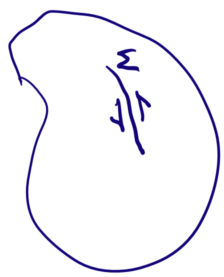
Seismic sources

Point source approximation



We can approximate an earthquake as being a point source when looking at

- wavelengths $\lambda \gg \Sigma$ fault surface, or
- period $T \gg t_r$ rupture time

Moment tensor:

For point sources, we can introduce a symmetric moment tensor $\underline{\underline{M}}$ such that

$$\underline{\underline{M}} = \int_0^{t_{final}} \int_\Sigma \partial_t \underline{\underline{S}}\, d^3x\, dt = \int_\Sigma \underline{\underline{S}}^{final} d^3x$$

$$= \int_\Sigma \underline{\underline{m}}^{final} d^2x$$

is related to the final stress glut $\underline{\underline{S}}^{final}$ and surface moment-density tensor $\underline{\underline{m}}^{final}$

For planar faults and isotropic media, the moment tensor $\underline{\underline{M}}$ scales with the final slip offset on the fault:

$$\underline{\underline{M}} = \int_{\Sigma} \mu \, \Delta u^{final} \overbrace{\left(\hat{\underline{\nu}} \hat{\underline{\sigma}} + \hat{\underline{\sigma}} \hat{\underline{\nu}} \right)}^{\text{double couples}} d^2x$$

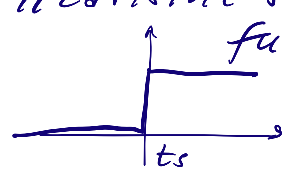
and

$$M_0 = \int_{\Sigma} \mu \, \Delta u^{final} \, d^2x \qquad \text{scalar moment}$$

The scalar moment $M_0 = \frac{1}{\sqrt{2}} \left(\underline{\underline{M}} : \underline{\underline{M}} \right)^{1/2}$ has become a standard measure of the size of an earthquake.

For point sources, we can now write the earthquake in terms of equivalent body force

$$\underline{f} = - \underbrace{\underline{\underline{M}}}_{\text{moment tensor}} \cdot \nabla_{\underline{x}} \, \delta(x - x_s) \underbrace{H(t - t_s)}_{\text{Heaviside step function}}$$



To separate volumetric and deviatoric effects:

$$\underline{\underline{M}} = \frac{1}{3} \big(\underbrace{\mathrm{tr}\, \underline{\underline{M}}}_{=0 \text{ for hydrostatic stress}} \big) \underline{\underline{I}} + \underline{\underline{M}}^{dev}$$

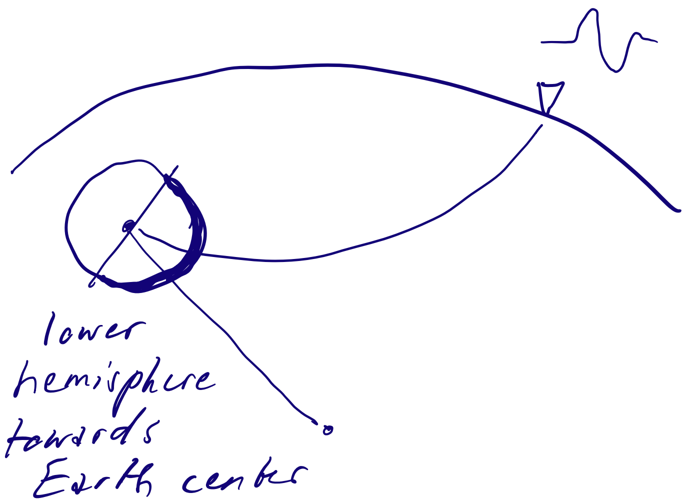
or

$$\underline{\underline{M}} = \underbrace{\underline{\underline{M}}^{iso}}_{\substack{\text{isotropic} \\ \hat{=}\ \text{volume change}}} + \underbrace{\underline{\underline{M}}^{dev}}_{\text{deviatoric}}$$

where

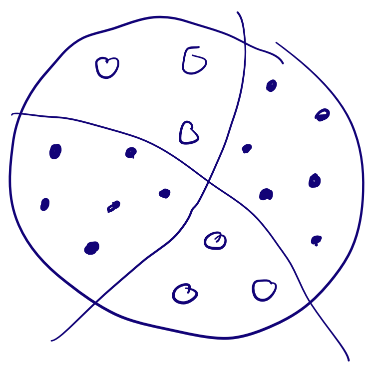
$$\underline{\underline{M}}^{dev} = \underbrace{\underline{\underline{M}}^{DC}}_{\substack{\text{double couple} \\ \hat{=}\ \text{earthquake}}} + \underbrace{\underline{\underline{M}}^{CLVD}}_{\substack{\text{compensated linear vector dipole} \\ \hat{=}\ \text{magma dyke intrusion}}}$$

Beachball representation: Depicts the radiation pattern of an earthquake



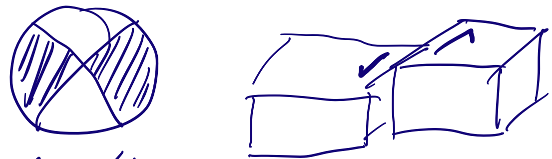
• black: positive first upswing

○ white: negative

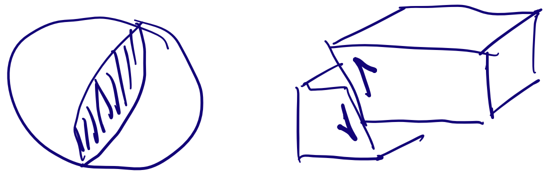


We can always find 2 planes as solution to double couples (true & auxiliary), which leads to the fault plane ambiguity.

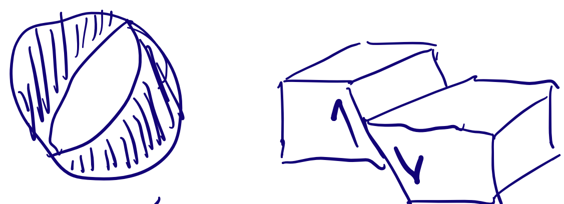
Examples:



strike-slip



thrust



normal

Volterra's theorem: Writes displacement as convolution in time of moment tensor $\underline{\underline{M}}$ and Green's functions $\underline{\underline{G}}$

$$u_n(\underline{x},t) = \int_{\Sigma} u_l(\underline{\xi},\tau)\, c_{ijkl}\, \hat{\nu}_j \overbrace{*}^{\text{convolution}} \partial_k G_{ln}\, d^2\xi$$

$$= \int_{\Sigma} m_{kl} * \partial_k G_{ln}\, d^2\xi$$

$$= M_{kl} * \underbrace{G_{ln,k}}_{\partial_k}$$

Source-time function: What is the time dependence?

stress glut rate $\dot{\underline{\underline{S}}} = \dot{\underline{\underline{M}}}\, \delta(x - x_s)$

moment-rate tensor $\dot{\underline{\underline{M}}} = \int_{\Sigma} \partial_t \underline{\underline{S}}\, d^2x$

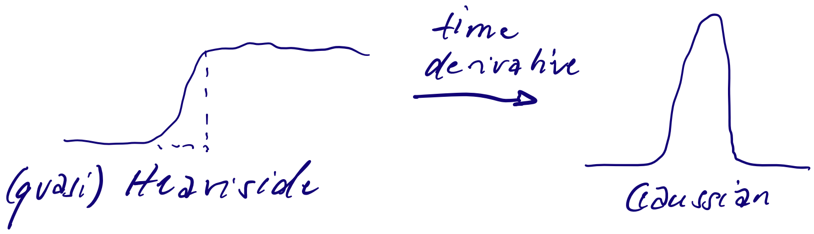
assuming synchronous in all components of $\underline{\underline{M}}$, we find

$$\dot{\underline{\underline{M}}}(t) = \sqrt{2}\, \underbrace{M_0}_{\substack{\text{scalar moment} \\ \hat{=}\ \text{size}}}\, \hat{\underline{\underline{M}}}\, \overbrace{\dot{m}(t)}^{\text{normalized source time function}}$$

$\hat{\underline{\underline{M}}}$: directions $\hat{=}$ radiation pattern

For a planar fault and non-deviatoric stress, the source time (rate) function is

$$\dot{m}(t) = \frac{1}{M_0} \int_\Sigma \mu \, \underbrace{\partial_t \Delta u}_{\text{time behaviour of slip across fault}} \, d^2x$$



Analytic solution:

For an elastic, homogeneous, whole space, the far-field compressional displacement writes as [Aki & Richards, 2002]

$$\underbrace{u_r}_{\text{radial}}(\underline{x}, t) = \underbrace{\frac{1}{4\pi \rho v_p^3 r}}_{\text{amplitude}} \; \underbrace{\dot{M}\left(t - \frac{r}{v_p}\right)}_{\text{temporal pulse}} \; \underbrace{\sin 2\vartheta \cos \varphi}_{\text{radiation pattern}}$$

with position $\underline{x} = (r, \vartheta, \varphi)$ spherical coord.

the shear displacement writes as

$$u_\vartheta = \frac{1}{4\pi \rho v_s^3 r} \dot{M}\left(t - \frac{r}{v_s}\right) \cos 2\vartheta \cos \varphi$$

$$u_\varphi = \frac{1}{4\pi \rho v_s^3 r} \dot{M}\left(t - \frac{r}{v_s}\right) (-\cos \vartheta \sin \varphi)$$

Note that for these far-field displacements:

- $\frac{1}{r}$ factor due to geometrical spreading

- shear displacement has no nodal planes, therefore P-waves are used for beachballs

- higher amplitudes in slow media (sediments) and S-waves due to $\frac{1}{v_p^3}$ resp. $\frac{1}{v_S^3}$ factor

- $\left(\frac{v_p}{v_s}\right)^3 \approx 5$ since for a Poisson solid $v_p = \sqrt{3}\, v_s$

$\longrightarrow$ S-waves are about 5x stronger