Numerical Born kernels to account for finite-frequency effects in surface wave tomography

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Summary: In order to conduct seismic tomography, one should use the most adequate description of wave propagation; thus far, almost all tomographic models rely on ray theory, a high-frequency approximation. Particularly for surface waves, Born theory, a single scattering theory, improves it by taking first order scattering effects into account. The resulting sensitivity kernels are typically derived from analytical far-field Green's tensors, which lead to singularities at source and receiver locations; numerical derivation of kernels in contrast can avoid this, but full numerical integration of the equations of motion in 3D is expensive.

If we restrict ourselves to the case of a smooth, laterally heterogeneous Earth, we can use a zero-thickness membrane as an analogue for Love and Rayleigh wave propagation, speeding up dramatically the numerical integration. We implement "membrane waves" with a new finite-difference algorithm, leading to the numerical deviation of Born sensitivity kernels and apply them to surface wave phase velocity tomography.

A Membrane waves as an analogue for surface waves

For a smooth, laterally heterogeneous Earth, membrane waves are a valid alternative to Love and Rayleigh waves at distinct frequencies (Tanimoto, 1990; Tromp & Dahlen, 1993). Comparisons of the numerical scheme with an analytical solution can be done in case of a homogeneous phase velocity map and applying a simple "gaussian source".

analytical solution —
numerical —

$$\rho \ddot{\vec{u}} = \nabla \cdot \tau + \mathbf{f}$$

$$\mathbf{u}_L = W(\mathbf{r})(-\hat{\mathbf{r}} \times \nabla_1)\chi(\vartheta,\varphi)$$

$$\nabla_1^2 \chi = \frac{\omega^2}{\mathbf{c}(\vartheta,\varphi)^2}\chi$$

model time [s]

The accuracy of the surface Laplacian is influenced by grid distortion. Plotted on the left is the cell distortion induced by the refinement mechanism (for a grid level 4). On the right, differences to the analytical solution for a spherical harmonic function (L=6,M=1) are plotted. The percentages are still small when compared to the maximum analytical Laplacian.

The error of the unfiltered (red) and filtered (green) numerical solution for different levels of grid refinement, compared with the analytical solution in terms of relative phase delay. Filtering is done around the corner frequency for Love waves at 150 s. Notice that the numerical displacements arrive earlier (negative delays) compared to the analytical ones.

unfiltered
filtered
relative phase delay [%]
grid level

Calculation time for the finite-difference code in case of Love waves at 150 s period with a propagation time of ~40 min for different processors or grid spacings.
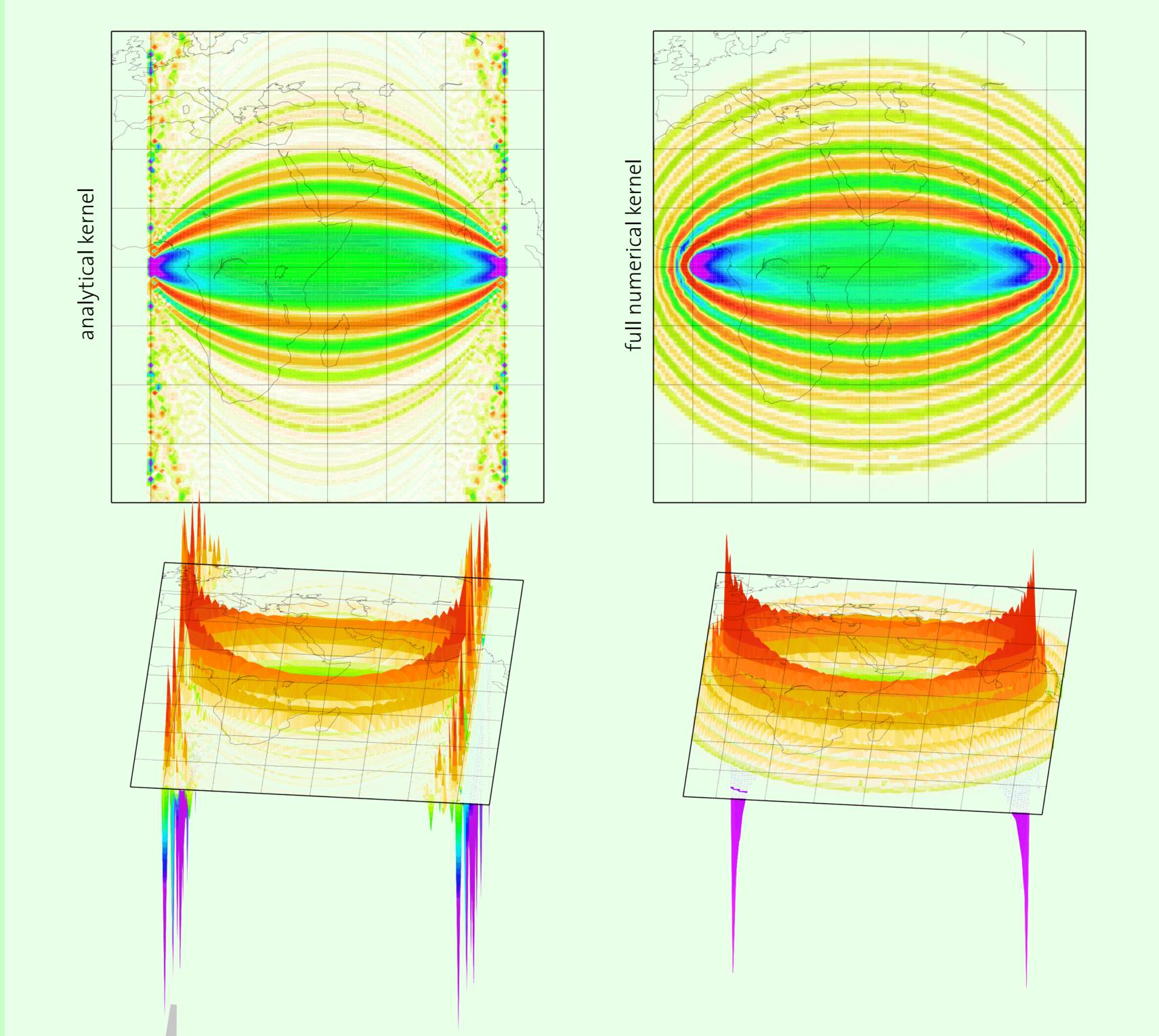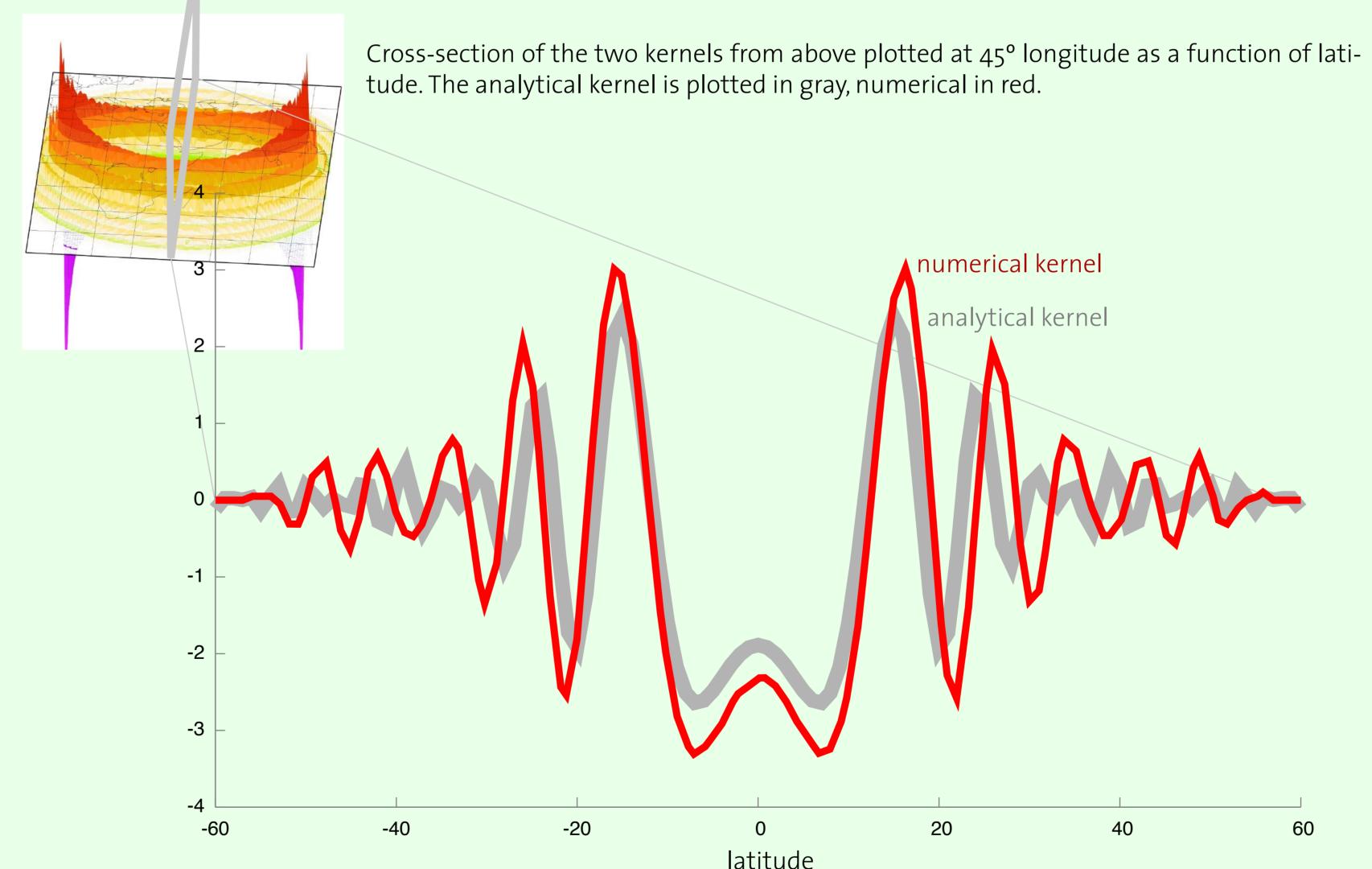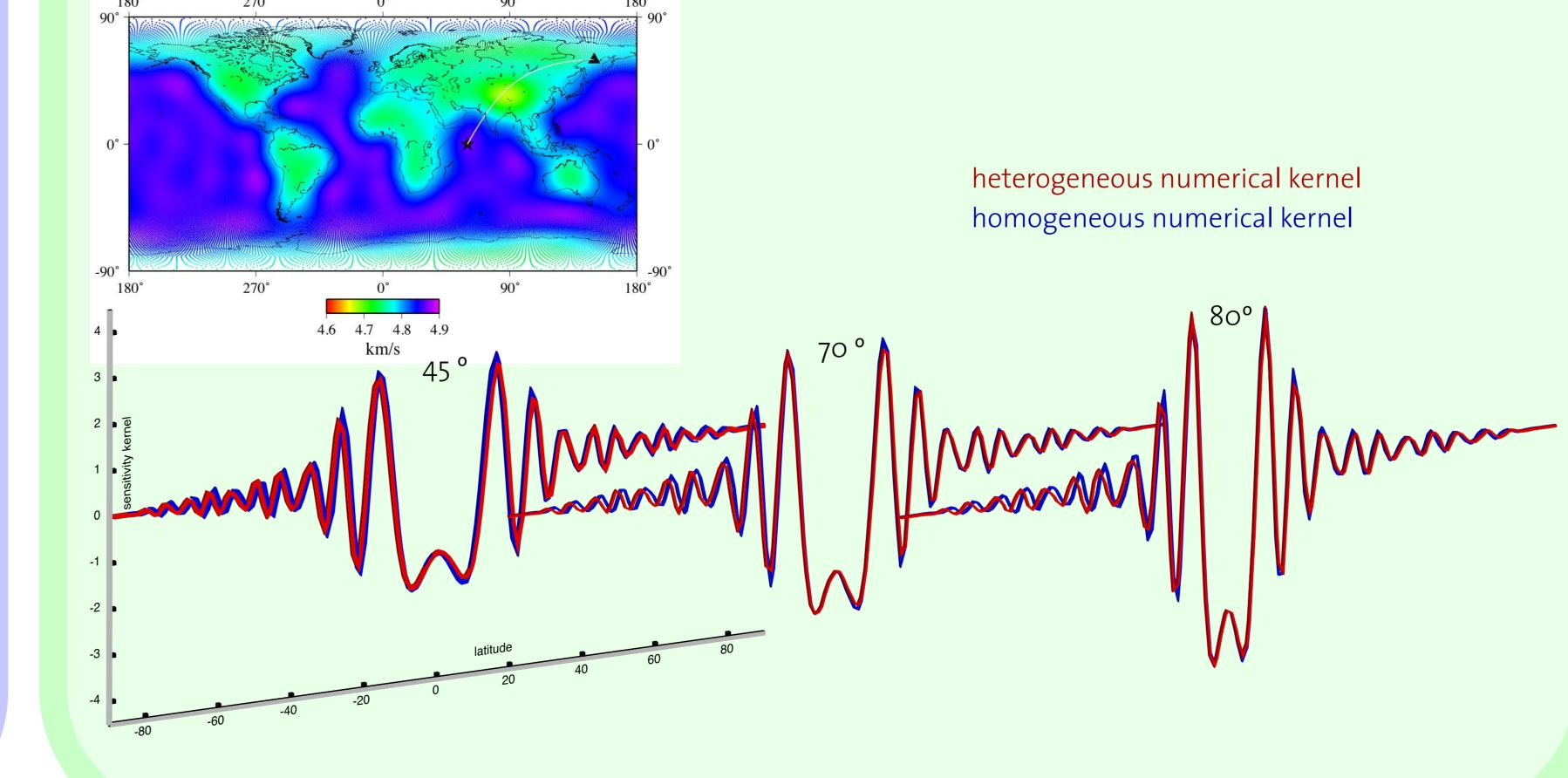
seconds
number of processors
grid spacing (km)

B Born sensitivity kernels

Finite-frequency sensitivity kernels for Love waves at 150 s period found analytically involve a far-field approximation for the Green's tensor thus leading to singularities at source and receiver position and producing artefacts in the surroundings (plotted on the left); using a straight forward numerical implementation with the help of membrane waves, one finds numerical kernels without additional far-field approximation (on the right).

analytical kernel
full numerical kernel

Cross-section of the two kernels from above plotted at 45° longitude as a function of latitude. The analytical kernel is plotted in gray, numerical in red.

numerical kernel
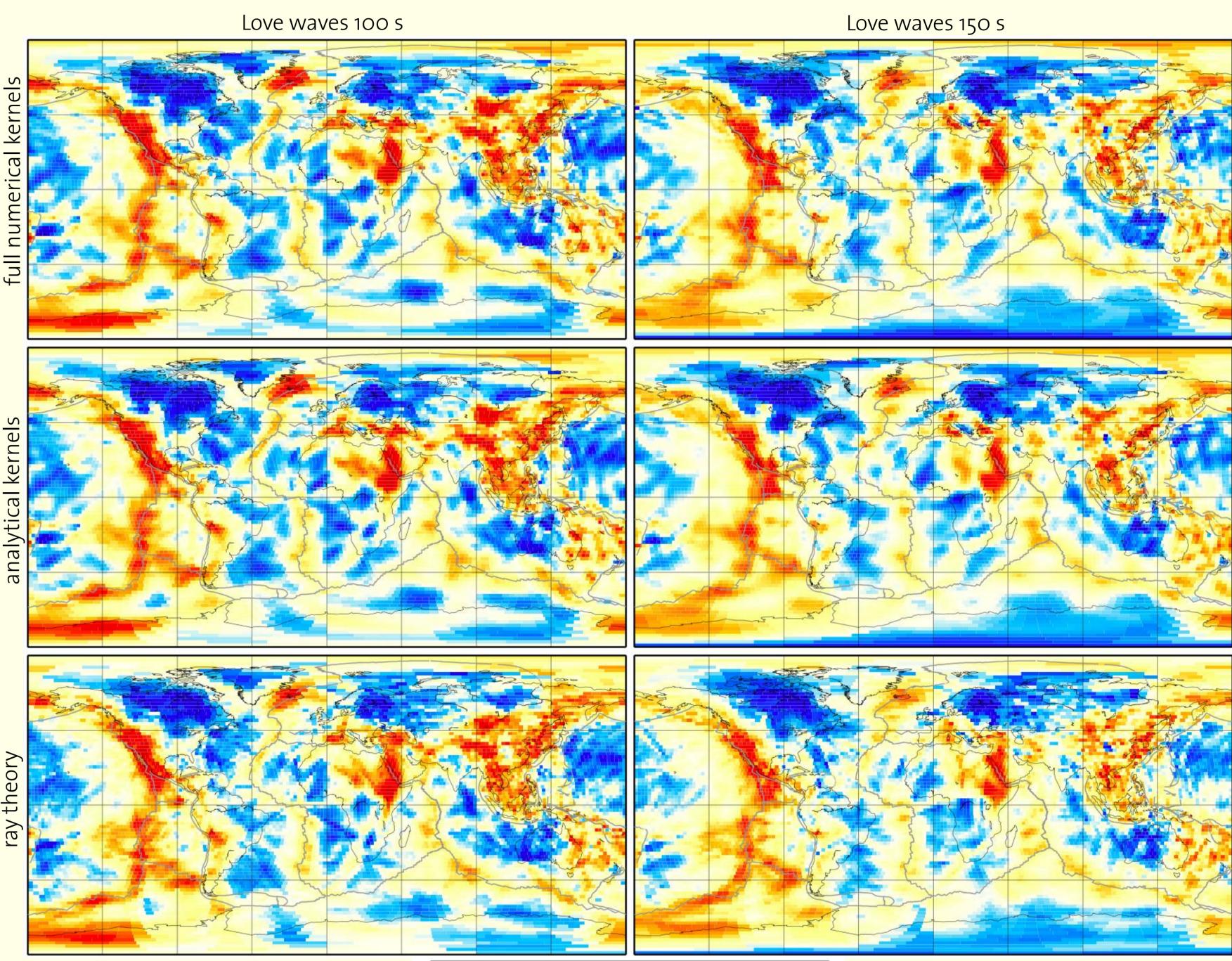analytical kernel
latitude

Effects of a heterogeneous crustal phase velocity map (based on CRUST 2.0; smoothed to harmonic degree 12). Profiles of numerical kernels for 150 s Love waves, taken perpendicular to the source-receiver great circle which crosses the Tibetan anomaly, at epicentral distances of 45°, 70° and 80°. "Heterogeneous" kernel (red) compared to "homogeneous" one (blue).
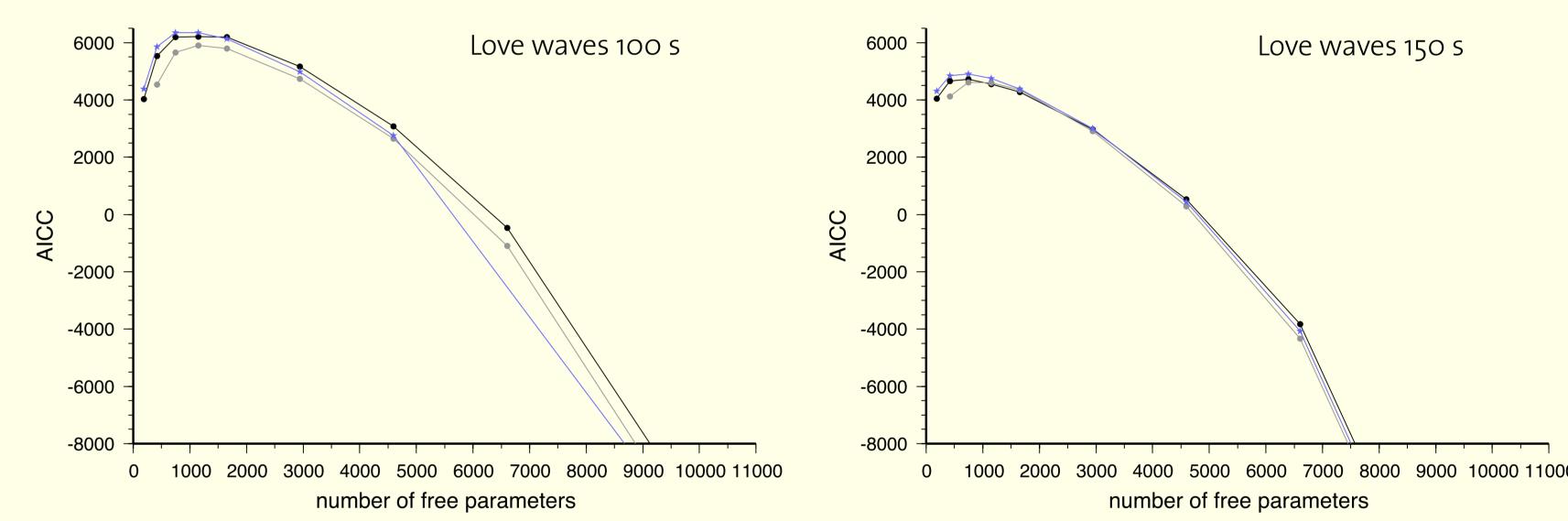
heterogeneous numerical kernel
homogeneous numerical kernel
km/s
45° 70° 80°

C Surface wave tomography from Harvard database

Surface wave phase velocity maps for Love waves inverted from traveltimes in the Harvard database by using full numerical kernels from membrane waves with back-scattering, analytical kernels without back-scattering and ray paths. LSQR inversion uses the same damping for all images.

Love waves 100 s
Love waves 150 s
full numerical kernels
analytical kernels
ray theory

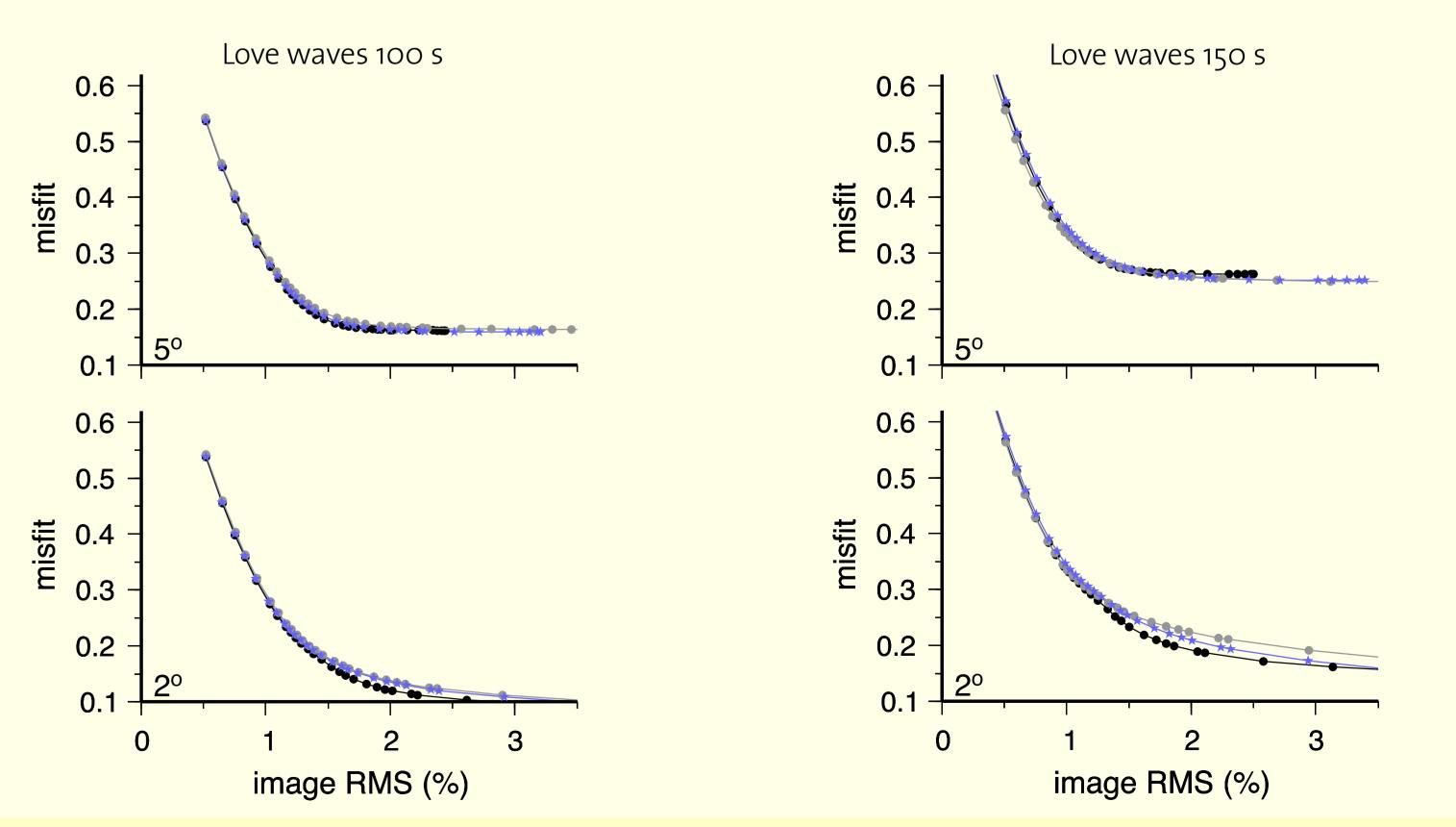-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

The corrected Akaike-criterion gives an estimate of the best fit by the number of free model parameters. The inversions with the numerical Born kernels including back-scattering are plotted in blue, inversions with analytical Born kernels without back-scattering in gray and inversions using ray paths in black.

Love waves 100 s
Love waves 150 s
AICC
number of free parameters

The L-curves help finding coherent values for the damping parameters. Curves are shown for two regular inversion grids using pixel sizes of 5°×5° and 2°×2°. Misfit values for the inversions with the full numerical Born kernels including back-scattering are plotted in blue, with analytical Born kernels without back-scattering in gray and for inversions using ray paths in black. We see that, at a given level of model complexity, ray-theory-based maps in general achieve a better fit to the data.

Love waves 100 s
Love waves 150 s
misfit
image RMS (%)
5°
2°

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あなたの注意のためにあなたにあなたに感謝しなさい。著者へコメントを送りなさい。