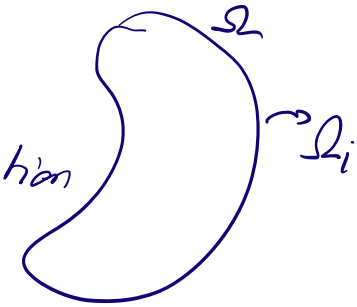
Numerical methods

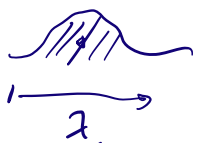
Fully numerical solutions, have 3 characteristics:

1) Divide domain into grid Ω_i
2) Ω_i defines functional representation
3) define spatial / temporal differential / integral operators $\nabla u \rightarrow \tilde{\nabla} u_i$

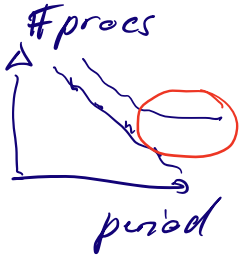


What to look for?

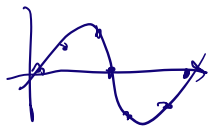
1) fix maximum error at maximum distance
2) model complexity ($\rightarrow$ meshing)



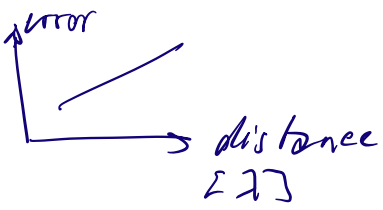
3) practical needs: seismograms, wavefield
4) how many processors to solve solution
   $\rightarrow$ cost-accuracy analysis decides what method is best



5) optimize input parameter
6) run at higher resolution to validate fit distance



numerical dispersion



Strong vs. weak form of the wave equation:

$$\rho \, \partial_t^2 \underline{u} = \underline{\nabla} \cdot \underline{\underline{T}} \qquad \underline{\text{strong}} \text{ form (FD, PS)}$$

$$\int_\Omega \underline{w} \cdot \rho \, \partial_t^2 \underline{u} \, dV = \int_\Omega \underline{w} \cdot \underline{\nabla} \cdot \underline{\underline{T}} \, dV \qquad \underline{\text{weak}} \text{ form (FEM, FV)}$$

Let's look at

$$\int_\Omega \underline{w} \cdot \underline{\nabla} \cdot \underline{\underline{T}} \, dV = \int_\Omega \underline{w} \cdot \underline{\nabla} \cdot (\underline{\underline{\underline{c}}} : \underline{\nabla} \underline{u}) \, dV$$

$$= -\int_\Omega \underline{\nabla} \underline{w} : \underline{\underline{\underline{c}}} : \underline{\nabla} \underline{u} \, dV$$

$$+ \int_\Omega \underline{\nabla} \cdot (\underline{w} \cdot \underline{\underline{T}}) \, dV$$

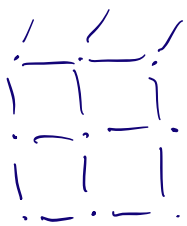
$$= \int_{\partial\Omega} \hat{\underline{n}} \cdot \underline{\underline{T}} \cdot \underline{w} \, dS \overset{!}{=} 0$$

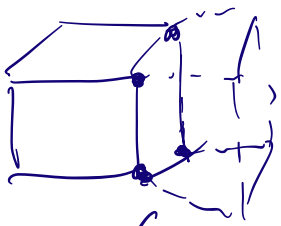
with stress-free boundary condition

$\hat{\underline{n}} \cdot \underline{\underline{T}} = 0$ on $\partial\Omega$

Finite volume methods: $\int_{\Omega_i} \nabla \cdot \underline{f} \, dV = \int_{\partial\Omega} \underline{f} \cdot \hat{n} \, dS$

Ω_i: volume; $\partial\Omega$: surface for each element





conforming mesh

non-conforming mesh

Finite difference (FD) → Finite element (SEM) → Finite volume (DG)

→ increasing (geometrical) flexibility & costs