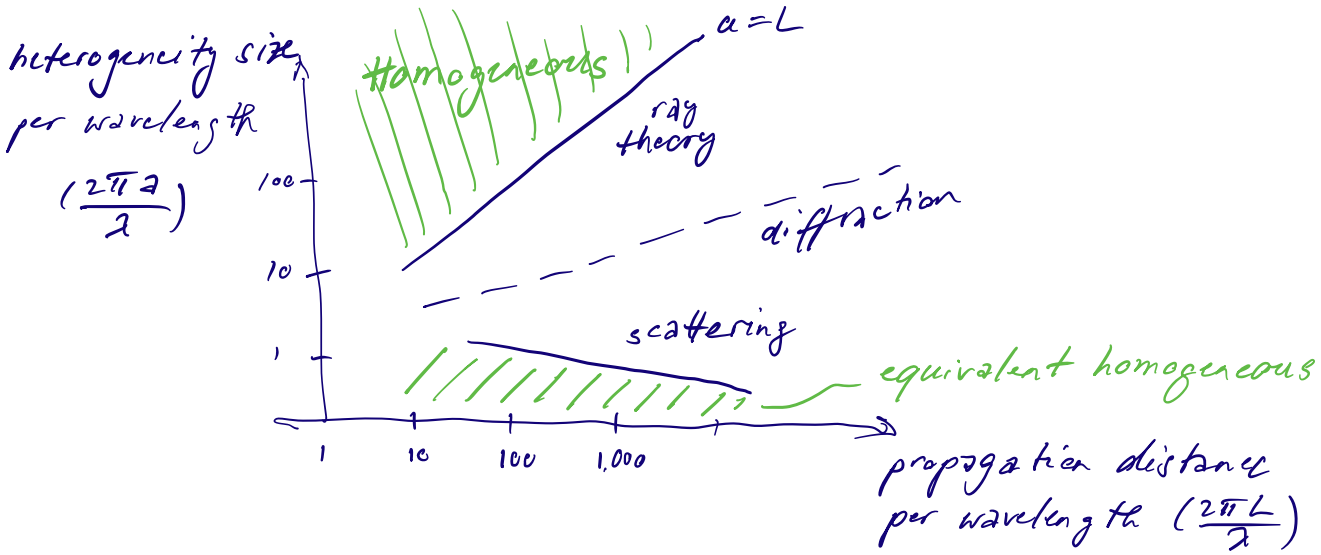
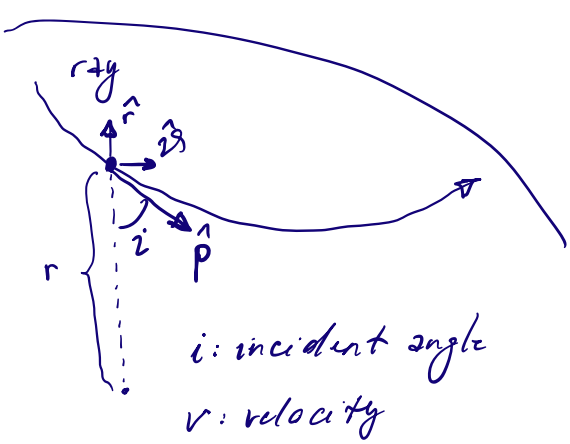
Body waves

Ray theory - Primer



Ray theory is an infinite-frequency approximation and considers the medium being smooth for wavelength observed.



Consider the unit slowness vector (in a spherical Earth)

$$\hat{p} = \hat{r}\cos i + \hat{\beta}\sin i$$

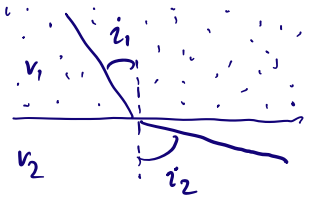
which varies along the ray. However, the parameter

$$p = \frac{(r \sin i)}{v} = \text{const.}$$

and called "ray parameter"

From optics, Snell's law

$$\frac{r_1 \sin i_1}{v_1} = \frac{r_2 \sin i_2}{v_2}$$

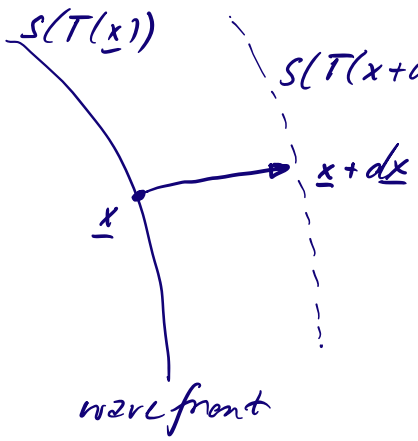


obeys Fermat's principle

$$T = \int_{ray} \frac{1}{c}\, ds \qquad T: \text{traveltime}$$

Traveltime with slowness $\frac{1}{c(x)}$ is stationary for rays, i.e., minimizes T.

Eikonal equation:



A wavefront S at time $t = T(\underline{x})$ reaches the point $\underline{x}+d\underline{x}$ at a time dt later. Then,

$$t+dt = T(\underline{x}+d\underline{x}) \quad \text{and} \quad dt = \underline{\nabla} T \cdot d\underline{x}$$

For a ray parameterized as $\underline{x} = \underline{x}(\xi)$, perpendicular to S, the Eikonal equation states

$$(\nabla T)^2 = \frac{1}{c^2}$$

and

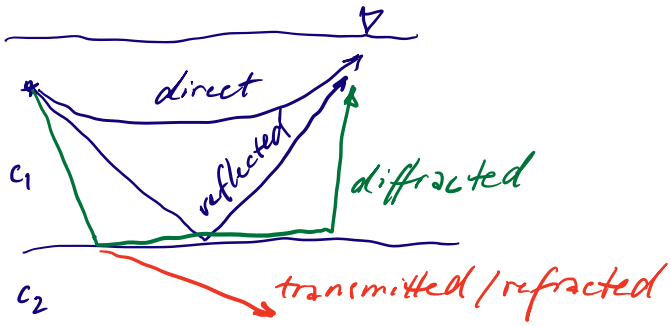
$$\frac{dx}{d\xi} = c^2 \nabla T \qquad \xi: \text{traveltime}$$

or $\frac{dx}{ds} = c\,\nabla T$ $\quad$ s: distance along ray

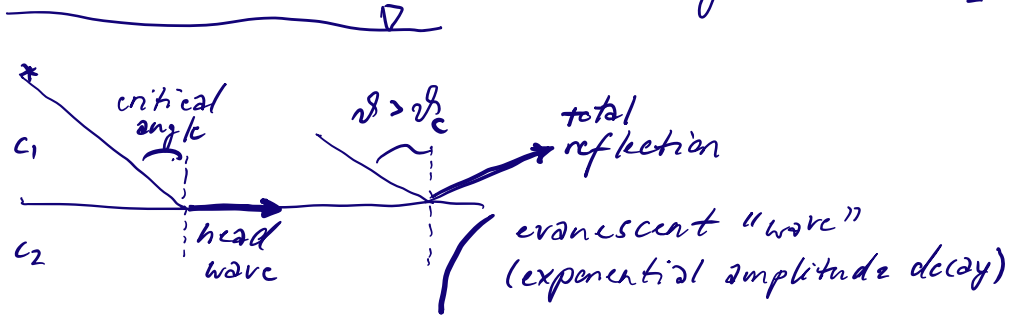
Reflection/transmission coefficients for layered media can be found analytically, as demonstrated for SH waves, assuming that displacement is continuous at the boundary as well as the traction (vertical component of stress at interface).

Nomenclature

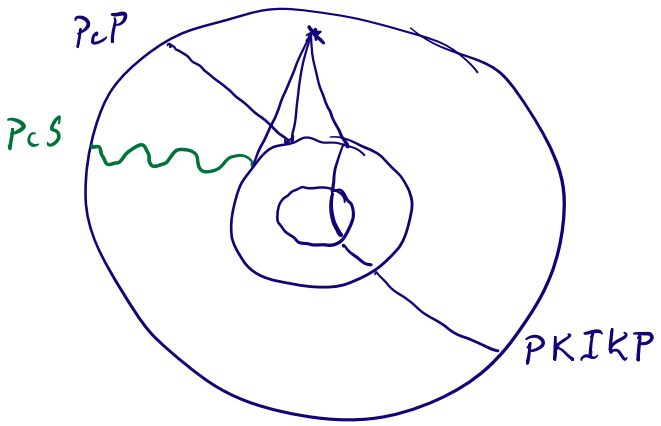
Local



critical angle $\vartheta_c = \sin^{-1}\left(\frac{c_1}{c_2}\right)$



Global



Global phases:

P – P-wave in mantle

K – P-wave in outer core

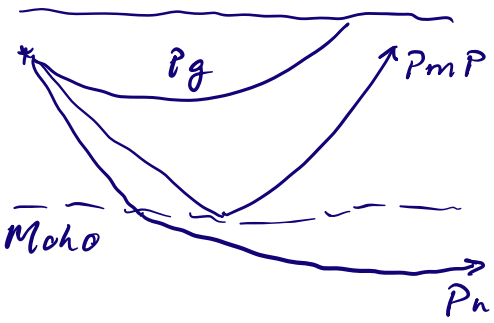
I – P-wave in inner core

S – S-wave in mantle

J – S-wave in inner core

c – reflection off the core-mantle boundary

i – reflection off the inner-core boundary



Local phases:

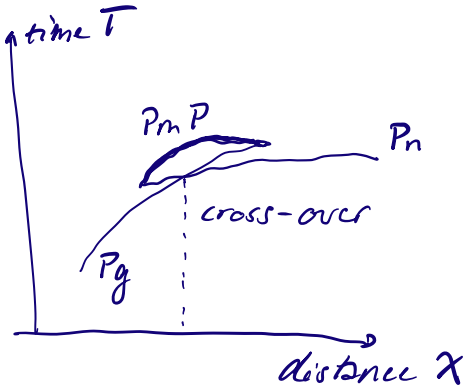
g – wave turning in crust

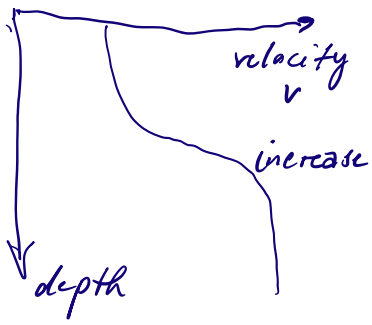
m – reflection at Moho

n – wave traveling in uppermost mantle

Traveltime curves

triplication





cross-over ~ 150 km distance

continental crust
(~ 30–50 km Moho)

~ 30 km distance

oceanic crust
(~ 6km Moho)

shadow zone