Attenuation & Scattering

ErSE390 Seismic waves

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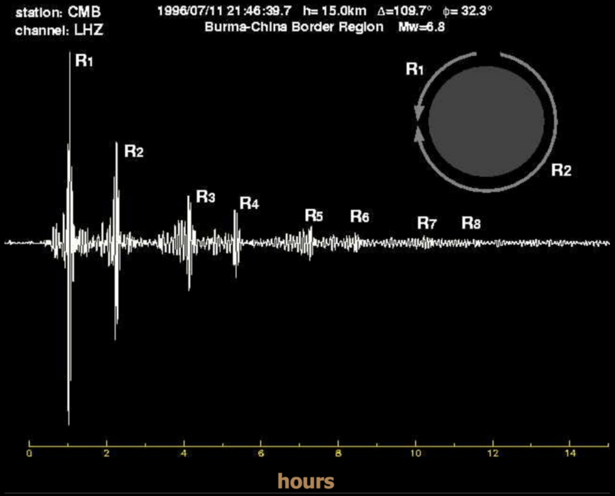
Seismic amplitudes

How come the Earth quiets down again after big earthquakes?



What affects the amplitude of a seismic signal at a given station. We can think of:

- source magnitude & radiation pattern
- directivity effects
- geometrical spreading
- seismic velocities (e.g., low velocities in sediment basins)
- reflections/refractions due to heterogeneous velocities
- focussing/defocussing
- scattering
- topography
- anelastic or viscoelastic media
- ...

Let us sort these like:

Source effects

- source magnitude & radiation pattern
- directivity effects
- off-fault plasticity

Elastic (path) effects

- geometrical spreading
- seismic velocities (e.g., low velocities in sediment basins)
- focussing/defocussing
- **scattering**

Station effects

- instrument response
- (non-linear) site effects (e.g., liquefaction)

Anelastic (path) effects

- **anelastic/viscoelastic media**

For this lecture, we will focus on attenuation. In particular, we separate attenuation into:

Intrinsic attenuation

leads to energy dissipation:

- volatiles
- crystal defects
- grain boundaries
- thermoelastic processes

Scattering attenuation

leads to energy redistribution:

- small-scale heterogeneities (wrt/ wavelength)

observed, e.g., as coda waves

From here on, when we speak of attenuation, we usually refer to **intrinsic attenuation**. We will consider scattering attenuation later on.

Attenuation

On **microscopic** scales, fundamental processes related to crystal defects, grain boundaries and thermoelastic processes transform elastic energy into heat.

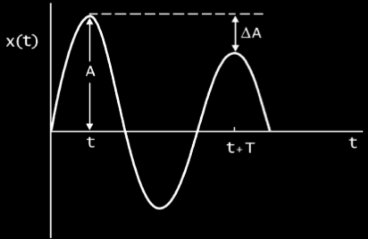
On **macroscopic** scales, the gross effect gets captured by the **quality factor Q**. This will become a material property for anelastic, or viscoelastic, media. A common definition in seismology of Q is given by:

$$Q^{-1} = \frac{1}{Q} = \frac{1}{2\pi} \frac{\Delta E}{E}$$

with ΔE the elastic energy lost per cycle, and E the maximum elastic energy contained in a cycle.

A main task consists of measuring the frequency dependence of Q over a broad-range of seismic frequencies and quantifying it for different regions. Let us look in more detail how attenuation affects seismic signals, and how we can incorporate attenuation into seismic wave propagation.

Attenuation can be measured for a given position as the amplitude decay ΔA over a time period cycle T:



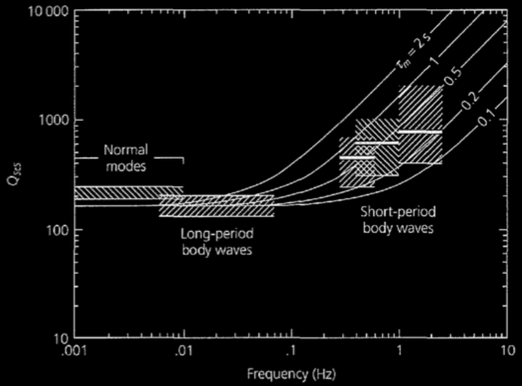
The quality factor is related to the amplitude decay by

$$Q^{-1} = \frac{1}{\pi}\frac{\Delta A}{A} \qquad \text{(approximation valid for } Q \gg 1\text{)}$$

Measuring attenuation for a single location, **spectral decay measurements** are often used.

Instead of fixing position and measuring over a time cycle, one could fix time and measure over a wavelength distance, i.e., measuring at 2 different locations. This technique can be used in **2-station measurements**, where 2 seismic stations on a great-circle path are chosen to measure attenuation.

Frequency-dependence of Q in the mantle as measured by Sipkin & Jordan (1979) on multiple ScS phases from deep-focus earthquakes:



Q is almost frequency-independent in the range between $[0.001, 0.1]$ Hz, but with a rapid increase at high frequencies (> 0.1 Hz).

There are strong regional variations, making the determination of global average mantle attenuation values difficult.

Let's find out how we can model the attenuation behavior of a body.

Elastic media

For purely elastic media, Hooke's law relates stress σ to strain ϵ:

$$\sigma = \mathbf{c} : \epsilon$$

This elastic behavior can be represented by a spring.

Viscous media

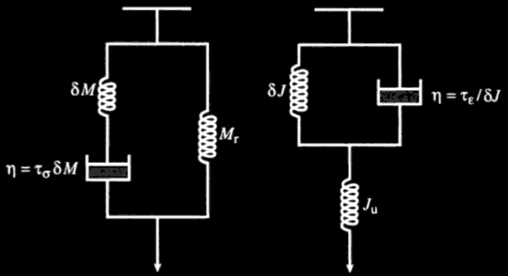
A different type of mechanical behavior is shown by viscous media, where Stokes' law applies:

$$\sigma = \eta \, \dot{\epsilon}$$

using the viscosity coefficient η and strain rate $\dot{\epsilon}$. The viscous component can be represented by a dashpot.

Imperfect elastic material can be considered as having properties intermediate between elastic and viscous media, therefore called **viscoelastic**. For such anelastic media a new stress-strain relation must be defined, represented by a mechanical model of springs and dashpots connected with each other.

It turns out that a mechanical model named **standard linear solid** or **Zener body**, mimicks the attenuation behavior of anelastic media:



Equivalent representations of a standard linear solid, or Zener body: (Left) Maxwell body and a spring connected in parallel, (Right) Kelvin-Voigt body and a spring connected in series. M is a complex elastic modulus, $J = 1/M$ the compliance. Dashpot elements have an associated relaxation time τ.

The constitutive relationship for a standard linear solid becomes

$$\dot{\sigma} + \frac{1}{\tau_\sigma}\sigma = M_u\left(\dot{\epsilon} + \frac{1}{\tau_\epsilon}\epsilon\right)$$

with τ_σ and τ_ϵ the stress and strain relaxation times, respectively, and M_u the unrelaxed modulus.

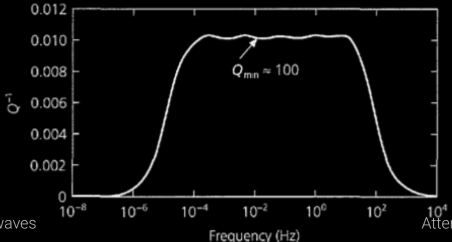
How do we get a flat attenuation response $Q(\omega)$ within a limited absorption frequency-band?

We can superimpose multiple standard linear solids to approximate a frequency-independent Q model. The complex modulus M can be written as:

$$M(\omega) = \sum_{l=1}^{L} M_r^l \frac{1 + i\,\omega\tau_\epsilon^l}{1 + i\,\omega\tau_\sigma^l}$$

where total relaxed modulus $M(0) = \sum M_r^l \equiv M_R$, the unrelaxed modulus $M(\infty) = \sum M_r^l \frac{\tau_\epsilon^l}{\tau_\sigma^l} \equiv M_U$.

Note that the modulus $M(\omega)$ is frequency dependent, thus the standard linear solid model implies physical dispersion.



Superposition of absorption peaks by multiple standard linear solids leads to a roughly constant attenuation Q^{-1} model.

For seismic wave propagation in anelastic media, the constitutive relationship can be written as

$$\sigma(t) = \int_{-\infty}^{t} M(t-t')\,\dot{\epsilon}(t')dt'$$

where stress σ depends on the past history of strain ϵ and a time-dependent modulus M. The modulus M can be seen as the stress response to a step in strain. The instantaneous response $M(t=0) = M_u$ is called the unrelaxed modulus, and $M(t=\infty) = M_r$ the relaxed modulus after having reached its final equilibrium (note that $M_u > M_r$). $M(t)$ is also called a stress relaxation function. The difference $\delta M = M_u - M_r$ is the modulus defect.

The standard linear solid model defines

$$M(t) = M_u - \delta M\left[1 - e^{-t/\tau_\sigma}\right] \qquad \text{and} \qquad \frac{M_r}{M_u} = \frac{\tau_\sigma}{\tau_\epsilon}$$

For a combination of L standard linear solids, in frequency domain we can separate the complex modulus M into its real and imaginary parts $M(\omega) = M_1(\omega) + i\, M_2(\omega)$ with

$$M_1(\omega) = \frac{M_R}{L} \sum_{l=1}^{L} \frac{1 + \omega^2 \tau_\sigma^l \tau_\epsilon^l}{1 + (\omega \tau_\sigma^l)^2} \qquad \text{and} \qquad M_2(\omega) = \frac{M_R}{L} \sum_{l=1}^{L} \frac{\omega(\tau_\epsilon^l - \tau_\sigma^l)}{1 + (\omega \tau_\sigma^l)^2}$$

assuming that all solids have the same relaxed modulus $M_r^l = \frac{M_R}{L}$.

The quality factor Q then is defined via

$$Q^{-1}(\omega) = \frac{1}{Q(\omega)} = \frac{\mathrm{Im}\{\mathrm{M}(\omega)\}}{\mathrm{Re}\{\mathrm{M}(\omega)\}} = \frac{M_2(\omega)}{M_1(\omega)}$$

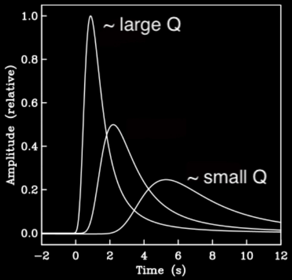
In time domain, we can substitute the modulus

$$M(t) = \frac{M_R}{L} \sum_{l=1}^{L} \left[1 - \left(1 - \frac{\tau_\epsilon^l}{\tau_\sigma^l}\right) e^{-t/\tau_\sigma^l} \right]$$

back into the constitutive relationship which can then be solved using a memory variables approach.

An obvious effect of attenuation is the diminishing of seismic amplitudes. Given an almost frequency-independent quality factor Q, it also means that attenuation, and therefore **energy dissipation is stronger at higher frequencies**. Or in simple words, the more wavelengths a signal has to travel to reach a seismic station, the more it gets attenuated. This leads to frequencies above 1 Hz being damped enough below noise levels such that they cannot be observed anymore on teleseismic distances.

An additional effect of attenuation is pulse broadening, where an initial pulse containing high frequencies is being emptied of its high-frequency content. The pulse broadens.



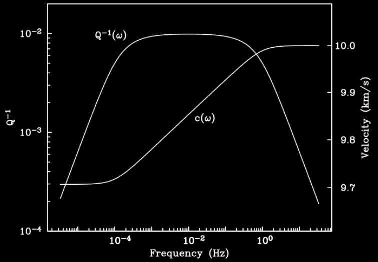
Effect of attenuation

Another important aspect of attenuation is that it introduces **physical dispersion** due to causality. The value of P- or S-wave speed c depends now on the frequency ω:

$$c(\omega) = c_0 \left[1 + \frac{1}{\pi}\frac{1}{Q}\ln(\frac{\omega}{\omega_0})\right]$$

where ω_0 is some reference frequency (e.g., PREM is defined at 1 Hz, thus $\omega_0 = 2\pi$) with its associated reference wave speed c_0. Note that for elastic moduli, e.g., shear modulus μ, the dispersion term contains a factor 2:

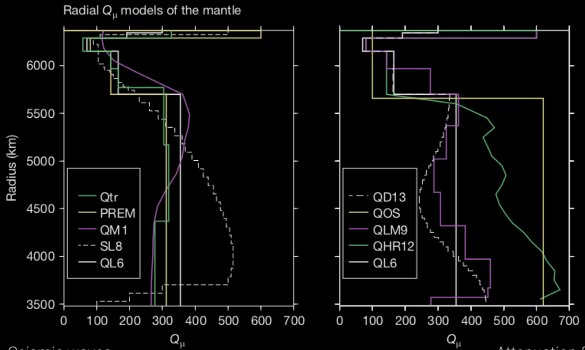
$$\mu(\omega) = \mu_0 \left[1 + \frac{2}{\pi}\frac{1}{Q_\mu}\ln(\frac{\omega}{\omega_0})\right]$$



In the Earth, we observe that shear attenuation is much stronger than bulk attenuation. That is, Q_κ is often magnitudes larger than Q_μ. It is believed that intrinsic attenuation happens mostly in shear, associated to lateral movements of lattice effects and grain boundaries. Setting $Q_\kappa = \infty$ leads to P-wave attenuation Q_α^{-1} being related to S-wave attenuation Q_β^{-1} by:

$$Q_\alpha^{-1} = \frac{4}{9} Q_\beta^{-1}$$

A ratio often close to what is being observed for different rock types.

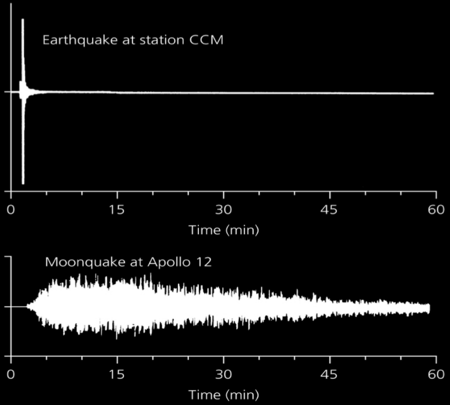


Unfortunately, even today there is still little consent about the radial 1D attenuation structure in the Earth.
Left: mantle Q-models based on normal modes & surface wave measurements.
Right: mantle Q-models based on body wave measurements.
Studies suggest that also the inner core has attenuation, with bulk and shear attenuation of the same magnitudes.

Q dependence

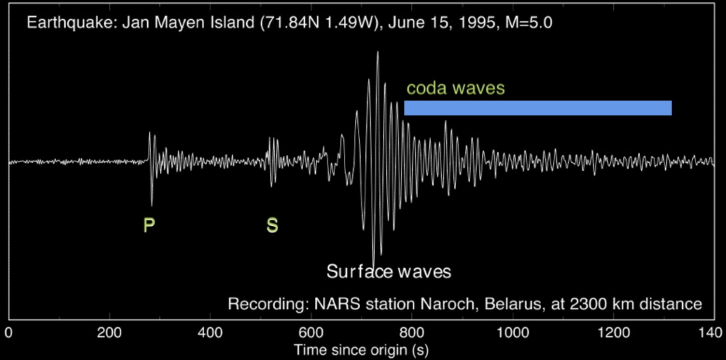
As a final note, Q also depends on physical parameters, and in particular on temperature (even more so than velocities), as well as on partial melting and water content. At shorter periods however the effect of scattering becomes increasingly important and it becomes difficult to separate these.

Having a better constraint on Q^{-1}, i.e., on attenuation, however would greatly help to provide independent constraints on Earth properties.



Scattering

Crustal scattering can be observed, e.g., as coda waves arriving after surface waves:



Most popular, scattering gets observed on P, S, and Lg waves.

The total attenuation effect of scattered waves can be summarized as:

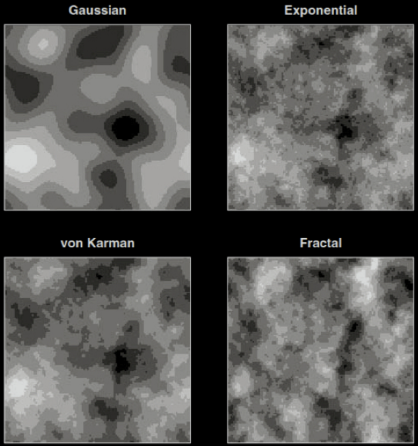
$$\frac{1}{Q} = \underbrace{\frac{1}{Q_{intrinsic}}}_{\text{energy dissipation}} + \underbrace{\frac{1}{Q_{scattering}}}_{\text{energy redistribution}}$$

That is a combination of intrinsic and scattering attenuation. Especially for higher-frequency signals, it is often difficult to separate intrinsic from scattering attenuation. Scattering attenuation is thought of being strongly frequency dependent. A formula proposed is:

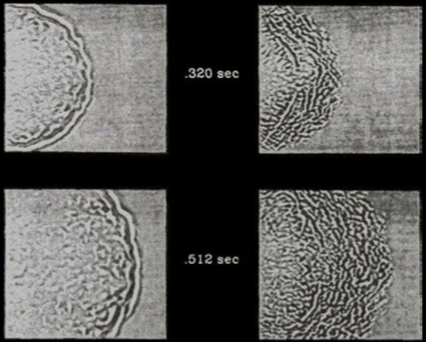
$$\frac{1}{Q_{scattering}} = \frac{g\,v}{\omega}$$

with v and ω the velocity and frequency, and g a dispersion coefficient (depending on the relative energy loss through a heterogeneous layer with a given thickness).

Numerical simulations can help investigating the net effect of scattering. The stochastic nature of heterogeneous velocities can be characterized by different spatial random fields:



The effect of stochastic velocities upon wave propagation is clearly visible, although P-waves remain fairly coherent with a complex set of later arrivals. These will appear at a station as coda scattered from all directions.



Left: P-waves, Right: SV-waves

References i

More details can be found in these reference books:

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