

Question 1 Recurrence

$$T(n) = 3T\left(\frac{n}{4}\right) + 4n$$

$$= 3\left(3T\left(\frac{n}{16}\right) + n\right) + 4n = 9T\left(\frac{n}{16}\right) + 4n + 3n$$

$$= 3\left(3\left(3T\left(\frac{n}{64}\right) + \frac{n}{4}\right) + n\right) + 4n = 27T\left(\frac{n}{64}\right) + 4n + 3n + \frac{9n}{4}$$

$$= 3\left(3\left(3\left(3T\left(\frac{n}{256}\right) + \frac{n}{16}\right) + \frac{n}{4}\right) + n\right) + 4n = 81T\left(\frac{n}{256}\right) + 4n + 3n + \frac{9n}{4} + \frac{27n}{16}$$

After the j th substitution for all $j \geq 1$

$$T(n) = 3^j T\left(\frac{n}{4^j}\right) + \sum_{i=1}^j \frac{(4)(3^{j-i})n}{4^{i-1}}$$

$$f(n) = 4n = \theta(n^1)$$

Thus $a=3, b=4, d=1$

Then case 1 applies since $3 < 4^1$

Hence,

$$T(n) = \theta(n)$$

Question 4 Double Hashing

Assuming Reverse(6) = 60

25 – initial 0, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25												

14 – initial 4, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25				14								

9 – initial 7, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25				14			9					

7 – initial 12, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25				14			9					7

5 – initial 4, 1 collision, sequence 4→5

0	1	2	3	4	5	6	7	8	9	10	11	12
25				14	5		9					7

3 – initial 8, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25				14	5		9	3				7

0 – initial 13, 0 collision, 1 resize

0	1	2	3	4	5	6	7	8	9	10	11	12
25				14	5		9	3				7

13	14	15	16	17	18	19	20	21	22	23	24	25
0												

21 – initial 2, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25		21		14	5		9	3				7

13	14	15	16	17	18	19	20	21	22	23	24	25
0												

6 – initial 8, 1 collision, sequence 8→17

0	1	2	3	4	5	6	7	8	9	10	11	12
25		21		14	5		9	3				7

13	14	15	16	17	18	19	20	21	22	23	24	25
0				6								

0	1	2	3	4	5	6	7	8	9	10	11	12
25		21	33	14	5		9	3				7

0	1	2	3	4	5	6	7	8	9	10	11	12
25		21	33	14	5		9	3				7

13	14	15	16	17	18	19	20	21	22	23	24	25
0				6								

[illegible]

39	40	41	42	43	44	45	46	47	48	49	50	51
0				6								

0	1	2	3	4	5	6	7	8	9	10	11	12
25		21	33	14	5		9	3				7

13	14	15	16	17	18	19	20	21	22	23	24	25
0				6								

[illegible]

39	40	41	42	43	44	45	46	47	48	49	50	51
0				6						42		

24 – initial 20, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25		21	33	14	5		9	3				7

13	14	15	16	17	18	19	20	21	22	23	24	25
0				6			24					

26	27	28	29	30	31	32	33	34	35	36	37	38
25												

39	40	41	42	43	44	45	46	47	48	49	50	51
0				6						42		

107 – initial 6, 0 collisions

0	1	2	3	4	5	6	7	8	9	10	11	12
25		21	33	14	5	107	9	3				7

13	14	15	16	17	18	19	20	21	22	23	24	25
0				6			24					

26	27	28	29	30	31	32	33	34	35	36	37	38
25												

39	40	41	42	43	44	45	46	47	48	49	50	51
0				6						42		

Question 7 Algorithm Analysis

Question 4 was done by hand

#5: time complexity – $O(n*k)$ where n is the number of strings in the given array and k is the maximum amount characters in the strings. This is because we iterate and compare each string in the array k times comparing the k th element from the right each iteration for n iterations.

Space complexity – $O(n)$ since the 256 buckets can hold a total of n strings where n is the length of the given string array

#6 time complexity – $O(m + n)$ where m is the length of the given string and n is the length of the given pattern. Goes through each character in the given string and then does another comparison depending on how many characters there are in the pattern.

Space complexity – $O(m + n)$ where m is the length of the given string and n is the length of the given pattern. This is because since each substring takes new space there can be a maximum of m space created for the substrings and a max of n spaces can be created for the pattern if each character in the pattern is different