A Scalable Approach to Minimize Charging Cost for Electric Bus Fleets

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***Abstract*—Insert abstract here**

***Index Terms*—Insert keywords here**

I. Introduction

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II. Overview

Previous solutions to the bus charge problem have all struggled with scalability where scalable is defined as a solution where the runtime and cost increase linearly with the number of buses. In this work, we propose a solution which scales to a large number of buses by segmenting the problem into a series of sub-problems belonging to one of three groups as shown in Fig. 1.

Each sub-problem is solved using a linear, quadratic, or integer program and when used together the series of programs provides a near optimal charge plan. Each sub-problem addresses elements from one of three areas: energy allocation and bus grouping, session length and bus-to-charger assignments, and second-by-second optimization.

A. Energy Allocation and Group Assignment

The first set of problems answers two primary questions: At which time should energy be delivered to each bus and which buses' are most able to share chargers and contains three sub-problems: Unconstrained charge schedule, Smooth charge schedule, and group separation.

The unconstrained schedule problem from Section III computes an optimal charge schedule which minimises the monthly cost of power in the presence of uncontrolled loads under the assumption that each bus maintains a dedicated charger.

The smooth schedule problem from Section IV has the same form as the unconstrained scheduling problem with two differences: The monthly cost is required to match the optimal cost from the solution to the unconstrained scheduling problem and the objective for the smooth schedule problem penalizes change in the scheduled charge rates.

The group assignment problem from Section V uses the resulting charge schedules from the solution to the smooth schedule problem to separate buses into groups where each bus's schedule overlaps as little as possible with the other schedules for buses in the same group so that each group can be addressed separately to manage the number of computations in succeeding problems.

B. Session Time and Charger Assignment

The problems in the session time and charger assignment section are computed on a per-group basis to reduce the number of computations, address the questions of when should charge sessions start and stop, and on which charger should each session occur and are comprised of three sub-problems: De-Fragmentation, charger assignment, and session refinement.

The defragmentation problem from Section VI attempts to consolidate charge sessions with small amounts of energy to reduce the number of charge sessions and serves to both decrease the computational complexity of the charger assignment problem by reducing the number of charge sessions and simplify the charge schedule to make it more operationally feasible.

After consolidation, each charge session is defined by a minimum/maximum start/stop time as given by the bus's arrival and departure times and an energy requirement in kW. The charger assignment problem from Section VII uses the availability and energy constraints to assign chargers to charge sessions.

Once charge sessions are placed, the final step is to ensure each session makes the most of each charger's availability. Many times, especially when using non-optimal gaps in the charger assignment problem, the charge schedules may not use all available time on a charger. The charger refinement problem expands each charge session to fill unused time on either side and prioritizes sessions with higher energy demands for adjacent sessions.

C. Final Optimization

Solutions to the previous problems provide us with a set of charge sessions, energy requirements, and time schedules for specific chargers. The final question to be answered is how should the energy for each session will be delivered. The two sub-problems in the final optimization section mirror the first two problems from the Energy Allocation and Group Assignment section. The first uses the energy and time constraints from previos solutions to compute an optimal charge schedule in Section IX and is analagous to the unconstrained charge problem. The second computes a smoothed charge schedule with the same cost as the constrained schedule solution in Section X and is analagous to the Smooth charge schedule problem in Section IV.

III. Unconstrained Schedule

This section describes a program that finds an optimal charge schedule where buses are allowed to charge without

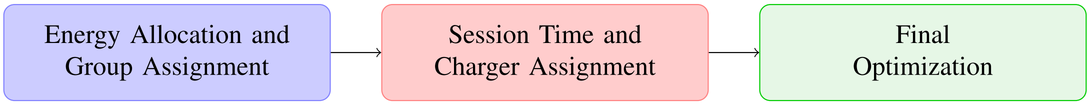

Fig. 1: Overall Processing Chain

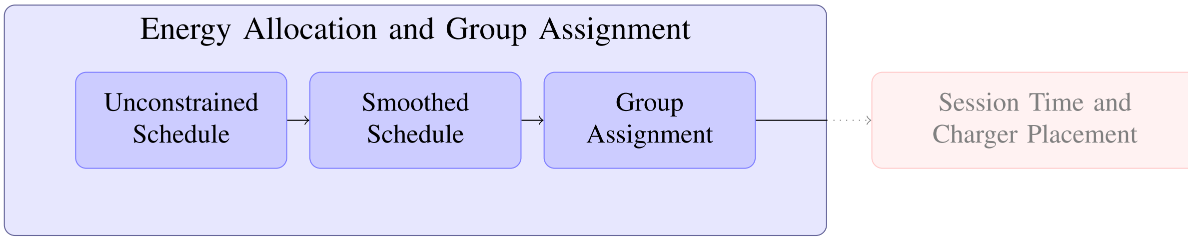

Fig. 2: Processing chain for the energy allocation and group assignment problems

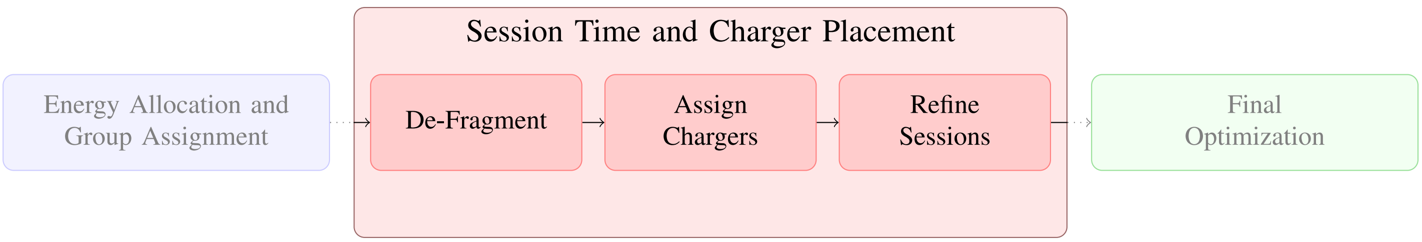

Fig. 3: Processing chain for each group

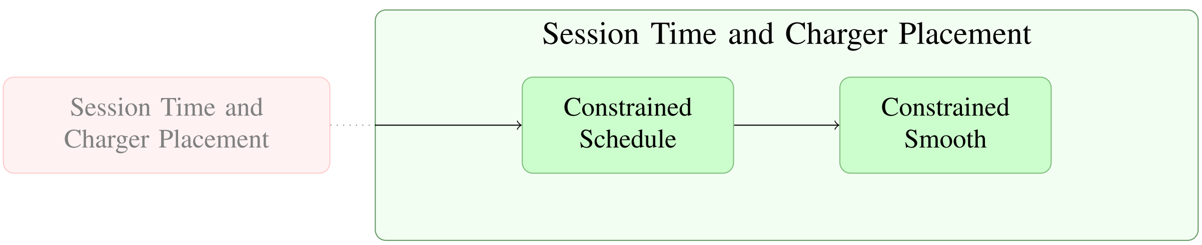

Fig. 4: Processing chain for the Final Optimization set

regard to the number of available chargers. This solution is considered "optimal" and will be used in later sections to formulate a feasible solution that accounts for the number of chargers.

A. Formulation

The cost objective that we desire to minimize is modeled after [1], which contains two primary elements: the cost of energy, and power demand. Energy is billed per kWh for on-peak and off-peak hours. The on-peak rate is more expensive because there is generally more demand for power during this time, whereas off-peak hours tend to be less expensive. The demand is covered in two separate chargers. The first is a facilities charge which is billed per kW for the highest 15-minute average power use over the course of the month. The second is a demand charge, which is also billed per kW, but is only billed for the highest 15-minute average power usesd during on-peak hours. The rates for each component are given in Table I.

Before we may compute the total monthly cost of electricity, we must define expressions for the average power and energy over time. Let each day be divided into 15-minute intervals for each bus where the average power expended for bus i during time j is denoted $p(i,j)$ as shown in Fig. 5. The resulting solution of the program we will develop will yield the average power expended by each bus during each period of time.

One constraint for which the solution must account is bus availability. When a bus is out of the station, the maximum average power for that time must be zero. For example, if bus 1 were out on route for t_5, t_6, and t_7, then the average power for those periods would be equal to zero as shown in Fig. 6. Let $\boldsymbol{b}_{p(i,j)}$ be the average power used by bus i at time index j, and $\boldsymbol{b}$ be a vector which contains $b_{p(i,j)}$ for each bus and time index. Also let $\mathcal{A} \subset i \times j$ be the set of all indices where bus i is in the station during time t_j and $\tilde{\mathcal{A}}$ be its complement. Furthermore, let p_{max} be the maximum power that a charger can deliver.

We define a set of constraints so that buses do not use power when not in the station by letting

$$\begin{aligned} b_{p(i,j)} &= 0 \ \forall i,j \in \tilde{\mathcal{A}} \\ b_{p(i,j)} &\leq p_{\text{max}} \ \forall i,j \in \mathcal{A} \\ -b_{p(i,j)} &\leq 0 \ \forall i,j \in \mathcal{A} \end{aligned} \tag{1}$$

TABLE I: Description of the billing structure

	On-Peak	Off-Peak	Facilities (Both)
Energy Rate	5.8282 ¢/kWh	2.9624 ¢/kWh	None
Energy Rate Symbol	$\mu_{\text{e-on}}$	$\mu_{\text{e-off}}$	None
Power Rate	\$ 15.73 /kW	None	\$ 4.81 /kW
Power Rate Symbol	$\mu_{\text{p-on}}$	None	$\mu_{\text{p-all}}$

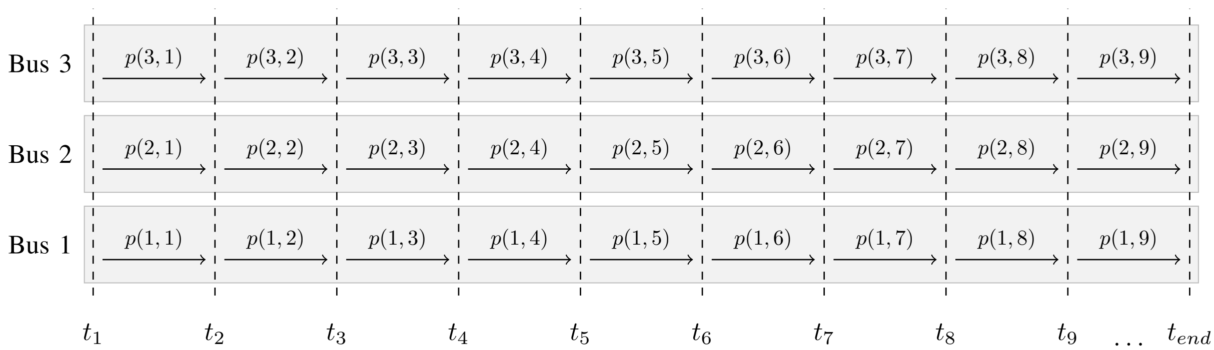

Fig. 5: Demonstrates how bus power use is conceptualized

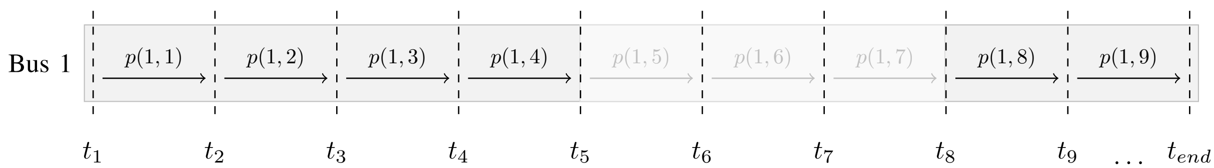

Fig. 6: Bus schedule with availability

The constraints in Eq. 1 however to not account for buses that must charge for partial periods. For example, if the day were divided into 15-minute time blocks, but a bus began charging at 10:07, then an average power of p_{max} for that time slot would be inaccurate. Therefore, Eq. 1 must be modified so that the average power for each block correctly reflects partial availability. Let $\alpha(i,j)$ give the percentage of time that bus i is available during time j. Eq. 1 can be rewritten as

$$\begin{aligned} -b_{p(i,j)} &\leq 0 \ \forall i,j \\ b_{p(i,j)} &\leq p_{\text{max}} \cdot \alpha(i,j) \ \forall i,j \end{aligned} \tag{2}$$

B. Battery

Each bus must also maintain its state of charge above acceptable levels throughout the day. When buses leave the station, each bus discharges some quantity of energy throughout the course of the route. Let $\delta(i,j)$ be the amount of charge lost by bus i at time j and let $h(i,j)$ be the state of charge of bus i at time j. The state of charge for each bus can be defined as

$$\begin{aligned} &h(i,j) = h(i,j-1) + b_p(i,j-1) \cdot \Delta T - \delta(i,j) \ \forall i, j > 1 \\ &h(i,1) = \eta_i \ \forall i \end{aligned} \tag{3}$$

where η_i is the initial state of charge for bus i and ΔT is the difference in time between $t_{i,j}$ and $t_{i,j+1}$. Now that each value for the state of charge is defined, each value for h must be constrained so that it is greater than a given threshold, h_{min} but does not exceed the maximum battery capacity h_{max}. This yields

$$\begin{aligned} -h(i,j) &\leq -h_{\text{min}} \ \forall i,j \\ h(i,j) &\leq h_{\text{max}} \ \forall i,j. \end{aligned} \tag{4}$$

The final battery related constraint has to do with how we are planning for the bus. The expenses that come from power are computed monthly, but we desire to simulate the movements of the bus for only a day, and use this to extrapolate what the monthly cost may be. Therefore, the state of charge for a bus at the end of the day must reflect its starting value. This yields the following constraint:

$$h_{i,\text{end}} = h(i,1) \ \forall i$$

so that

$$h_{i,\text{end}} - h(i,1) = 0 \ \forall i \tag{5}$$

C. Cumulative Load Management

While this solution does not directly account for the number of available chargers, we do account for the cumulative load capacities of all chargers. Let the number of chargers be denoted n_{charger}. We desire to maintain the average cumulative power for each time step at a level that is serviceable given n_{charger}. We define a slack variable $p_c(j)$ which represents the total average power consumed by all buses at time j. The variable $p_c(j)$ is computed as the sum of average bus powers so that

$$p_c(j) = \sum_i b_{p(i,j)}$$

or also as

$$p_c(j) - \sum_i b_{p(i,j)} = 0 \; \forall j. \tag{6}$$

D. Objective

Now that the relevent constraints have been addressed, we must work towards computing the total objective function. We do so by first computing the total average power for the complete system. This total power is comprised of power used by the buses, and power used by external sources such as lights, ice melt, electric trains, etc which we refer to as "uncontrolled loads", where the average power for the uncontrolled loads at time step j is denoted $u(j)$. We compute the total power as the sum of power used by the buses, $p_c(j)$ and the power consumed by uncontrolled loads $u(j)$ so that the total power, denoted $p_t(j)$ is computed as

$$p_t(j) = p_c(j) + u(j)$$

or

$$p_t(j) - p_c(j) - u(j) = 0 \; \forall j. \tag{7}$$

The next step is to compute the fifteen minute average power use for each time step, denoted p_{15}. We do this by letting

$$p_{15}(j) = \frac{1}{n} \sum_{l \in \{j_{15}\}} p_t(l) \tag{8}$$

where $\{j_{15}\}$ is the set of all indices 15 minutes prior to j and n is the cardinality of $\{j_{15}\}$. Next, note that the rate schedule requires both the maximum overall average power, denoted $p_{\text{facilities}}$, and the maximum average power during on-peak hours, or p_{demand}. Let $\mathcal{S}_{\text{on}}$ be the set of time indices belonging to on-peak hours, and recall that the max over all average power values is greater than or equal to $p_{15}(j)$ for all j. We can express this constraint is

$$p_{\text{facilities}} \geq p_{15}(j) \; \forall j$$

or alternatively as

$$p_{15}(j) - p_{\text{facilities}} \leq 0 \; \forall j \tag{9}$$

Because $p_{\text{facilities}}$ will be used in the objective function, the value for $p_{\text{facilities}}$ will be minimised until it is equal to the largest value in p_{15}. Following a similar logic, we also define a set of constraints for the maximum average on-peak power, p_{demand} so that

$$p_{15}(i) - p_{\text{demand}} \leq 0 \; \forall i \in \mathcal{S}_{\text{on}} \tag{10}$$

The next step in computing the objective function is to compute the total *energy* consumed during on and off-peak hours respectively. Let e_{on} be the total energy consumed during on-peak hours and e_{off} be the energy consumed during off-peak hours. We can compute energy as the product of average power and time. In our case, we compute this as

$$\begin{aligned} e_{\text{on}} &= \Delta T \cdot \sum_{i \in g} p_t(i) \\ e_{\text{off}} &= \Delta T \cdot \sum_{i \in \tilde{g}} p_t(i) \end{aligned} \tag{11}$$

where $\tilde{g}$ is the complement of g. We can now compute the total monthly charge as

$$J_{\text{cost}} = \begin{bmatrix} e_{\text{on}} \\ e_{\text{off}} \\ p_{\text{facilities}} \\ p_{\text{demand}} \end{bmatrix}^T \begin{bmatrix} \mu_{\text{e-on}} \\ \mu_{\text{e-off}} \\ \mu_{\text{p-all}} \\ \mu_{\text{p-on}} \end{bmatrix} \tag{12}$$

IV. Unconstrained Smooth Schedule

Because the optimizer tends to work along boundaries to find optimal solutions, the schedule produced by the previous formulation will tend to either contain values equal to the maximum charge rate, or zero. This results in many instances of off-to-on and on-to-off scenarios which is not well tolerated by hardware. For example, if the chargers have a maximum charge rate of 350 kW, then going from zero to 350 several times a minute may decrease the life of the hardware and be difficult to achieve in practice.

It is therefore necessary to implement a smoothing criteria so that the 'on-off' patterns from the first solution are softened. This is done by first solving the un-constrained charge problem as given. Next, the same problem is solved again but with two primary differences. The first is that the demand, facilities, on-peak energy, and off-peak energy are constrained so that they are equal to the values obtained in the previous problem, resulting in an equivalent solution. Next, we define an alternative objective which incentivizes "smooth" transitions between time steps.

This objective is defined as

$$J_{\text{thrash}} = \frac{1}{n} \sum_{i,j,\in \mathcal{K}} \|b(i,j) - b(i,j-1)\|_2^2, \tag{13}$$

where $\mathcal{K}$ is the set of all i,j where bus i may charge during time j and $j-1$.

V. Group Assignment

Before the bus charge problem can be solved, we need to address how the problem will scale. Historically, when buses are routed to chargers it requires a program which will evaluate all possible combinations to select an optimal, contention-free schedule.

Because contention increases on the order of $O(n^2)$ with the number of charge sessions and requires that each combination be evaluated to find an optimal solution, the placement problem is NP-hard==insert citation of knapsack problem==. Before we can formulate a scalable solution to the bus problem, we need a method to separate buses into groups to reduce the coupling between charge sessions.

The group assignment problem separates buses into n_{group} groups, where group m is allocated n^m_{charger} chargers and n^m_{bus} buses. Each group must have sufficient chargers to fill it's needs and prefer buses with dissimilar schedules to better avoid contention.

We know that the number of cross-terms in future problems will be reduced when each group has the same number of buses. Therefore, let n^m_{bus} be described as

$$n^m_{\text{bus}} \geq \left\lfloor \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rfloor$$
$$n^m_{\text{bus}} \leq \left\lceil \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rceil$$

or alternatively as

$$\begin{aligned} -n^m_{\text{bus}} &\leq - \left\lfloor \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rfloor \\ n^m_{\text{bus}} &\leq \left\lceil \frac{n_{\text{bus}}}{n_{\text{group}}} \right\rceil . \end{aligned} \tag{14}$$

We must also ensure that the number of chargers assigned to each group is exactly equal to the number of available chargers so that

$$n_{\text{charger}} = \sum_m n^m_{\text{charger}} \tag{15}$$

where n^m_{charger} is the number of chargers assinged to group m.

The next set of constraints ensures that each bus is is part of a group exactly once. Let $\beta(i,m)$ be a binary variable which is one when bus i is in group m. We constrain each bus to be a member of exactly one group by letting

$$\sum_m \beta(i,m) = 1 \ \forall i$$

We must also ensure that buses are assigned to groups where the power delivered to each bus can be achieved with the number of chargers assigned to that group. First, we define a slack variable which gives the total power used in group m at time step j as $p(m,j)$. Recall, we also know the expected power use for each bus as this is a result of the linear program described in sections III-A through III-D as $b_{p(i,j)}$, which allows us to describe the total power for any one group as

$$p(m,j) = \sum_i \beta(i,m) b_{p(i,j)}$$

or alternatively as

$$p(m,j) - \sum_i \beta(i,m) b_{p(i,j)} = 0 \ \forall m,j. \tag{16}$$

Next, we know that the total load of each group must be less than or equal to the collective capability of that group's chargers, which can be expressed as

$$n^m_{\text{charger}} \cdot p_{\max} \geq p(m,j) \ \forall m,j$$

or in standard form as

$$p(m,j) - n^m_{\text{charger}} \cdot p_{\max} \leq 0 \ \forall m,j \tag{17}$$

so that the number of chargers is sufficient to charge the collective load of the group.

We also desire to group buses together who's routes are most orthogonal. If two buses contain no overlap, they will be easiest to schedule and the inner product of their schedules from the first program will be equal to zero. Let

$$\phi(i,i') = \mathbf{b}(i,:)^T \mathbf{b}(j,:),$$

where $\mathbf{b}(i,:)$ is the charge schedule for bus i as computed in the first program. We desire to minimize the total cross terms $\phi(i,i')$ for all buses in the same group. Define a slack variable $v(i,i',m)$ which is equal to $\phi(i,i')$ if buses i and i' are both in group m and zero otherwise so that

$$\begin{cases} v(i,i',m) = \phi(i,i') & \beta(i,m) = 1, \beta(i',m) = 1 \\ v(i,i',m) = 0 & \text{otherwise} \end{cases}$$

which can also be expressed by letting

$$\begin{aligned} &v(i,i',m) \leq \phi(i,i') \\ &v(i,i',m) \geq \phi(i,i') - M\left(2 - \beta(i,m) - \beta(i',m)\right) \\ &v(i,i',m) \leq 0 + M\beta(i,m) \\ &v(i,i',m) \leq 0 + M\beta(i',m) \\ &v(i,i',m) \geq 0 \end{aligned}$$

or alternatively as

$$\begin{aligned} v(i,i',m) &\leq \phi(i,i') \\ -v(i,i',m) + M\beta(i,m) + M\beta(i',m) &\leq -\phi(i,i') + 2M \\ v(i,i',m) - M\beta(i,m) &\leq 0 \\ v(i,i',m) - M\beta(i',m) &\leq 0 \\ -v(i,i',m) &\leq 0. \end{aligned} \tag{18}$$

The final objective can then be expressed as

$$J_{\text{select}} = \sum_{i,i',m} v(i,i',m). \tag{19}$$

Because this objective is more of a preference then a requirement for the bus charging problem, it is not always necessary to solve completely. Experimental results have shown that increasing the MIP Gap to upwards of 50% can be sufficient for this problem and helps reduce the computational complexity of this step.

VI. De-Fragmentation

A minimum charge session length is another operational constraint that must be considered. We also consider constraints on minimum energy delivered per session. The intent of these constraints is to avoid charging for small durations or for small amounts of energy so that charge sessions are consolidated for convenience.

To solve this program, assume there exists a "smoothed" solution from the second program which has been appropriately placed in a group from program two. Next, let the preliminary solution be subdivided into charge sessions, each with a specific amount of energy, a minimum start time, and a maximum stop time. If the energy for any charge session is less than the allowed, then this session is marked as "fragmented". The remaining sessions are either marked as "used" or "unused", where a used session delivers more power than specified in the "fragmentation-threshold", and an unused session delivers zero power.

The purpose of this program is to determine which sessions will be "active" in deployment while adhering to minimum charge thresholds. The sessions in question are the "fragmented" sessions. Let $\theta(i,r)$ be a binary variable which indicates if session r from bus i will be active. Because the only sessions in question are fragmented, we only need to define $\theta(i, r)$ for fragmented sessions. Limiting the binary variables in this fashion significantly reduces the computational complexity of this step. The charge problem will be resolved using the same constraints and objective as the first program, but with two primary changes.

The first change constrains the minimum power delivery for each "active" charge session to be *at least* as large as the original power delivery. Let $\rho(i, r)$ be a vector which is ΔT, in hours, during the times bus i charges during session r and zero otherwise so that

$$\mathbf{b}(i,:)\rho(i,r) \geq \psi(i,j) \tag{20}$$

where $\psi(i, j)$ is the minimum energy for session i, r and session i, r is considered "active". For inactive sessions, the energy is constrained so that it is equal to zero. Finally, for fragmented sessions, the session energy must be greater than the minimum threshold, ω when active and zero otherwise which can be expressed as

$$\begin{aligned} \mathbf{b}(i,:)\rho(i,r) &\geq \omega - \omega(1-\theta(i,r)) \\ \mathbf{b}(i,:)\rho(i,r) &\leq 0 + \theta(i,r)e_{\text{max}} \end{aligned} \tag{21}$$

where e_{max} is the maximum energy delivered in a session.

VII. Charger Assignment

The results from the Linear Program defined in the previous section give us a general estimate of how much and when buses should charge, however we must still address two primary issues. The first is defining concrete start and stop times for each charge session. The second is limiting the charge sessions to a finite number of chargers. After solving the first program, we take the results and derive *preliminary* intervals and average power consumptions for each charge session.

For example, consider a solution to a three bus, two charger scenario given in Fig. 7. Note that there appears to be three buses charging at the same time from t_5 to t_6 even though there are only two chargers. We can reformulate this solution in terms of continuous start and stop variables and a variable charge rate so that the *duration* of each charge session may be relaxed. The objective is to store the given energy in the corresponding bus within the given charge interval.

Note how few of the charge sessions utilize the chargers to full capacity. This implies that there exists a smaller charge window in which equivalent power can be delivered. This allow us to use the charge durations from the solution from Fig. 7 as bounds on *allowable* charge windows instead of absolute truth.

An example of how Fig. 7 may be reformulated is given in Fig. 9. Note how the actual charge sessions don't necessarily need to take up all the time they were initially allocated in the first solution and that these times can fluctuate if the average charge rate is less than the maximum charger capacity. In this example, we assume a maximum charge capacity of 350kW.

Note how the third charge session does have to be exactly where it was scheduled because the average is equal to the maximum charge rate. If we examin just the schedule for Bus 1, we note that there are four essential variables for the corresponding charge session: $a(i, r)$, $b(i, r)$, $f(i, r)$ and $d(i, r)$ which represent the minimum start time, actual start time, actual end time, and maximum end time respectively.

The problem we must now solve is one of arranging these "rectangles" such that each one is larger than it's minimum width (or charge time). We must also account for the number of chargers. It can be helpful to view the problem as a bin packing problem, where each session must fit within the "swim lane" of a charger. For example, taking the charge sessions given in Fig. 9 and arranging them so that there is no overlap between sessions will yield a valid solution as shown in Fig. 8.

From Fig. 10, we know that $a(i, r), b(i, r), f(i, r)$ and $d(i, r)$ must be such that

$$\begin{aligned} a(i,r) &\leq b(i,r) \\ b(i,r) &\leq f(i,r) \\ f(i,r) &\leq d(i,r). \end{aligned}$$

or alternatively as

$$\begin{aligned} -b(i,r) &\leq -a(i,r) \\ b(i,r) - f(i,r) &\leq 0 \\ f(i,r) &\leq d(i,r) \end{aligned} \tag{22}$$

Where $a(i, r)$ and $d(i, r)$ are known from the previous optimization problem, and $b(i, r)$ and $f(i, r)$ are optimization variables.

We must differentiate between chargers and so, define $\sigma(i, r, k)$ as a binary selector variable which is one if charger k services bus i for session r and zero otherwise. We know that only one charger can charge each bus at a time. We also know that each charge session *must* be serviced, which implies that

$$\sum_k \sigma(i,r,k) = 1 \;\forall i,r. \tag{23}$$

Next, we also know that during each session a certain amount of energy must be transfered from the charger to the battery. The amount of energy that must be transfered to bus i during session r can be computed from the results of the first linear program and is denoted $e(i, r)$. We can compute a minimum time window from this value as

$$w(i,r)_{\text{min}} = \frac{e(i,r)}{p_{\text{max}}}.$$

If we include constraints for a minimum time per session, then the previous expression becomes

$$w(i,r)_{\text{min}} = \max\left(w_{\text{min}}, \frac{e(i,r)}{p_{\text{max}}}\right)$$

Because this is the minimum time window, we must ensure that the difference between the start and stop times is at least this large so that

$$f(i,r) - b(i,r) \geq w(i,r) \;\forall i,r$$

or alternatively,

$$b(i,r) - f(i,r) \leq -w(i,r) \;\forall i,r. \tag{24}$$

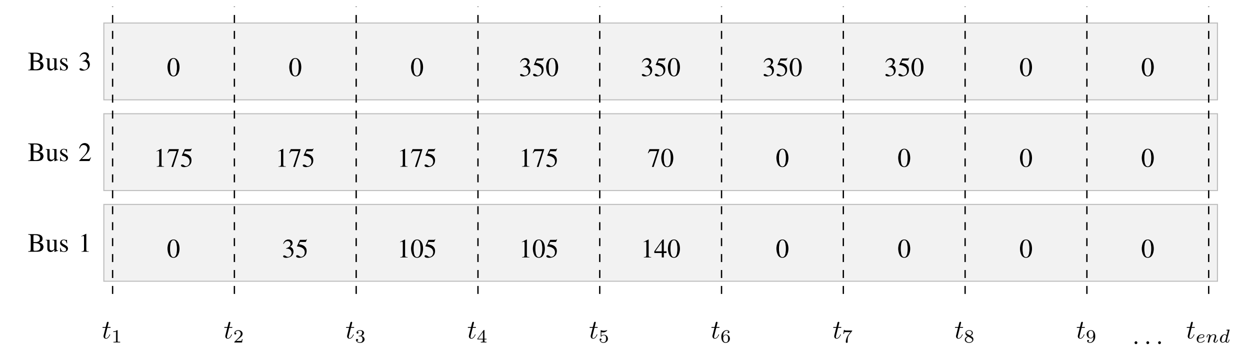

Fig. 7: An example solution to a 3-bus, 2-charger scenario from the first QP

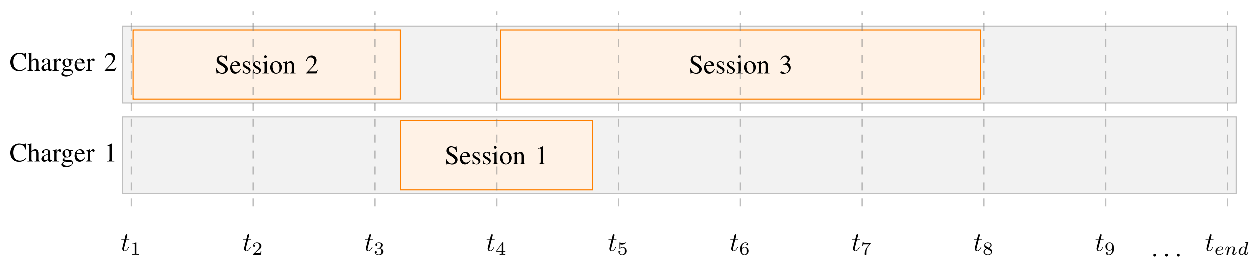

Fig. 8: Demonstrates the solution to the second program

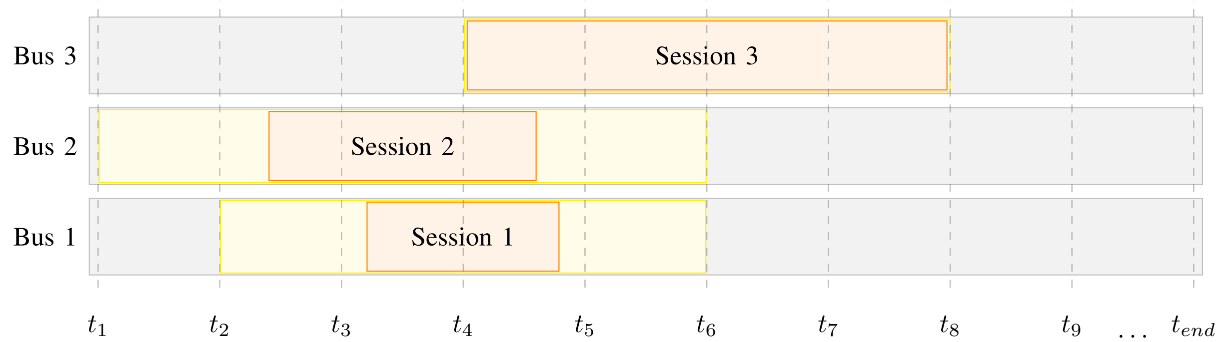

Fig. 9: Demonstrates how the problem from the first QP can be reformulated in terms of continuous variables

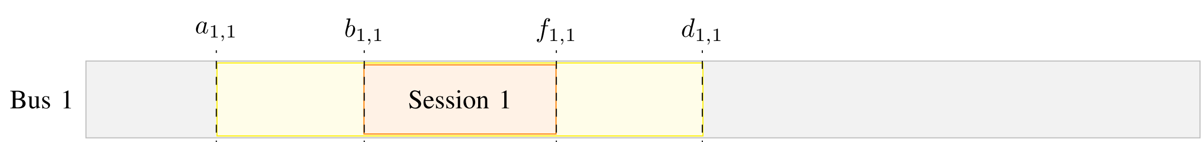

Fig. 10: Gives variables of optimization for the second program

The final set of constraints deals with contention so that no charger can be scheduled for two sessions that overlap. let $\mathcal{L} = \{(i, r) \times (i', r')\}$ where charge sessions i, r and i', r' have the potential to overlap. Before we can prevent overlap, we must define a binary variable $l(i, r, i', r')$ which is equal to one when session i, r is scheduled before session i', r' and zero otherwise so that

$$\begin{cases} f(i,r) \leq b(i',r') & l(i,r,i',r') = 1 \\ f(i',r') \leq b(i',r') & l(i,r,i',r') = 0 \end{cases}$$

Here we can expand this thought through use of the "big-M" technique. Let M be large. In this case, we can set it equal to the number of seconds in a day. We know what the top constraint must be trivially satisfied when $l(i, r, i', r') = 0$ and the bottom must also when $l(i, r, i', r') = 1$. This leads to a reformulation so that

$$f(i,r) - b(i',r') \leq M(1 - l(i,r,i',r')$$
$$f(i',r') - b(i,r) \leq l(i,r,i',r')M$$

However, this constraint *only* needs to hold when sessions i, r and i', r' are scheduled to charge on the same charger or that $\sigma(i, r, k) = \sigma(i', r', k) = 1$. We can reformulate the above constraint to satisfy this condition by letting

$$\begin{aligned} f(i,r) - b(i',r') &\leq M(3 - \sigma(i,r,k) - \sigma(i',r',k) - l(i,r,i',r')) \\ f(i',r') - b(i,r) &\leq M(2 - \sigma(i,r,k) - \sigma(i',r',k) + l(i,r,i',r')) \end{aligned} \tag{25}$$

Finally, we desire the power profile to closly match the profile given in the first linear program, which would occure if each charge session matched the durations given in the first solution. We formulated an objective function which minimized the largest difference between

$$\min_{f,b} \sum_{i,r} \|b(i,r) - a(i,r)\|_2^2 + \|f(i,r) - d(i,r)\|_2^2$$

which has the effect of driving each variable to the desired value and more heavily penalizing values that are further from their optimal.

VIII. Optimizing Charge Schedules

Many times it is not feasible to compute the optimal set of charge schedules given in the previous sections. As the number of buses and charge sessions becomes large, computing a small-MIPGap solution becomes untractable. Using a large MIPGap resolves issues related to computational complexity, but results in sub-optimal charge-time windows.

We compute a more optimal set of charge windows by using the results from the previous program to infer charger assignment, and ordering for each charge session. We also know that the optimal solution will expand the charge windows to use any available time where a charger is unused, implying that the "stop" time for each session will either be equal to it's buse's departure time, or the start time of the next window which can be expressed as

$$\begin{cases} c(s,i,r+1) = c(f,i,r) & c(d,i,r) > c(a,i,r+1) \\ \begin{aligned} c(s,i,r+1) &= c(a,i,r+1) \\ c(f,i,r) &= c(d,i,r) \end{aligned} & c(d,i,r) <= c(a,i,r+1) \end{cases}$$

where $c(s,i,r)$ is the start time for charger i's r^{th} charge session, $c(f,i,r)$ is the stop time for charger i's r^{th} charge session, $c(d,i,r)$ is the departure time for the bus scheduled for charger i's r^{th} charge session, and $c(a,i,r)$ is the arrival time for the bus scheduled for charger i's r^{th} charge session. The minimum charge length must also be used so that energy can be properly delivered, so that

$$c(f,i,r) - c(s,i,r) \geq w(i,r)$$

where $w(i,r)$ is the corresponding minimum charge time corresponding the session.

The final step to optimizing the charge windows is to give preference to windows with larger power deliveries. Let the objective for the optimization program be

$$J_{\text{window}} = \frac{1}{n} \sum_{i,r} \left\| \frac{c(f,i,r) - c(s,i,r)}{e(i,r)} \right\|_2^2. \tag{26}$$

When the function J contains windows with equal amounts of energy, the minimum will be found where each charge interval is the same width. As the amount of energy increases, the objective penalizes less for larger window sizes and thus gives preference to high energy sessions.

IX. Constrained Schedule

Up to this point, we have computed the "optimal" schedule which assumes any bus can charge without regard to the number of chargers. We then separate buses into groups to reduce the scope of the problem and treat each sub-problem separately while we defragment and assign each charge session to specific chargers before determining the final start and stop times for each bus's charge session.

The final step in this process is to determine how the energy will be delivered so that cost is minimised. Begin with constraints for bus power, energy, and cost from Section **??** that are expressed as equations 2, 6, 7, 8, 9, 10 and 11. Next, include constraints for energy so that the energy for each charge session is properly delivered using a modified version of Eqn. 20 so that

$$\mathbf{b}(i,:)\rho(i,r) = \psi(i,r) \tag{27}$$

where $\psi(i,r)$ is the required energy for bus i during rest period r as computed from the solution of the De-Fragmentation problem.

X. Constrained Smooth Schedule

The results of the Charger-Constrained problem will contain the same on-off defects as the solution from the Unconstrained program which can be managed using a similar method as before where we re-run the same program as before but make two changes. The first change constrains the objective so that it achieves the optimal cost. The second minimises differences between charge rates at adjacent time steps using the smoothing objective from Eqn. 13.

XI. Results

The results given in this section aim to demonstrate how the proposed method can be used to find a scalable solution to the bus charge problem. Because the proposed solution contains various sub-problems, optimization parameters for each sub-problem may be tuned to best meet the demands of a given scenario, allowing for a wide degree of flexability that is not present in prior works which formulate solutions to the bus charge problem as a single program.

A. Overall Performance

In this section, we compare the proposed method with a baseline algorithm and a method developed be insert reference to He. et al here. The baseline method models how bus drivers charge their electric vehicles at the Utah Transit Authority in Salt Lake City, Utah. At UTA, when bus drivers arrive at the station, they refuel their electric buses whenever a charger is available so that the number of charge sessions is maximized. The method from He et al. works somewhat differently by minimising the cost of energy with respect to the time of use tarrifs μ_{e-on} and μ_{e-off}.

The comparison we observe is given for a 10-bus, 10-charger scenario and a single group. Each method was used to compute a charage schedule and the costs from demand, facilities, and energy charges are given in Fig. 11. Note how

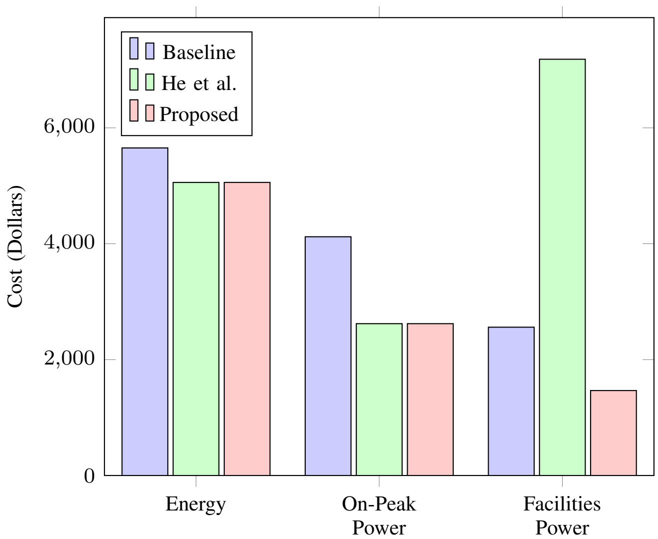

Fig. 11: Cost comparison with prior work

the baseline algorithm suffers significantly from the demand charges associated with On-Peak Power, and He et al. incurres additional cost from the facilities charges, indicating that an emphasis on energy charges and habitual charging patterns can be improved.

We observe where the differences in cost originate in Fig. 13. Observe how the baseline charge profile achieved the largest 15-minute average power between 19:12 adn 21:36 which is during on-peak hours and consequently yielded the large On-Peak Power charges given in Fig. 11. Additionally, note how the proposed method maintains a relatively flat power profile so that the load is balanced throughout the day which we investigate in Fig. 12.

In Fig. 12, note how the proposed method produces a bus load that mirrors the uncontrolled load, yielding the flat load profile from Fig. 13 which is especially prevalent from 7:12 to 14:24. The results show that the proposed method works well, outperforming both historical patterns at UTA as well and improves upon prior academic techniques.

B. Optimality Gaps and Contention

In the previous section, we discussed performance of the proposed method when each program is solved to an optimal solution. In general, the most computationally demanding solution addressed bus-to-charger placement and generally requries a gap of 1×10^{-5} for optimality. This work also seeks to address how to compute a solution in a scalable manner and so this section reviews computational time as the number of buses increases.

This section considers a 7-charger scenario and compares runtime results for 8, 9, and 10 buses to illustrate how runtimes for set optimality gaps change as contention increases. Fig. 15 shows how the computational time increases as the optimality gap decreases. Note how the computational time suddenly increases as the gap decreases, a phenomena which is exacerbated as contention increases. Additionally, note that the optimality gaps are small even before the sudden increase in runtime indicating that it may not be necessary to solve past the inflection point where the runtime suddently increases.

C. Contention: Sub-Optimal Schedules

In the previous section we observed that a scalable solution to the bus charge problem must remain below the inflection point where runtime suddently increases which can be accomplished by either relaxing the optimality gap, or decreasing the computational complexity of the charger placement problem.

In this section, we show that a relaxed optimality gap in the charger placement problem may result in an undesirable solution and consequently that there exist scenarios that require small optimality gaps which normally lie beyond the inflection point shown in Fig. 15, indicating the need to reduce the charger assignment problem's computational complexity.

Fig. 14 displays the charge session durations as a function of average charge rate for two 18 bus 6 charger scenarios where the first was computed using a small optimality gap and the second resulted when the gap was relaxed. Note how the charge sessions from the optimal solution tend to have larger session durations and lower charge rates than the relaxed solution which is desired because sessions with low charge rates and long durations are simpler to carry out in practice.

Figures 16 and 17 show the corresponding optimal and relaxed charge plans by letting the color at the i, j location represent the charge rate for bus i at time j and show why an optimal solution to the charger assignment problem yields better charge sessions. Observe how the first sessions for buses $1 - 4$ and $6 - 13$ are assigned to a single charger, which compresses the charge sessions to accomodate the large number of buses while the remaining chargers appear to have one session at most. In comparison, the optimal solution in Fig. 17 have a more evenly distributed session load for each charger so that each session is lengthened, leading to lower charge rates.

It is also interesting to note that the monthly costs of each solution may or may not be equivalent even though an optimal solution is clearly superior. Therefore, a small gap is required to consistently achieve optimal session placement. We also know from Fig. 15 that small optimality gaps may increase the number of computations so that the charger assignment problem becomes untractable for large numbers of buses.

D. The Importance of Groups

One contribution this work provides is a *scalable* way to compute cost-oriented charge schedules. We know from the previous section that the charger assignment problem will not scale for small optimality gaps. This section describes how the computational complexity of the charger assignment problem can be managed by separating the buses into groups so that the charger assignment problem can be solved for each group independently.

In this section, we consider a 18 bus, 12 charger scenario with a 0.13% gap in the charger assignment problem. Fig. 18, shows the respective runtimes for a one and two group scenario as computed in Section V. Note how the runtime for the two group scenario is several orders of magnitude less then

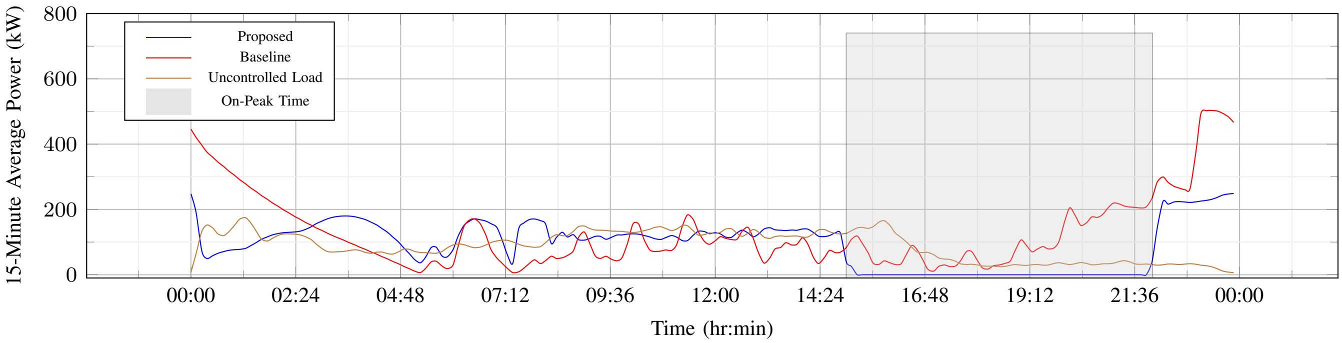

Fig. 12: Comparison between uncontrolled and bus loads

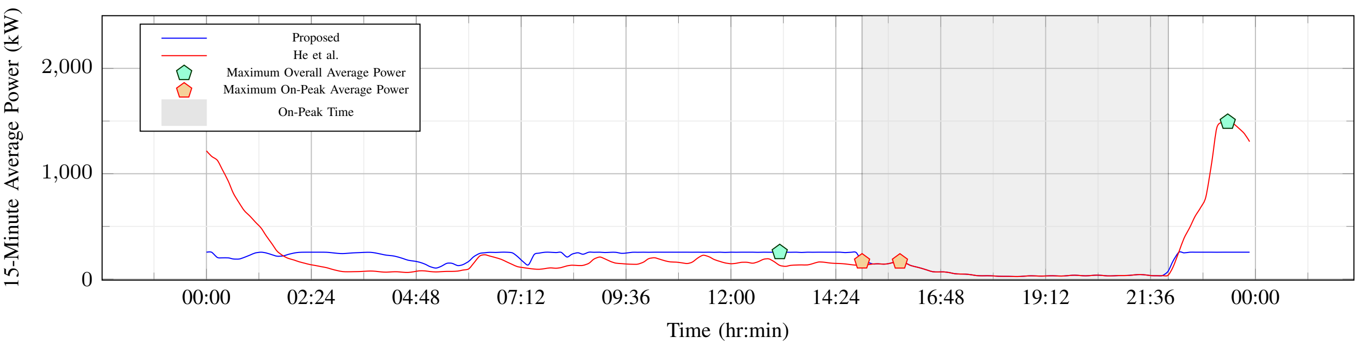

Fig. 13: 15-Minute average power for one day

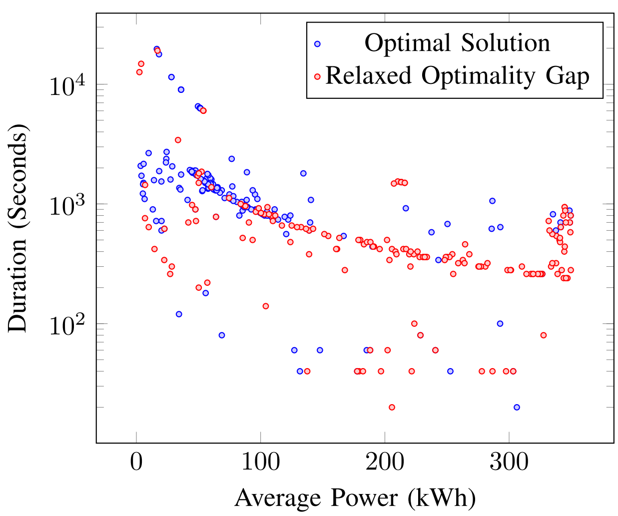

Fig. 14: Comparison of charge session duration vs average charge rate

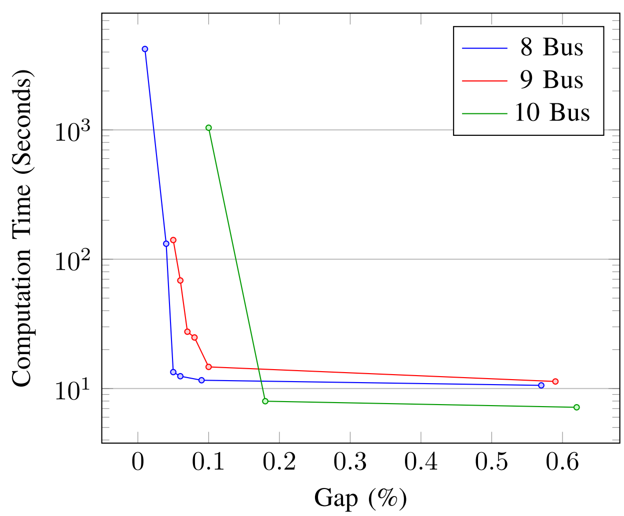

Fig. 15: Comparison of Runtime for a 7-Charger Scenario

the runtime for the single group case which demonstrats how a small number of groups can manage the runtime for optimal charger assignment solutions.

E. Effects of De-Fragmentation

This paper also addresses the operational preference to consolidate charge sessions when possible. This section demonstrates the effectiveness of the defragmentation method given in Section VI and how consolidation affects the monthly cost. In section VI, the threshold for defragmentation is given by the minimum allowable energy per charge session. In this section we compare two 40 bus 7 charger scenarios where the first contains results without defragmentation and the second consolidates charge sessions so that each session delivers at least 30 kWh. The results for each session are presented in Figures 20 and 19 where the color of i, j element of a figure represents the charge rate for bus i during time j. Note how Fig. 20 contains *many* small an inconsequential charge sessions and requires each bus to charge each time it enters

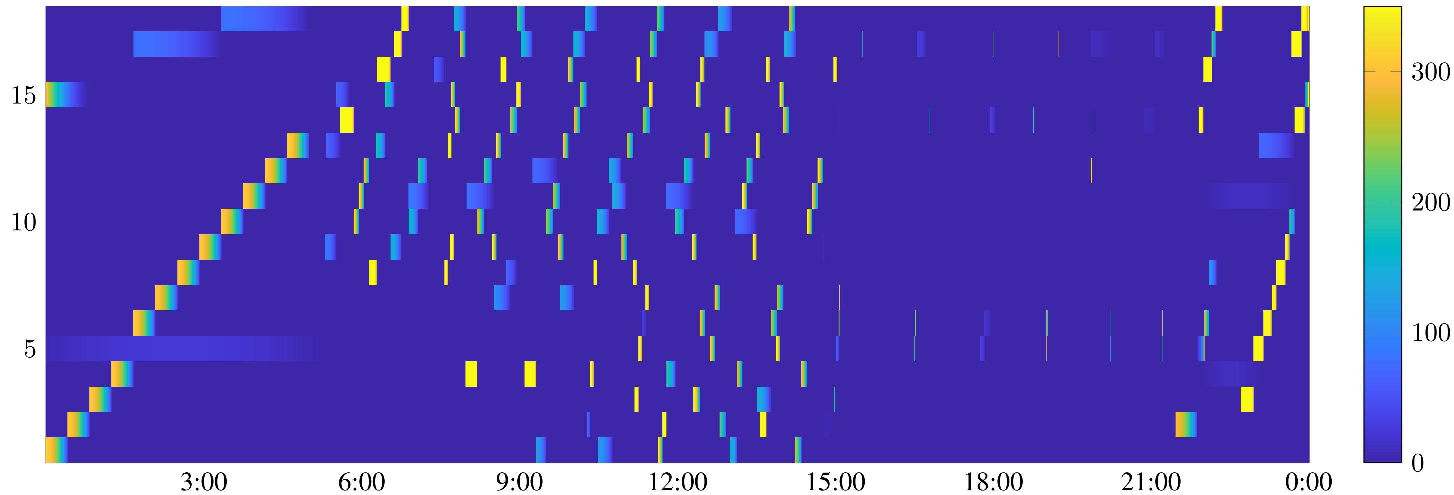

Fig. 16: Routes with a large gap in the route placement problem

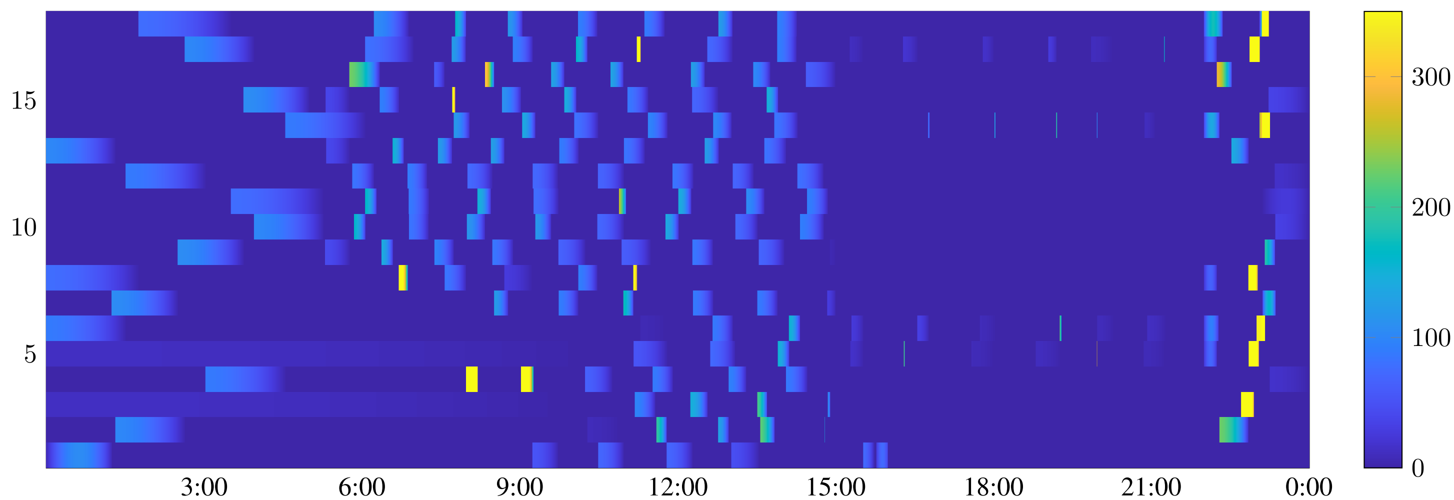

Fig. 17: Routes with a small gap in the route placement problem

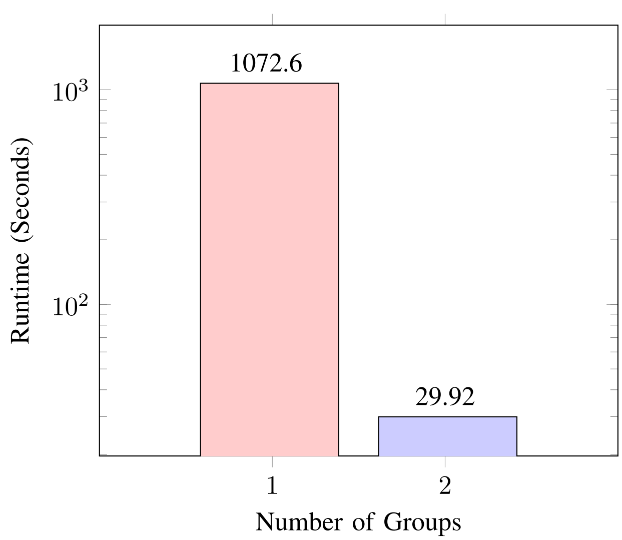

Fig. 18: Runtimes for a 18 bus 12 charger scenario at a 0.13% gap

the station. In comparison, Fig. 19 contains only a handful of charge sessions so that each bus only need charge 4 – 5 times throughout the day.

Furthermore, Fig. 21 demonstrates that despite the additional constraints associated with consolidation, the monthly cost remains consistent over a large window of thresholds. As the minimum allowable energy per session increases, the number of binary variables in the defragmentation problem increases, resulting in significant runtimes for the defragmentation problem as shown in Fig. 22. However, because buses are divided into groups prior to defragmentation, the smaller groups decrease the computational complexity for defragmentation so that larger consolidation thresholds can be applied in a scalable manner.

F. Scalability

In this section, we consolidate what we have learned in the previous sections to demonstrate how the proposed framework can be used to compute a scalable and cost effective solution for large numbers of buses. This section focuses on a scenario

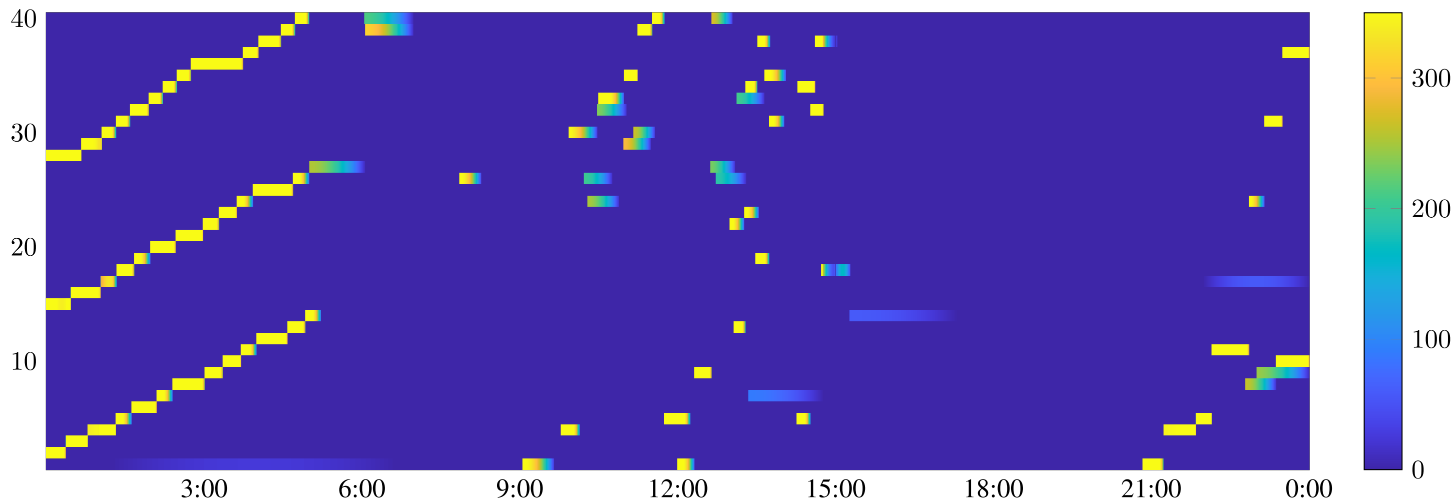

Fig. 19: Routes with De-Fragmentation

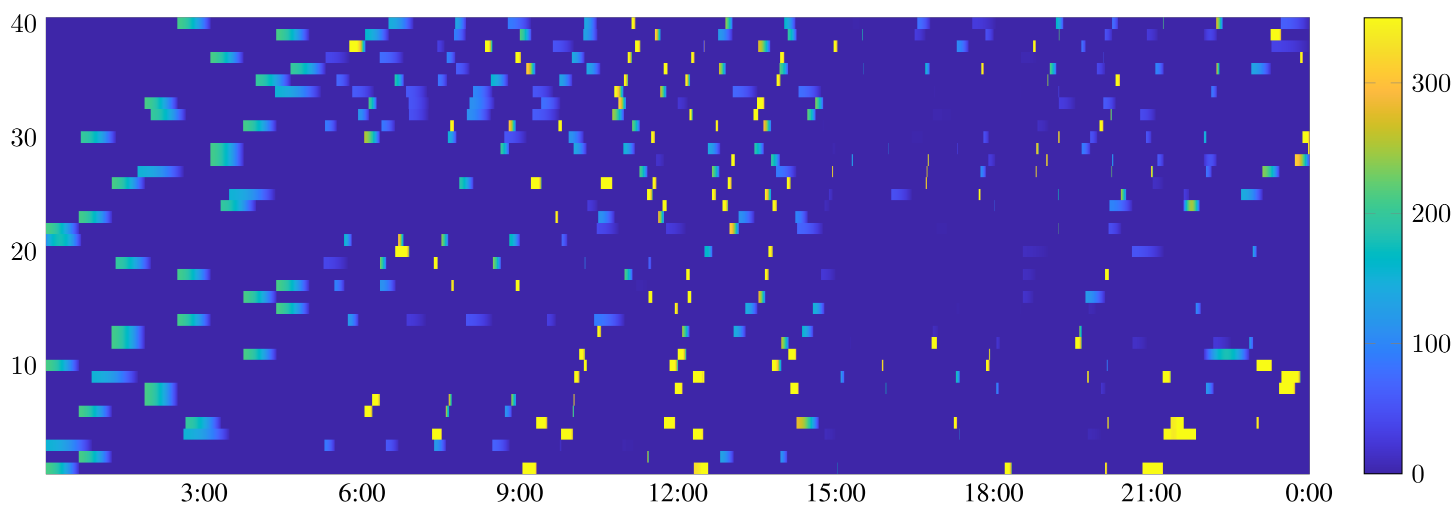

Fig. 20: Routes without De-Fragmentation

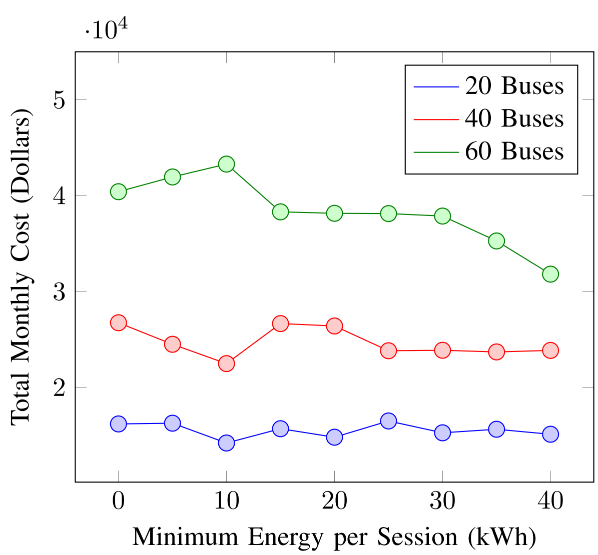

Fig. 21: Cost comparison of different degragmentation thresholds in a pro-time optimization scheme.

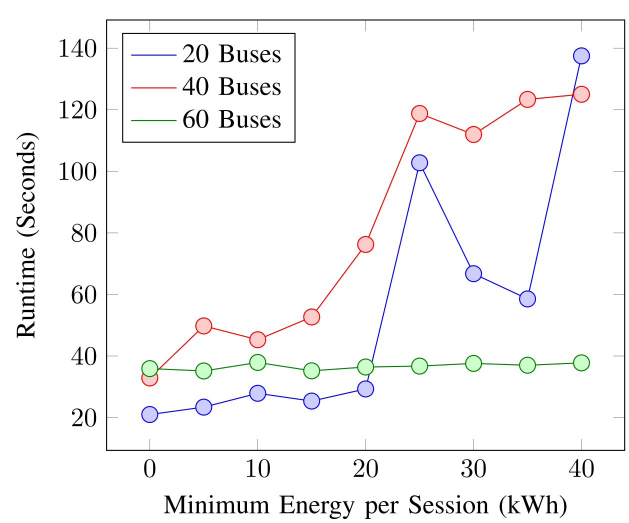

Fig. 22: Comparison of runtime for the uncontested and contested scenarios over different de-fragmentation criteria

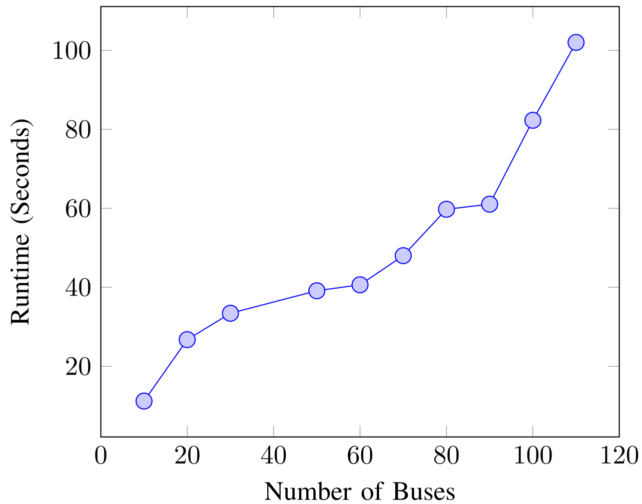

Fig. 23: Runtime comparison for different numbers of buses

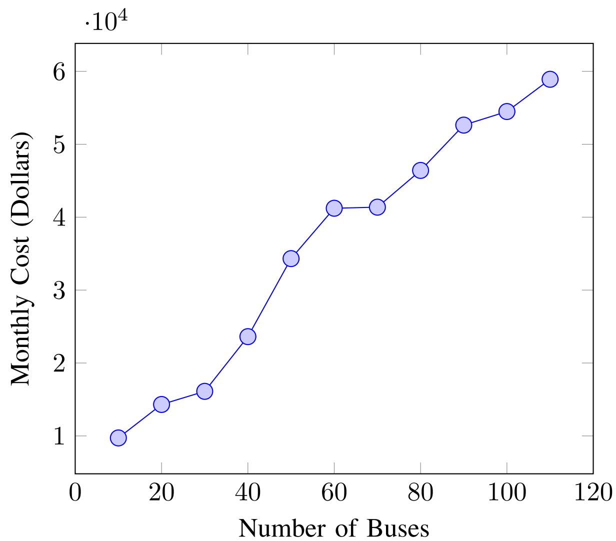

Fig. 24: Cost comparison for different numbers of buses

with a minimum energy per session of 20 kWh, a relaxed gap for the charger assignment solution, and a single group.

The results given in Fig. 23 show a runtime that generally increases by one second per bus from 10 to 110 buses. One would expect the runtime to increase at least on the order of $O(n^2)$ for a globally optimal solution because of the coupling between bus variables. The fact that the proposed method appears linear on the given range indicates a scalable solution.

Generally, one would also expect such savings to come with significant increases to the monthly cost. The results in Fig. 24 however demonstrate how the proposed solution yields a quasi-linear increase of approximately $404.10 dollars per bus per month.

XII. Conclusions

In summary, this paper proposes a method to compute cost-oriented charge schedules for large numbers of battery electric buses by dividing the charge problem into several sub-problems which focus on energy placement and group separation, charge session length and assignment, and cost optimization. The proposed method has been shown to scale as both the runtime and monthly cost increase linearly with the number of buses.

Furthermore, because the proposed method contains a number of sub-problems, setting the optimization criteria for each sub-problem gives the user flexability so that the proposed method can be adapated to solve a variety of scenarios and optimization preferences.

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Variable	Description	Range	Variable	Description	Range
Indices					
i	Bus index	$\mathbb{N}$	j	Time Index	$\mathbb{N}$
k	Charger index	$\mathbb{N}$	r	Route Index	
m	group index	$\mathbb{N}$			
Optimal Solution — Formulation					
n_{bus}	The number of buses in the optimization framework.	$\mathbb{Z}$	n_{time}	The number of time indices in a day.	$\mathbb{Z}^+$
$b_{p(i,j)}$	The average power consumed by bus i during time period j.	$\mathbb{R}$	t_j	The time at time index j. This paper also refers to the period of time from t_j to t_{j+1} as "period t_j".	$\mathbb{R}$
$\boldsymbol{b}$	A vector containing each value for $b_{p(i,j)}$.	$\mathbb{R}^{n_{\text{bus}} \cdot n_{\text{time}}}$	$\alpha(i,j)$	the percentage of time bus i spends in the station from t_j to t_{j+1}.	$[0,1]$
$\mathcal{A}$	The set of all $i \times j$ elements where bus i can charge at time index j	$i \times j$	p_{max}	The maximum power a bus charger can deliver to a bus in kW. This paper assumes a value of 350 for most examples and results.	$\mathbb{R}^+$
$\tilde{\mathcal{A}}$	The complement of $\mathcal{A}$.	$i \times j$			
Optimal Solution — Battery					
h_{min}	The minimum allowable state of charge	$(0, h_{\text{max}})$	h_{max}	The maximum state of charge	$\mathbb{R}^+$
η_i	The beginning state of charge for bus i	$(h_{\text{min}}, h_{\text{max}})$	$h(ij)$	The state of charge for bus i at time t_j.	$(h_{\text{min}}, h_{\text{max}})$
ΔT	The change in time from t_j to t_{j+1}	$\mathbb{R}^+$	$\boldsymbol{h}$	A vector containing all state of charge values.	$\mathbb{R}_+^{n_{\text{bus}} \cdot n_{\text{time}}}$

$\delta(ij)$	The battery discharge for bus i during time period j.	$\mathbb{R}_+$	$h(i, \text{end})$	Bus i's final state of charge.	$(h_{\text{min}}, h_{\text{max}})$
Optimal Solution — Cumulative Load Management					
n_{charger}	The time index for the start of bus i's j^{th} stop	$\mathbb{Z}_+$	$p_c(j)$	The average power consumed by all buses during time period j.	$\mathbb{R}$
$\boldsymbol{p_c}$	A vector containing all values of $p_c(j)$.	$\mathbb{R}_+^{n_{\text{time}}}$	J_{thrash}	A secondary objective function which penalizes multiple plug-in instances per charge session.	$\mathbb{R}_+$
$g(i,j)$	A slack variable used to compute the absolute value of $\lvert b_{p(i,j)} - b_{p(i,j-1)} \rvert$	$\mathbb{R}_+$			
Optimal Solution — Objective					
$\mu_{\text{e-on}}$	On-Peak Energy Rate	$\mathbb{R}_+$	$\mu_{\text{e-off}}$	Off-Peak Energy Rate	$\mathbb{R}_+$
$\mu_{\text{p-on}}$	On-Peak Demand Power Rate	$\mathbb{R}_+$	$\mu_{\text{p-all}}$	Facilities Power Rate	$\mathbb{R}_+$
$\mathcal{S}_{\text{on}}$	The set of on-peak time indices	$\{1, ..., n_{\text{time}}\}$	p_{demand}	Maximum average power during on-peak periods	$\mathbb{R}$
$p_{\text{facilities}}$	Maximum average power over all time instances.	$\mathbb{R}_+$	$p_t(j)$	The total average power co nsumed by both the bus chargers and the uncontrolled loads.	$\mathbb{R}_+^{n_{\text{time}}}$
$u(j)$	The average power over time j consumed by the uncontrolled loads	$\mathbb{R}_+^{n_{\text{time}}}$	$\boldsymbol{p}_t$	a vector containing $p_t(i)$ for all i.	$\mathbb{R}_+^{n_{\text{time}}}$
e_{on}	The total amount of energy consumed by the bus chargers and uncontrolled loads during off-peak hours.	$\mathbb{R}_+$	e_{off}	The total energy consumed by the bus chargers and uncontrolled loads during on-peak hours.	$\mathbb{R}_+$
J_{cost}	The section of the objective function pertaining to the fiscal expense of charging buses.	$\mathbb{R}$	J_{all}	The expression for the complete objective function.	$\mathbb{R}$

Scalability					
n_{group}	The number of groups in which to divide the buses and available chargers in preparation for the third program.	$\mathbb{Z}_+$	n^m_{charger}	The number of chargers assigned to group m.	$\mathbb{Z}_+$
n^m_{bus}	The number of buses in group m.	$\mathbb{Z}_+$	$p(j,m)$	The total power used during time index j by all buses in group m.	$\mathbb{R}_+$
$\beta(i,m)$	A binary selector variable which is one when bus i is in gropu m and zero otherwise.	$\{0,1\}$	n^m_{charger}	The number of chargers assigned to group m	$\mathbb{Z}_+$
$\phi(i,i')$	The inner product of the optimal charge schedules for buses i and i' respectively.	$\mathbb{R}_+$	$v(i,i',g)$	A variable that is $w(i,i')$ when buses i and i' are in group g and zero otherwise.	$\mathbb{Z}_+$
M_s	The maximum value for $\phi(i,i')$.	$\mathbb{R}_+$	J_{select}	The objective function for the group-selection problem	$\mathbb{R}_+$
De-Fragmentation					
$\theta(i,r)$	A binary variable which is one when charge session r from bus i will be used in a defragmented solution.	$\{0,1\}$	$\rho(i,r)$	A vector whose elements are equal to ΔT during time indices when bus i is charging during charge session r and zero otherwise.	$\mathbb{R}^{n_{\text{time}}}$
$\psi(i,j)$	The minimum allowable energy delivered to bus i during charge session r where the session in question is considered “active”.	$\mathbb{R}$	ω	The minimum allowable energy for any charge session.	$\mathbb{R}$
$e_{\max}$	The maximum allowable energy delivered in any session.	$\mathbb{R}$			
Charge Schedules					
$a(i,r)$	The beginning of the allowable charge interval for bus i's r^{th} charge session.	$\mathbb{R}_+$	$b(i,r)$	The commanded start time for bus i's r^{th} charge session	$\mathbb{N}$
$f(i,r)$	The commanded end time for bus i's r^{th} charge session.	$\mathbb{R}_+$	$d(i,r)$	The end time of the allowable charge interval for bus i's r^{th} charge session.	$\mathbb{R}_+$
$\sigma(i,r,k)$	A selector variable which is one when bus i charges at charger k for session r.	$\{0,1\}$	M	The number of seconds in a day	$\mathbb{Z}_+$

$l(i, r, i', r')$	A selector variable which is one when bus i charges before bus i' during the r and r' sessions respectively.	$\{0, 1\}$			
Optimizing Charge Schedules					
$c(s, i, r)$	The start time for bus i's r^{th} charge session.	$\mathbb{R}$	$c(f, i, r)$	The stop time for bus i's r^{th} charge session.	$\mathbb{R}$
$c(a, i, r)$	The arrival time of bus i for charge session r.	$\mathbb{R}$	$c(d, i, r)$	The departure time for bus i after having completed the r^{th} charge session	$\mathbb{R}$
J_{window}	The loss function which drives charge windows to the desired length.	$\mathbb{R}$			
Multi-Rate Charging					
$x(i, j)$	The final charge schedule for bus i at time j, yielding the power at which bus i will charge.	$\mathbb{R}_+$	$z(j)$	The total power used by all buses at time j. $\mathbb{R}_+$	
$\gamma(i, d)$	A binary vector which is one at all time steps where bus i charges during charge session d.	$\{0, 1\}^{n_{\text{time}}}$	$e(i, r)$	The amount of energy to be delivered to bus i during charge session r.	$\mathbb{R}_+$
$J_{\text{multi-rate}}$	The objective function over which we minimize to solve the multi-rate section of the bus charge problem.	R_+			

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[1] Rocky Mountain Power. *Rocky Mountain Power Electric Service Schedule No. 8*. 2021. URL: https://www.rockymountainpower.net/content/dam/pcorp/documents/en/rockymountainpower/rates-regulation/utah/rates/008_Large_General_Service_1_000_kW_and_Over_Distribution_Voltage.pdf.