A Continuous Approach to Minimize Cost for Charging Electric Bus Fleets

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***Abstract*—**

***Index Terms*—Battery Electric Buses, Cost Minimization, Multi-Rate Charging, Mixed Integer Linear Program**

Refocus on scheduling & financials. Intro should help me understand contributions of the paper. What are we doing that is novel?

I. Introduction

Battery powered electric motors for buses have long been a desired alternative to the internal combustion engine [15]. The reduced maintenance [1], zero emissions [12], and access to renewable energy [6] are but some of the benefits that have caused transit authorities to begin incorporating them into their bus fleets.

Despite their benefits, transitioning to BEBs must address additional challenges, in particular, their extended refuel times. If a diesal or CNG bus runs low on fuel, refueling takes five to ten minutes. An electric bus on the other hand may require several hours, causing the bus to fall behind schedule. Won't "fall behind" but needs to be considered in plan

reword and combine

Maintaining a schedule while staying charged is one of the main challenges that BEBs face, and requires careful planning which must account for the battery discharge along routes, charge times, and a limited number of chargers.

Way too much on charging methodologies. Focus this on state of the art for BEB charger scheduling

Charging while a bus is in motion, or dynamic charging, is one way to simplify a charge plan. There are a number of ways to do this including overhead [7] and inductive charging [11] [3]. An overhead charging scenario allows the bus to charge on overhead power lines while in motion while inductive charging relies on specialized hardware in the roads to inductively transfer energy when a bus passes overhead. Both methods remove the need to stop for service and allow an electrical vehicle to stay in service indefinately. They also require extensive infrastructure that may not be available.

In the absence of infrastructure, [10] and [27] have proposed methods that exchange depleted batteries for fresh ones. Such a method would eliminate both the logistical challenges of planning and the infrastructure dependence of dynamic charging. The only drawback, is that BEBs are not built with battery exchanges in mind, therefore the task can require specialized hardware, technical expertise, or automation, all of which add complexity and cost.

One charge option that avoids both the infrastructural demands of dynamic charging and the technical difficulties of battery swapping is stationary charging, which plans rest periods into a bus's schedule during which that bus can charge [26]. Stationary charging is the least invasive form of bus charging because it only requires charging hardware at specific locations and makes no changes to bus batteries. Prior work in this area addresses a number of problems including distributed charging networks [17], bus availability, environmental impact [28], route scheduling [21], battery health [9], the cost of electricity [13] and the cost of charging infrastructure [25].

One drawback to using a stationary charging solution is that it does require significant rest periods for charging. One way to decrease the charge intervals is to use high power chargers, which deliver more energy in a smaller period of time. Large power demands however do increase the overall cost of energy because they must be supported by highly capable infrastructure [23], [8], [4]. An effective charge plan must therefore balance the need to charge quickly with the desire to maintain a low power profile [6], [18], [20], [2], [24] which includes power used by BEBs and the power needed by other consumers.

Because the additional power users are outside the control of the charge plan, their power requirements are referred to in this paper as "uncontrolled loads". Uncontrolled loads complicate the problem of finding an optimal charge plan. If a bus charges in the presence of a large uncontrolled load, the overall power profile is heightened, increasing cost for energy.

Tell us more about utility considerations. As is, this paragraph is out of place.

A significant contribution of [16] was how the authors minimized the the financial impact of charging in the presence of uncontrolled loads by formulating the charge problem as a graph and solving for the optimal path. The graph based approach represented time discretely, which lent itself well to integrating uncontrolled loads, which are sampled discretely in practice.

This is what we care about

← Important concept. Make sure to explain well.

Unfortunately, more temporal precision decreases the time step between sections in the graph, which leads to a larger graph and significantly increases the computational complexity. The authors of [5] compute the charge schedule continuously by formulating the charge problem as a bin packing problem [14], yielding a precise time schedule for charging, but finding a continuous-time schedule that incorporates uncontrolled loads and minimizes a comprehensive cost function remains an open problem and is the focus of this paper.

[5] or [14]?

The rest of this paper is organized as follows: Section II discuses the basic problem formulation, Section III discuses linear constraints that govern the behavior and limitations of the sate of charge. Section IV discuses how to incorporate uncontrolled loads into the optimization framework. Section V explaines how the objective function is formed, and Section VI discusses performance.

I really don't have a grasp on this yet. Intro should be clarified.

II. Availability and Resource Contention

The charge scheduling framework described in this paper is formulated as a constrained optimization problem that can

Strange section heading

It is unclear what is in this section. From 1st sentence it would seem that that the entire formulation is, but then only still don't see the purpose of this plot

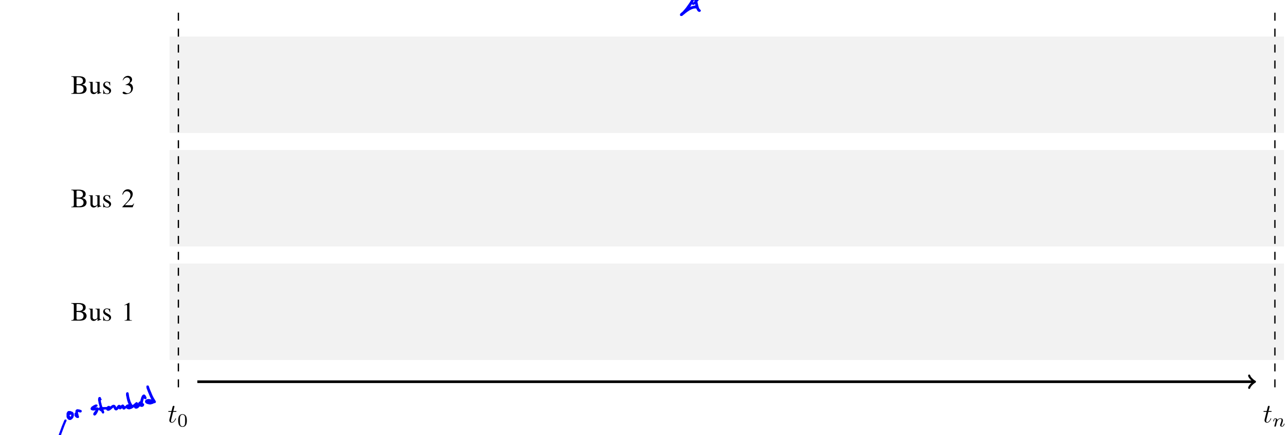


Fig. 1: Description of the bus and time axis

The expression of constraints in matrix (or standard) form obfuscates the constraint in many cases. Maybe the matrix form is not needed.

be solved as a Mixed Integer Linear Program (MILP) of the form

$$\min_{\mathbf{y}} \mathbf{y}^T\mathbf{v} \text{ \sout{subject to}} \\ \text{s.t. } \tilde{A}\mathbf{y} = \tilde{\mathbf{b}},\ A\mathbf{y} \le \mathbf{b}, \tag{1}$$

where $\mathbf{y}$, $\tilde{A}$, A, and $\mathbf{g}$ represent the solution vector, equality and inequality constraints, and cost vector respectively. In this paper, $\mathbf{y}$ is comprised of several variables, and is expressed as

$$\mathbf{y} = \begin{bmatrix} \boldsymbol{\sigma} \\ \mathbf{c} \\ \mathbf{s} \\ \mathbf{h} \\ \mathbf{k} \\ \mathbf{r} \\ \mathbf{g} \\ \mathbf{p} \\ q_{\text{on}} \\ q_{\text{all}} \end{bmatrix}, \tag{2}$$

Treat each subsection as a 5 paragraph essay, complete with an intro of what will be in the section

g is left out

Need to communicate that these variables will be explained in detail throughout the remainder of the section.

where $\boldsymbol{\sigma}$ describes on which charger a bus will charge, $\mathbf{c}$ and $\mathbf{s}$ describe time intervals over which buses charge, $\mathbf{h}$ gives the bus state of charge, $\mathbf{k}$, $\mathbf{r}$ and $\mathbf{p}$ are used to discretize the effects from $\mathbf{c}$ and $\mathbf{s}$, and q_{on} and q_{all} represent maximum average power values that are used to compute the monthly cost of power.

The cost function in (1) will be designed to model a realistic billing structure used by [19] and minimises the cost ~~even in the presence of~~ while coordinating known, yet uncontrolled loads. Additionally, the constraints are designed to encorporate bus schedules, limit bus state of charges (grammar) and include a linear charge model calibrated on data from the Utah Transit Authority.

A. Setup

I don't understand what the categories are. This is confusing. Could we describe as different queues?

A solution to the bus charge problem includes both temporal and categorical information. The temporal aspect shows when and for how long a bus should charge and the categorical (grammar) shows which bus must charge, indicating a solution with two dimensions. The first dimension represents time continuously from left to right, and the second describes the buses as shown in Fig. 1.

Each bus follows a schedule of arrival and departure times, where the i^{th} bus's j^{th} stop begins at arrival time a_{ij} and terminates at departure time d_{ij} (see Fig. 2). A bus can be assigned to charge anytime the bus is in the station such that the charge start time, c_{ij}, is greater than or equal to a_{ij}, and the charge stop time, s_{ij}, (zc_{ij}) is less than the departure time d_{ij} as shown in Fig. 2. In the context of a MILP, the arrival and departure times a_{ij} and d_{ij} are known ahead of time and charge times c_{ij} and s_{ij} are optimization variables.

Seems like this paragraph should be with

B. Constraints

Be more descriptive (Scheduling Constraints)

The relationship between the arrival, departure, and charge intervals for the i^{th} bus at the j^{th} stop can be expressed as a set of inequality constraints such that

$$\begin{aligned} a_{ij} &< c_{ij} \\ c_{ij} &< s_{ij} \\ s_{ij} &< d_{ij}. \end{aligned} \tag{3}$$

These constraints can be rewritten such that the optimization variables are on the left, the known parameters are on the right, and the relationship is "less than" (or standard form) such that

$$\begin{aligned} -c_{ij} &< -a_{ij} \\ c_{ij} - s_{ij} &< 0 \\ s_{ij} &< d_{ij}. \end{aligned} \tag{4}$$

Standard form is preferred ~~because it provides a standard way to represent equations~~ solvers. Having the optimization variables on the left also allows the expression to be written using matrix notation as

$$\begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{ij} \\ s_{ij} \end{bmatrix} \le \begin{bmatrix} -a_{ij} \\ 0 \\ d_{ij} \end{bmatrix}. \tag{5}$$

However, because all constraints must follow the form $A\mathbf{y} = \mathbf{b}$ as shown in (1), (5) is expressed in terms of $\mathbf{y}$ such that

$$\begin{gathered} \begin{bmatrix} -1^c_{ij} & 0 & \dots & 0 \\ 1^c_{ij} & 0 & \dots & -1^d_{ij} \\ 0 & 0 & \dots & 1^d_{ij} \end{bmatrix} \mathbf{y} \le \begin{bmatrix} -a_{ij} \\ 0 \\ d_{ij} \end{bmatrix} \quad \forall i,j \\ A_1\mathbf{y} \le \mathbf{b}_1, \end{gathered} \tag{6}$$

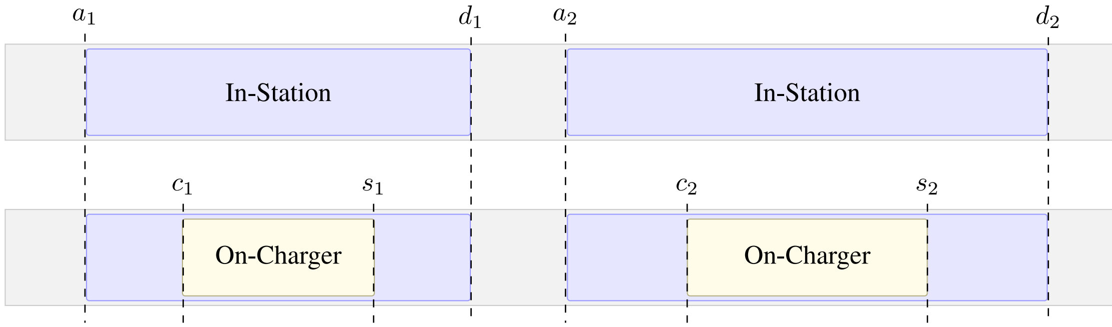


Fig. 2: Bus Charging

Not sure what this means (I know it means in y, but not stated)

where 1^c_{ij} is 1 at the location corresponding to c_{ij}, 1^d_{ij} is 1 at the location corresponding to d_{ij}, and A_1 and $\mathbf{b}_1$ stack the constraints given in (5) for all i, j.

The decision variables s_{ij} and c_{ij} from (5) show when a bus must start and finish charging, but do not indicate on which charger. The variable $\boldsymbol{\sigma}$ from (2) is a vector of binary variables. Each element of $\boldsymbol{\sigma}$ is denoted σ_{ijk} and is 1 when bus i charges during the j^{th} stop at charger k. Because a bus can only charge at one charger at a time, the values in $\boldsymbol{\sigma}$ must be constrained such that

$$\sum_k \sigma_{ijk} \leq 1 \ \forall i, j \tag{7}$$

This may be a bit confusing for some readers as you are meaning to say for any fixed i & j. Someone may incorrectly read this.

notation mismatch

or in standard form as

$$\begin{bmatrix} 1_{ij1} & 0 & \dots & 0 & 1_{ijk} \end{bmatrix} \mathbf{y} \leq \mathbf{1} \ \forall i, j$$
$$A_2 \mathbf{y} \leq \mathbf{b}_2, \tag{8}$$

Not clear how you progress from 1 to k here

From (7) this corresponds to (8)

where 1_{ijk} represents a 1 at the location corresponding to σ_{ijk}. The variable σ_{ijk} is used in several scenarios. The first is to ensure that buses without charge assignments have a charge time of zero by constraining s_{ij} and c_{ij} to be the same value. This is done by letting

$$s_{ij} - c_{ij} \leq M \sum_k \sigma_{ijk}$$
$$s_{ij} - c_{ij} - \sum_k \sigma_{ijk} M \leq 0$$
$$\begin{bmatrix} 1_s & -1_c & -M_\sigma \end{bmatrix} \begin{bmatrix} s_{ij} \\ c_{ij} \\ \sigma_{ij1} \\ \vdots \\ \sigma_{ijk} \end{bmatrix} \leq 0 \ \forall i, j \tag{9}$$

Confusing at first pass because it isn't stated that each line is actually the same constraint stated in a different way

undefined subscripts

This is a vector?

used elsewhere?

where M is the maximum difference between s_{ij} and c_{ij}, or the number of seconds in a day, denoted nTime and M_σ represents multiple values of M at locations corresponding to each σ_{ijk}. The constraints in (9) can be appropriately zero padded and stacked for all i, j to form the linear expression

$$A_3 \mathbf{y} \leq \mathbf{b}_3 \tag{10}$$

The values in $\boldsymbol{\sigma}$, $\mathbf{c}$, and $\mathbf{s}$ form a complete charge plan representation were c_{ij} and s_{ij} describe time periods when a bus will charge and σ_{ijk} gives which charger to use. (see Fig. 3). The variable σ_{ijk} is also necessary to eliminate situations where more then one bus is assigned to a charger at the same time. Note that this can only happen when a_{ij} for bus i is less than $d_{i'j'}$ for bus i' as shown in Fig. 4. Let $\mathcal{S}$ be the set of all bus break pairs such that $(ij,\ i^{`}j^{`}) \in \mathcal{S}$ if overlap is possible between bus i and bus $i^{`}$ during the $j$ and $j^{`}$ stops respectively. Charging overlap can be avoided by constraining

Do you mean to the same charger?

Makes it feel like a run-on section since the concept wasn't introduced previously.

The overlap?

It is not clear to me what S is.

wrong direction for prime

what is a "break pair"?

i & j are integers. Is ij here integer multiplication perhaps ((i,j), (i',j')) ∈ S?

Put this all on one line

$$\left.\begin{matrix} c_{i^{`}j^{`}} > s_{ij} \\ \text{or} \\ c_{ij} > s_{i^{`}j^{`}} \end{matrix}\right\} \forall \left(ij,\ i^{`}j^{`}\right) \in \mathcal{S} \tag{11}$$

Why; I thought you got rid of total ordering. If you have two chargers then why can't two buses charge at same time? Confusing to wait until to say so.

Let $l_{(ij,\ i^{`}j^{`})}$ be a binary decision variable that is 1 when $c_{i'j'} > s_{ij}$, and 0 when $c_{ij} > s_{i'j'}$. The expression from (11) can be rewritten as

$$\begin{aligned} c_{i^{`}j^{`}} - s_{ij} &> -M l_{(ij,\ i^{`}j^{`})} \\ c_{ij} - s_{i^{`}j^{`}} &> -M(1 - l_{(ij,\ i^{`}j^{`})}) \end{aligned} \tag{12}$$

However, this constraint is only necessary when buses i and $i^{`}$ must use the same charger. This can be remedied as

$$\begin{aligned} c_{i'j'} - s_{ij} &> M\left[(\sigma_{i'j'k} + \sigma_{ijk}) - 2\right] - M l_{(ij,\ i^{`}j^{`})} \ \forall k \\ c_{ij} - s_{i'j'} &> M\left[(\sigma_{i'j'k} + \sigma_{ijk}) - 2\right] - M(1 - l_{(ij,\ i^{`}j^{`})}) \ \forall k \end{aligned} \tag{13}$$

When $(\sigma_{i'j'k} + \sigma_{ijk}) < 2$, (1~~4~~3) is trivially satisfied for all values of $c_{i'j'}$ and s_{ij}. When $\sigma_{i'j'k} = \sigma_{ijk} = 1$, (1~~4~~3) simplifies to (12). Equation (1~~4~~3) can be expressed in standard form as

mismatch

$$\begin{aligned} -c_{i^{`}j^{`}} + s_{ij} + M\sigma_{i'j'k} + M\sigma_{ijk} - M l_{iji^{`}j^{`}} &\leq 2M \ \forall k \\ -c_{ij} + s_{i^{`}j^{`}} + M\sigma_{i'j'k} + M\sigma_{ijk} + M l_{iji^{`}j^{`}} &\leq 3M \ \forall k \end{aligned} \tag{14}$$

and finally as

$$\begin{bmatrix} -1 & 0 & 0 & 1 & M & M & -M \\ 0 & 1 & -1 & 0 & M & M & M \end{bmatrix} \begin{bmatrix} c_{i'j'} \\ s_{i'j'} \\ c_{ij} \\ s_{ij} \\ \sigma_{i'j'k} \\ \sigma_{ijk} \\ l_{iji^{`}j^{`}} \end{bmatrix} \leq \begin{bmatrix} 2M \\ 3M \end{bmatrix} \forall k \tag{15}$$

The constraints in (14) can be repeated for all $\left(ij,\ i^{`}j^{`}\right) \in \mathcal{S}$ and concatenated into a single matrix expression

$$A_4 \mathbf{y} \leq \mathbf{b}_4 \tag{16}$$

Too trivial, Should have fewer chargers than busses.

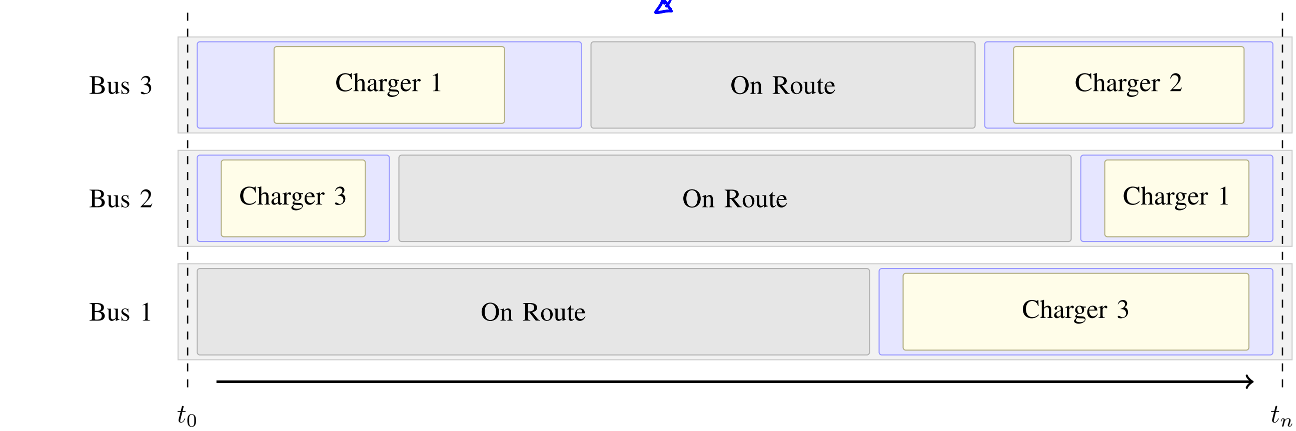


Fig. 3: Reserving time slots on chargers

If you used chargers on the y-axis this would be much more clear.

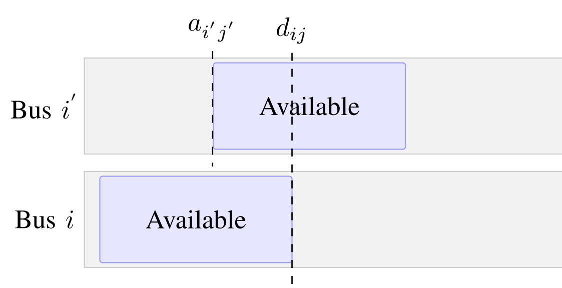


Fig. 4: Potential Overlap

III. Battery State of Charge

tell me what constraints we'll go over

BEBs must also maintain their state of charge above a minimum threshold, denoted h_{min}. ¶ Let h_{ij} be the state of charge for bus i at the beginning of stop j as shown in ~~figure~~ Fig. 5. The initial value for bus i, denoted h_{i0}, is equal to some constant such that

If this is h, we are missing many zero padded variables

$$
\begin{aligned}
h_{i0} &= \eta_i \ \forall i \\
\begin{bmatrix} 0 & 0 & \dots & 0 & 1_i & 0 \end{bmatrix} \mathbf{y} &= \eta_i \ \forall i \\
\tilde{A}_1 \mathbf{y} &= \tilde{\mathbf{b}}_1
\end{aligned}
\tag{17}
$$

Should this have a 0 index?

and is otherwise computed as the the sum of incoming and outgoing energy where incoming energy comes from charging, and outgoing energy comes from the battery discharge. The discharge from operating bus i over route j is denoted δ_{ij}. ¶ The increase in battery state of charge follows a linear charge model such that the increase is equal to the energy rate, denoted p_i, times the time spent charging, denoted Δ_{ij}[22]. The total change from h_{ij} to h_{ij+1} can be expressed as

$$h_{ij+1} = h_{ij} + \Delta_{ij} \cdot p_i - \delta_{ij}. \tag{18}$$

The value for Δ_{ij} can also be expressed in terms of the difference between a_{ij} and d_{ij} such that

$$
\begin{aligned}
h_{ij} + p_i \cdot (s_{ij} - c_{ij}) - \delta_i &= h_{ij+1} \\
h_{ij+1} - h_{ij} - p_i s_{ij} + p_i c_{ij} &= -\delta_i \\
\begin{bmatrix} 1 & -1 & -p_i & p_i \end{bmatrix} \begin{bmatrix} h_{ij+1} \\ h_{ij} \\ s_{ij} \\ c_{ij} \end{bmatrix} &= -\delta_i \ \forall i,j
\end{aligned}
\tag{19}
$$

The constraints for each i,j outlined in (19) can be vertically concatenated to form

$$
\begin{aligned}
A_{ij} \mathbf{y} &= \mathbf{b}_{ij} \ \forall i,j \\
\tilde{A}_2 \mathbf{y} &= \tilde{\mathbf{b}}_2
\end{aligned}
\tag{20}
$$

Run-on section

Now that the state of charge is defined, the next constraint ensures that the minimum battery state of charge remains both above the minimum threshold, h_{min}, and below the battery capacity, h_{max}. These constraints are given as

Should be mentioned in intro

$$
\begin{aligned}
-h_{ij} &\le -h_{\text{min}} \\
h_{ij} &\le h_{\text{max}}
\end{aligned}
\quad \forall i,j
\tag{21}
$$

or

$$
\begin{aligned}
\begin{bmatrix} 0 & \dots & 0 & -1_h & 0 & \dots & 0 \\ 0 & \dots & 0 & 1_h & 0 & \dots & 0 \end{bmatrix} \mathbf{y} &\le \begin{bmatrix} h_{\text{min}} \\ h_{\text{max}} \end{bmatrix} \quad \forall ij \\
A_5 \mathbf{y} &\le \mathbf{b}_5
\end{aligned}
\tag{22}
$$

The final constraint has to do with the assumption that we desire to use the model for one day to predict the expected cost over a month. ← Not previously mentioned. To do this, the state of charge at the end of the day must equal the state of charge at the beginning. Let $h_{i,\text{end}}$ be the final daily state of charge for bus i. This is constrained to be the same as the beginning state of charge as

$$
\begin{aligned}
h_{i0} &= h_{i,\text{end}} \ \forall i \\
h_{i0} - h_{i,\text{end}} &= 0 \ \forall i.
\end{aligned}
\tag{23}
$$

However, because equality for two continuous variables is computationally demanding, the constraint in (23) can also be expressed as

$$h_{i0} - h_{i,\text{end}} \le 0. \tag{24}$$

Because the final state of charge is dependent on the amount of power used to charge, and power/energy use is penalized

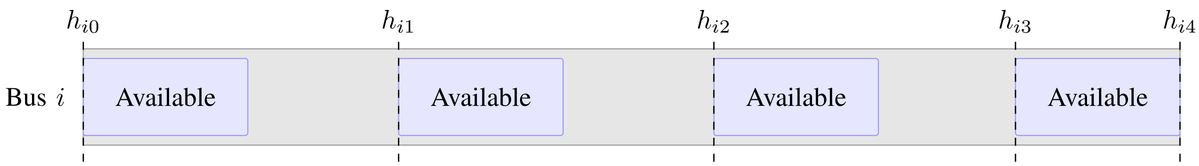


Fig. 5: State of Charge Variables

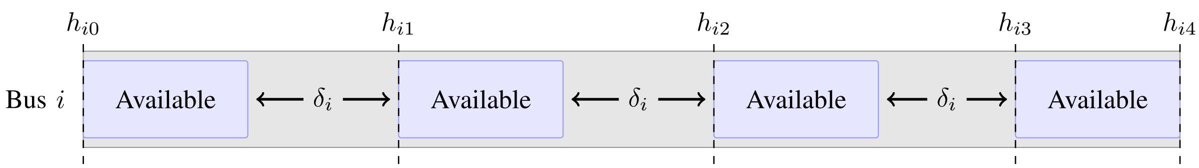


Fig. 6: Placement for δ_i

If you go to optimal

(see section V), the optimization process will force the final state of charge down until is nearly equal to the initial.

Would a word like "Including" or "Considering" be better since "Integrating" can have other meanings (ie Calculus)

IV. Integrating Uncontrolled Loads

You have done a great job at explaining the variables in this section. It may be good, however, to take a step back and see if you can simplify anything.

A monthly power bill is made up of several charges, two of which depend on the maximum energy consumed over 15 minutes. This 15-minute average power includes energy that is consumed by loads other than bus chargers, or "uncontrolled loads". In practice, data for uncontrolled loads is sampled and therefore discrete. The representations for how buses use power in Section III are continuous, making their effects difficult to integrate with a discrete uncontrolled load. This section integrates these uncontrolled loads into the planning framework by converting the continuous start and end points, c_{ij} and s_{ij} from Section II, to a vector $\mathbf{p}_{ij}$, where the n^{th} element of $\mathbf{p}_{ij}$ represents the average power over the interval t_{i-1} to t_i from bus i during route j. These route power vectors can be added together to form a discrete profile for the buses.

n^{th} corresponds to i,j?

what is this?

Let the day be divided into time segments, each of duration ΔT. The first step is to determine the index of each segment that a bus begins charging, k^{start}_{ij}, and the index of the segement that a bus finishes charging, k^{end}_{ij}. Each index can be computed as an integer multiple of ΔT that satisfies

not yet defined

$$
\begin{aligned}
k^{\text{start}}_{ij} \cdot \Delta T + r^{\text{start}}_{ij} &= c_{ij} \\
k^{\text{end}}_{ij} \cdot \Delta T + r^{\text{end}}_{ij} &= s_{ij} \\
k^{\text{start}}_{ij}, k^{\text{end}}_{ij} &\in \mathbb{Z} \\
0 < r^{\text{start}}_{ij}, r^{\text{end}}_{ij} &< \Delta T.
\end{aligned}
\tag{25}
$$

Equation (25) yields the discrete indices k^{start}_{ij} and k^{end}_{ij} along with corresponding remainder values r^{start}_{ij} and r^{end}_{ij} which will be used later in this section to calculate the average power for time segments in which buses only charge part of the time. Equation 25 can be rewriten in standard form and zero padded such that

$$
\begin{bmatrix} \Delta T & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta T & 1 & -1 \end{bmatrix}
\begin{bmatrix} k^{\text{start}}_{ij} \\ r^{\text{start}}_{ij} \\ c_{ij} \\ k^{\text{end}}_{ij} \\ r^{\text{end}}_{ij} \\ s_{ij} \end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \forall i,j
\tag{26}
$$

$$\tilde{A}_2 \mathbf{y} = \tilde{\mathbf{b}}_2.$$

and

$$
\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} k^{\text{start}}_{ij} \\ r^{\text{start}}_{ij} \\ c_{ij} \\ k^{\text{end}}_{ij} \\ r^{\text{end}}_{ij} \\ s_{ij} \end{bmatrix}
\leq \begin{bmatrix} 0 \\ \Delta T \\ 0 \\ \Delta T \end{bmatrix} \quad \forall i,j
\tag{27}
$$

$$A_6 \mathbf{y} \leq \mathbf{b}_6.$$

The next step is to use k^{start}_{ij} and k^{end}_{ij} to compute three sets of binary vectors, denoted $\mathbf{g}^{\text{start}}_{ij}$, $\mathbf{g}^{\text{on}}_{ij}$, and $\mathbf{g}^{\text{end}}_{ij}$, which act as selectors for indices which correspond to charge times. The values in $\mathbf{g}^{\text{start}}_{ij}$ and $\mathbf{g}^{\text{end}}_{ij}$ are equal to 1 during intervals that contain energy from the remainders r^{start}_{ij} and r^{end}_{ij}. For example, the values for $\mathbf{g}^{\text{start}}_{ij}$ and $\mathbf{g}^{\text{end}}_{ij}$ from the scenario in Fig. 7b would be

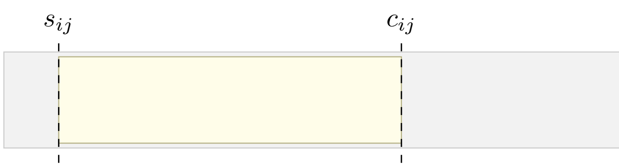
not sure what this means.

$$
\mathbf{g}^{\text{start}}_{ij} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{g}^{\text{end}}_{ij} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.
\tag{28}
$$

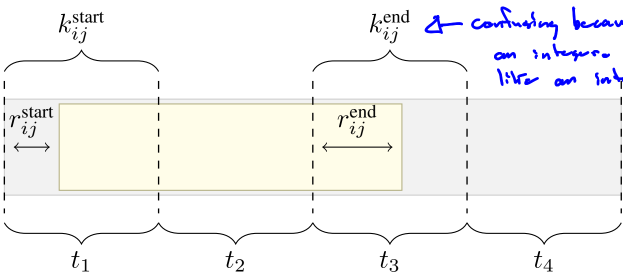
The values in $\mathbf{g}^{\text{on}}_{ij}$ will be equal to 1 for all time indices where buses charges the entire time. For example, the values in g^{on}_{ij} that correspond to Fig. 7b would be

$$
\mathbf{g}^{\text{on}}_{ij} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.
\tag{29}
$$

Let $\mathbf{f}$ be a vector of one-based integer indices such that $f_w = w \ \forall w \in (1, \text{nPoint})$, where nPoint is the desired number



(a) Continuous Charging Segment



confusing because $k_{ij}^{\text{start/end}}$ is an integer. Here it looks like an interval.

(b) Discrete Charging Segments

Is t_i an interval or a point in time?

Fig. 7: Discretization of continuous charging intervals

of discrete samples. For example, if the day was discretized into 4 periods, then $\mathbf{f}$ would be

$$\mathbf{f} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}. \tag{30}$$

Defining the index as an element of $\mathbf{f}$ allows us to convert from the single indices k_{ij}^{start} and k_{ij}^{end} to the binary vectors $\mathbf{g}_{ij}^{\text{start}}$ and $\mathbf{g}_{ij}^{\text{end}}$ by letting

$$\begin{aligned} k_{ij}^{\text{start}} &= \mathbf{f}^T \mathbf{g}_{ij}^{\text{start}} \\ k_{ij}^{\text{end}} &= \mathbf{f}^T \mathbf{g}_{ij}^{\text{end}} \\ 1 &= \mathbf{1}^T \mathbf{g}_{ij}^{\text{start}} \\ 1 &= \mathbf{1}^T \mathbf{g}_{ij}^{\text{end}} \\ \mathbf{g}_{ij}^{\text{start}} &\in \{0,1\}^{\text{nPoint}} \\ \mathbf{g}_{ij}^{\text{end}} &\in \{0,1\}^{\text{nPoint}}, \end{aligned} \tag{31}$$

which can be expressed in standard form and zero padded to form a set of linear constraints.

$$\begin{bmatrix} 0 & \mathbf{0}^T & -1 & \mathbf{f}^T \\ 0 & \mathbf{1}^T & 0 & 0 \\ -1 & \mathbf{f}^T & 0 & \mathbf{0}^T \\ 0 & 0 & 0 & \mathbf{1}^T \end{bmatrix} \begin{bmatrix} k_{ij}^{\text{start}} \\ \mathbf{g}_{ij}^{\text{start}} \\ k_{ij}^{\text{end}} \\ \mathbf{g}_{ij}^{\text{end}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \forall i,j$$
$$\tilde{A}_3 \mathbf{y} = \tilde{\mathbf{b}}_3. \tag{32}$$

The values of $\mathbf{g}_{ij}^{\text{on}}$ can be computed by first noticing that indices that correspond to complete charge intervals must remain between k_{ij}^{start} and k_{ij}^{end}, implying that

$$\left.\begin{aligned} g_w f_w &\le k^{\text{end}} - 1 \\ g_w f_w &\ge k^{\text{start}} + 1 \end{aligned} \quad g_w = 1 \right\}, \tag{33}$$

What is this bracket for?

which can be expressed as a set of linear constraints such that

$$\begin{aligned} g_w \cdot f_w &\le k_{ij}^{\text{end}} + M(1-g_w) - 1 \\ g_w \cdot f_w &\ge k_{ij}^{\text{start}} - M(1-g_w) + 1 \end{aligned} \tag{34}$$

M seems to change values quite frequently.

where M is $2 \cdot$ nPoint. The constraints in (34) do not require that all values between k_{ij}^{start} and k_{ij}^{end} be set to one however, only that if a value is equal to one, that it must be between k_{ij}^{start} and k_{ij}^{end}. For all values between k_{ij}^{start} and k_{ij}^{end} to be 1, the sum of $\mathbf{g}_{ij}^{\text{on}}$ must be equal to the difference between k_{ij}^{end} and k_{ij}^{start} such that

$$\begin{aligned} g_w \cdot f_w &\le k_{ij}^{\text{end}} + M(1-g_w) - 1 \\ g_w \cdot f_w &\ge k_{ij}^{\text{start}} - M(1-g_w) + 1 \\ \mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} &= k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - 1. \end{aligned} \tag{35}$$

The constraints in (35) work well for a general use case, however when k_{ij}^{end} is equal to k_{ij}^{start}, the last constraint in (35) ~~equation 35~~ becomes

$$\mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} = -1, \tag{36}$$

which leads to an empty feasible set because all of the elements of $\mathbf{g}_{ij}^{\text{on}}$ are binary. Let k_{ij}^{eq} be a binary variable which is equal to 0 when k_{ij}^{end} is not equal to k_{ij}^{start}. Equation (35) can be modified to incorporate k_{ij}^{eq} to switch between the cases where k_{ij}^{end} is equal, and not equal to k_{ij}^{start} by letting

$$\begin{aligned} g_w \cdot f_w &\le k_{ij}^{\text{end}} + M(1-g_w) - 1 \\ g_w \cdot f_w &\ge k_{ij}^{\text{start}} - M(1-g_w) + 1 \\ \mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} &= k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - k_{ij}^{\text{eq}}. \end{aligned} \tag{37}$$

and constraining k_{ij}^{eq} such that

$$\begin{aligned} k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - M k_{ij}^{\text{eq}} &\le 0 \\ -k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + M k_{ij}^{\text{eq}} &\le M. \end{aligned} \tag{38}$$

The constraints from (37) and (38) can be expressed in standard form as

$$\begin{aligned} \mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} - k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + k_{ij}^{\text{eq}} &= 0 \\ k_{ij}^{\text{end}} - k_{ij}^{\text{start}} - M k_{ij}^{\text{eq}} &\le 0 \\ -k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + M k_{ij}^{\text{eq}} &\le M \\ g_w\left(f_w + M\right) - k_{ij}^{\text{end}} &\le M - 1 \\ g_w\left(M - f_w\right) + k_{ij}^{\text{start}} &\le M - 1. \end{aligned} \tag{39}$$

The inequality constriants from equation 39 imply that

$$\begin{bmatrix} f_w + M & -1 & 0 \\ M - f_w & 0 & 1 \end{bmatrix} \begin{bmatrix} g_w \\ k_{ij}^{\text{end}} \\ k_{ij}^{\text{start}} \end{bmatrix} \le \begin{bmatrix} M-1 \\ M-1 \end{bmatrix} \forall g_w \in \mathbf{g}_{ij}^{\text{on}} \tag{40}$$

and that

$$\begin{bmatrix} 1 & -1 & -M \\ -1 & 1 & M \end{bmatrix} \begin{bmatrix} k_{ij}^{\text{end}} \\ k_{ij}^{\text{start}} \\ k_{ij}^{\text{eq}} \end{bmatrix} \le \begin{bmatrix} 0 \\ M \end{bmatrix} \forall i,j, \tag{41}$$

which can be concatenated for all i,j and zero padded to form a joint matrix, satisfying

$$A_7 \mathbf{y} \le \mathbf{b}_7. \tag{42}$$

Similarly, the equality constraint from equation 39 can also be concatenated and zero padded such that

$$\mathbf{1}^T \mathbf{g}_{ij}^{\text{on}} - k_{ij}^{\text{end}} + k_{ij}^{\text{start}} + k_{ij}^{\text{eq}} = 0 \;\forall i, j$$
$$\begin{bmatrix} \mathbf{1}^T & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{g}_{ij}^{\text{on}} \\ k_{ij}^{\text{end}} \\ k_{ij}^{\text{start}} \\ k_{ij}^{\text{eq}} \end{bmatrix} = 0 \qquad (43)$$
$$\tilde{A}_4 \mathbf{y} = \tilde{\mathbf{b}}_4.$$

The next step is to define the average power during intervals that only charge for part of the time. These intervals correspond to the remainder values r_{ij}^{start} and r_{ij}^{end} and, as with previous constraints, maintain different behavior when $k_{ij}^{\text{eq}} = 0$ and $k_{ij}^{\text{eq}} = 1$. The average power that corresponds to r_{ij}^{start} and r_{ij}^{end} can be computed as

grammar

what is P

$$\left.\begin{aligned} p_{ij}^{\text{start}} &= \frac{p \cdot (\Delta T - r_{ij}^{\text{start}})}{\Delta T} \\ p_{ij}^{\text{end}} &= \frac{p \cdot r_{ij}^{\text{end}}}{\Delta T}. \end{aligned}\right\} k_{ij}^{\text{eq}} = 1$$
$$\left.\begin{aligned} p_{ij}^{\text{start}} &= \frac{p \cdot (r_{ij}^{\text{end}} - r_{ij}^{\text{start}})}{\Delta T} \\ p_{ij}^{\text{end}} &= 0 \end{aligned}\right\} k_{ij}^{\text{eq}} = 0 \qquad (44)$$

Usually case is an opening bracket, not a closing

what is r_{ij}^{end} when $k_{ij}^{\text{eq}} = 0$?

Equation (44) can also be expressed as a set of linear inequality constraints such that

$$\begin{aligned} p_{ij}^{\text{start}} &\le p - \frac{p}{\Delta T} r_{ij}^{\text{start}} + M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{start}} &\ge p - \frac{p}{\Delta T} r_{ij}^{\text{start}} - M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{start}} &\le \frac{p}{\Delta T} r_{ij}^{\text{end}} - \frac{p}{\Delta T} r_{ij}^{\text{start}} + M k_{ij}^{\text{eq}} \\ p_{ij}^{\text{start}} &\ge \frac{p}{\Delta T} r_{ij}^{\text{end}} - \frac{p}{\Delta T} r_{ij}^{\text{start}} - M k_{ij}^{\text{eq}} \\ p_{ij}^{\text{end}} &\le \frac{p}{\Delta T} r_{ij}^{\text{end}} + M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{end}} &\ge \frac{p}{\Delta T} r_{ij}^{\text{end}} - M\left(1 - k_{ij}^{\text{eq}}\right) \\ p_{ij}^{\text{end}} &\le M k_{ij}^{\text{eq}} \\ p_{ij}^{\text{end}} &\ge -M k_{ij}^{\text{eq}}, \end{aligned} \qquad (45)$$

where M is the battery capacity, and can be expressed in standard form as

$$\begin{aligned} p_{ij}^{\text{start}} + \frac{p}{\Delta T} r_{ij}^{\text{start}} + M k_{ij}^{\text{eq}} &\le M + p \\ -p_{ij}^{\text{start}} - \frac{p}{\Delta T} r_{ij}^{\text{start}} + M k_{ij}^{\text{eq}} &\le M - p \\ p_{ij}^{\text{start}} - \frac{p}{\Delta T} r_{ij}^{\text{end}} + \frac{p}{\Delta T} r_{ij}^{\text{start}} - M k_{ij}^{\text{eq}} &\le 0 \\ -p_{ij}^{\text{start}} + \frac{p}{\Delta T} r_{ij}^{\text{end}} - \frac{p}{\Delta T} r_{ij}^{\text{start}} - M k_{ij}^{\text{eq}} &\le 0 \\ p_{ij}^{\text{end}} - \frac{p}{\Delta T} r_{ij}^{\text{end}} + M k_{ij}^{\text{eq}} &\le M \\ -p_{ij}^{\text{end}} + \frac{p}{\Delta T} r_{ij}^{\text{end}} + M k_{ij}^{\text{eq}} &\le M \\ p_{ij}^{\text{end}} - M k_{ij}^{\text{eq}} &\le 0 \\ -p_{ij}^{\text{end}} - M k_{ij}^{\text{eq}} &\le 0 \end{aligned} \qquad (46)$$

and by using matrix multiplication such that

$$\begin{bmatrix} 1 & 0 & \frac{p}{\Delta T} & 0 & M \\ -1 & 0 & -\frac{p}{\Delta T} & 0 & M \\ 1 & 0 & \frac{p}{\Delta T} & -\frac{p}{\Delta T} & -M \\ -1 & 0 & -\frac{p}{\Delta T} & \frac{p}{\Delta T} & -M \\ 0 & 1 & 0 & -\frac{p}{\Delta T} & M \\ 0 & -1 & 0 & \frac{p}{\Delta T} & M \\ 0 & 1 & 0 & 0 & -M \\ 0 & -1 & 0 & 0 & -M \end{bmatrix} \begin{bmatrix} p_{ij}^{\text{start}} \\ p_{ij}^{\text{end}} \\ r_{ij}^{\text{start}} \\ r_{ij}^{\text{end}} \\ k_{ij}^{\text{eq}} \end{bmatrix} \le \begin{bmatrix} M+p \\ M-p \\ 0 \\ 0 \\ M \\ M \\ 0 \\ 0 \end{bmatrix} \;\forall i, j$$
$$A_8 \le \mathbf{b}_8 \qquad (47)$$

where p_{ij}^{start}, p_{ij}^{end}, and p represent the average power that corresponds to r_{ij}^{start}, r_{ij}^{end}, and full charging intervals respectively. The total average power use is calculated as

$$\mathbf{p}_{\text{total}} = \bar{\mathbf{p}}_{\text{load}} + \sum_{ij} \mathbf{g}_{ij}^{\text{start}} \cdot p_{ij}^{\text{start}} + \mathbf{g}_{ij}^{\text{on}} \cdot p + \mathbf{g}_{ij}^{\text{end}} \cdot p_{ij}^{\text{end}} \qquad (48)$$

where $\bar{\mathbf{p}}_{\text{load}}$ is the average power of the uncontrolled loads. Note, however that (48) contains the bilinear terms $\mathbf{g}_{ij}^{\text{start}} \cdot p_{ij}^{\text{start}}$ and $\mathbf{g}_{ij}^{\text{end}} \cdot p_{ij}^{\text{end}}$. The expression $\mathbf{g}_{ij}^{\text{start}} \cdot p_{ij}^{\text{start}}$ from (48) can be thought of as a vector, $\mathbf{p}_{ij}^{\text{start}}$ which contains values for p_{ij}^{start} whenever g_{ij}^{start} is not equal to 0 such that

$$\left.\begin{aligned} p_w &= p^{\text{start}} & g_w &= 1 \\ p_w &= 0 & g_w &= 0 \end{aligned}\right\} \forall p_w \in \mathbf{p}_{ij}^{\text{start}}, \qquad (49)$$

which can be rewritten as a set of linear inequality constraints such that

$$\begin{aligned} p_w &\ge p_{ij}^{\text{start}} - M(1 - g_w) \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \\ p_w &\le p_{ij}^{\text{start}} + M(1 - g_w) \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \\ p_w &\ge -M g_w \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \\ p_w &\le M g_w \forall p_w \in \mathbf{p}_{ij}^{\text{start}} \end{aligned} \qquad (50)$$

The same approach can be taken to replace $\mathbf{g}_{ij}^{\text{end}} \cdot p_{ij}^{\text{end}}$ with the vector $\mathbf{p}_{ij}^{\text{end}}$ by letting

$$\begin{aligned} p_w &\ge p_{ij}^{\text{end}} - M(1 - g_w) \;\forall p_w \in \mathbf{p}_{ij}^{\text{end}} \\ p_w &\le p_{ij}^{\text{end}} + M(1 - g_w) \;\forall p_w \in \mathbf{p}_{ij}^{\text{end}} \\ p_w &\ge -M g_w \;\forall p_w \in \mathbf{p}_{ij}^{\text{end}} \\ p_w &\le M g_w \;\forall p_w \in \mathbf{p}_{ij}^{\text{end}}, \end{aligned} \qquad (51)$$

which can be written in standard form, stacked to accomodate the constraints for all i, j, and zero padded in the usual fashion as

$$\begin{bmatrix} -1 & 1 & M \\ 1 & -1 & M \\ -1 & 0 & -M \\ 1 & 0 & -M \end{bmatrix} \begin{bmatrix} p_w \\ p_{ij}^{\text{start}} \\ g_w \end{bmatrix} \le \begin{bmatrix} M \\ M \\ 0 \\ 0 \end{bmatrix} \;\forall p_w \in \mathbf{p}_{ij}^{\text{start}} \qquad (52)$$
$$A_9 \le \mathbf{b}_9.$$

Equation (51) can be expressed in standard form, stacked for all i, j, and zero padded in a similar fashion such that

$$\begin{bmatrix} -1 & 1 & M \\ 1 & -1 & M \\ -1 & 0 & -M \\ 1 & 0 & -M \end{bmatrix} \begin{bmatrix} p_w \\ p_{ij}^{\text{end}} \\ g_w \end{bmatrix} \le \begin{bmatrix} M \\ M \\ 0 \\ 0 \end{bmatrix} \;\forall p_w \in \mathbf{p}_{ij}^{\text{end}} \qquad (53)$$
$$A_{10} \mathbf{y} \le \mathbf{b}_{10}.$$

	On-Peak	Off-Peak	Both
Energy	On-Peak Energy Charge	Off-Peak Energy Charge	None
Energy Rate	$u_{\text{e-on}}$	$u_{\text{e-off}}$	None
Power	Demand Charge	None	Facilities Charge
Power Rate	$u_{\text{p-on}}$	None	$u_{\text{p-all}}$

Fig. 8: Description of the assumed billing structure

An expression for the total power used can then be expressed as

$$\mathbf{p}^{\text{total}} = \mathbf{p}^{\text{load}} + \sum_{ij} \mathbf{p}_{ij}^{\text{start}} + \mathbf{p}_{ij}^{\text{end}} + \mathbf{g}_{ij}^{\text{on}} \cdot p \tag{54}$$

and in standard form as

$$\begin{aligned} \begin{bmatrix} 1 & -1^{\text{start}} & -1^{\text{end}} & -1^{\text{on}} \cdot p \end{bmatrix} \begin{bmatrix} \mathbf{p}_w^{\text{total}} \\ \mathbf{p}_w^{\text{start}} \\ \mathbf{p}_w^{\text{end}} \\ \mathbf{g}_w^{\text{on}} \end{bmatrix} &= \mathbf{p}_w^{\text{load}} \\ \tilde{A}_4 \mathbf{y} &= \tilde{\mathbf{b}}_4 \end{aligned} \tag{55}$$

V. Objective Function

This work adopts the objective function developed in [16], which implements the rate schedule from [19]. The rate schedule in [19] is based off of two primary components: power, and energy.

Power is billed per kW for the highest 15 minute average power over a fixed period of time. It is common practice for power providers to use a higher rate during "on-peak" periods when power is in higher demand and use a lower rate during "off-peak" hours, which account for all other time periods.

The rate schedule given in [19] assesses a fee for a users maximum average power during on-peak hours called the On-Peak Power charge, and a user's overall maximum average power, called a facilities charge as shown in figure 8.

Energy fees are also assessed per kWh of energy consumed with a higher rate for energy consumed during on-peak hours and a lower rate for energy consumed during off-peak hours.

A. Power Charges

It is necessary to compute the maximum power both overall and for on-peak periods. Section IV adopted the convention that ΔT denotes the time offset between power samples and that each power reading would reflect the average power used in the previous interval. Now let us set ΔT to 15 minutes, making $\mathbf{p}_{\text{total}}$ an expression of the 15 minute average power. Next, let $\mathcal{S}_{\text{on}}$ be the set of all indices belonging to on-peak time periods such that $j \in \mathcal{S}_{\text{on}}$ implies that the j^{th} element of $\mathbf{p}^{\text{total}}$, p_j^{total}, represents a 15 minute average during an on-peak interval and let q_{on} be the maximum on-peak average power. With these definitions, constriants for determining the maximum on-peak average are defined as

$$\begin{aligned} p_j^{\text{total}} &\le q_{\text{on}} \ \forall j \in \mathcal{S}_{\text{on}} \\ \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} p_j^{\text{total}} \\ q_{\text{on}} \end{bmatrix} &\le 0 \ \forall j \in \mathcal{S}_{\text{on}} \\ A_{11}\mathbf{y} &\le \mathbf{0} \\ A_{11}\mathbf{y} &\le \mathbf{b}_{11} \end{aligned} \tag{56}$$

Because an increased value in q_{on} is directly related to an increase in cost, the optimizer will minimize q_{on} until it is equal to the maximum value in $\{p_j^{\text{total}} \ \forall j \in \mathcal{S}_{\text{on}}\}$. A similar proceedure can be used to derive a set of constraints for the overall maximum average power, denoted q_{all}, and is represented as

$$\begin{aligned} A_{12}\mathbf{y} &\le \mathbf{0} \\ A_{12}\mathbf{y} &\le \mathbf{b}_{12}. \end{aligned} \tag{57}$$

The charges for power are then expressed as

$$\begin{aligned} \text{power cost} &= q_{\text{on}} \cdot u_{\text{p-on}} + q_{\text{all}} \cdot u_{\text{p-all}} \\ &= \begin{bmatrix} u_{\text{p-on}} & u_{\text{p-all}} \end{bmatrix} \begin{bmatrix} q_{\text{on}} \\ q_{\text{all}} \end{bmatrix} \\ &= \mathbf{u}_{\text{p}}^T \mathbf{y} \end{aligned} \tag{58}$$

where $u_{\text{p-on}}$ is the rate per kW for on-peak power use, or the demand charge and $u_{\text{p-all}}$ is the rate per kW for the overall maximum 15 minute average.

B. Energy Charges

Energy is defined as the integral of power over a length of time. Because the values for power given in this work reflect an average power, the energy over a given period can be computed by multiplying the average power by the change in time, or ΔT such that

$$\text{Total Energy} = \mathbf{1}^T \mathbf{p}_{\text{total}} \cdot \Delta T. \tag{59}$$

However, because the energy is billed for on-peak and off-peak time periods, we define two binary vectors $\mathbf{1}_{\text{on}}$ and $\mathbf{1}_{\text{off}}$ such that $1_j^{\text{on}} = 1 \ \forall j \in S_{\text{on}}$ and zero otherwise. Similarly, $1_{\text{off}} = \mathbf{1} - 1_{\text{on}}$. The on-peak and off-peak energy can then be computed as

$$\begin{aligned} \text{On-Peak Energy} &= \mathbf{1}_{\text{on}}^T \mathbf{p}_{\text{total}} \cdot \Delta T \\ \text{Off-Peak Energy} &= \mathbf{1}_{\text{off}}^T \mathbf{p}_{\text{total}} \cdot \Delta T. \end{aligned} \tag{60}$$

Let $u_{\text{e-on}}$ and $u_{\text{e-off}}$ represent the on-peak and off-peak energy rates respectively. The total cost for energy is computed as

$$\begin{aligned} \text{Energy Cost} &= (\mathbf{1}_{\text{on}} \cdot u_{\text{e-on}} \cdot \Delta T)^T \mathbf{p}_{\text{total}} + (\mathbf{1}_{\text{off}} \cdot u_{\text{e-off}} \cdot \Delta T)^T \mathbf{p}_{\text{total}} \\ &= (\mathbf{u}_{\text{e-on}} + \mathbf{u}_{\text{e-off}})^T \mathbf{p}_{\text{total}} \\ &= \mathbf{u}_{\text{e}}^T \mathbf{y} \end{aligned} \tag{61}$$

C. Cost Function and Final Problem

The entire cost function is given as the sum of the energy and power costs such that

$$\begin{aligned} \text{Cost} &= \mathbf{u}_{\text{p}}^T \mathbf{y} + \mathbf{u}_{\text{e}}^T \mathbf{y} \\ &= (\mathbf{u}_{\text{p}} + \mathbf{u}_{\text{e}})^T \mathbf{y} \\ &= \mathbf{v}^T \mathbf{y} \end{aligned} \tag{62}$$

The complete problem can now be formulated as

$$\begin{gathered}\min_{\mathbf{y}} \mathbf{y}^T\mathbf{v} \text{ subject to} \\ \begin{bmatrix}\tilde{A}_1\\ \tilde{A}_2\\ \tilde{A}_3\end{bmatrix}\mathbf{y} = \begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\\ \tilde{\mathbf{b}}_3\end{bmatrix}, \begin{bmatrix}A_1\\ A_2\\ A_3\\ A_4\\ A_5\\ A_6\\ A_7\\ A_8\\ A_9\\ A_{10}\\ A_{11}\\ A_{12}\end{bmatrix}\mathbf{y} \le \begin{bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\\ \mathbf{b}_3\\ \mathbf{b}_4\\ \mathbf{b}_5\\ \mathbf{b}_6\\ \mathbf{b}_7\\ \mathbf{b}_8\\ \mathbf{b}_9\\ \mathbf{b}_{10}\\ \mathbf{b}_{11}\\ \mathbf{b}_{12}\end{bmatrix}\end{gathered} \tag{63}$$

or

$$\begin{gathered}\min_{\mathbf{y}} \mathbf{y}^T\mathbf{g} \text{ subject to} \\ \tilde{A}\mathbf{y} = \tilde{\mathbf{b}},\ A\mathbf{y} \le \mathbf{b},\end{gathered} \tag{64}$$

VI. Results

A. comparison with baseline

B. Cost increase per bus

C. Computation Time

D. Comparison with Prior Work

compare continuous variables to discrete

VII. Conclusions and Future Work

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