. The first is to constrain a zero-second charge time when not in use. This is done as

??

just second?

Not defined

$$\begin{aligned} s_{ij} - c_{ij} &\leq M \sum_k \sigma_{ijk} \\ s_{ij} - c_{ij} - \sum_k \sigma_{ijk} M &\leq 0 \\ \begin{bmatrix} 1_s & -1_c & -M_\sigma \end{bmatrix} \begin{bmatrix} s_{ij} \\ c_{ij} \\ \sigma_{ij1} \\ \vdots \\ \sigma_{ijk} \end{bmatrix} &\leq 0 \ \forall i, j \\ A_3 \mathbf{y} &\leq \mathbf{b}_3 \end{aligned} \tag{9}$$

This is a change from previous formulations which have zero padding

The variable σ_{ijk} is also necessary to eliminate situations where more then one bus is assigned to a charger at the same time. Note that this can only happen when a_{ij} for bus i is less

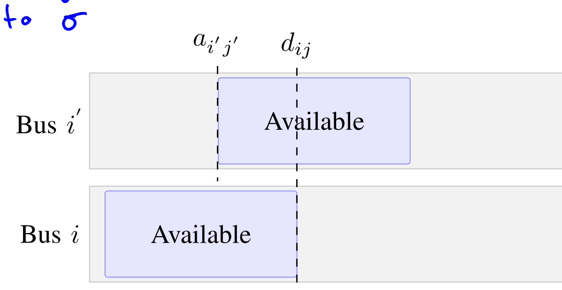

Fig. 4: Potential Overlap

This assumes a total ordering which could be really bad. Why should buses be constrained to always start charging after a bus that comes in before when there are multiple chargers?

not defined

than $d_{i'j'}$ for bus i' as shown in Fig. 4. Charging overlap can be avoided by constraining

Should this have a $\forall$ statement, or is it just for one bus in front. If the latter, then this is problematic. Needs to at least be for a set of buses.

$$c_{i'j'} > s_{ij}. \tag{10}$$

However, this constraint is only necessary when both bus stops are designated for charging. This can be remedied as

$$c_{i'j'} - s_{ij} > M \left[(\sigma_{i'j'k} + \sigma_{ijk}) - 2 \right] \ \forall k \text{ and } \forall\ i',j' \text{ in some set} \tag{11}$$

not defined

use \eqref{} in Latex. Then you should be correct regardless of the style

Where $M = 2 \cdot$ nTime. When $(\sigma_{i'j'k} + \sigma_{ijk}) < 2$, ~~equation~~ (11) is trivially satisfied for all values of $c_{i'j'}$ and s_{ij}. When $\sigma_{i'j'k} = \sigma_{ijk} = 1$, ~~equation~~ (11) simplifies to ~~equation~~ (10). Equation 11 can be expressed in standard form as

$$-c_{i'j'} + s_{ij} + M\sigma_{i'j'k} + M\sigma_{ijk} \leq 2M \ \forall k \tag{12}$$

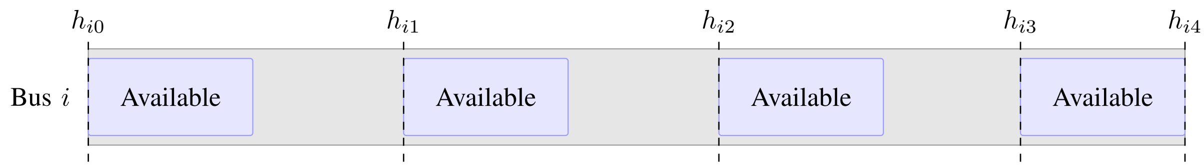

Fig. 5: State of Charge Variables

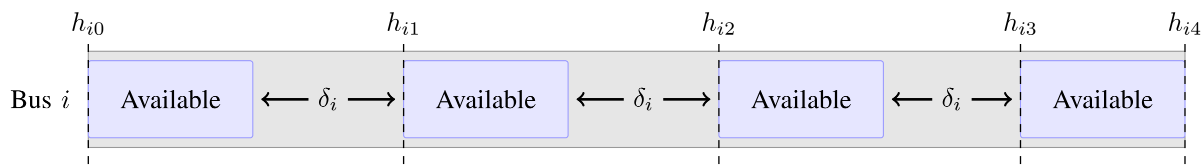

Fig. 6: Placement for δ_i

and finally as

$$\begin{bmatrix}-1 & 1 & M & M\end{bmatrix}\begin{bmatrix}c_{i'j'}\\ s_{ij}\\ \sigma_{i'j'k}\\ \sigma_{ijk}\end{bmatrix} \leq 2M\ \forall k \tag{13}$$

Need to define this as a set and allow i and j' to come from this set.

The constraints in equation 12 can be repeated for all instances where overlap is possible and concatenated into a single matrix such that

$$\begin{aligned} A_4\mathbf{y} &\leq \mathbf{1}\cdot 2M \\ A_4\mathbf{y} &\leq \mathbf{b}_4 \end{aligned} \tag{14}$$

IV. Battery State of Charge

motivate / describe "+1"

BEBs must also maintain their state of charge above a minimum threshold, denoted h_{min}. Let h_{ij+1} be the state of charge for bus i at the beginning of stop j. Each bus has an initial state of charge defined by h_{i0} as shown in figure 6. This can be constrained as

F proper noun

$$h_{i0} = \eta_i\ \forall i. \tag{15}$$

Seems redundant

where η_i is the initial state of charge for bus i. Equation 15 can be expressed in standard form such that

$$\begin{aligned} \begin{bmatrix}0 & 0 & \dots & 0 & 1_i & 0\end{bmatrix}\mathbf{y} &= \eta_i\ \forall i \\ \tilde{A}_1\mathbf{y} &= \tilde{\mathbf{b}}_1 \end{aligned} \tag{16}$$

Why switch to tildes

Buses do not always execute same loop, may want to index to be the n^{th} route

The transition from h_{ij} to h_{ij+1} is computed as the sum of battery effects due to charging and discharging. The discharge from operating bus i over one route loop is denoted δ_i. The increase in battery state of charge follows a linear charge model such that the increase is equal to the energy rate, denoted p, times the time spent charging, denoted Δ_{sc}. The total change from h_{ij} to h_{ij+1} can be expressed as

References to support

$$h_{ij+1} = h_{ij} + \Delta T \cdot p - \delta_i. \tag{17}$$

confusing as ΔT could be different for each stop. Just use your optimization variables as in (18)

Should be P_i to be generalized to individual buses

Needs to have indices as it will be different based on bus & visit

The value for ΔT can also be expressed in terms of the difference between a_{ij} and d_{ij} such that

$$\begin{aligned} h_{ij+1} &= h_{ij} + p\cdot(s_{ij} - c_{ij}) - \delta_i \\ h_{ij+1} - h_{ij} - ps_{ij} + pc_{ij} &= -\delta_i \\ \begin{bmatrix}1 & -1 & -p & p\end{bmatrix}\begin{bmatrix}h_{ij+1}\\ h_{ij}\\ s_{ij}\\ c_{ij}\end{bmatrix} &= -\delta_i\ \forall i,j \end{aligned} \tag{18}$$

The constraints for each i,j outlined in equation 18 can be vertically concatenated to form

$$\begin{aligned} A_{ij}\mathbf{y} &= \mathbf{b}_{ij}\ \forall i,j \\ \tilde{A}_2\mathbf{y} &= \tilde{\mathbf{b}}_2 \end{aligned} \tag{19}$$

Now that the state of charge is defined, the final constraint ensures that the minimum battery state of charge is kept above the minimum threshold, h_{min}. These constraints are given as

$$-h_{ij} \leq -h_{\text{min}}\ \forall i,j \tag{20}$$

or

$$\begin{aligned} \begin{bmatrix}0 & \dots & 0 & -1_h & 0 & \dots & 0\end{bmatrix}\mathbf{y} &\leq \mathbf{1}\cdot h_{\text{min}} \\ A_5\mathbf{y} &\leq \mathbf{b}_5 \end{aligned} \tag{21}$$

The final constraint has to do with the assumption that we are modeling one day and want to repeat the cost to predict the expected cost over a month. ~~In order to~~ To do this, the state of charge at the end of the day must equal the state of charge at the beginning. Let $h_{i,\text{end}}$ be the final daily state of charge for bus i. This is constrained to be the same as the beginning state of charge as

You might simplify this and state that $h_{i0} \geq h_{i,\text{end}}$

$$\begin{aligned} h_{i0} &= h_{i,\text{end}}\ \forall i \\ h_{i0} - h_{i,\text{end}} &= 0\ \forall i. \end{aligned} \tag{22}$$

However, because equality for two continuous variables is computationally demanding, the constraint in equation 22 can also be expressed as

$$h_{i0} - h_{i,\text{end}} \leq 0. \tag{23}$$

Because the final state of charge is dependent on the amount of power used to charge, and power/energy use is penalized only guaranteed at the optimal, so you'll need to ensure optimality reached

(see section VI), the optimization process will force the final state of charge down until is nearly equal to the initial. TODO:

1) Add section that talks about constraints to keep the battery SOC below the maximum SOC

The entire section needs more explanation of what is going on prior to giving each portion of the formulation.

- Why is discretization needed?
- Why do we need selector variables?
- What is role of f?

V. Integrating Uncontrolled Loads

Motivate more what we are doing: Finding a discrete representation of power?

A monthly power bill is made up of several charges, two of which depend on the maximum energy consumed over 15 minutes. This 15-minute average power includes energy that is consumed by loads other than bus chargers. This section integrates these uncontrolled loads into the planning framework. Measurements for uncontrolled loads are available at discrete time instants. The primary challenge addressed in this section is how to relate the continuous time model of power use from section III with a discrete time representation of the uncontrolled loads be (grammar) converting the continuous start and end points c_{ij} and s_{ij} from Section III to a vector $\mathbf{p}$, where p_i represents the average power over the interval from t_{i-1} to t_i.

long, run-on sentence

Explain in more detail

The first step is to separate each value for c_{ij} and s_{ij} into an integer-remainder pair such that.

Different ΔT than in (17) (ΔT not defined)

$$
\begin{aligned}
k_{ij}^{\text{start}} \cdot \Delta T + r_{ij}^{\text{start}} &= c_{ij} \\
k_{ij}^{\text{end}} \cdot \Delta T + r_{ij}^{\text{end}} &= s_{ij} \\
k_{ij}^{\text{start}}, k_{ij}^{\text{end}} &\in \mathbb{Z} \\
0 < r_{ij}^{\cancel{\text{start}}} \; r_{ij}^{\cancel{\text{end}}} &< \Delta T. \quad ; r \in \{\text{start}, \text{end}\}
\end{aligned} \tag{24}
$$

What is meant by this? The description sounds like if $c_{ij} = 4.32$, $k^{\text{start}} = 4$ & $r^{\text{start}}_{ij} = 0.32$ } This is not correct

where k_{ij}^{start} and k_{ij}^{end} are the integer portions of c_{ij} and s_{ij}, r_{ij}^{start} and r_{ij}^{end} are the remainders, and ΔT is the step (of what?) size. Equation 24 can be rewriten in standard form and zero padded such that

$$
\begin{bmatrix} \Delta T & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta T & 1 & -1 \end{bmatrix}
\begin{bmatrix} k_{ij}^{\text{start}} \\ r_{ij}^{\text{start}} \\ c_{ij} \\ k_{ij}^{\text{end}} \\ r_{ij}^{\text{end}} \\ s_{ij} \end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \forall i, j \tag{25}
$$
$$
\tilde{A}_2 \mathbf{y} = \tilde{\mathbf{b}}_2.
$$

Back to zero padding. Need to be consistent.

why Ã vs A? Be consistent

and

$$
\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} k_{ij}^{\text{start}} \\ r_{ij}^{\text{start}} \\ c_{ij} \\ k_{ij}^{\text{end}} \\ r_{ij}^{\text{end}} \\ s_{ij} \end{bmatrix}
\le \begin{bmatrix} 0 \\ \Delta T \\ 0 \\ \Delta T \end{bmatrix} \quad \forall i, j \tag{26}
$$
$$
A_6 \mathbf{y} \le \mathbf{b}_6.
$$

The next step is to use k and r to compute three sets of binary vectors, denoted $\mathbf{g}_{ij}^{\text{start}}$, $\mathbf{g}_{ij}^{\text{on}}$, and $\mathbf{g}_{ij}^{\text{end}}$, which act as selectors for times when buses are charged. The values in $\mathbf{g}_{ij}^{\text{start}}$ and $\mathbf{g}_{ij}^{\text{end}}$ are equal to 1 during intervals that contains energy from the remainders r_{ij}^{start} and r_{ij}^{end} and $\mathbf{g}_{ij}^{\text{on}}$ is equal to 1 for all time indices that buses charges the entire time.

confusing as f is a vector and w is a scalar.

not defined

Let $\mathbf{f}$ be a vector of one-based integer indices such that $\mathbf{f}_w = w \; \forall w \in (1, \text{nTime})$. The values in $\mathbf{g}_{ij}^{\text{start}}$ and $\mathbf{g}_{ij}^{\text{end}}$ are defined as

$$
\begin{aligned}
k_{ij}^{\text{start}} &= \mathbf{f}^T \mathbf{g}_{ij}^{\text{start}} \\
k_{ij}^{\text{end}} &= \mathbf{f}^T \mathbf{g}_{ij}^{\text{end}} \\
1 &= \mathbf{1}^T \mathbf{g}_{ij}^{\text{start}} \\
1 &= \mathbf{1}^T \mathbf{g}_{ij}^{\text{end}} \\
\mathbf{g}_{ij}^{\text{start}} &\in \{0, 1\}^{\text{nTime}} \\
\mathbf{g}_{ij}^{\text{end}} &\in \{0, 1\}^{\text{nTime}}.
\end{aligned} \tag{27}
$$

I don't understand f, so I don't follow from here on down

Equation 27 can be expressed in standard form and zero padded to form additional constraints in terms of $\mathbf{y}$.

$$
\begin{bmatrix} 0 & \mathbf{0}^T & -1 & \mathbf{f}^T \\ 0 & \mathbf{1}^T & 0 & 0 \\ -1 & \mathbf{f}^T & 0 & \mathbf{0}^T \\ 0 & 0 & 0 & \mathbf{1}^T \end{bmatrix}
\begin{bmatrix} k_{ij}^{\text{start}} \\ \mathbf{g}_{ij}^{\text{start}} \\ k_{ij}^{\text{end}} \\ \mathbf{g}_{ij}^{\text{end}} \end{bmatrix}
= \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \forall i, j \tag{28}
$$
$$
\tilde{A}_2 \mathbf{y} = \tilde{\mathbf{b}}_2.
$$

Different from (25)?

The final piece is to define $\mathbf{g}^{\text{on}}$. The values in $\mathbf{g}^{\text{on}}$ must be both greater than k_{ij}^{start}, and less than k_{ij}^{end} when they are not zero such that

$$
\left.
\begin{aligned}
g_w f_w &\le k^{\text{end}} \\
g_w f_w &\ge k^{\text{start}}
\end{aligned}
\quad g_w = 1 \right\}. \tag{29}
$$

Equation 29 can be expressed as a set of linear constraints such that

$$
\begin{aligned}
g_w \cdot f_w &\le k_{ij}^{\text{end}} + M(1 - g_w) \\
g_w \cdot f_w &\ge k_{ij}^{\text{start}} - M(1 - g_w)
\end{aligned} \tag{30}
$$

where M is $2 \cdot \text{nTime}$. The constraints in equation 30 do not require that all values between k_{ij}^{start} and k_{ij}^{end} be set to one however, only that if a value is equal to one, that it must be between k_{ij}^{start} and k_{ij}^{end}. For all values between k_{ij}^{start} and k_{ij}^{end} to be 1, the sum of $\mathbf{g}_{ij}^{\text{on}}$ must be equal to the difference between k_{ij}^{end} and k_{ij}^{start} such that

$$
\begin{aligned}
g_w \cdot f_w &\le k_{ij}^{\text{end}} + M(1 - g_w) \\
g_w \cdot f_w &\ge k_{ij}^{\text{start}} - M(1 - g_w) \\
\mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} &= k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - 1.
\end{aligned} \tag{31}
$$

The constraints in equation 31 work well for a general use case, however when k_{ij}^{end} is equal to k_{ij}^{start}, the last constraint in equation 31 becomes

$$
\mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} = -1 \tag{32}
$$

which leads to an empty feasible set because all the elements of $\mathbf{g}_{ij}^{\text{on}}$ are binary. Let k_{ij}^{eq} be a binary variable which is equal to 0 when k_{ij}^{end} is not equal to k_{ij}^{start}. Equation 31 can be modified to handle the case where k_{ij}^{end} is equal to k_{ij}^{start} as

$$
\begin{aligned}
g_w \cdot f_w &\le k_{ij}^{\text{end}} + M(1 - g_w) \\
g_w \cdot f_w &\ge k_{ij}^{\text{start}} - M(1 - g_w) \\
\mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} &= k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - k_{ij}^{\text{eq}}.
\end{aligned} \tag{33}
$$

The variable k_{ij}^{eq} is defined as

$$
\begin{aligned}
k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - M k_{ij}^{\text{eq}} &\le 0 \\
-k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + M k_{ij}^{\text{eq}} &\le M.
\end{aligned} \tag{34}
$$

The constraints from equations 33 and 34 can be expressed in standard form as

$$\begin{aligned} \mathbf{1}^T\mathbf{g}_{ij}^{\text{on}} - k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + k_{ij}^{\text{eq}} &= 0 \\ k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - Mk_{ij}^{\text{eq}} &\leq 0 \\ -k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + Mk_{ij}^{\text{eq}} &\leq M \\ g_w\left(f_w + M\right) - k_{ij}^{\text{end}} &\leq M \\ g_w\left(M - f_w\right) + k_{ij}^{\text{start}} &\leq M. \end{aligned} \tag{35}$$

The inequality constriants from equation 35 imply that

$$\begin{bmatrix} f_w + M & -1 & 0 \\ M - f_w & 0 & 1 \end{bmatrix} \begin{bmatrix} g_w \\ k_{ij}^{\text{end}} \\ k_{ij}^{\text{start}} \end{bmatrix} \leq \begin{bmatrix} M \\ M \end{bmatrix} \forall g_w \in \mathbf{g}_{ij}^{\text{on}}. \tag{36}$$

and that

$$\begin{bmatrix} 1 & -1 & -M \\ -1 & 1 & M \end{bmatrix} \begin{bmatrix} k_{ij}^{\text{end}} \\ k_{ij}^{\text{start}} \\ k_{ij}^{\text{eq}} \end{bmatrix} \leq \begin{bmatrix} 0 \\ M \end{bmatrix} \quad \forall i,j \tag{37}$$

which can be concatenated for all i, j and zero padded to form a joint matrix which has the form

$$A_7\mathbf{y} \leq \mathbf{b}_7. \tag{38}$$

Similarly, the equality constraint from equation 35 can also be concatenated and zero padded such that

$$\begin{aligned} \mathbf{1}^T\mathbf{s}_{ij}^{\text{on}} - k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + k_{ij}^{\text{eq}} &= 0 \ \forall i,j \\ \begin{bmatrix} \mathbf{1}^T & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{g}_{ij}^{\text{on}} \\ k_{ij}^{\text{end}} \\ k_{ij}^{\text{start}} \\ k_{ij}^{\text{eq}} \end{bmatrix} &= 0 \\ \tilde{A}_3\mathbf{y} &= \tilde{\mathbf{b}}_3. \end{aligned} \tag{39}$$

I don't follow because of

The next step is to define the average power during intervals that only charge for part of the time. These intervals correspond to the remainder values r_{ij}^{start} and r_{ij}^{end} and, as with previous constraints, distinguish between behavior for $k_{ij}^{\text{eq}} = 0$ and $k_{ij}^{\text{eq}} = 1$. The average power that corresponds to r_{ij}^{start} and r_{ij}^{end} can be computed as ??

$$\left.\begin{aligned} p_{ij}^{\text{start}} &= \frac{p \cdot \left(\Delta T - r_{ij}^{\text{start}}\right)}{\Delta T} & \\ p_{ij}^{\text{end}} &= \frac{p \cdot r_{ij}^{\text{end}}}{\Delta T}. & k_{ij}^{\text{eq}} = 1 \\ p_{ij}^{\text{start}} &= \frac{p \cdot \left(r_{ij}^{\text{end}} - r_{ij}^{\text{start}}\right)}{\Delta T} & k_{ij}^{\text{eq}} = 0 \\ p_{ij}^{\text{end}} &= 0 & \end{aligned}\right\}. \tag{40}$$

Equation 40 can also be expressed as a set of linear inequality constraints such that

$$\begin{aligned} p_{ij}^{\text{start}} &\leq p - \frac{p}{\Delta T}r_{ij}^{\text{start}} + M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{start}} &\geq p - \frac{p}{\Delta T}r_{ij}^{\text{start}} - M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{start}} &\leq \frac{p}{\Delta T}r_{ij}^{\text{end}} - \frac{p}{\Delta T}r_{ij}^{\text{start}} + Mk_{ij}^{\text{eq}} \\ p_{ij}^{\text{start}} &\geq \frac{p}{\Delta T}r_{ij}^{\text{end}} - \frac{p}{\Delta T}r_{ij}^{\text{start}} - Mk_{ij}^{\text{eq}} \\ p_{ij}^{\text{end}} &\leq \frac{p}{\Delta T}r_{ij}^{\text{end}} + M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{end}} &\geq \frac{p}{\Delta T}r_{ij}^{\text{end}} - M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{end}} &\leq Mk_{ij}^{\text{eq}} \\ p_{ij}^{\text{end}} &\geq -Mk_{ij}^{\text{eq}} \end{aligned} \tag{41}$$

where M $\cancel{=}$ is the battery capacity and can be expressed in standard form as

$$\begin{aligned} p_{ij}^{\text{start}} + \frac{p}{\Delta T}r_{ij}^{\text{start}} + Mk_{ij}^{\text{eq}} &\leq M + p \\ -p_{ij}^{\text{start}} - \frac{p}{\Delta T}r_{ij}^{\text{start}} + Mk_{ij}^{\text{eq}} &\leq M - p \\ p_{ij}^{\text{start}} - \frac{p}{\Delta T}r_{ij}^{\text{end}} + \frac{p}{\Delta T}r_{ij}^{\text{start}} - Mk_{ij}^{\text{eq}} &\leq 0 \\ -p_{ij}^{\text{start}} + \frac{p}{\Delta T}r_{ij}^{\text{end}} - \frac{p}{\Delta T}r_{ij}^{\text{start}} - Mk_{ij}^{\text{eq}} &\leq 0 \\ p_{ij}^{\text{end}} - \frac{p}{\Delta T}r_{ij}^{\text{end}} + Mk_{ij}^{\text{eq}} &\leq M \\ -p_{ij}^{\text{end}} + \frac{p}{\Delta T}r_{ij}^{\text{end}} + Mk_{ij}^{\text{eq}} &\leq M \\ p_{ij}^{\text{end}} - Mk_{ij}^{\text{eq}} &\leq 0 \\ -p_{ij}^{\text{end}} - Mk_{ij}^{\text{eq}} &\leq 0 \end{aligned} \tag{42}$$

and finally as

$$\begin{aligned} \begin{bmatrix} 1 & 0 & \frac{p}{\Delta T} & 0 & M \\ -1 & 0 & -\frac{p}{\Delta T} & 0 & M \\ 1 & 0 & \frac{p}{\Delta T} & -\frac{p}{\Delta T} & -M \\ -1 & 0 & -\frac{p}{\Delta T} & \frac{p}{\Delta T} & -M \\ 0 & 1 & 0 & -\frac{p}{\Delta T} & M \\ 0 & -1 & 0 & \frac{p}{\Delta T} & M \\ 0 & 1 & 0 & 0 & -M \\ 0 & -1 & 0 & 0 & -M \end{bmatrix} \begin{bmatrix} p_{ij}^{\text{start}} \\ p_{ij}^{\text{end}} \\ r_{ij}^{\text{start}} \\ r_{ij}^{\text{end}} \\ k_{ij}^{\text{eq}} \end{bmatrix} &\leq \begin{bmatrix} M + p \\ M - p \\ 0 \\ 0 \\ M \\ M \\ 0 \\ 0 \end{bmatrix} \forall i,j \\ A_8 &\leq \mathbf{b}_8 \end{aligned} \tag{43}$$

where p_{ij}^{start}, p_{ij}^{end}, and p represent the average power that corresponds to r_{ij}^{start}, r_{ij}^{end}, and full charging intervals respectively. The total average power use is calculated as

$$\mathbf{p}_{\text{total}} = \bar{\mathbf{p}}_{\text{load}} + \sum_{ij} \mathbf{g}_{ij}^{\text{start}} \cdot p_{ij}^{\text{start}} + \mathbf{g}_{ij}^{\text{on}} \cdot p + \mathbf{g}_{ij}^{\text{end}} \cdot p_{ij}^{\text{end}} \tag{44}$$

where $\bar{\mathbf{p}}_{\text{load}}$ is the average power of the uncontrolled loads.

Note, however that the results from equation 44 contain a bilinear term. The first bilnear expression in equation 44 must be rewritten as a vector containing values for p_{ij}^{start} whenever $g_{ij}^{\text{start}} \neq 0$. The resulting vector is denoted $\mathbf{p}_{ij}^{\text{start}}$ such that

$$\left.\begin{aligned} p_w &= p^{\text{start}} & g = 1 \\ p_w &= 0 & g = 0 \end{aligned}\right\} \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \tag{45}$$

The constraints for equation 45 can be rewritten as a set of linear inequality constraints such that

$$\begin{aligned} p_w &\geq p_{ij}^{\text{start}} - M(1-g_w) \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \\ p_w &\leq p_{ij}^{\text{start}} + M(1-g_w) \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \\ p_w &\geq -M g_w \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \\ p_w &\leq M g_w \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \end{aligned} \tag{46}$$

The same approach can be taken to replace the other bilinear form $\mathbf{g}_{ij}^{\text{end}} \cdot p_{ij}^{\text{end}}$ with the vector $\mathbf{p}_{ij}^{\text{end}}$ as

$$\begin{aligned} p_w &\geq p_{ij}^{\text{end}} - M(1-g_w)\ \forall p_w \in \mathbf{p}_{ij}^{\text{end}} \\ p_w &\leq p_{ij}^{\text{end}} + M(1-g_w)\ \forall p_w \in \mathbf{p}_{ij}^{\text{end}} \\ p_w &\geq -M g_w\ \forall p_w \in \mathbf{p}_{ij}^{\text{end}} \\ p_w &\leq M g_w\ \forall p_w \in \mathbf{p}_{ij}^{\text{end}}. \end{aligned} \tag{47}$$

Equation 46 can be written in standard form, stacked to accomodate the constraints for all i, j, and zero padded appropriately as

$$\begin{gathered} \begin{bmatrix} -1 & 1 & M \\ 1 & -1 & M \\ -1 & 0 & -M \\ 1 & 0 & -M \end{bmatrix} \begin{bmatrix} p_w \\ p_{ij}^{\text{start}} \\ g_w \end{bmatrix} \leq \begin{bmatrix} M \\ M \\ 0 \\ 0 \end{bmatrix} \forall p_w \in \mathbf{p}_{ij}^{\text{start}}. \\ A_9 \leq \mathbf{b}_9 \end{gathered} \tag{48}$$

Equation 47 can be expressed in standard form, stacked for all i, j, and zero padded in a similar fashion such that

$$\begin{gathered} \begin{bmatrix} -1 & 1 & M \\ 1 & -1 & M \\ -1 & 0 & -M \\ 1 & 0 & -M \end{bmatrix} \begin{bmatrix} p_w \\ p_{ij}^{\text{end}} \\ g_w \end{bmatrix} \leq \begin{bmatrix} M \\ M \\ 0 \\ 0 \end{bmatrix} \forall p_w \in \mathbf{p}_{ij}^{\text{end}} \\ A_{10}\mathbf{y} \leq \mathbf{b}_{10}. \end{gathered} \tag{49}$$

An expression for the total power used can then be expressed as

$$\mathbf{p}^{\text{total}} = \mathbf{p}^{\text{load}} + \sum_{ij} \mathbf{p}_{ij}^{\text{start}} + \mathbf{p}_{ij}^{\text{end}} + \mathbf{g}_{ij}^{\text{on}} \cdot p \tag{50}$$

and in standard form as

$$\begin{gathered} \begin{bmatrix} 1 & -1^{\text{start}} & -1^{\text{end}} & -1^{\text{on}} \cdot p \end{bmatrix} \begin{bmatrix} \mathbf{p}_w^{\text{total}} \\ \mathbf{p}_w^{\text{start}} \\ \mathbf{p}_w^{\text{end}} \\ \mathbf{g}_w^{\text{on}} \end{bmatrix} = \mathbf{p}_w^{\text{load}} \\ \tilde{A}_4\mathbf{y} = \tilde{\mathbf{b}}_4 \end{gathered} \tag{51}$$

VI. Objective Function

This work adopts the objective function developed in insert reference to our prior paper here, which implements the rate schedule from insert reference to rocky mountain power. The rate schedule in insert reference to rocky mountain power rate schedule is based off of two primary components: power, and energy.

Power is billed per kW for the highest 15 minute average power over a fixed period of time. It is common practice for power providers to use a higher rate during ”on-peak” periods when power is in higher demand and use a lower rate during ”off-peak” hours, which accoutn for all other time periods.

2 apostrophes

Latex: use two forward apostrophes (located below escape key)

Is demand charge for on-peak hours? I thought demand was on-peak seasons.

The rate schedule given in insert reference to rocky mountain here assesses a fee for a users maximum average power during on-peak hours called the demand charge, and a user's overall maximum average power, called a facilities charge as shown in figure 7.

Energy fees are also assessed per kWh of energy consumed with a higher rate for energy consumed during on-peak hours and a lower rate for energy consumed during off-peak hours.

A. Power Charges

Discretizing to 15 minute intervals?

It is necessary to compute the maximum power both overall and for on-peak periods. Section V adopted the convention that ΔT denotes the time offset between power samples and that each power reading would reflect the average power used in the previous interval. Now let us set ΔT to 15 minutes, making $\mathbf{p}_{\text{total}}$ an expression of the 15 minute average power. Next, let $\mathcal{S}_{\text{on}}$ be the set of all indices belonging to on-peak time periods such that $j \in \mathcal{S}_{\text{on}}$ implies that p_j^{total} not defined represents a 15 minute average during an on-peak interval and let q_{on} be the maximum on-peak average power. With these definitions, constriants for determining the maximum on-peak average may be stated

$$\begin{gathered} p_j^{\text{total}} \leq q_{\text{on}}\ \forall j \in \mathcal{S}_{\text{on}} \\ \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} p_j^{\text{total}} \\ q_{\text{on}} \end{bmatrix} \leq 0\ \forall j \in \mathcal{S}_{\text{on}} \\ A_{11}\mathbf{y} \leq \mathbf{0} \\ A_{11}\mathbf{y} \leq \mathbf{b}_{11} \end{gathered} \tag{52}$$

Because an increased value in q_{on} is directly related to an increase in cost, the optimizer will minimize q_{on} until it is equal to the maximum value in $\{p_j^{\text{total}}\ \forall j \in \mathcal{S}_{\text{on}}\}$. A similar proceedure can be used to derive a set of constraints for the overall maximum average power, denoted q_{all}, and is represented as

$$\begin{aligned} A_{12}\mathbf{y} &\leq \mathbf{0} \\ A_{12}\mathbf{y} &\leq \mathbf{b}_{12}. \end{aligned} \tag{53}$$

The charges for power are then expressed as

$$\begin{aligned} \text{power cost} &= q_{\text{on}} \cdot u_{\text{p-on}} + q_{\text{all}} \cdot u_{\text{p-all}} \\ &= \begin{bmatrix} u_{\text{p-on}} & u_{\text{p-all}} \end{bmatrix} \begin{bmatrix} q_{\text{on}} \\ q_{\text{all}} \end{bmatrix} \\ &= \mathbf{u}_{\text{p}}^T \mathbf{y} \end{aligned} \tag{54}$$

where $u_{\text{p-on}}$ is the rate per kW for on-peak power use, or the demand charge and $u_{\text{p-all}}$ is the rate per kW for the overall maximum 15 minute average.

B. Energy Charges

Energy is defined as the integral of power over a length of time. Because the values for power given in this work reflect an average power, the energy over a given period can be computed by multiplying the average power by the change in time, or ΔT such that

$$\text{Total Energy} = \mathbf{1}^T \mathbf{p}_{\text{total}} \cdot \Delta T. \tag{55}$$

However, because the energy is billed for on-peak and off-peak time periods, we define two binary vectors $\mathbf{1}_{\text{on}}$ and $\mathbf{1}_{\text{off}}$

	On-Peak	Off-Peak	Both
Energy	On-Peak Energy Charge	Off-Peak Energy Charge	None
Energy Rate	$u_{\text{e-on}}$	$u_{\text{e-off}}$	None
Power	Demand Charge	None	Facilities Charge
Power Rate	$u_{\text{p-on}}$	None	$u_{\text{p-all}}$

Fig. 7: Description of the assumed billing structure

such that $1^{\text{on}}_j = 1 \ \forall j \in S_{\text{on}}$ and zero otherwise. Similarly, $1_{\text{off}} = \mathbf{1} - 1_{\text{on}}$. The on-peak and off-peak energy can then be computed as

$$\begin{aligned} \text{On-Peak Energy} &= \mathbf{1}_{\text{on}}^T \mathbf{p}_{\text{total}} \cdot \Delta T \\ \text{Off-Peak Energy} &= \mathbf{1}_{\text{off}}^T \mathbf{p}_{\text{total}} \cdot \Delta T. \end{aligned} \tag{56}$$

Let $u_{\text{e-on}}$ and $u_{\text{e-off}}$ represent the on-peak and off-peak energy rates respectively. The total cost for energy is computed as

$$\begin{aligned} \text{Energy Cost} &= \left(\mathbf{1}_{\text{on}} \cdot u_{\text{e-on}} \cdot \Delta T\right)^T \mathbf{p}_{\text{total}} + \left(\mathbf{1}_{\text{off}} \cdot u_{\text{e-off}} \cdot \Delta T\right)^T \mathbf{p}_{\text{total}} \\ &= \left(\mathbf{u}_{\text{e-on}} + \mathbf{u}_{\text{e-off}}\right)^T \mathbf{p}_{\text{total}} \\ &= \mathbf{u}_{\text{e}}^T \mathbf{y} \end{aligned} \tag{57}$$

C. Cost Function and Final Problem

The entire cost function is given as the sum of the energy and power costs such that

$$\begin{aligned} \text{Cost} &= \mathbf{u}_{\text{p}}^T \mathbf{y} + \mathbf{u}_{\text{e}}^T \mathbf{y} \\ &= \left(\mathbf{u}_{\text{p}} + \mathbf{u}_{\text{e}}\right)^T \mathbf{y} \\ &= \mathbf{g}^T \mathbf{y} \end{aligned} \tag{58}$$

The complete problem can now be formulated as

$$\min_{\mathbf{y}} \ \mathbf{y}^T \mathbf{g} \text{ subject to}$$
$$\begin{bmatrix} \tilde{A}_1 \\ \tilde{A}_2 \\ \tilde{A}_3 \end{bmatrix} \mathbf{y} = \begin{bmatrix} \tilde{\mathbf{b}}_1 \\ \tilde{\mathbf{b}}_2 \\ \tilde{\mathbf{b}}_3 \end{bmatrix}, \quad \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \\ A_{11} \\ A_{12} \end{bmatrix} \mathbf{y} \le \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \\ \mathbf{b}_4 \\ \mathbf{b}_5 \\ \mathbf{b}_6 \\ \mathbf{b}_7 \\ \mathbf{b}_8 \\ \mathbf{b}_9 \\ \mathbf{b}_{10} \\ \mathbf{b}_{11} \\ \mathbf{b}_{12} \end{bmatrix} \tag{59}$$

or

$$\begin{aligned} &\min_{\mathbf{y}} \ \mathbf{y}^T \mathbf{g} \text{ subject to} \\ &\tilde{A}\mathbf{y} = \tilde{\mathbf{b}}, \ A\mathbf{y} \le \mathbf{b}, \end{aligned} \tag{60}$$

VII. Results

VIII. Conclusions and Future Work