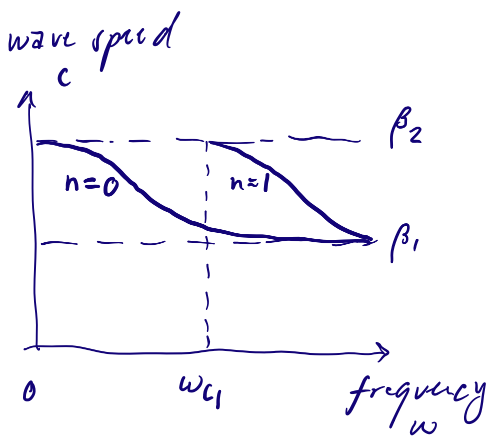
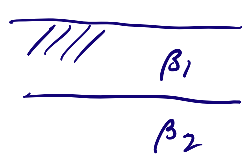
Surface waves

Dispersion: $\boxed{\text{Waves at different frequency travel at different (phase) velocities}}$

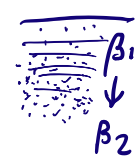


We already saw that for Loves waves to have a solution to the equations of motion, the media has to satisfy: $\beta_1 < \beta_2$ and

Love-wave velocity: $\beta_1 < c < \beta_2$



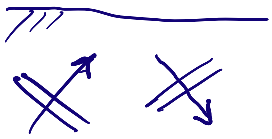
or gradual increase



For a given frequency ω, there exist many solutions, called "modes", with c of the fundamental mode $(n=0)$ and overtones $(n=1,..)$ given by:

$$\tan(n\pi) = \tan\left(\omega H\sqrt{\frac{1}{\beta_1^2}-\frac{1}{c^2}}\right) = \frac{\mu_2\sqrt{\frac{1}{c^2}-\frac{1}{\beta_2^2}}}{\mu_1\sqrt{\frac{1}{\beta_1^2}-\frac{1}{c^2}}}$$

for a layer over halfspace model,
β: SH velocities



For Rayleigh waves, solutions exist for a homogeneous halfspace. Rayleigh waves are the constructive interference of P & SV waves reflected at the free surface.

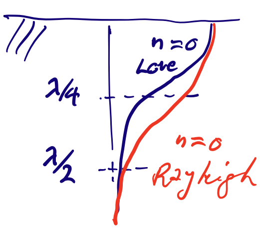
The dispersion of surface waves can be described as:

Love waves — are always dispersive, as wave speed c is a function of frequency ω, once they exist...

Rayleigh waves — in a homogeneous halfspace, they are non-dispersive (c independant of ω)

— in layered media, they are dispersive

Skin depth: Depth of a surface wave which it penetrates into the medium ("sees" perturbations)



rule of thumb:

	$z_R \approx \lambda/2$	$z_R' \approx \lambda$	Rayleigh
	$z_L \approx \lambda/4$	$z_L' \approx \frac{3}{2}\lambda$	Love
	fundamental	1. overtone	

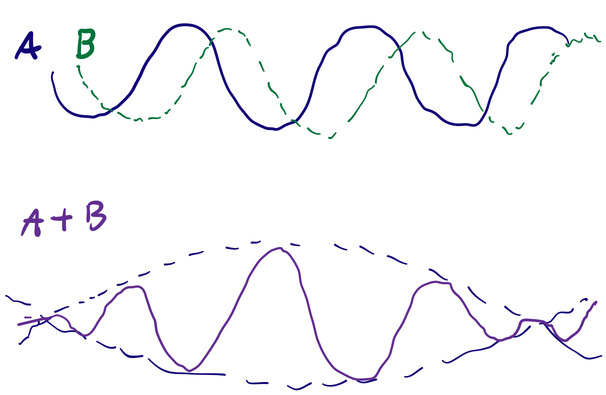
Group velocity / phase velocity:

$$\boxed{\begin{array}{l} \text{phase velocity} \quad c = \dfrac{\omega}{k} \\[2ex] \text{group velocity} \quad g = \dfrac{d\omega}{dk} \end{array}}$$

For 2 surface waves at $\omega+\delta\omega$ and $\omega-\delta\omega$, i.e., wavenumbers $k+\delta k$ and $k-\delta k$, their superposition is:

$$u = \underbrace{\cos[(\omega-\delta\omega)t-(k-\delta k)x]}_{A} + \underbrace{\cos[(\omega+\delta\omega)t-(k+\delta k)x]}_{B}$$

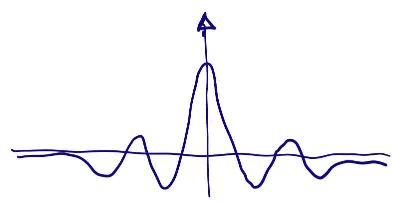
$$= 2\underbrace{\cos(\omega t - kx)}_{\text{wave at speed } c}\underbrace{\cos(\delta\omega t - \delta k x)}_{\text{modulation}}$$



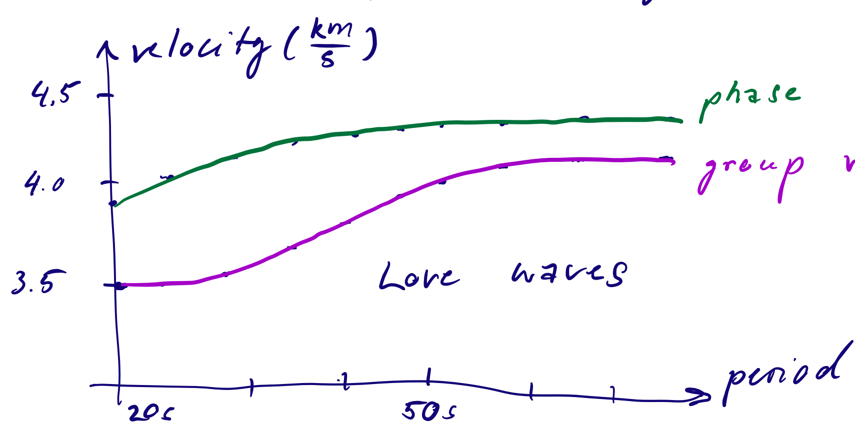
For a surface-wave packet consisting of a set of waves with discrete frequencies ω_i, the wavefield can be written as

$$u(x,t) = \sum_{i=1}^{\infty} \underbrace{S(\omega_i)}_{\text{excitation}} \underbrace{\cos\left(\omega_i\left[\frac{x}{c(\omega_i)} - t\right]\right)}_{\text{wave at speed } c(\omega_i)} \underbrace{\varepsilon \, \frac{2\sin\left(\varepsilon\left[\frac{x}{g(\omega_i)} - t\right]\right)}{\varepsilon\left[\frac{x}{g(\omega_i)} - t\right]}}_{\text{modulation} \sim \text{speed } g(\omega_i)}$$

with $\varepsilon \ll \omega_i$, the modulation will have a shape of a sinc-function traveling



with "group velocity" $g(\omega) = \dfrac{c(\omega)}{1 - \frac{\omega}{c(\omega)} \frac{dc}{d\omega}}$, which determines the dispersion of group velocity, if the dispersion of phase velocity $c(\omega)$ is known.



More "exotic" modes not treated here would be:

- Leaky modes – decay exponentially
- Airy phase – phase & group velocities are stationary
  $\frac{du}{d\omega} = 0 \rightarrow$ highest amplitudes