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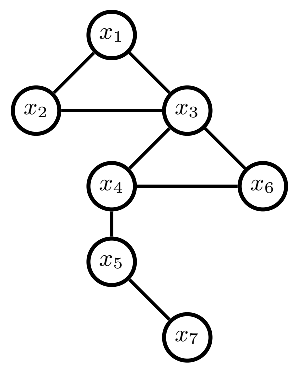
COMPUTER VISION

EXERCISE 3 – GRAPHICAL MODELS

1 Pen and Paper

1.1 Markov Random Fields



a) For the undirected graph $\mathcal{G}$ above, draw a circle around each maximal clique. What is the Markov Random Field $p(x_1, \ldots, x_7)$ of the graph?

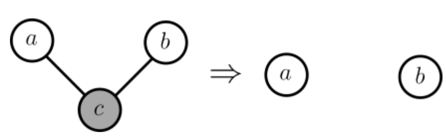
b) Gobal Markov Property: Given $\mathcal{S} = \{x_3, x_5\}$, define disjoint sets $\mathcal{A}, \mathcal{B}$ such that $\mathcal{A} \perp\!\!\!\perp \mathcal{B} \mid \mathcal{S}$. How many of such pairs of sets can you find?
Further, what is the Markov Blanket of x_4?

Instructions on how to count pairs of $\mathcal{A}$ and $\mathcal{B}$:
$\mathcal{A}$ and $\mathcal{B}$ cannot be empty.
Swapping $\mathcal{A}$ and $\mathcal{B}$ does not count as a new pair.
$\mathcal{A} \cup \mathcal{B} \cup \mathcal{S}$ must contain all variables in the graph.

c) Local Markov Property: Which of the following statements are correct, which are wrong and why?

- $x_1 \perp\!\!\!\perp x_5 \mid x_2$
- $x_2 \perp\!\!\!\perp x_7 \mid \{x_3, x_5\}$
- Marginalizing over x_3 makes x_1 and x_4 dependent.
- $p(x_4 \mid x_1, x_2, x_3, x_5, x_6, x_7) = p(x_4 \mid x_3, x_5, x_6)$

d) Prove the conditional independence property $a \perp\!\!\!\perp b \mid c$



by showing that $p(a, b \mid c) = p(a \mid c)p(b \mid c)$.

1.2 Factor Graphs

a) Draw the factor graph for function

$$f(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = f_1(x_1, x_3) f_2(x_1, x_2, x_5) f_3(x_2, x_5) f_4(x_3) f_5(x_4, x_5, x_6) f_6(x_6) f_7(x_6, x_7).$$

b) Given $\mathcal{X} = \{x_1, \ldots, x_6\}$ with $x_i \in \{0, 1\}$ $\forall i$, a Markov Random Field is given by the distribution

$$p(x_1, \ldots, x_6) = \frac{1}{Z} \phi_1(x_1, x_3) \phi_2(x_1, x_2, x_5) \phi_3(x_4, x_5, x_6).$$

Define the potentials as

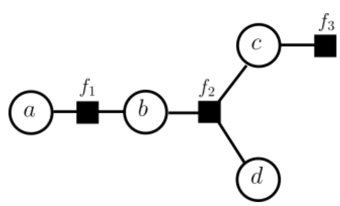
$$\begin{aligned} \phi_1(x_1, x_3) &= [x_1 = x_3] \\ \phi_2(x_1, x_2, x_5) &= [x_2 = x_5] \cdot [x_1 = 1] \\ \phi_3(x_4, x_5, x_6) &= 2\, x_4 x_5 + 3\, x_6. \end{aligned}$$

Calculate the value of the partition function Z. Is $p(x_1, \ldots, x_6)$ a Gibbs distribution? Explain your answer.

1.3 Belief Propagation

a) In one or two sentences, what is the nugget or key observation behind Belief Propagation?

b) **Marginal Inference.** Consider the Markov Random Field defined on binary variables $a, b, c, d \in \{0, 1\}$ with the following factor graph:



The potentials are given by $f_1(a, b) = [a = b]$, $f_2(b, c, d) = 0.6b + 0.2c + 0.3d$ and $f_3(c) = [c = 1]$. Compute all marginal distributions using the *Sum-Product Algorithm*. Here, please do not use the log representation for the messages μ.

c) **Maximum-A-Posteriori.** Consider a distribution defined over binary variables $a, b, c \in \{0, 1\}$:

$$p(a, b, c) = \frac{1}{Z} \, f_1(a, b) f_2(b, c) f_3(c)$$

The factors are given as

$$f_1(a, b) = \begin{pmatrix} 0.4 & 0.15 \\ 0.12 & 0.25 \end{pmatrix}, \qquad f_2(b, c) = \begin{pmatrix} 0.3 & 0.5 \\ 0.12 & 0.22 \end{pmatrix}, \qquad f_3(c) = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$$

so e.g. $f_2(b = 0, c = 1) = 0.5$. What is the most likely joint configuration $a^*, b^*, c^* = \operatorname{argmax}_{a,b,c}\, p(a, b, c)$? Again, please use *Max-Product Algorithm* and do not use the log representation.

d) **Loop Cut.** Consider a pairwise Markov Random Field defined on a ring (a chain that is connected at both ends) with 100 binary variables:

$$p(x) = \phi_0(x_1, x_{100}) \prod_{i=1}^{99} \phi_i(x_i, x_{i+1})$$

Is it possible to compute $\operatorname{argmax}_{x_1, \ldots, x_{100}}\, p(x)$ efficiently?

Hint: Think about what happens if you condition on one of the nodes.

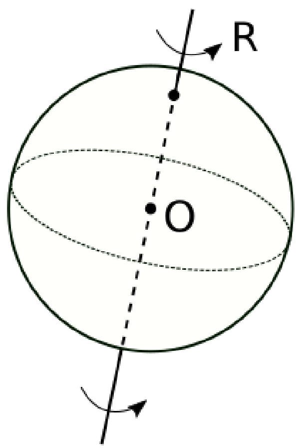
1.4 Graphical Models for Multi-View Reconstruction

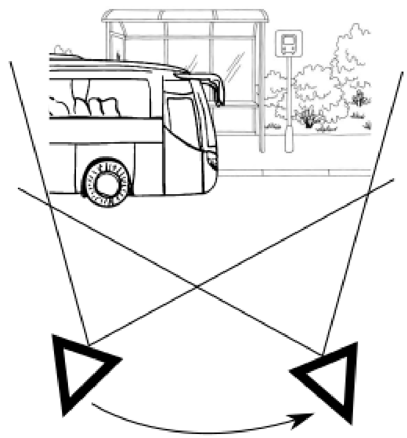
a) What are the advantages of using Probabilistic Graphical Models for Computer Vision tasks like Multi-View Reconstruction? And what are the challenges or drawbacks of those models?

b) In the lecture, we discussed a probabilistic dense multi-view reconstruction method using an MRF on a voxel grid. Which assumption(s) is this model based on? And in which cases are those violated?

1.5 Graphical Models for Optical Flow

a) **Flow fields.** Draw a sketch of the optical flow fields for the following two scenarios: The first is a sphere rotating around the shown axis. Please assume the sphere to be textured. The second shows a static bus stop scene with the camera moving from left to right. Indicate with arrows the magnitude of the optical flow for different points in the scene.



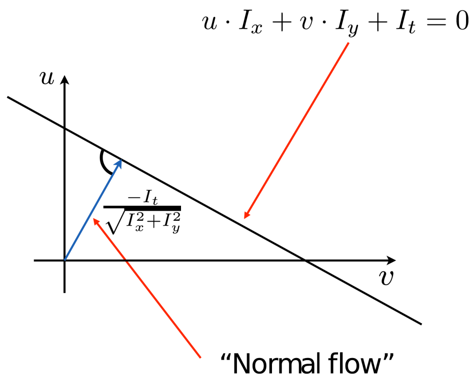


b) In the lecture the Barber Pole was discussed as an example for the aperture problem. Answer the question from the slides concerning the Barber Pole: What is the motion field? What is the optical flow field?

c) **Aperture problem.** The aperture problem arises from the brightness consistency contraint / optical flow constraint equation

$$u_{x,y} \cdot I_x(x, y) + v_{x,y} \cdot I_y(x, y) + I_t(x, y) = 0$$

This constraint can be graphically represented as follows:



Understand the relationship and explain the aperture problem using the figure above. Additionally, derive the term for the normal flow.

2 Coding Exercises

This exercise is split into two parts: `localization.ipynb` and `denoising.ipynb`. In the first part you will implement max-product belief propagation for chain structured Markov Random Field to solve a pre-specified vehicle localization problem. In the second part the belief propagation algorithm for general Markov random fields is already implemented. You will apply this algorithm to an image denoising problem and experiment with the strength of the prior to obtain the best possible denoising result.