Seismic sources

Excitation by an earthquake

Remember the equations of motion and constitutive relation, without external body forces

$$\boxed{\begin{aligned} \rho\, \partial_t^2 \underline{u} &= \underline{\nabla} \cdot \underline{\underline{\tau}} \qquad (1) \\ \underline{\underline{\tau}} &= \underline{\underline{\underline{\underline{c}}}} : \underline{\underline{\varepsilon}} \qquad (2) \end{aligned}}$$

$\underline{\underline{\tau}}$: stress tensor

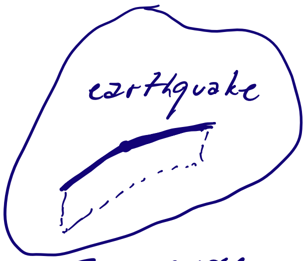
$\underline{\underline{\varepsilon}}$: strain tensor

Where is the earthquake?

The conservation of momentum described by (1) must always be valid, as it describes the physics!

$\rightarrow$ therefore, equation (2) must fail...

Backus & Mulcahy (GJRAS, 1976):
"Earthquakes are represented by a break down of Hooke's law"



Σ source volume

Let's separate

$\underline{\underline{\tau}}^{true}$ true physical stress in medium

$\underline{\underline{\tau}}^{model}$ model stress (by Hooke's model)

We write then

$$\rho\, \partial_t^2 \underline{u} = \underline{\nabla} \cdot \underline{\underline{\tau}}^{true}$$

$$= \underline{\nabla} \cdot \underline{\underline{\tau}}^{model} - \underline{\nabla} \cdot \underbrace{(\underline{\underline{\tau}}^{model} - \underline{\underline{\tau}}^{true})}_{\text{defines stress glut } \underline{\underline{S}}}$$

and find

$$\boxed{\begin{aligned} \rho\, \partial_t^2 \underline{u} &= \underline{\nabla} \cdot \underline{\underline{\tau}}^{model} + \underline{f} \\ \underline{f} &= -\underline{\nabla} \cdot \underline{\underline{S}} \end{aligned}}$$

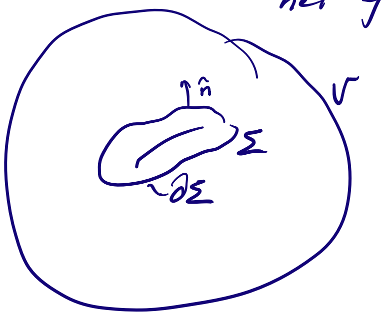
$\underline{f}$: equivalent body force

where $\underline{\underline{\tau}}^{model}$ obeys Hooke's law and stress glut $\underline{\underline{S}}$ will be:

- zero outside of the "source volume" Σ
- symmetric (since τ^{model}, τ^{true} are symm.)

We assumed Hooke's law fails and thus introduced the differential stress, i.e. stress glut, without any further need of modifications (such as internal structure, ...).

Did we introduce a net force or net torque, which would push the Earth away or rotate it?

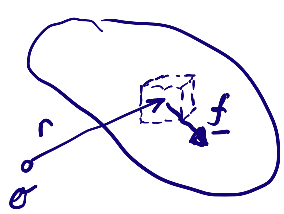


net force: $\int_V \underline{f}(\underline{x},t)\, d^3x = -\int_V \underline{\nabla}\cdot \underline{\underline{S}}\, d^3x$

$= -\int_\Sigma \underline{\nabla}\cdot\underline{\underline{S}}\, d^3x$

$= -\int_{\partial\Sigma} \underline{\hat{n}}\cdot\underline{\underline{S}}\, d^2x = 0$ ✓

because $\underline{\underline{S}}$ vanishes on $\partial\Sigma$

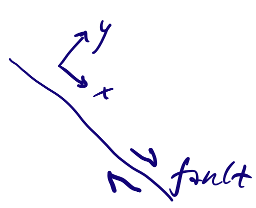


net torque: $\int_V \underline{r} \times \underline{f}(\underline{x},t)\, d^3x = \int_V \underline{r} \times \underline{\nabla}\cdot\underline{\underline{S}}\, d^3x$

$= 0$ ✓

because $\underline{\underline{S}} = \underline{\underline{S}}^T$ symmetric

Example: displacement across the fault

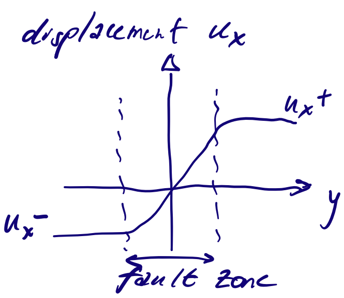


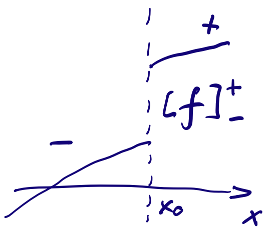
As we make the fault ~~zone~~ thinner,

stress glut $S_{xy} = \tau_{xy}^{model} - \tau_{xy}^{true}$

becomes $\mu\, \Delta u\, \delta(y)$ with $\Delta u = u_x^+ - u_x^- = [u_x]_-^+$

offset on the fault

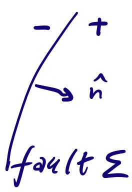




Consider the equivalent body force $\underline{f} = -\underline{\nabla} \cdot \underline{\underline{S}}$, what is $\frac{df}{dx}$?

We need to regard f as a distribution Df

$$\frac{d}{dx} Df = \underbrace{D \frac{df}{dx}}_{\substack{\text{well behaved} \\ \text{on one side} \\ \text{\& on the other of } x_0}} + \underbrace{[f]^+_- \, \delta(x - x_0)}_{\text{jump}}$$



Related to the displacement on the fault

$$\underline{\nabla} D\underline{u} = D(\underline{\nabla}\underline{u}) + \underline{\hat{n}} [\underline{u}]^+_- \, \delta_\Sigma$$

For the stresses

$$D \underline{\underline{\tau}}^{true} = D(\underline{\underline{\underline{\underline{c}}}} : \underline{\nabla}\underline{u}) = \underline{\underline{\underline{\underline{c}}}} : D(\underline{\nabla}\underline{u})$$

$$D \underline{\underline{\tau}}^{model} = \underline{\underline{\underline{\underline{c}}}} : \underbrace{\underline{\nabla} D\underline{u}}_{D(\underline{\nabla}\underline{u}) + \hat{n}[u]\delta_\Sigma} = D \underline{\underline{\tau}}^{true} + \underline{\underline{\underline{\underline{c}}}} : \underline{\hat{n}} [\underline{u}]^+_- \, \delta_\Sigma$$

assuming $\underline{\underline{\underline{\underline{c}}}}$ is continuous across Σ.

Therefore, the stress glut

$$\underline{\underline{S}} = D \underline{\underline{\tau}}^{model} - D \underline{\underline{\tau}}^{true} = \underline{\underline{\underline{\underline{c}}}} : \underline{\hat{n}} [\underline{u}]^+_- \, \delta_\Sigma$$

(in our 1D example becomes $\mu \Delta u \, \delta(y)$)

Ideal fault:

For an ideal fault, we assume:

- infinitesimally thin Σ
- tangential slip discontinuity $\Delta u = [u]_-^+$
- walls obey $\hat{\underline{n}} \cdot \Delta \underline{u} = 0$ (normal component vanishes)
- $\Delta \underline{u} = 0$ on $\partial \Sigma$ (no slip on fault rim)

We then have:

- $\underline{\underline{\tau}}^{true}$ discontinuous across Σ
- $D \underline{\underline{\tau}}^{true} = \underline{\underline{\underline{\underline{c}}}} : D(\underline{\nabla} \underline{u})$
- $\underline{\underline{\tau}}^{model} = \underline{\underline{\underline{\underline{c}}}} : \underline{\nabla}(D \underline{u})$

and the stress glut for an idealized fault

$$\underline{\underline{S}} = \underline{\underline{\tau}}^{model} - \underline{\underline{\tau}}^{true}$$

$$= \underbrace{\underline{\underline{\underline{\underline{c}}}} : \hat{\underline{n}} \, \Delta \underline{u}}_{\underline{\underline{m}}} \; \delta_\Sigma = \underline{\underline{m}} \, \delta_\Sigma$$

with surface moment-density tensor

$$\underline{\underline{m}} = \underline{\underline{\underline{\underline{c}}}} : \underbrace{\hat{\underline{n}} \, \Delta \underline{u}}_{slip}$$

assuming $\underline{\underline{\underline{\underline{c}}}}$ is continuous across Σ (i.e., $[\underline{\underline{\underline{\underline{c}}}}]_-^+ = 0$)

Equivalent body force: $\underline{f} = -\underline{\underline{m}} \cdot \underline{\nabla} \delta_\Sigma$ in V

Equivalent surface force: $\underline{t} = (\underline{\hat{n}} \cdot \underline{\underline{m}}) \delta_\Sigma$ on ∂V

Note that tensor $\underline{\underline{m}}$ can be decomposed by eigenvectors & values which leads to the concept of double couples for the equivalent body force.