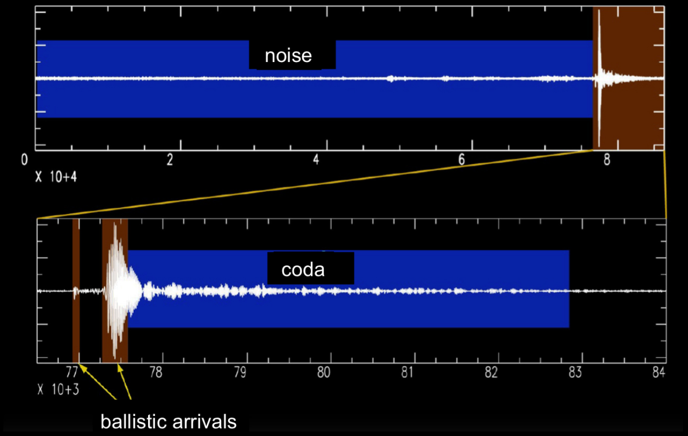
Ambient noise & diffuse wavefields

ErSE390 Seismic waves

Table of contents

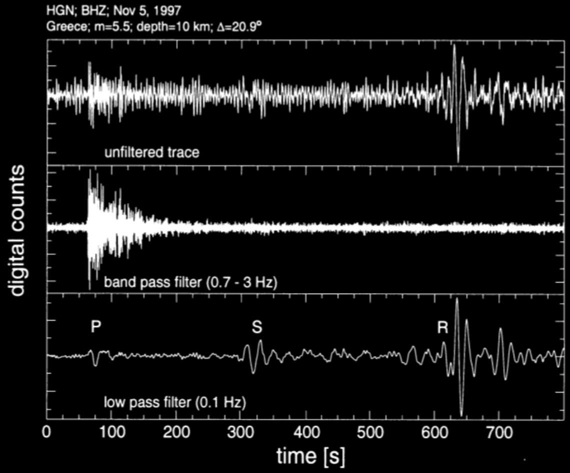
Diffuse wavefields

Let's start with a typical seismic record, highlighting its different sections

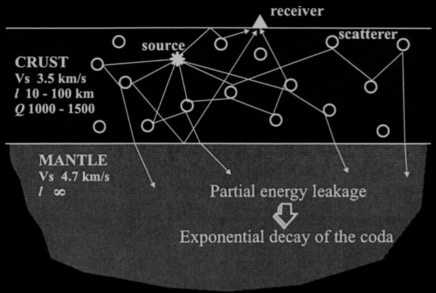


We will first look into the decay of coda waves and then turn to the ambient noise records.

The propagation of seismic waves through the Earth can be complex. For example, a regional earthquake recording filtered at different frequency ranges reveals a complex high-frequency P-wave coda signal, with a completely different nature than the low-frequency signal underlying it [Snieder, 1999]:



Coda wave decay



Let's recall the decay of coda waves, characterized by the total quality factor Q_c as:

$$\frac{1}{Q_c} = \underbrace{\frac{1}{Q_{intrinsic}}}_{\text{energy dissipation}} + \underbrace{\frac{1}{Q_{scattering}}}_{\text{energy redistribution}}$$

that is, in general Q_c is a combination of intrinsic and scattering attenuation.

[Aki and Chouet, 1975] proposes the following frequency-dependent energy decay formula for coda waves:

$$E(t,f) = S(f)\, t^{-\alpha}\, e^{-2\pi f t / Q_c(f)}$$

where $S(f)$ a frequency-dependent source term, t lapse time, f frequency, α a positive exponent and Q_c the quality factor of coda waves. In a diffuse regime, where energy is equipartitioned around the stations, one assumes that

$$Q_c = Q_{intrinsic}$$

Coda wave scattering

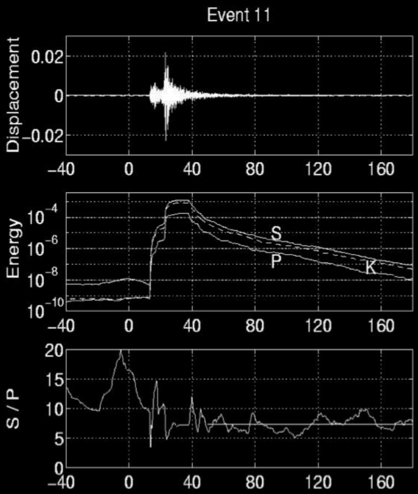
For weak velocity and density perturbations, one usually relies on single-scattering theory (Born approximation). For very strong scattering, the diffusion equation can be applied.

Most current approaches to synthesizing multiple scattering use radiative transfer theory to model energy transport [Shearer, 2007, Sato et al., 2012]. A theoretical result for elastic waves and no intrinsic attenuation predicts the P- to S-wave energy ratio

$$\frac{E_P}{E_S} = \frac{1}{2}\left(\frac{v_s}{v_p}\right)^3$$

given the wave speeds v_p and v_s. For a Poisson solid, this predicts about 10 times more S-wave energy than P-wave energy at equilibrium.

Diffuse wavefields are composed of waves with random amplitudes and phases, propagating in all possible directions. It is equal to the assumption of spatially and temporally uncorrelated sources.



When entering the diffusive regime due to multiple scattering, energy becomes uniform in phase space, reaching so-called **equipartition**. Equipartition means that for a diffuse wavefield in average all the modes of propagation are excited to equal energy. In practice, the diffuse regime is reached when the P- to S-wave energy ratio stabilizes.

The extraction of Green's functions from diffuse wavefields has been successfully applied in helioseismology, ultrasonics, marine acoustics and seismology [Campillo and Paul, 2003, Shapiro and Campillo, 2004].

Seismic interferometry

The *cross-correlation* of two continuous seismic signals $s_A(t)$ and $s_B(t)$ (of lengths $[0, T]$) is defined as

$$C_{AB}(t) = s_A(t) \otimes s_B(t) = \int_0^T s_A(\tau)\, s_B(t+\tau)\mathrm{d}\tau$$

The correlation of a signal with itself is the auto-correlation. Often, the cross-correlation is normalized by the auto-correlation functions

$$\tilde{C}_{AB}(t) = \frac{\int_0^T s_A(\tau)\, s_B(t+\tau)\mathrm{d}\tau}{\sqrt{C_{AA}(t)\, C_{BB}(t)}}$$

Note that cross-correlations have the symmetry $C_{AB}(t) = C_{BA}(-t)$ and a time range $[-T, T]$ with the positive branch (t > 0), called causal, and negative branch, called acausal branch.

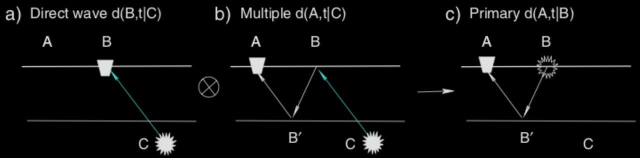
In general, applications of seismic interferometry can be divided into [Schuster, 2014]:

controlled-source interferometry

used mostly in exploration, based on ballistic record sections.

ambient-noise interferometry

used mostly in seismology, based on noise record sections.

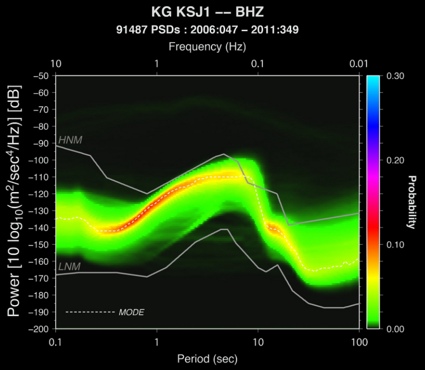


From here on, we will focus on **ambient noise** interferometry, although the underlying principle of cross-correlations remains the same.

Ambient noise seismology

In seismograms from earthquake recordings, we typically see that surface waves at 20 s period have the largest amplitudes. If periods are shorter (< 10-15 s), energy is lost due to scattering in the crust. If periods are larger (> 25 s), energy is lost in the asthenosphere [Aki and Richards, 2002].

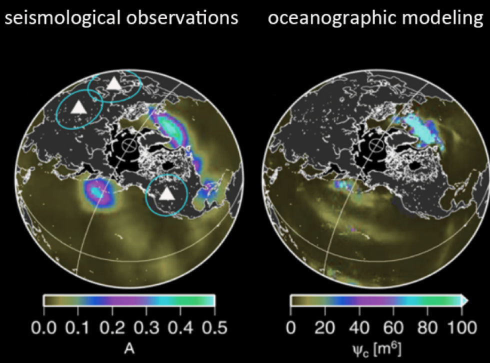
Removing the ballistic record section however and focussing on the noise record only, we find a very different energy distribution in seismic recordings:



The standard method for quantifying seismic background noise is the power spectral density (PSD), based on amplitudes squared of the Fourier transform of a signal. Its discrete form can be written as:

$$S(\omega) = \frac{\Delta t}{N} \left| \sum_{n=1}^{N} x_n e^{-i\,\omega n \Delta t} \right|^2$$

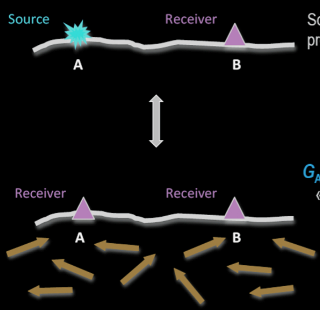
The origin of ambient noise in different period ranges can be attributed to:



Hillers et al., 2012

< 0.1 s:	cultural origin, rivers, wind on buildings/trees
$\sim$ 5-8 s:	secondary microseisms: pressure fluctuations of standing ocean swell on ocean floors
$\sim$ 12-15 s:	primary microseisms: coupling between ocean swell and continental shelves
> 40-50 s:	hum: long-periodic ocean wave coupling (oceanic infragravity waves) and/or atmospheric disturbances

The basic concept of ambient noise seismology:



Source in A $\Rightarrow$ the signal recorded in B characterizes the propagation between A and B.

➡ **Green function** between A and B: G_{AB}

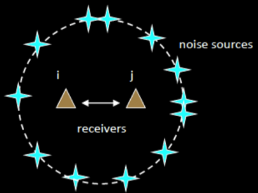
G_{AB} can be reconstructed by the correlation of noise or « diffuse » (equipartitioned) fields recorded at A and B (C_{AB})

Experimentally verified with seismological data (Campillo and Paul, 2003; Shapiro and Campillo, 2004,........)

Equipartition implies that the average cross-correlation of the wavefield is proportional to the imaginary part of the Green's function in the frequency domain. The extraction of the Green's function from the noise field between two stations allows to investigate the subsurface medium surrounding them.

In practice, even when energy is not fully equipartitioned, cross-correlations can still be used to infer Green’s functions:

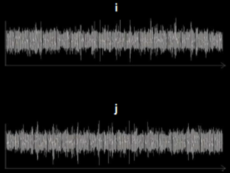
Extraction of Green’s functions from correlations of seismic noise

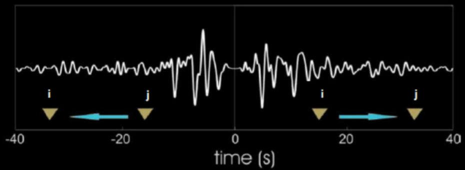


$$\frac{d}{d\tau}C(\tau,\vec{r}_A,\vec{r}_B) = \frac{-\sigma^2}{4a}\left(G_a(\tau,\vec{r}_A,\vec{r}_B) - G_a(-\tau,\vec{r}_A,\vec{r}_B)\right)$$

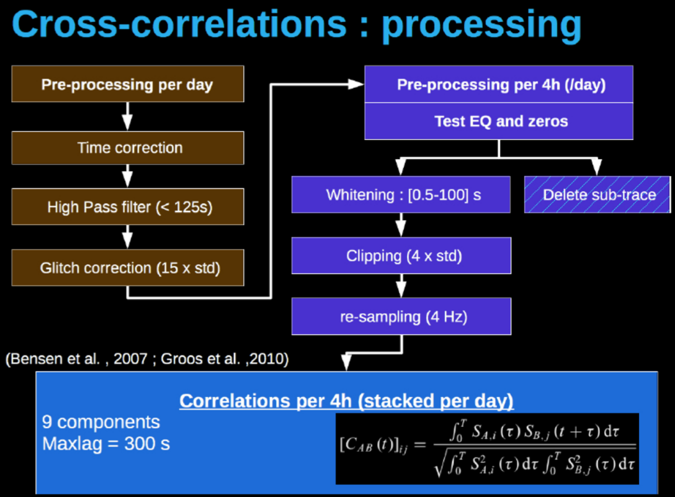
Campillo. 2006

$$G_{i,j} = \begin{pmatrix} Z_iZ_j & Z_iR_j & Z_iT_j \\ R_iZ_j & R_iR_j & R_iT_j \\ T_iZ_j & T_iR_j & T_iT_j \end{pmatrix}$$

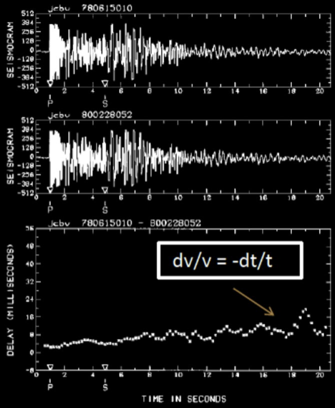




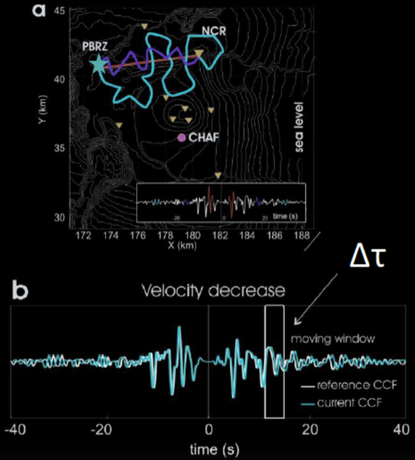
Data pre-processing becomes an important aspect of any ambient noise study. For example, a suggested extraction workflow could look like:



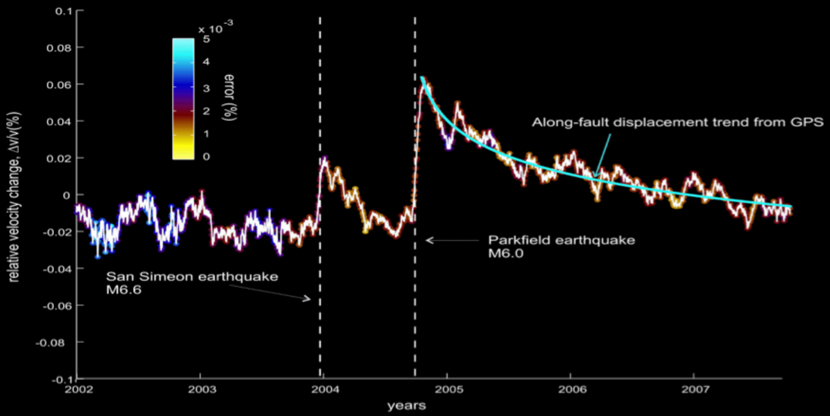
Time-dependent measurements to detect changing media properties, e.g., on the coda of cross-correlations [Poupinet et al., 1984]:



Poupinet et al., 1984



Time-dependent measurements to detect changing media properties [Brenguier et al., 2008]:



More details can be found in these references:

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