A Light Weight Approach to Minimize Charging Cost for Electric Bus Fleets

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***Abstract*—Insert abstract here**

***Index Terms*—Insert keywords here**

I. Introduction

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II. Scheduling an Optimal Solution

This section describes a program that finds an optimal charge schedule where buses are allowed to charge without regard to the number of available chargers. This solution is considered "optimal" and will be used in later sections to formulate a feasible solution that accounts for the number of chargers.

A. Formulation

The cost objective that we desire to minimize is modeled after [1], which contains two primary elements: the cost of energy, and power demand. Energy is billed per kWh for on-peak and off-peak hours. The on-peak rate is more expensive because there is generally more demand for power during this time, whereas off-peak hours tend to be less expensive. The demand is covered in two separate chargers. The first is a facilities charge which is billed per kW for the highest 15-minute average power use over the course of the month. The second is a demand charge, which is also billed per kW, but is only billed for the highest 15-minute average power usesd during on-peak hours. The rates for each component are given in Table I.

Before we may compute the total monthly cost of electricity, we must define expressions for the average power and energy over time. Let each day be divided into 15-minute intervals for each bus where the average power expended for bus i during time j is denoted $p(i,j)$ as shown in Fig. 1. The resulting solution of the program we will develop will yield the average power expended by each bus during each period of time.

One constraint for which the solution must account is bus availability. When a bus is out of the station, the maximum average power for that time must be zero. For example, if bus 1 were out on route for t_5, t_6, and t_7, then the average power for those periods would be equal to zero as shown in Fig. 2. Let $\boldsymbol{b}_{p(i,j)}$ be the average power used by bus i at time index j, and $\boldsymbol{b}$ be a vector which contains $b_{p(i,j)}$ for each bus and time index. Also let $\mathcal{A} \subset i \times j$ be the set of all indices where bus i is in the station during time t_j and $\tilde{\mathcal{A}}$ be its complement. Furthermore, let p_{max} be the maximum power that a charger can deliver.

We define a set of constraints so that buses do not use power when not in the station by letting

$$\begin{aligned} b_{p(i,j)} &= 0 \ \forall i,j \in \tilde{\mathcal{A}} \\ b_{p(i,j)} &\leq p_{\text{max}} \ \forall i,j \in \mathcal{A} \\ -b_{p(i,j)} &\leq 0 \ \forall i,j \in \mathcal{A} \end{aligned} \tag{1}$$

The constraints in Eq. 1 however to not account for buses that must charge for partial periods. For example, if the day were divided into 15-minute time blocks, but a bus began charging at 10:07, then an average power of p_{max} for that time slot would be inaccurate. Therefore, Eq. 1 must be modified so that the average power for each block correctly reflects partial availability. Let $\alpha(i,j)$ give the percentage of time that bus i is available during time j. Eq. 1 can be rewritten as

$$\begin{aligned} -b_{p(i,j)} &\leq 0 \ \forall i,j \\ b_{p(i,j)} &\leq p_{\text{max}} \cdot \alpha(i,j) \ \forall i,j \end{aligned} \tag{2}$$

B. Battery

Each bus must also maintain its state of charge above acceptable levels throughout the day. When buses leave the station, each bus discharges some quantity of energy throughout the course of the route. Let $\delta(i,j)$ be the amount of charge lost by bus i at time j and let $h(i,j)$ be the state of charge of bus i at time j. The state of charge for each bus can be defined as

$$\begin{aligned} &h(i,j) = h(i,j-1) + b_p(i,j-1) \cdot \Delta T - \delta(i,j) \ \forall i, j > 1 \\ &h(i,1) = \eta_i \ \forall i \end{aligned} \tag{3}$$

where η_i is the initial state of charge for bus i and ΔT is the difference in time between $t_{i,j}$ and $t_{i,j+1}$. Now that each value for the state of charge is defined, each value for h must be constrained so that it is greater than a given threshold, h_{min} but does not exceed the maximum battery capacity h_{max}. This yields

$$\begin{aligned} -h(i,j) &\leq -h_{\text{min}} \ \forall i,j \\ h(i,j) &\leq h_{\text{max}} \ \forall i,j. \end{aligned} \tag{4}$$

The final battery related constraint has to do with how we are planning for the bus. The expenses that come from power are computed monthly, but we desire to simulate the movements of the bus for only a day, and use this to extrapolate what the monthly cost may be. Therefore, the state of charge for a bus at the end of the day must reflect its starting value. This yields the following constraint:

$$h_{i,\text{end}} = h(i,1) \ \forall i$$

so that

$$h_{i,\text{end}} - h(i,1) = 0 \ \forall i \tag{5}$$

TABLE I: Description of the billing structure

	On-Peak	Off-Peak	Facilities (Both)
Energy Rate	5.8282 ¢/kWh	2.9624 ¢/kWh	None
Energy Rate Symbol	$\mu_{\text{e-on}}$	$\mu_{\text{e-off}}$	None
Power Rate	\$ 15.73 /kW	None	\$ 4.81 /kW
Power Rate Symbol	$\mu_{\text{p-on}}$	None	$\mu_{\text{p-all}}$

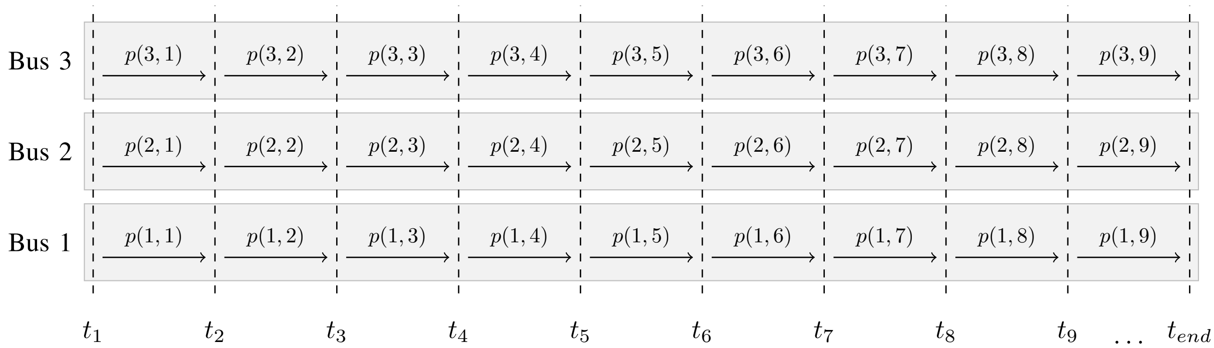

Fig. 1: Demonstrates how bus power use is conceptualized

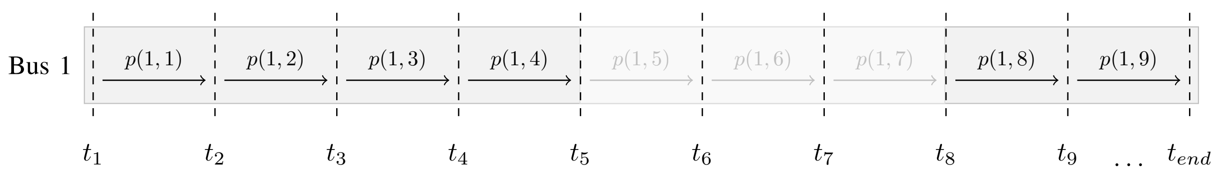

Fig. 2: Bus schedule with availability

C. Cumulative Load Management

While this solution does not directly account for the number of available chargers, we do account for the cumulative load capacities of all chargers. Let the number of chargers be denoted n_{charger}. We desire to maintain the average cumulative power for each time step at a level that is serviceable given n_{charger}. We define a slack variable $p_c(j)$ which represents the total average power consumed by all buses at time j. The variable $p_c(j)$ is computed as the sum of average bus powers so that

$$p_c(j) = \sum_i b_{p(i,j)}$$

or also as

$$p_c(j) - \sum_i b_{p(i,j)} = 0 \ \forall j. \tag{6}$$

D. Objective

Now that the relevent constraints have been addressed, we must work towards computing the total objective function. We do so by first computing the total average power for the complete system. This total power is comprised of power used by the buses, and power used by external sources such as lights, ice melt, electric trains, etc which we refer to as "uncontrolled loads", where the average power for the uncontrolled loads at time step j is denoted $u(j)$. We compute the total power as the sum of power used by the buses, $p_c(j)$ and the power consumed by uncontrolled loads $u(j)$ so that the total power, denoted $p_t(j)$ is computed as

$$p_t(j) = p_c(j) + u(j)$$

or

$$p_t(j) - p_c(j) - u(j) = 0 \ \forall j. \tag{7}$$

The next step is to compute the fifteen minute average power use for each time step, denoted p_{15}. We do this by letting

$$p_{15}(j) = \frac{1}{n} \sum_{l \in \{j_{15}\}} p_t(l) \tag{8}$$

where $\{j_{15}\}$ is the set of all indices 15 minutes prior to j and n is the cardinality of $\{j_{15}\}$. Next, note that the rate schedule requires both the maximum overall average power, denoted $p_{\text{facilities}}$, and the maximum average power during on-peak hours, or p_{demand}. Let $\mathcal{S}_{\text{on}}$ be the set of time indices belonging to on-peak hours, and recall that the max over all average power values is greater than or equal to $p_{15}(j)$ for all j. We can express this constraint is

$$p_{\text{facilities}} \geq p_{15}(j) \ \forall j$$

or alternatively as

$$p_{15}(j) - p_{\text{facilities}} \leq 0 \ \forall j \tag{9}$$

Because $p_{\text{facilities}}$ will be used in the objective function, the value for $p_{\text{facilities}}$ will be minimised until it is equal to the largest value in p_{15}. Following a similar logic, we also define a set of constraints for the maximum average on-peak power, p_{demand} so that

$$p_{15}(i) - p_{\text{demand}} \leq 0 \ \forall i \in \mathcal{S}_{\text{on}} \tag{10}$$

The next step in computing the objective function is to compute the total *energy* consumed during on and off-peak hours respectively. Let e_{on} be the total energy consumed during on-peak hours and e_{off} be the energy consumed during off-peak hours. We can compute energy as the product of average power and time. In our case, we compute this as

$$\begin{aligned} e_{\text{on}} &= \Delta T \cdot \sum_{i \in g} p_t(i) \\ e_{\text{off}} &= \Delta T \cdot \sum_{i \in \tilde{g}} p_t(i) \end{aligned} \tag{11}$$

where $\tilde{g}$ is the complement of g. We can now compute the total monthly charge as

$$J_{\text{cost}} = \begin{bmatrix} e_{\text{on}} \\ e_{\text{off}} \\ p_{\text{facilities}} \\ p_{\text{demand}} \end{bmatrix}^T \begin{bmatrix} \mu_{\text{e-on}} \\ \mu_{\text{e-off}} \\ \mu_{\text{p-all}} \\ \mu_{\text{p-on}} \end{bmatrix} \tag{12}$$

E. Pre-Smoothing

Because the optimizer tends to work along boundaries to find optimal solutions, the schedule produced by the previous formulation will tend to either contain values equal to the maximum charge rate, or zero. This results in many instances of off-to-on and on-to-off scenarios which is not well tolerated by hardware. For example, if the chargers have a maximum charge rate of 350 kW, then going from zero to 350 several times a minute may decrease the life of the hardware and be difficult to achieve in practice.

It is therefore necessary to implement a smoothing criteria so that the 'on-off' patterns from the first solution are softened. This is done by first solving the un-constrained charge problem as given. Next, the same problem is solved again but with two primary differences. The first is that the demand, facilities, on-peak energy, and off-peak energy are constrained so that they are equal to the values obtained in the previous problem, resulting in an equivalent solution. Next, we define an alternative objective which incentivizes "smooth" transitions between time steps.

This objective is defined as

$$J_{\text{thrash}} = \frac{1}{n} \sum_{i,j,\in\mathcal{K}} \|b(i,j) - b(i,j-1)\|_2^2, \tag{13}$$

where $\mathcal{K}$ is the set of all i, j where bus i may charge during time j and $j-1$.

III. Scalability

Before the bus charge problem can be solved, we need to address how the problem will scale. Historically, when buses are routed to chargers it requires a program which will evaluate all possible conflicts and selects a contention-free schedule, which tends to increase on the order of $0(n^2)$ and is NP-hard==insert citation of knapsack problem==. Before the bus problem can scale, we need a method to separate buses into groups to manage the problem's scope.

The group-selection problem separates buses into n_{group} groups, where group m is allocated n^m_{charger} chargers and n^m_{bus} buses. Each group must have sufficient chargers to fill it's needs and prefers buses with dissimilar routes.

We know that the number of cross-terms in future problems will be reduced when each group has the same number of buses. Therefore, let n^m_{bus} be described as

$$\begin{aligned} n^m_{\text{bus}} &\geq \left\lfloor \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rfloor \\ n^m_{\text{bus}} &\leq \left\lceil \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rceil \end{aligned}$$

or alternatively as

$$\begin{aligned} -n^m_{\text{bus}} &\leq -\left\lfloor \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rfloor \\ n^m_{\text{bus}} &\leq \left\lceil \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rceil. \end{aligned} \tag{14}$$

The next set of constraints ensures that there are sufficient charging resources in each group. First, we define a slack variable which gives the total power used in group m at time step j as $p(j,m)$. Recall, we also know the expected power use for each bus as this is a result of the linear program described in sections II-A through II-D as $b_{p(i,j)}$, which allows us to describe the total power for any one group as

$$p(j,m) = \sum_m \beta(i,m) b_{p(i,j)}$$

or alternatively as

$$p(j,m) - \sum_m \beta(i,m) b_{p(i,j)} = 0 \;\forall i,j,m \tag{15}$$

where $\beta(i,m)$ is a binary indicator variable that is one when bus i is in group m and serves as part of the solution to this problem.

Next, we know that the total load of each group must be less than or equal to the collective capability of that group's chargers, which can be expressed as

$$n^m_{\text{charger}} \cdot p_{\max} \leq p(j,m) \;\forall j,m$$

or in standard form as

$$n^m_{\text{charger}} \cdot p_{\max} - p(j,m) \leq 0 \;\forall j,m \tag{16}$$

so that the number of chargers is sufficient to charge the collective load of the group.

We also desire to group buses together who's routes are most orthogonal. If two buses contain no overlap, they will be easiest to schedule and the inner product of their schedules from the first program will be equal to zero. Let

$$\phi(i,i') = \mathbf{b}(i,:)^T \mathbf{b}(j,:),$$

where $\mathbf{b}(i,:)$ is the charge schedule for bus i as computed in the first program. We desire to minimize the total cross terms $\phi(i,i')$ for all buses in the same group. Define a slack variable $v(i,i',g)$ which is equal to $\phi(i,i')$ if buses i and i' are in the same group, and zero otherwise so that

$$\begin{cases} v(i,i',g) = \phi(i,i') & \beta(i,m) = 1, \beta(i',m) = 1 \\ v(i,i',g) = 0 & \text{otherwise} \end{cases}$$

which can also be expressed by letting

$$\begin{aligned}
&v(i,i',g) \leq \phi(i,i') \\
&v(i,i',g) \geq \phi(i,i') - M\left(2 - \beta(i,m) - \beta(i',m)\right) \\
&v(i,i',g) \leq 0 + M\beta(i,m) \\
&v(i,i',g) \leq 0 + M\beta(i',m) \\
&v(i,i',g) \geq 0
\end{aligned}$$

or alternatively as

$$\begin{aligned}
v(i,i',g) &\leq \phi(i,i') \\
-v(i,i',g) + M\beta(i,m) + M\beta(i',m) &\leq -\phi(i,i') + 2M \\
v(i,i',g) - M\beta(i,m) &\leq 0 \\
v(i,i',g) - M\beta(i',m) &\leq 0 \\
-v(i,i',g) &\leq 0.
\end{aligned} \tag{17}$$

The final objective can then be expressed as

$$J_{\text{select}} = \sum_{i,i',g} v(i,i',g). \tag{18}$$

Because this objective is more of a preference then a requirement for the bus charging problem, it is not always necessary to solve completely. Experimental results have shown that increasing the MIP Gap to upwards of 50% can be sufficient for this problem and helps reduce the computational complexity of this step.

IV. De-Fragmentation

A minimum charge session length is another operational constraint that must be considered. We also consider constraints on minimum energy delivered per session. The intent of these constraints is to avoid charging for small durations or for small amounts of energy so that charge sessions are consolidated for convenience.

To solve this program, assume there exists a "smoothed" solution from the second program which has been appropriately placed in a group from program two. Next, let the preliminary solution be subdivided into charge sessions, each with a specific amount of energy, a minimum start time, and a maximum stop time. If the energy for any charge session is less than the allowed, then this session is marked as "fragmented". The remaining sessions are either marked as "used" or "unused", where a used session delivers more power than specified in the "fragmentation-threshold", and an unused session delivers zero power.

The purpose of this program is to determine which sessions will be "active" in deployment while adhering to minimum charge thresholds. The sessions in question are the "fragmented" sessions. Let $\theta(i,r)$ be a binary variable which indicates if session r from bus i will be active. Because the only sessions in question are fragmented, we only need to define $\theta(i,r)$ for fragmented sessions. Limiting the binary variables in this fashion significantly reduces the computational complexity of this step. The charge problem will be resolved using the same constraints and objective as the first program, but with two primary changes.

The first change constrains the minimum power delivery for each "active" charge session to be *at least* as large as the original power delivery. Let $\rho(i,r)$ be a vector which is ΔT, in hours, during the times bus i charges during session r and zero otherwise so that

$$\mathbf{b}(i,:)\rho(i,r) \geq \psi(i,j) \tag{19}$$

where $\psi(i,j)$ is the minimum energy for session i,r and session i,r is considered "active". For inactive sessions, the energy is constrained so that it is equal to zero. Finally, for fragmented sessions, the session energy must be greater than the minimum threshold, ω when active and zero otherwise which can be expressed as

$$\begin{aligned}
\mathbf{b}(i,:)\rho(i,r) &\geq \omega - \omega(1 - \theta(i,r)) \\
\mathbf{b}(i,:)\rho(i,r) &\leq 0 + \theta(i,r)e_{\text{max}}
\end{aligned} \tag{20}$$

where e_{max} is the maximum energy delivered in a session.

V. Charge Schedules

The results from the Linear Program defined in the previous section give us a general estimate of how much and when buses should charge, however we must still address two primary issues. The first is defining concrete start and stop times for each charge session. The second is limiting the charge sessions to a finite number of chargers. After solving the first program, we take the results and derive *preliminary* intervals and average power consumptions for each charge session.

For example, consider a solution to a three bus, two charger scenario given in Fig. 3. Note that there appears to be three buses charging at the same time from t_5 to t_6 even though there are only two chargers. We can reformulate this solution in terms of continuous start and stop variables and a variable charge rate so that the *duration* of each charge session may be relaxed. The objective is to store the given energy in the corresponding bus within the given charge interval.

Note how few of the charge sessions utilize the chargers to full capacity. This implies that there exists a smaller charge window in which equivalent power can be delivered. This allow us to use the charge durations from the solution from Fig. 3 as bounds on *allowable* charge windows instead of absolute truth.

An example of how Fig. 3 may be reformulated is given in Fig. 5. Note how the actual charge sessions don't necessarily need to take up all the time they were initially allocated in the first solution and that these times can fluctuate if the average charge rate is less than the maximum charger capacity. In this example, we assume a maximum charge capacity of 350kW.

Note how the third charge session does have to be exactly where it was scheduled because the average is equal to the maximum charge rate. If we examin just the schedule for Bus 1, we note that there are four essential variables for the corresponding charge session: $a(i,r)$, $b(i,r)$, $f(i,r)$ and $d(i,r)$ which represent the minimum start time, actual start time, actual end time, and maximum end time respectively.

The problem we must now solve is one of arranging these "rectangles" such that each one is larger than it's minimum width (or charge time). We must also account for the number of chargers. It can be helpful to view the problem as a bin

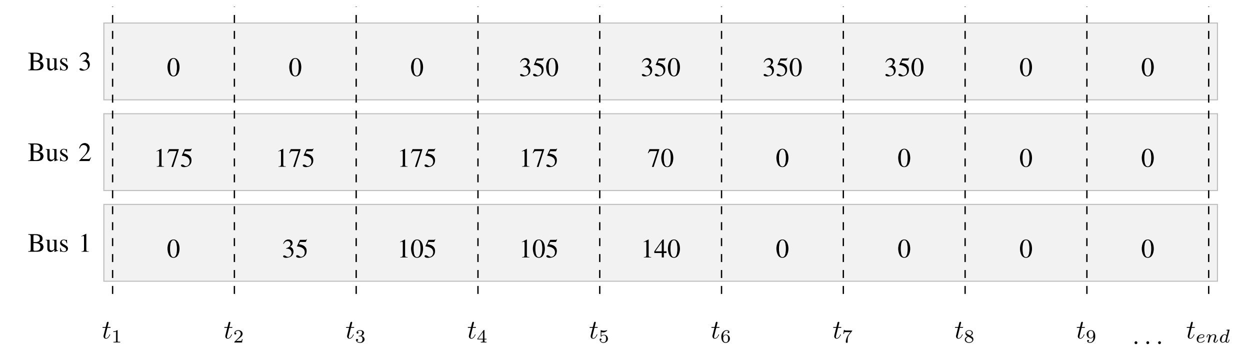

Fig. 3: An example solution to a 3-bus, 2-charger scenario from the first QP

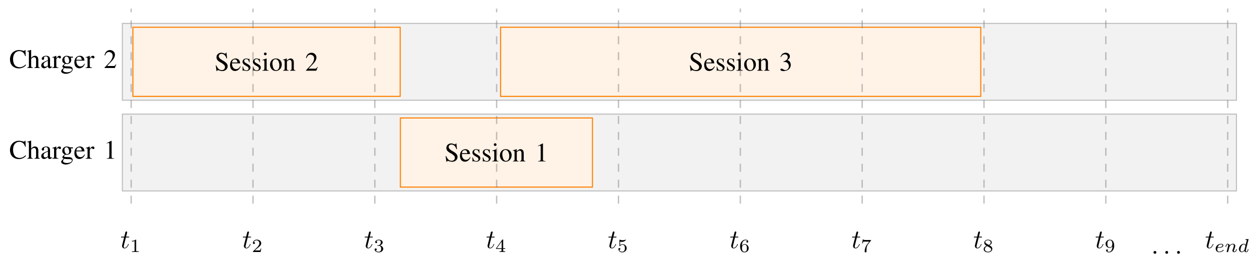

Fig. 4: Demonstrates the solution to the second program

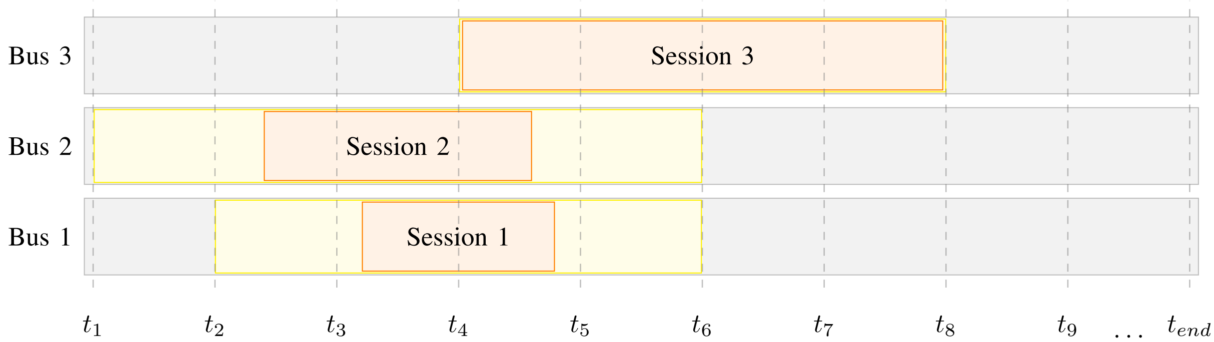

Fig. 5: Demonstrates how the problem from the first QP can be reformulated in terms of continuous variables

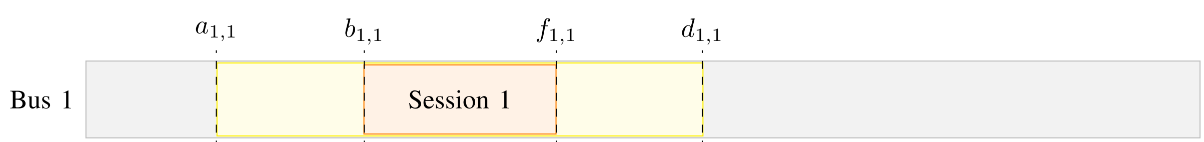

Fig. 6: Gives variables of optimization for the second program

packing problem, where each session must fit within the "swim lane" of a charger. For example, taking the charge sessions given in Fig. 5 and arranging them so that there is no overlap between sessions will yield a valid solution as shown in Fig. 4.

From Fig. 6, we know that $a(i,r), b(i,r), f(i,r)$ and $d(i,r)$ must be such that

$$\begin{aligned} a(i,r) &\leq b(i,r) \\ b(i,r) &\leq f(i,r) \\ f(i,r) &\leq d(i,r). \end{aligned}$$

or alternatively as

$$\begin{aligned} -b(i,r) &\leq -a(i,r) \\ b(i,r) - f(i,r) &\leq 0 \\ f(i,r) &\leq d(i,r) \end{aligned} \tag{21}$$

Where $a(i,r)$ and $d(i,r)$ are known from the previous optimization problem, and $b(i,r)$ and $f(i,r)$ are optimization variables.

We must differentiate between chargers and so, define $\sigma(i,r,k)$ as a binary selector variable which is one if charger k services bus i for session r and zero otherwise. We know that only one charger can charge each bus at a time. We also know that each charge session *must* be serviced, which implies that

$$\sum_k \sigma(i,r,k) = 1 \;\forall i,r. \tag{22}$$

Next, we also know that during each session a certain amount of energy must be transfered from the charger to the battery. The amount of energy that must be transfered to bus i during session r can be computed from the results of the first linear program and is denoted $e(i,r)$. We can compute a minimum time window from this value as

$$w(i,r)_{\text{min}} = \frac{e(i,r)}{p_{\text{max}}}.$$

If we include constraints for a minimum time per session, then the previous expression becomes

$$w(i,r)_{\text{min}} = \max\left(w_{\text{min}}, \frac{e(i,r)}{p_{\text{max}}}\right)$$

Because this is the minimum time window, we must ensure that the difference between the start and stop times is at least this large so that

$$f(i,r) - b(i,r) \geq w(i,r) \;\forall i,r$$

or alternatively,

$$b(i,r) - f(i,r) \leq -w(i,r) \;\forall i,r. \tag{23}$$

The final set of constraints deals with contention so that no charger can be scheduled for two sessions that overlap. let $\mathcal{L} = \{(i,r) \times (i',r')\}$ where charge sessions i,r and i',r' have the potential to overlap. Before we can prevent overlap, we must define a binary variable $l(i,r,i',r')$ which is equal to one when session i,r is scheduled before session i',r' and zero otherwise so that

$$\begin{cases} f(i,r) \leq b(i',r') & l(i,r,i',r') = 1 \\ f(i',r') \leq b(i',r') & l(i,r,i',r') = 0 \end{cases}$$

Here we can expand this thought through use of the "big-M" technique. Let M be large. In this case, we can set it equal to the number of seconds in a day. We know what the top constraint must be trivially satisfied when $l(i,r,i',r') = 0$ and the bottom must also when $l(i,r,i',r') = 1$. This leads to a reformulation so that

$$\begin{aligned} f(i,r) - b(i',r') &\leq M(1 - l(i,r,i',r') \\ f(i',r') - b(i,r) &\leq l(i,r,i',r')M \end{aligned}$$

However, this constraint *only* needs to hold when sessions i,r and i',r' are scheduled to charge on the same charger or that $\sigma(i,r,k) = \sigma(i',r',k) = 1$. We can reformulate the above constraint to satisfy this condition by letting

$$\begin{aligned} f(i,r) - b(i',r') &\leq M(3 - \sigma(i,r,k) - \sigma(i',r',k) - l(i,r,i',r')) \\ f(i',r') - b(i,r) &\leq M(2 - \sigma(i,r,k) - \sigma(i',r',k) + l(i,r,i',r')) \end{aligned} \tag{24}$$

Finally, we desire the power profile to closly match the profile given in the first linear program, which would occure if each charge session matched the durations given in the first solution. We formulated an objective function which minimized the largest difference between

$$\min_{f,b} \sum_{i,r} \|b(i,r) - a(i,r)\|_2^2 + \|f(i,r) - d(i,r)\|_2^2$$

which has the effect of driving each variable to the desired value and more heavily penalizing values that are further from their optimal.

VI. Optimizing Charge Schedules

Many times it is not feasible to compute the optimal set of charge schedules given in the previous sections. As the number of buses and charge sessions becomes large, computing a small-MIPGap solution becomes untractable. Using a large MIPGap resolves issues related to computational complexity, but results in sub-optimal charge-time windows.

We compute a more optimal set of charge windows by using the results from the previous program to infer charger assignment, and ordering for each charge session. We also know that the optimal solution will expand the charge windows to use any available time where a charger is unused, implying that the "stop" time for each session will either be equal to it's buse's departure time, or the start time of the next window which can be expressed as

$$\begin{cases} c(s,i,r+1) = c(f,i,r) & c(d,i,r) > c(a,i,r+1) \\ \begin{aligned} c(s,i,r+1) &= c(a,i,r+1) \\ c(f,i,r) &= c(d,i,r) \end{aligned} & c(d,i,r) <= c(a,i,r+1) \end{cases}$$

where $c(s,i,r)$ is the start time for charger i's r^{th} charge session, $c(f,i,r)$ is the stop time for charger i's r^{th} charge session, $c(d,i,r)$ is the departure time for the bus scheduled for charger i's r^{th} charge session, and $c(a,i,r)$ is the arrival time for the bus scheduled for charger i's r^{th} charge session. The minimum charge length must also be used so that energy can be properly delivered, so that

$$c(f,i,r) - c(s,i,r) \geq w(i,r)$$

where $w(i,r)$ is the corresponding minimum charge time corresponding the session.

The final step to optimizing the charge windows is to give preference to windows with larger power deliveries. Let the objective for the optimization program be

$$J_{\text{window}} = \frac{1}{n} \sum_{i,r} \left\| \frac{c(f,i,r) - c(s,i,r)}{e(i,r)} \right\|_2^2. \tag{25}$$

When the function J contains windows with equal amounts of energy, the minimum will be found where each charge interval is the same width. As the amount of energy increases, the objective penalizes less for larger window sizes and thus gives preference to high energy sessions.

VII. Multi-Rate Charging

Up to this point, we have computed the "optimal" schedule which assumes any bus can charge without regard to the number of chargers. We then separate buses into groups to reduce the scope of the problem and treat each sub-problem separately while we defragment and assign each charge session to specific chargers before determining the final start and stop times for each bus's charge session so that each bus maintains charge sessions at various times and chargers with specific energy requirements.

The final step in this process is to determine how the energy will be delivered so that cost is minimised. Begin with constraints for bus power, energy, and cost from Section II that are expressed as equations 2, 6, 7, 8, 9, 10 and 11. Next, include constraints for energy so that the energy for each charge session is properly delivered (see equation 19). Once this program is successfully run, it must be run again using the smoothing objective from Eqn. 13.

Acknowledgment

This material is based in part upon work supported by the National Science Foundation through the ASPIRE Engineering Research Center under Grant No. EEC-1941524, the Department of Energy through a prime award with ABB under Grant No. DE-EE0009194, and PacifiCorp under contract number 3590. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation, the Department of Energy, or Pacificorp.

Variable	Description	Range	Variable	Description	Range
Indices					
i	Bus index	$\mathbb{N}$	j	Time Index	$\mathbb{N}$
k	Charger index	$\mathbb{N}$	r	Route Index	
m	group index	$\mathbb{N}$			
Optimal Solution — Formulation					
n_{bus}	The number of buses in the optimization framework.	$\mathbb{Z}$	n_{time}	The number of time indices in a day.	$\mathbb{Z}^{+}$
$b_{p(i,j)}$	The average power consumed by bus i during time period j.	$\mathbb{R}$	t_j	The time at time index j. This paper also refers to the period of time from t_j to t_{j+1} as "period t_j".	$\mathbb{R}$
$\boldsymbol{b}$	A vector containing each value for $b_{p(i,j)}$.	$\mathbb{R}^{n_{\text{bus}} \cdot n_{\text{time}}}$	$\alpha(i,j)$	the percentage of time bus i spends in the station from t_j to t_{j+1}.	$[0,1]$
$\mathcal{A}$	The set of all $i \times j$ elements where bus i can charge at time index j	$i \times j$	p_{max}	The maximum power a bus charger can deliver to a bus in kW. This paper assumes a value of 350 for most examples and results.	$\mathbb{R}^{+}$
$\tilde{\mathcal{A}}$	The complement of $\mathcal{A}$.	$i \times j$			
Optimal Solution — Battery					
h_{min}	The minimum allowable state of charge	$(0, h_{\text{max}})$	h_{max}	The maximum state of charge	$\mathbb{R}^{+}$
η_i	The beginning state of charge for bus i	$(h_{\text{min}}, h_{\text{max}})$	$h(ij)$	The state of charge for bus i at time t_j.	$(h_{\text{min}}, h_{\text{max}})$
ΔT	The change in time from t_j to t_{j+1}	$\mathbb{R}^{+}$	$\boldsymbol{h}$	A vector containing all state of charge values.	$\mathbb{R}_{+}^{n_{\text{bus}} \cdot n_{\text{time}}}$

$\delta(ij)$	The battery discharge for bus i during time period j.	$\mathbb{R}_+$	$h(i, \text{end})$	Bus i's final state of charge.	$(h_{\text{min}}, h_{\text{max}})$
Optimal Solution — Cumulative Load Management					
n_{charger}	The time index for the start of bus i's j^{th} stop	$\mathbb{Z}_+$	$p_c(j)$	The average power consumed by all buses during time period j.	$\mathbb{R}$
$\boldsymbol{p_c}$	A vector containing all values of $p_c(j)$.	$\mathbb{R}_+^{n_{\text{time}}}$	J_{thrash}	A secondary objective function which penalizes multiple plug-in instances per charge session.	$\mathbb{R}_+$
$g(i,j)$	A slack variable used to compute the absolute value of $\lvert b_{p(i,j)} - b_{p(i,j-1)}\rvert$	$\mathbb{R}_+$			
Optimal Solution — Objective					
$\mu_{\text{e-on}}$	On-Peak Energy Rate	$\mathbb{R}_+$	$\mu_{\text{e-off}}$	Off-Peak Energy Rate	$\mathbb{R}_+$
$\mu_{\text{p-on}}$	On-Peak Demand Power Rate	$\mathbb{R}_+$	$\mu_{\text{p-all}}$	Facilities Power Rate	$\mathbb{R}_+$
$\mathcal{S}_{\text{on}}$	The set of on-peak time indices	$\{1, ..., n_{\text{time}}\}$	p_{demand}	Maximum average power during on-peak periods	$\mathbb{R}$
$p_{\text{facilities}}$	Maximum average power over all time instances.	$\mathbb{R}_+$	$p_t(j)$	The total average power co nsumed by both the bus chargers and the uncontrolled loads.	$\mathbb{R}_+^{n_{\text{time}}}$
$u(j)$	The average power over time j consumed by the uncontrolled loads	$\mathbb{R}_+^{n_{\text{time}}}$	$\boldsymbol{p}_t$	a vector containing $p_t(i)$ for all i.	$\mathbb{R}_+^{n_{\text{time}}}$
e_{on}	The total amount of energy consumed by the bus chargers and uncontrolled loads during off-peak hours.	$\mathbb{R}_+$	e_{off}	The total energy consumed by the bus chargers and uncontrolled loads during on-peak hours.	$\mathbb{R}_+$
J_{cost}	The section of the objective function pertaining to the fiscal expense of charging buses.	$\mathbb{R}$	J_{all}	The expression for the complete objective function.	$\mathbb{R}$

Scalability					
n_{group}	The number of groups in which to divide the buses and available chargers in preparation for the third program.	$\mathbb{Z}_+$	n^m_{charger}	The number of chargers assigned to group m.	$\mathbb{Z}_+$
n^m_{bus}	The number of buses in group m.	$\mathbb{Z}_+$	$p(j,m)$	The total power used during time index j by all buses in group m.	$\mathbb{R}_+$
$\beta(i,m)$	A binary selector variable which is one when bus i is in gropu m and zero otherwise.	$\{0,1\}$	n^m_{charger}	The number of chargers assigned to group m	$\mathbb{Z}_+$
$\phi(i,i')$	The inner product of the optimal charge schedules for buses i and i' respectively.	$\mathbb{R}_+$	$v(i,i',g)$	A variable that is $w(i,i')$ when buses i and i' are in group g and zero otherwise.	$\mathbb{Z}_+$
M_s	The maximum value for $\phi(i,i')$.	$\mathbb{R}_+$	J_{select}	The objective function for the group-selection problem	$\mathbb{R}_+$
De-Fragmentation					
$\theta(i,r)$	A binary variable which is one when charge session r from bus i will be used in a defragmented solution.	$\{0,1\}$	$\rho(i,r)$	A vector whose elements are equal to ΔT during time indices when bus i is charging during charge session r and zero otherwise.	$\mathbb{R}^{n_{\text{time}}}$
$\psi(i,j)$	The minimum allowable energy delivered to bus i during charge session r where the session in question is considered “active”.	$\mathbb{R}$	ω	The minimum allowable energy for any charge session.	$\mathbb{R}$
e_{max}	The maximum allowable energy delivered in any session.	$\mathbb{R}$			
Charge Schedules					
$a(i,r)$	The beginning of the allowable charge interval for bus i’s r^{th} charge session.	$\mathbb{R}_+$	$b(i,r)$	The commanded start time for bus i’s r^{th} charge session	$\mathbb{N}$
$f(i,r)$	The commanded end time for bus i’s r^{th} charge session.	$\mathbb{R}_+$	$d(i,r)$	The end time of the allowable charge interval for bus i’s r^{th} charge session.	$\mathbb{R}_+$
$\sigma(i,r,k)$	A selector variable which is one when bus i charges at charger k for session r.	$\{0,1\}$	M	The number of seconds in a day	$\mathbb{Z}_+$

$l(i, r, i', r')$	A selector variable which is one when bus i charges before bus i' during the r and r' sessions respectively.	$\{0, 1\}$			
Optimizing Charge Schedules					
$c(s, i, r)$	The start time for bus i's r^{th} charge session.	$\mathbb{R}$	$c(f, i, r)$	The stop time for bus i's r^{th} charge session.	$\mathbb{R}$
$c(a, i, r)$	The arrival time of bus i for charge session r.	$\mathbb{R}$	$c(d, i, r)$	The departure time for bus i after having completed the r^{th} charge session	$\mathbb{R}$
J_{window}	The loss function which drives charge windows to the desired length.	$\mathbb{R}$			
Multi-Rate Charging					
$x(i, j)$	The final charge schedule for bus i at time j, yielding the power at which bus i will charge.	$\mathbb{R}_+$	$z(j)$	The total power used by all buses at time j. $\mathbb{R}_+$	
$\gamma(i, d)$	A binary vector which is one at all time steps where bus i charges during charge session d.	$\{0, 1\}^{n_{\text{time}}}$	$e(i, r)$	The amount of energy to be delivered to bus i during charge session r.	$\mathbb{R}_+$
$J_{\text{multi-rate}}$	The objective function over which we minimize to solve the multi-rate section of the bus charge problem.	R_+			

REFERENCES

[1] Rocky Mountain Power. *Rocky Mountain Power Electric Service Schedule No. 8*. 2021. URL: https://www.rockymountainpower.net/content/dam/pcorp/documents/en/rockymountainpower/rates-regulation/utah/rates/008_Large_General_Service_1_000_kW_and_Over_Distribution_Voltage.pdf.