Anisotropy

ErSE390 Seismic waves

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Primer

During this course, many phenomena that affect seismic waves haven't been looked at so far. Some problems arise due to the multi-scale nature of these phenomena, where microscopic processes influence wave propagation on macroscopic scales. Furthermore, the interaction of seismic waves with complex structures leads to a plethora of effects to investigate. Finally, we mostly focused on elastic media, although acoustic and poroelastic media are equally important.

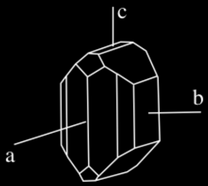
In the following, we will mention a few topics related to anisotropy, mostly with the intent to provoke curiosity and motivate you to further dive into these fields and subjects of studies.

Anisotropy

Seismic wave propagation in the Earth is influenced by anisotropic rocks (or media). First, let us distinguish between:

intrinsic anisotropy

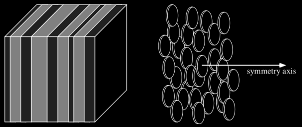
anisotropic rocks with lattice-preferred orientation (LPO) minerals, e.g., olivine



olivine (orthorhombic)

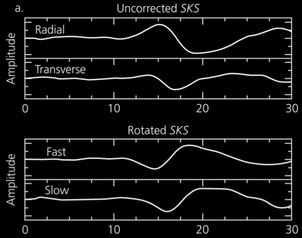
apparent anisotropy

anisotropic rocks with shape-preferred orientation (SPO), fine layering, fractures, etc., which causes seismic anisotropy at longer wavelengths compared to the scalelength of the heterogeneity

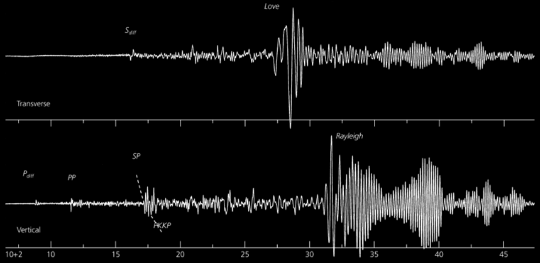


The two major observations of seismic anisotropy are:

Shear-wave splitting:
for example on SKS, PKS (polarized in radial direction at the CMB) for crust/mantle anisotropy

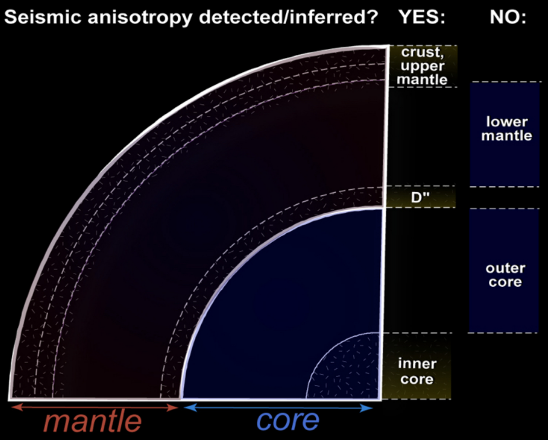


Love/Rayleigh discrepancy:
fast Love waves, slower Rayleigh waves.



Both observations, although more clearly in SKS splitting, come with a dependency on azimuth (e.g., in the Pacific).

Earth structure



Observed anisotropy is assumed to come from the crust, upper mantle, D″ region and the inner core.
Inner core anisotropy is thought to have a symmetry axis in the direction of the Earth's rotation axis.

Stress-strain relationship

For seismic wave propagation, Hooke's law for a fully anisotropic elastic media describing the stress-strain relationship can be written as

$$\tau_{ij} = c_{ijkl}\, \epsilon_{ij}$$

with stress tensor τ, strain tensor ϵ and the elastic tensor $\mathbf{c}$ characterizing the anisotropic media. Due to the symmetries $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}$, the fully anisotropic elastic tensor has 21 free parameters.

Parameterizations

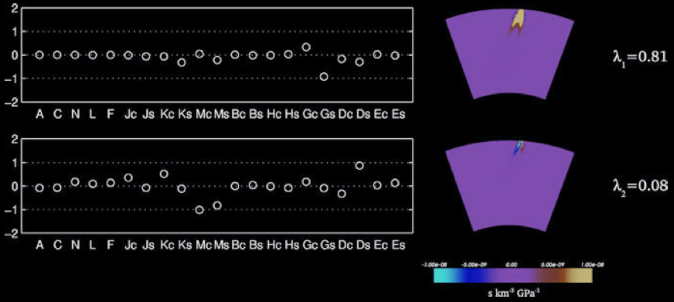
Often, an additional hexagonal symmetry is assumed for seismological problems. This symmetry has a principal axis with any direction normal to this axis having the same properties.

In case the symmetry axis is radial, its called radial anisotropy or **transverse isotropy** in global seismology. In exploration, if the axis is vertical it is called **vertical transverse isotropy** (VTI). Transversely isotropic media can be characterized by 5 independent parameters, e.g., $(\alpha_v, \alpha_h, \beta_v, \beta_h, \eta)$ or by a Love parameterization (A, C, L, N, F). As example, vertically polarized shear-wave speed $\beta_v = \sqrt{\frac{L}{\rho}}$ and horizontally polarized $\beta_h = \sqrt{\frac{N}{\rho}}$.

Parameterizations

General hexagonal anisotropy is additionally specified by a direction of its symmetry axis, i.e., by including two angles for the orientation. It has thus a total of 7 independent parameters. Such a tilted principal axis leads to **tilted transverse isotropy** (TTI), mostly used in exploration. This also leads to **azimuthal anisotropy**, often used in global seismology, with a dependency on the azimuthal orientation.

Constraining all anisotropic parameters by seismic measurements remains a major challenge:



A principal component analysis for example of SKS-measurements reveals mostly a sensitivity towards only the G_c and G_s parameters [Sieminski et al., 2009], which are simply linked to Thomson's parameter γ.

Backus averaging & upscaling

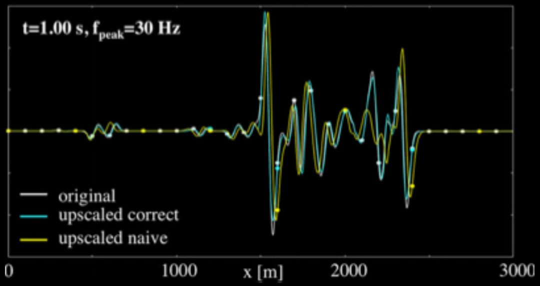
For finely layered isotropic media, [Backus, 1962] showed that by using a moving average the "effective" media properties seen by a longer wavelength signal become transversely isotropic:

$$
\begin{aligned}
C &= < \frac{1}{\lambda + 2\mu} >^{-1} \\
L &= < \frac{1}{\mu} >^{-1} \\
F &= C < \frac{\lambda}{\lambda + 2\mu} >^{-1} \\
M &= < \mu > \\
A &= F^2/C + 4M - 4 < \frac{\mu^2}{\lambda + 2\mu} >
\end{aligned}
$$

and $B = A - 2M$. Finding the effective medium at longer wavelengths refers to a concept called upscaling.

Assume we want to solve the wave equation for a general medium with small-scale heterogeneities, however, we are interested only in the propagation of longer-wavelength signals.

Separating large- and fine-scale heterogeneities, a two-scale homogenization method can be used to compute the "effective" media properties and seismic wave equations equivalent to the initial problem. This involves solving a non-trivial "cell problem" to find the effective medium. However, it can drastically reduce the computational costs of calculating accurate seismograms for a medium with small-scale heterogeneities [Capdeville et al., 2015, Fichtner and Hanasoge, 2017]:



More details can be found in these references:

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